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# Identification of a potential to model the behavior of fiber-reinforced composites

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**Abstract.** In this Note, the derivation of the state potential to model the mechanical behavior of fiber-reinforced composites is presented. It allows us to account for the statistical distribution of matrix cracks, interfacial debonding and sliding in the framework of Continuum Mechanics.

## *Identification d'un potentiel pour modéliser le comportement de composites à fibres*

**Résumé.** Dans cette Note, le comportement mécanique de composites est étudié à l'aide de la modélisation de la distribution aléatoire de fissures matricielles, de la décohésion et du glissement interfacial dans le cadre de la mécanique des milieux continus.

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## *Version française abrégée*

Une loi de comportement de composites est écrite dans le cadre de la thermodynamique des processus irréversibles (Coleman et Gurtin, 1967). Dans la modélisation, il s'agit d'introduire les variables internes qui décrivent le comportement de tels matériaux et le potentiel d'état associé  $\bar{\psi}$ . Le potentiel d'état est constitué d'une partie réversible  $\bar{\psi}_e$  et d'une partie stockée  $\bar{\psi}_s$ . Lorsque le comportement microscopique est élastique l'énergie stockée correspond à la somme de toutes les densités d'énergies élastiques associées aux champs de contraintes résiduelles. Ainsi en considérant deux étapes élastiques, on peut obtenir une expression de la densité d'énergie libre totale (Volterra, 1907). Lors d'un chargement, un composite renforcé par des fibres subit une multi-fissuration matricielle s'accompagnant de décohésion et de glissement à l'interface. Les résultats obtenus dans le cas de la multi-fissuration matricielle peuvent être aisément étendus au cas de la multi-fragmentation de monofilaments. Le comportement à la fatigue peut également être inclus dans ce cadre.

La cellule élémentaire (fig. 1), base de l'étude, contient des fissures matricielles distribuées de manière aléatoire et caractérisées par une densité de probabilité  $F(L)$  qui évolue en fonction de la contrainte macroscopique  $\bar{\sigma}$ . Les fissures matricielles engendrent un accroissement de densité d'énergie potentielle  $\Delta\bar{\varphi}_e$  donné par l'équation (1), où  $\Delta\varphi_e(L)$  est la variation de densité d'énergie potentielle due à la présence d'une fissure dans une cellule de longueur  $L$ . La décohérence et le glissement interfacial induisent des déformations inélastiques et des boucles d'hystérésis (Aveston *et al.*, 1971; Hutchinson et Jensen, 1990). Par application du principe des travaux virtuels (Burr *et al.*, 1995), la déformation inélastique  $\bar{\varepsilon}_{in}$  est donnée par l'équation (2), où  $\Delta\varepsilon_f(z, L)$  est la déformation additionnelle dans les fibres,  $z$  est la position courante dans une cellule de longueur  $L$ . Le champ de contrainte correspondant est auto-équilibré et permet donc un stockage d'énergie  $\bar{\psi}_s$ . L'expression de  $\bar{\psi}_s$  est donnée par l'équation (3), où  $\Delta\varepsilon_m(z, L)$  est la déformation additionnelle dans la matrice,  $f$  est la fraction volumique de fibres,  $E_f$  et  $E_m$  sont les modules d'Young respectifs de la fibre et de la matrice. Ces mécanismes élémentaires sont décrits par des variables internes. La fissuration matricielle est modélisée par une variable interne d'endommagement  $D$  (Lemaitre et Dufailly, 1977) définie par l'équation (4). Les équations (1) et (4) montrent que cette variable est fonction, *a priori*, de tous les moments de la distribution  $F$  de longueurs  $L$ . La déformation inélastique  $\bar{\varepsilon}_{in}$  est une des variables internes modélisant l'endommagement interfacial (*i.e.*, la décohérence et le glissement). La seconde variable,  $d$ , est obtenue à partir de l'expression de la densité d'énergie libre stockée  $\bar{\psi}_s$  (équation (5)). Cette expression a également été introduite dans l'étude des bétons et des roches (Andrieux, 1981). La densité d'énergie libre totale  $\bar{\psi}$  est ainsi définie par l'équation (6), et les forces associées aux quatre variables d'état sont définies par les équations (7). L'équation (7.1) permet de définir la contrainte macroscopique, l'équation (7.2) permet de définir le taux de restitution de densité d'énergie de fissuration, l'équation (7.3) permet de définir le taux de restitution de densité d'énergie d'auto-contrainte, et l'équation (7.4) permet de définir la contrainte de rappel associée au mécanisme d'endommagement interfacial. Les lois d'évolution des variables internes peuvent être obtenues soit par analyse micromécanique (Feillard *et al.*, 1994), soit par essais macroscopiques (fig. 2). En particulier la mesure de la variable interne  $d$  passe par la quantification de la densité d'énergie libre stockée  $\bar{\psi}_s$  : *cf.* Chrysochoos (1987) ou Cho *et al.* (1991). L'équation (8) permet de définir une longueur équivalente  $\bar{L}_c$  associée à la fissuration matricielle. Lorsque  $\Delta\varphi_e(\bar{L}_c)$  est inversement proportionnelle à  $\bar{L}_c$ , la longueur équivalente à considérer est la distance moyenne entre fissures et une description locale de l'endommagement est appropriée. Dans le cas contraire, ce n'est qu'une approximation, et une description non locale de l'endommagement peut être justifiée. En utilisant le modèle de Cox (1952) et les équations (4) et (8),  $D$  est lié à  $\bar{L}_c$  et  $\bar{L}$  par l'équation (9), où  $\beta$  est une constante dépendant des propriétés élastiques et géométriques de la fibre et de la matrice. En supposant que la ténacité interfaciale est faible et que le glissement interfacial est approximé par une résistance au cisaillement constante  $\tau$  (Kelly et Tyson, 1965), la longueur équivalente de décohérence  $\bar{l}_d$  est définie à partir de l'équation (10), où  $R$  est le rayon de fibre. La déformation inélastique est ainsi une fonction des distributions de fissures matricielles et de longueurs de décohérence. Le calcul de la variable d'endommagement d'interface  $d$  est obtenu à partir du calcul de la densité d'énergie stockée (*cf.* équation (3)). Dans le cas où la séparation minimale entre deux fissures est supérieure à la longueur de décohérence, celle-ci prend une unique valeur et la variable d'endommagement de l'interface est définie par l'équation (11). Ce résultat n'est valable que lorsque le comportement interfacial vérifie les hypothèses de ce paragraphe. Lorsque le comportement interfacial est plus complexe, la variable  $d$  n'est proportionnelle au rapport  $\bar{l}_d/\bar{L}$  qu'à la condition que le champ de contrainte à l'avant de la fissure est le champ de contrainte élastique du matériau sain.

## 1. Introduction

A Continuum Mechanics formulation applied to fiber-reinforced composites is written within the framework of the Thermodynamics of Irreversible Processes (Coleman and Gurtin, 1967). The first step in establishing such a model is to identify the internal variables which define the state of the material. The second step is to derive the expression of the state potential  $\bar{\psi}$  in terms of the state variables and the third one is to determine the evolution laws of the internal variables.

The state potential  $\bar{\psi}$  is made up of the sum of two terms: viz. the elastic energy density  $\bar{\psi}_e$  and the stored energy density  $\bar{\psi}_s$ . The elastic part is directly related to the applied load. The stored energy density is concerned with residual stress fields giving rise to macroscopic strains with no applied load. When the local behavior is elastic, the stored energy density is expressed as the total elastic energy density associated with the residual stress field. Therefore by considering two elastic steps, the total free energy density can be evaluated following a so-called "cut and paste" technique introduced by Volterra (1907). This approach will be used to model the behavior of fiber-reinforced composites.

Loading a composite consisting of a brittle matrix supported by stronger fibers usually causes multiple matrix-cracking which is accompanied by interfacial debonding and sliding. In the following, we assume that the whole matrix-cracking process occurs at load levels lower than the fiber breakage mechanism, and therefore only the former mechanism will be analyzed. The aim of this Note is to derive a state potential and to introduce the relevant internal variables to model matrix-cracking, debonding and sliding. It is worth noting that the same formalism can be used to analyze fragmentation tests on single filaments for which the only relevant mechanisms to consider are fiber breakage and subsequent debonding and pull-out. Similarly, the study of fatigue enters the same framework.

## 2. Microscopic description of the degradation mechanisms

The matrix cracks, which are assumed to be perpendicular to the fiber direction, cause a stiffness reduction when the stress is tensile. Furthermore it is the closure of the cracks which indicates the onset of increased stiffness when the specimen is subsequently loaded in compression. In the following, we will use elementary cells of length  $L$ , characterizing the crack spacing, and consisting of two different materials (m) and (f) as shown in figure 1, where  $E$  is the Young's modulus of the unbroken composite,  $E_m$  that of part (m), and  $E_f$  that of part (f). There is a matrix crack at the center of the cell. Because the matrix contains initial flaws of random strength, there is a distribution of matrix cracks characterized by a probability density function  $F(L)$  depending on the applied stress  $\bar{\sigma}$ . The presence of matrix cracks implies a potential energy density change  $\Delta\bar{\varphi}_e$ .

$$(1) \quad \Delta\bar{\varphi}_e = \frac{1}{L} \int_0^{+\infty} \Delta\varphi_e(L) LF(L) dL \quad \text{with} \quad \bar{L} = \int_0^{+\infty} LF(L) dL$$

where  $\Delta\varphi_e(L)$  is the average potential energy density change due to the presence of a crack in a cell of length  $L$ , and  $\bar{L}$  the average crack spacing. In the framework of Linear Elastic Fracture Mechanics, it is shown that the potential energy change  $\Delta\varphi_e(L)$  is equal to  $\bar{\sigma}^2 \omega_c/E$ , where  $\omega_c$  is dependent on the crack density (*i.e.*  $a/L$ , see *fig. 1*) as well as on the fact that cracks interact or not. This first basic mechanism is purely dissipative and therefore only influences  $\bar{\psi}_e$ .

Debonding followed by sliding gives rise to inelastic strains and hysteresis loops. To describe these phenomena, different models have been proposed (Aveston *et al.*, 1971; Hutchinson and Jensen, 1990). They all consider a friction length  $l_F$  here assumed to be equal to the debond length  $l_d$  (*fig. 1*). Similarly to cracking, which is a mode I mechanism, debonding *per se* is a purely dissipative mechanism as well. When debonding and slip occur simultaneously, a self-balanced microscopic stress

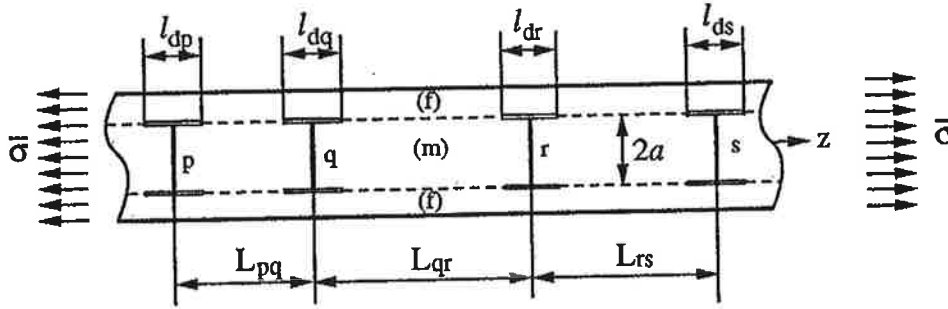


Fig. 1. - Distribution of crack spacing  $L$  and debond length  $l_d$ .

Fig. 1. - Distribution de distances  $L$  et de longueurs de décohésion  $l_d$ .

field is involved. The corresponding strains in the matrix and in the fiber are denoted by  $\Delta\varepsilon_m(z, L)$  and  $\Delta\varepsilon_f(z, L)$ , respectively, where  $z$  is the current coordinate. These strain differences result from the "cut and paste" procedure described previously. By application of the principle of virtual work (Burr *et al.*, 1995), the overall inelastic strain  $\bar{\varepsilon}_{in}$  is expressed as

$$(2) \quad \bar{\varepsilon}_{in} = \frac{1}{L} \int_0^{+\infty} \left[ \int_0^L \Delta\varepsilon_f(z, L) dz \right] F(L) dL.$$

Since the additional stress field is self-balanced the debonding and sliding processes are able to store energy. This result shows that to fully characterize both mechanisms knowledge of the stored energy density  $\bar{\psi}_s$  is crucial. It is expressed as the total elastic energy density associated with the residual stress field

$$(3) \quad \bar{\psi}_s = \frac{1}{L} \int_0^{+\infty} \left[ \int_0^L \{f E_f \Delta\varepsilon_f^2(z, L) + (1-f) E_m \Delta\varepsilon_m^2(z, L)\} dz \right] F(L) dL$$

where  $f$  is the volume fraction of part (f).

### 3. Macroscopic description of the degradation mechanisms

The matrix-cracking process described by an internal damage variable complies with the requirement of a fully dissipative mechanism. In a Continuum Mechanics framework, the presence of a crack results in a stiffness reduction defined by an internal damage variable  $D$  (Lemaitre and Dufailly, 1977). In the framework of Continuum Damage Mechanics, the Gibbs' elastic energy density of a damaged material is written as  $\bar{\varphi}_e(D) = \bar{\sigma}^2/2E(1-D)$  so that the potential energy density change can be written as follows

$$(4) \quad \Delta\bar{\varphi}_e = \bar{\varphi}_e(D) - \bar{\varphi}_e(D=0) = \frac{\bar{\sigma}^2}{2E} \frac{D}{1-D}.$$

Equation (4) shows that the correct internal variable modeling cracking is the damage variable  $D$  whereas the corresponding potential energy density change depends upon the damage variable  $D$  as well as the applied stress level, and therefore is not an acceptable measure of the degradation due to matrix-cracking. Equations (1) and (4) show that the damage variable  $D$  is a function of all the statistical moments of the crack spacing distribution. A 3D formulation requires an anisotropic

damage description. In the case of cracking perpendicular to the fiber direction, the generalization is straightforward since the only compliance change is given in the fiber direction, and therefore only one scalar damage variable is needed and the previous 1D analysis is relevant. For multilayered materials, anisotropic damage variables are needed at the level of each layer. To fully characterize the debonding and sliding mechanisms two variables are needed. The first one is the inelastic strain  $\bar{\epsilon}_{in}$ , and the second one denoted by  $d$  is introduced to define the stored energy density level

$$(5) \quad \bar{\psi}_s = \frac{E}{2} \frac{\bar{\epsilon}_{in}^2}{d}$$

This expression can be found in Andrieux (1981) to model concrete and rocks for which there is no conjunction between matrix-cracking and debonding, and sliding. However, in the study of ceramic-matrix composites there is a non-uniform stress field along the debond crack, thus equation (5) does not allow one to relate, *a priori*, the variable  $d$  to a debond crack density. In a 3D formulation, a second order tensor is needed to model the inelastic strains due to debonding and slip. Since each operative slip system can be integrated separately in terms of energetic contributions, the internal damage variables can be defined separately for each inelastic strain term. Therefore there are as many debond damage variables as non-vanishing inelastic strain terms (Burr *et al.*, 1995). In the case of cracking perpendicular to the fiber direction, only one inelastic strain component is different from zero, viz. the normal component along the fiber direction. Similarly, only one scalar debond damage variable is needed.

In addition to the total strain  $\bar{\epsilon}$ , the total free energy density  $\bar{\psi}$  is dependent on three internal variables: one damage variable  $D$  modeling matrix-cracking and related to the crack spacing distribution, two variables  $\bar{\epsilon}_{in}$  and  $d$  describing debonding and sliding, and related to the debond length distribution and to the crack opening distribution

$$(6) \quad \bar{\psi} = \frac{E(1-D)}{2} (\bar{\epsilon} - \bar{\epsilon}_{in})^2 + \frac{E}{2} \frac{\bar{\epsilon}_{in}^2}{d}$$

The associated forces are defined as follows

$$(7) \quad \bar{\sigma} = \frac{\partial \bar{\psi}}{\partial \bar{\epsilon}}, \quad Y = -\frac{\partial \bar{\psi}}{\partial D}, \quad y = -\frac{\partial \bar{\psi}}{\partial d}, \quad \bar{X} = \frac{\partial \bar{\psi}}{\partial \bar{\epsilon}_{in}}$$

Equation (7.1) defines the macroscopic stress  $\bar{\sigma}$ . Equation (7.2) defines the energy release rate density  $Y$  playing a similar role as the energy release rate  $\mathcal{G}$  in Linear Elastic Fracture Mechanics. Combining equations (7.1) and (7.2), one shows that the energy release rate density  $Y$  is proportional to the square of the "effective stress"  $\bar{\sigma}/(1-D)$ . Similarly, equation (7.3) defines the stored energy density  $y$  released during debonding and sliding. Since the variable  $d$  depends upon the details of the interfacial behavior, the definition of its associated force is also dependent upon the interfacial behavior. Lastly, equation (7.4) defines the back stress  $\bar{X}$  related to debonding and sliding. Again its exact value depends upon the interfacial behavior. To determine the evolution laws of the internal variables, two ways can be followed. The first one uses simulations of the complete micromechanical model along the lines developed for instance by Feillard *et al.* (1994) to get the distributions of crack spacing  $L$  and debond length  $l_d$ , and then the state potential. The second one is by performing experiments. The damage variable  $D$  is obtained by measuring the initial unloading modulus and the corresponding inelastic strain is  $\bar{\epsilon}_{in}$  (fig. 2). To measure the variable  $d$ , the stored energy  $\bar{\psi}_s$  has to be evaluated, for instance by using methods developed by Chrysochoos (1987) or Cho *et al.* (1991).

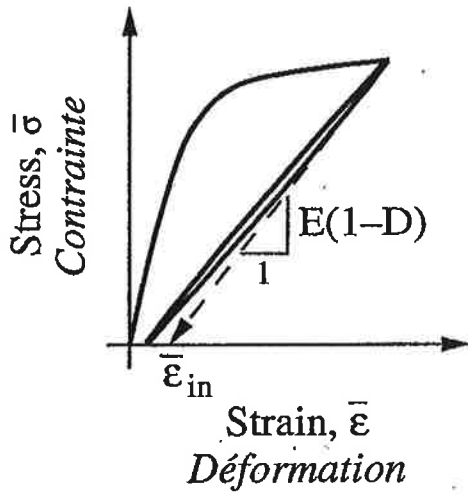


Fig. 2. - Schematic stress/strain curve.

Fig. 2. - Courbe contrainte/déformation.

#### 4. Examples

In this section, expressions of the three internal variables  $D$ ,  $\bar{\varepsilon}_{in}$  and  $d$  are derived in some particular cases. Equation (1) can be written in terms of an equivalent cracking length  $\bar{L}_c$  defined by

$$(8) \quad \bar{L} \Delta \bar{\varphi}_e = \bar{L}_c \Delta \varphi_c (\bar{L}_e).$$

It is worth noting that when  $\Delta \varphi_c (\bar{L}_c)$  is inversely proportional to  $\bar{L}_c$ , then  $\Delta \bar{\varphi}_c (L)$  is inversely proportional to  $L$  for any value of  $L$  and any distribution  $F$ . The relevant length to consider is the average crack spacing  $\bar{L}$ . In that case a local damage theory is applicable since the matrix cracks are not interacting with each other. Otherwise, choosing the equivalent cracking length equal to the average crack spacing is *a priori* only a crude approximation. The damage variable then depends upon the higher order moments of the crack spacing distribution and a non-local damage description seems more appropriate. Using a shear lag analysis (Cox, 1952) and equations (4) and (8), the damage variable  $D$  is linked to crack spacings  $\bar{L}_c$  and  $\bar{L}$  by

$$(9) \quad \frac{D}{1-D} = \frac{(1-f) E_m \tanh(\beta \bar{L}_c)}{f E_f \beta \bar{L}}$$

where the constant  $\beta$  is a function of the elastic and geometric properties of the fiber and the matrix. When  $\beta \bar{L}_c > 3$  (i.e.,  $\tanh(\beta \bar{L}_c) \cong 1$ ),  $D/(1-D)$  is inversely proportional to  $\beta \bar{L}$ . This last result can also be found in the framework of Linear Elastic Fracture Mechanics for non interacting cracks. The strain distribution  $\Delta \varepsilon_f(z, L)$  in the friction zone in the part (f) is assumed to be linear and characterized by a constant interfacial sliding strength  $\tau$  (Kelly and Tyson, 1965). If the debond toughness is negligible so that there is no longitudinal stress jump (Hutchinson and Jensen, 1990), the inelastic strain  $\bar{\varepsilon}_{in}$  is given by Burr *et al.* (1995).

$$(10) \quad \bar{\varepsilon}_{in} = \frac{\tau \bar{l}_d^2}{4 E_f R \bar{L}_c}$$

where  $R$  is the fiber radius, and  $\bar{l}_d$  is the equivalent debond length defined the same way as the equivalent cracking length  $\bar{L}_c$ . Equations (2) and (10) show that the inelastic strain is a function of the

crack spacing distribution as well as the debond length distribution. If the crack spacing distribution is such that the smallest spacing is greater than the debond length, there is a *unique* debond length value. The definition of the variable  $d$  is directly obtained from the evolution of the debond length  $\bar{l}_d$  as well as its definition in the expression of the stored energy  $\bar{\psi}_s$  given in equation (3)

$$(11) \quad d = \frac{3(1-f)E_m}{4fE_f} \frac{\bar{l}_d}{\bar{L}_c}$$

The damage variable  $d$  defines the size of the slip zone related to the crack spacing length  $\bar{l}_d/\bar{L}_c$ . If the stress redistribution is more complex (Feillard *et al.*, 1994), the previous results do not apply: the damage variable  $d$  is proportional to the ratio  $\bar{l}_d/\bar{L}_c$  only if the stress field ahead of the debond crack tip is assumed to be unaltered by the presence of sliding and cracking.

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