Achieving an $\mathcal{L}\_2$ string stable one vehicle look-ahead platoon with heterogeneity in time-delays

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Deesh Dileep[1], Mauro Fusco[2], Jan Verhaegh[2], Laurentiu Hetel[3], Jean-Pierre Richard[4], and Wim Michiels[1]

***Abstract*— A methodology is proposed to design stabilising and robust fixed-order decentralised controllers for heterogeneous vehicular platoons with Cooperative Adaptive Cruise Control (CACC). We consider Linear Time Invariant (LTI) models with constant time-delays at state, input and output. The closed-loop systems of (identical) local controllers and heterogeneous parameter vehicles are modelled by a system of delay differential algebraic equations. The proposed frequency domain approach uses the non-conservative direct optimisation approach towards stabilisation and robustness optimisation of delay systems. In this paper, the design problem of stabilising (identical) controllers achieving $\mathcal{L}_2$ string stability for one vehicle look-ahead platoon is reduced to a simultaneous controller design problem for a parameterised (sub)system, where the allowable values of the parameters correspond to heterogeneity (including time-delays) of the vehicles. By treating the heterogeneity in parameters as perturbations contained in specific intervals or regions, we determine the values for pseudo-spectral abscissa and robust induced-$\mathcal{L}_2$ norm. Hence, we ensure that the achieved exponential stability and string stability properties along with the overall computational complexity (of designing the controller) are independent of the number of vehicles. The application of CACC is simulated in MATLAB software.**

***Index Terms*— Decentralised control, Time-delay systems, H2/H-infinity methods, Linear systems, Large-scale systems.**

I. Introduction

Problems related to traffic jams, growing constraints in highway capacities and improving efficiency in road transport systems have caught the attention of researchers worldwide. Cooperative Adaptive Cruise Control (CACC) techniques are attractive as an automated vehicle following system based on inter-vehicular exchange of data through wireless communication, in addition to radar or lidar [1]. As a matter of fact, CACC is known to reduce the time gaps between vehicles significantly [1]. In this work, a scalable design method using frequency domain based technique for stabilising and robust (string stable) fixed-order (identical) local controllers for heterogeneous vehicular platoons with time-delays is presented. The heterogeneous time-delays could be present in these systems due to actuation, sensors or communication (see [2]).

For the case of CACC, it is impractical to employ centralised controllers (see [1], [3], [4]). One of the main objectives to be considered when designing controllers for CACC is the prevention of amplification of disturbances in the upstream direction of vehicles. This problem is generally represented using the notion of string stability. There are several definitions available for string stability in the literature, focusing on various aspects of cascaded systems (see [1], [3], [5], [6]). We consider sufficient conditions for (strict) string stability, based on the $\mathcal{L}_2$ gain, as in [1], [3]). The method proposed in [1] has been modified to incorporate the problem of heterogeneity and time-delays. Homogeneity in platoon is assumed at higher layer in [1] by considering the possibility of cancelling out heterogeneity using lower layer controllers. This might not be suitable for some scenarios which include multi-brand vehicular platoons with heterogeneity in time-delays. The one vehicle look-ahead topology is considered to design identical controllers for heterogeneous (parameter) vehicles, by optimising them for sufficient conditions of string stability in terms of (maximum) $\mathcal{L}_2$ gain. It is important to note that by considering the (energy based) $\mathcal{L}_2$ string stability conditions for heterogeneous vehicular platoon, there is no insight on the $\mathcal{L}_\infty$ string stability (the possible overshoot for signal in the time domain).

Generally, traditional design methods used for designing stabilising and $\mathcal{H}_\infty$ optimal controllers for Linear Time-Invariant (LTI) Multiple Input Multiple Output (MIMO) systems are grounded in the Riccati equation and Linear Matrix Inequality (LMI) framework (see [7], [8], and references therein). In most of these cases, the controllers designed by these methods may not be structured and their dimension could be equal to or larger than the order of the plant. In this paper, we consider non-conservative frequency domain based approaches proposed in [9], [10] to design structured and fixed-order controllers. Many researchers assume homogeneity of vehicles in large platoons (networks), however this might not be true in the real world scenario. In [11], sufficient conditions for designing string stable distributed controllers for heterogeneous platoon (solved using small gain theorem and LMI) was presented. In [5], a pole-zero cancellation method was proposed to cancel heterogeneity in engine time constants through post-compensation of the wireless feed-forward signal. In [6], the string stability of heterogeneous vehicular platoons in an adaptive cruise control configuration with non-connected automated vehicles has been considered. However, in this paper, we study the possibility to optimise, with reduced complexity, the stability and robustness/performance of (identical) local controllers for the large scale LTI heterogeneous vehicular platoons in an one vehicle look-ahead topology with numerous (constant) time-delays using frequency domain based direct optimisa-

[1]Department of Computer Science, KU Leuven, 3001 Leuven, Belgium. {deesh.dileep,wim.michiels}@cs.kuleuven.be

[2]TNO, Integrated Vehicle Safety Department, 5700 AT Helmond, The Netherlands. {mauro.fusco,jan.verhaegh}@tno.nl

[3]University of Lille, UMR 9189 - CRIStAL, CNRS, Centrale Lille, France. laurentiu.hetel@centralelille.fr

[4]University of Lille, UMR 9189 - CRIStAL, CNRS, Inria, Centrale Lille, France. jean-pierre.richard@centralelille.fr

tion techniques. Since we consider the frequency domain approach, we have necessary and sufficient stability conditions for the LTI system with time-delays. Additionally, it might be possible to add an upper layer of control to one vehicle look-ahead CACC, so as to include the possibility of information transfer from the last vehicle to the first vehicle (see [12], [13]). We refer to [14], [15], [16], [17] and references within for details on recent publications related to the subject.

The remainder of this paper is organised as follows. Section II provides a motivation for the problem considered in this paper. Section III presents the linearised vehicle plant model considered and the outputs available to the controller. Section IV describes the Delay Differential Algebraic Equation (DDAE) used to model the heterogeneous vehicular platoon with time-delays. Section V reviews the direct optimisation based approach available for designing robust fixed-order (identical) controllers for CACC. Section VI validates the proposed approach using MATLAB software. Finally, Section VII contains the concluding remarks.

II. Motivation

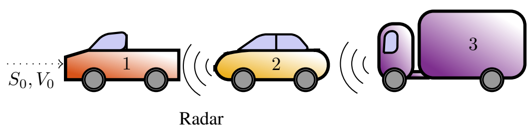

Fig. 1. Heterogeneous platoon of three vehicles in classical ACC one vehicle look-ahead topology.

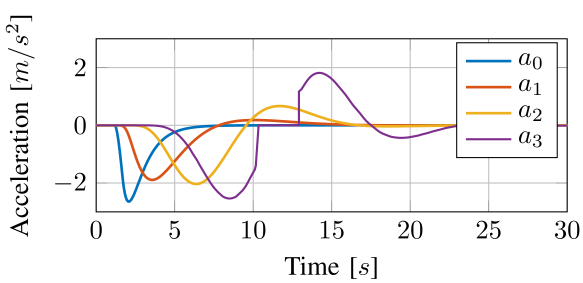

Fig. 2. Deceleration response, accelerations of heterogeneous (parameter) vehicles in ACC platoon which is exponentially stable but not string stable.

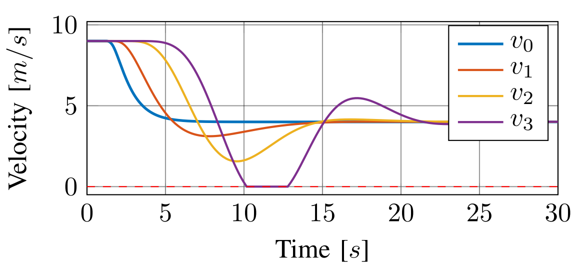

Fig. 3. Deceleration response, velocities of heterogeneous (parameter) vehicles in ACC platoon which is exponentially stable but not string stable.

We illustrate the importance of string stability (*performance*) for heterogeneous platoons using a classical Adaptive Cruise Control (ACC) configuration (see Fig. 1). We simulate the string instability phenomenon in heterogeneous vehicular platoons using third order LTI models with delays in the ACC configuration stabilised by a PD controller, and introducing a disturbance (deceleration input) in the reference vehicle. We can see in Figs. 2 and 3, a small deceleration signal in the reference vehicle (a_0), results in undesirable responses through the string. In Fig. 3, it can be seen that the deceleration reference signal results in negative velocity during simulation. However, in reality, it would be saturated (at zero). One way to interpret this effect would be that the lack of performance standards could result in undesirable stops (velocity $= 0\ m/s^2$) or traffic jams. The string stability *performance* of automated vehicles is necessary to be guaranteed for smooth traffic flows. Even though the undesirable effects on traffic flow are simulated in this section using a platoon of three vehicles, due to the nature of the problem and based on intuition, we can say that the performance worsens as the number of vehicles increase in the string. This *performance* problem is framed as a strict string stability problem in terms of induced-$\mathcal{L}_2$ norm for control design in Section V-C.

III. Plant model

In this section, we present the vehicle models used for the CACC problem considered in this paper. The LTI vehicle model has been taken from [1], however, we consider existing (constant) time-delays and some heterogeneous elements in the dynamics of the vehicles. The heterogeneity considered in this paper is confined to the parameters in Table I.

TABLE I
Heterogeneity in the CACC network.

Notation	Parameter of vehicle i
h_i	Head-way time constant
τ_i	Drive-line time constant
ϕ_{ai}	Actuation delay
ϕ_{bi}	Communication delay
ϕ_{ci}	Sensor delay

We assume all the parameters in Table I to be positive and real-valued. We consider the i^{th} vehicle model as,

$$\begin{pmatrix} \dot{S}_i(t) \\ \dot{V}_i(t) \\ \dot{F}_{ai}(t) \end{pmatrix} = \begin{pmatrix} V_i(t) \\ F_{ai}(t) \\ -\frac{1}{\tau_i}F_{ai}(t) + \frac{1}{\tau_i}u_i(t-\phi_{ai}) \end{pmatrix}, \quad (1)$$

for all $i = 1, ..., n$ where n is the total number of vehicles in the platoon, S_i is the position, V_i is the velocity, and F_{ai} is the acceleration of vehicle i. Given a reference trajectory, $S_{ref,0}(t) = V_{ref,0} \cdot t$, we stabilise and control the system around the stationary solution (when $u_i = 0$)

$$S_i(t) = S_{ref,i}(t) = V_{ref,0} \cdot t - \sum_{k=1}^{i}(h_k V_{ref,0} + L_k + r_k),$$

$i = 1, ..., n$, where L_i is the length, r_i is the standstill distance, and h_i is the head-way time constant of vehicle i. That is, we consider each vehicle to be associated with a reference trajectory with real-valued constant velocity $V_{ref,0}$.

Note that the reference distance for vehicle i from the vehicle ahead has a velocity component. We can describe the relative motion of the i^{th} vehicle as

$$\begin{aligned} &s_i(t) = S_i(t) - S_{ref,i}(t),\ v_i(t) = V_i(t) - \dot{S}_{ref,i}(t), \\ &a_i(t) = F_{ai}(t) - \ddot{S}_{ref,i}(t) = F_{ai}(t), \quad \forall\ i = 1, ..., n. \end{aligned} \tag{2}$$

For the virtual vehicle 0, we consider $V_0 = V_{ref,0},\ S_0 = S_{ref,0}$, then $s_0,\ v_0 = 0$. The equations in (1) change to

$$\begin{pmatrix} \dot{s}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{pmatrix} = \begin{pmatrix} v_i(t) \\ a_i(t) \\ -\frac{1}{\tau_i} a_i(t) + \frac{1}{\tau_i} u_i(t - \phi_{ai}) \end{pmatrix}, \tag{3}$$

and the corresponding transfer function from u_i to s_i can be written as

$$G_i(s) = \frac{\mathrm{e}^{-\phi_{ai} s}}{(\tau_i s + 1) s^2}. \tag{4}$$

We assume that the controller of vehicle i has access to the position error ($e_i(t)$), the velocity error ($\dot{e}_i(t)$), and the input signal transmitted from the vehicle ahead through wireless communication ($u_{i-1}(t)$), that is $y_i(t) = [e_i(t - \phi_{ci})\ \dot{e}_i(t - \phi_{ci})\ u_{i-1}(t - \phi_{b(i-1)})]^{\mathrm{T}}$. The position error is given by

$$\begin{aligned} e_i(t) &= S_{i-1}(t) - S_i(t) - h_i V_i(t) - L_i - r_i \\ &= s_{i-1}(t) - s_i(t) - h_i v_i(t), \end{aligned} \tag{5}$$

and the velocity error is given by

$$\begin{aligned} \dot{e}_i(t) &= V_{i-1}(t) - V_i(t) - h_i F_{ai}(t) \\ &= v_{i-1}(t) - v_i(t) - h_i a_i(t), \end{aligned} \tag{6}$$

for all vehicles $i = 1, ..., n$ (by definition, $s_0,\ v_0 = 0$).

IV. Closed-loop system model

The dynamics of the one vehicle look-ahead platoon without control is given (using the state $x_{pi}(t) = [e_i(t)\ v_i(t)\ a_i(t)\ \gamma_{ui}(t)\ \gamma_{yi}^{\mathrm{T}}(t)]^{\mathrm{T}}$) by

$$\begin{aligned} &E_p \dot{x}_{pi} = A_{pi0} x_{pi}(t) + A_{pi1} x_{pi}(t - \phi_{ai}) \\ &+ A_{pi2} x_{pi}(t - \phi_{ci}) + B_{pi1} u_i(t) + F_{pi0} x_{p(i-1)}(t) \\ &+ F_{pi1} x_{p(i-1)}(t - \phi_{b(i-1)}) + F_{pi2} x_{p(i-1)}(t - \phi_{ci}), \\ &y_i(t) = C_{pi0} x_{pi}(t), \qquad\qquad i = 1, ..., n, \quad x_{p0} = 0, \end{aligned} \tag{7}$$

where e_i is the position error, v_i is the relative velocity, and a_i is the acceleration of plant/vehicle i, whereas γ_{ui} and γ_{yi} are dummy variables for controlled input u_i and output to controller y_i respectively. Note that $x_{p0} = 0$ or vehicle 0 doesn't exist from analysis point of view (see (5), (6)), however, for simplicity of representation we consider $i = 1, ..., n$ in (7). In the matrices below

$$A_{pi0} = \begin{bmatrix} 0 & -1 & -h_i & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_i} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -I \end{bmatrix},\ B_{pi1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$[F_{pi0}]_{(j,k)} = \begin{cases} 1, & \text{if } (j,k) = (1,2) \\ 0, & \text{otherwise,} \end{cases}$$

$$[A_{pi1}]_{(j,k)} = \begin{cases} \frac{1}{\tau_i}, & \text{if } (j,k) = (3,4) \\ 0, & \text{otherwise,} \end{cases}$$

$$C_{pi0} = \begin{bmatrix} 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$A_{pi2} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -h_i \\ 0 & 0 & 0 \end{bmatrix} & 0 & \dots & 0 \end{bmatrix},$$

$$[F_{pi1}]_{(j,k)} = \begin{cases} 1, & \text{if } (j,k) = (7,4) \\ 0, & \text{otherwise,} \end{cases}$$

$$[F_{pi2}]_{(j,k)} = \begin{cases} 1, & \text{if } (j,k) = (6,2) \\ 0, & \text{otherwise,} \end{cases}$$

$$E_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

A_{pij} is given for $i = 1, ..., n$, F_{pij} is given for $i = 2, ..., n$, and $F_{p1j} = 0$ $(i = 1)$, where $j = 0, 1, 2$. We use I and 0 to denote the identity matrix and the matrix with zero entries of appropriate dimensions respectively, and $[\,\cdot\,]_{(j,k)}$ denotes the element at the j^{th} row and k^{th} column of a matrix. We also consider each subsystem to be controlled by a fixed-order LTI feedback controller (order n_c) of the form

$$\begin{cases} \dot{x}_{ci}(t) &= A_c x_{ci}(t) + B_c y_i(t), \\ u_i(t) &= C_c x_{ci}(t) + D_c y_i(t),\ \ i = 1, ..., n. \end{cases} \tag{8}$$

A static controller ($n_c = 0$) would have only the D_c component corresponding to $[k_p\ k_d\ k_{ff}]$ as in [1] (with $k_{ff} = 1$). We consider the scenario of the heterogeneous vehicles being controlled using identical local controllers $u_i(s) = K(s) y_i(s)\ \forall\ i = 1, ..., n$. We define the following state vector for the closed-loop system

$$x_i(t) = [x_{pi}^{\mathrm{T}}(t)\ u_i^{\mathrm{T}}(t)\ x_{ci}^{\mathrm{T}}(t)\ y_i^{\mathrm{T}}(t)]^{\mathrm{T}},$$

which includes plants, controllers, and network interconnections. We use $\mathbb{R}$ to denote the set of all real numbers. We re-write system equations using the new state $x_i \in \mathbb{R}^{n_{cl}}$, in the form of DDAE (see [10], [18] for more details on DDAE)

$$\begin{cases} E\dot{x}_i(t) &= A_{i0} x_i(t) + A_{i1} x_i(t - \phi_{ai}) + A_{i2} x_i(t - \phi_{ci}) \\ &\quad + F_{i0} x_{i-1}(t) + F_{i1} x_{i-1}(t - \phi_{b(i-1)}) \\ &\quad + F_{i2} x_{i-1}(t - \phi_{ci}) \qquad \forall\ i = 1, \dots, n,\ \ x_0 = 0, \end{cases} \tag{9}$$

where the matrices are

$$E = \begin{bmatrix} E_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_{i0} = \begin{bmatrix} A_{pi0} & B_{pi1} & 0 & 0 \\ C_{pi0} & 0 & 0 & -I \\ 0 & 0 & A_c & B_c \\ 0 & -I & C_c & D_c \end{bmatrix},$$

$A_{i1} = \mathrm{blkdiag}\{A_{pi1}, 0, 0, 0\}, F_{i0} = \mathrm{blkdiag}\{F_{pi0}, 0, 0, 0\},$
$A_{i2} = \mathrm{blkdiag}\{A_{pi2}, 0, 0, 0\}, F_{i1} = \mathrm{blkdiag}\{F_{pi1}, 0, 0, 0\},$
$F_{i2} = \mathrm{blkdiag}\{F_{pi2}, 0, 0, 0\},$

$i = 1, ..., n$, where the abbreviation $\mathrm{blkdiag}\{\cdot\}$ implies the block diagonal matrix. Notice in the above equation that the controller parameters are contained in matrix A_{i0}, as indicated with the dashed box. In the direct optimisation approach of [9], [10], [19], stability and performance measures expressed in terms of the spectral abscissa and $\mathcal{H}_\infty$ norms are optimised as a function of the elements of controller matrices A_c, B_c, C_c and D_c. We contain all the controller parameters in a vector

$$\bar{p} = \mathrm{vec}\left[\begin{array}{c:c} A_c & B_c \\ \hdashline C_c & D_c \end{array}\right]. \tag{10}$$

Whenever appropriate, we stress the dependence of A_{i0} on these parameters with the notation $A_{i0}(\bar{p})$.

V. Stability and performance objectives

We optimise the controller parameters in $\bar{p}$ for stability objectives in terms of spectral abscissa (the real part of the rightmost eigenvalue in the complex plane) and pseudo-spectral abscissa using algorithms in [9], [20] respectively as a starting point. Additionally, we tune controller parameters for robustness objectives in terms of induced-$\mathcal{L}_2$ norms, using a graphical frequency-gridding approach with the help of bode plot and `patternsearch`[1]. The objective functions considered in this paper are as follows.

A. Platoon stability: spectral abscissa

The linear models of vehicles and the platoon used in this paper are not stable. They have zero eigenvalues, therefore, as a first step we stabilise the platoon by computing local controllers that minimise the spectral abscissa. The spectral abscissa ($c(\bar{p})$) of the closed-loop system in (9) can be expressed as follows

$$c(\bar{p}) = \sup_{\lambda\in\mathbb{C}}\{\Re(\lambda) : \det\Delta(\lambda,\bar{p}) = 0\},$$

where,

$$\Delta(\lambda,\bar{p}) = \begin{bmatrix} \Delta_1 & 0 & 0 & \dots & 0 \\ \hat{F}_{21} & \Delta_2 & 0 & \dots & 0 \\ 0 & \hat{F}_{32} & \Delta_3 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{F}_{n(n-1)} & \Delta_n \end{bmatrix}. \tag{11}$$

We use the notation $\Re(\lambda)$ to indicate the real part of the complex number λ (eigenvalue), also, Δ_i corresponds to the characteristic matrix of individual vehicles,

$$\Delta_i(\lambda,\bar{p}) = \lambda E - A_{i0}(\bar{p}) - A_{i1}\mathrm{e}^{-\lambda\phi_{ai}} - A_{i2}\mathrm{e}^{-\lambda\phi_{ci}}, \tag{12}$$

for all $i = 1,...,n$, and $\hat{F}_{i(i-1)}$ appears due to the interaction between vehicles, where

$$\hat{F}_{i(i-1)}(\lambda) = -F_{i0} - F_{i1}\mathrm{e}^{-\lambda\phi_{b(i-1)}} - F_{i2}\mathrm{e}^{-\lambda\phi_{ci}}, \tag{13}$$

for all $i = 2,...,n$. Since the differences between $\Delta_i\ \forall\ i = 1,...,n$ lie in some parameters within the state coefficient matrices (see (12)), then we can rewrite it as

$$\bar{\Delta}(\lambda,\bar{p},\hat{p}_i) = \lambda E - \bar{A}_0(\bar{p},\hat{p}_i) - \bar{A}_1(\lambda,\hat{p}_i) - \bar{A}_2(\lambda,\hat{p}_i), \tag{14}$$

for all $i = 1,...,n$, where $\hat{p}_i = [\tau_i\ h_i\ \phi_{ai}\ \phi_{bi}\ \phi_{ci}]^{\mathrm{T}}$ is the plant parameter vector corresponding to vehicle i, $\bar{A}_0(\bar{p},\hat{p}_i) = A_{i0}(\bar{p})$, $\bar{A}_1(\lambda,\hat{p}_i) = A_{i1}\mathrm{e}^{-\lambda\phi_{ai}}$, and $\bar{A}_2(\lambda,\hat{p}_i) = A_{i2}\mathrm{e}^{-\lambda\phi_{ci}}$.

Theorem 1: For the systems given in (9), the spectral abscissa in (11) is equivalent to

$$c(\bar{p}) = \max_i \sup_{\lambda\in\mathbb{C}}\{\Re(\lambda) : \det\bar{\Delta}(\lambda,\bar{p},\hat{p}_i) = 0\} :\ i\in\{1,...,n\}. \tag{15}$$

Proof. The assertions for the Theorem 1 directly follow from the block-triangular structure of (11), then (15) arises from the diagonal blocks, and from the structure of the associated eigenvalue problem. □

The *exponential* stability (unlike *strict* string stability) of the whole CACC network in look-ahead topologies does not depend on the interaction/coupling of vehicles. We minimise the spectral abscissa of the platoon in (9) for a faster exponential decay rate of solutions,

$$\min_{\bar{p}} c(\bar{p}).$$

However, the computational complexity of this stabilisation approach still depends on the number of vehicles.

B. Platoon stability: pseudospectral abscissa

In this section, we solve the stabilisation problem of platoons in look-ahead topologies, using a method whose computational complexity is independent of the platoon size. We can formulate it as a problem of parameterised system, and compute the pseudospectral abscissa for some structured real-valued perturbations. Since the differences between the vehicles in (12) lie in some parameters within the state coefficient matrices (see (14)), a sufficient condition for stability is given by the *robust stability* of the corresponding *uncertain* system. Let us define the pseudospectrum of the perturbed system,

$$\Lambda := \bigcup_{\hat{p}_\delta\in\mathcal{R}} \left\{\lambda\in\mathbb{C}:\ \det(\bar{\Delta}(\lambda,\bar{p},\hat{p}_\delta)) = 0\right\}, \tag{16}$$

where the vector $\hat{p}_\delta$ (corresponding to the vehicle i) is the uncertainty belonging to some region $\mathcal{R}\in\mathbb{R}^5$. Note that $\bar{\Delta}$ in (16) can include all the characteristic matrices $\Delta_i\ \forall\ i = 1,...,n$ as in (12) by defining the heterogeneity in vehicle parameters to be contained within $\mathcal{R}$. For robust stability optimisation, we introduce the pseudospectral abscissa

$$\alpha := \sup\{\Re(\lambda) : \lambda\in\Lambda\}.$$

Since the matrices in (14) are affine in the uncertain parameters, the pseudospectral abscissa can be computed using the structure exploiting algorithm for real-valued perturbations in [20]. The objective would be to minimise the pseudospectral abscissa

$$\min_{\bar{p}} \alpha,$$

to obtain a stable system for all perturbations belonging to the region $\mathcal{R}$ with $\alpha < 0$.

[1]Built-in optimisation algorithm within MATLAB.

C. Platoon string stability

In this subsection, we focus on formulating a condition for the strict string stability of the vehicular platoon. We assume throughout this paper that we observe signals (inputs, accelerations, and velocities) for some exogenous disturbance ($w(s)$) at the input of the first vehicle such that $u_1(s) = w(s) + K(s)y_1(s)$. We define the transfer function of the closed-loop system (9) from $w(s)$ to the input observed at vehicle i as

$$P_i(s) := \frac{u_i(s)}{w(s)} \;\forall\; i = 1, ..., n.$$

The induced-$\mathcal{L}_2$ norm of the transfer function can be written as

$$||P_i||_{\mathcal{H}_\infty} = \sup_{w \neq 0} \frac{||u_i||_{\mathcal{L}_2}}{||w||_{\mathcal{L}_2}} \;\forall\; i = 1, ..., n,$$

where the $\mathcal{L}_2$ norm is defined on the interval $t \in [0, \infty)$

$$||u_i(t)||_{\mathcal{L}_2} = \sqrt{\int_0^\infty (u_i(t))^2 dt},$$

hence,

$$||u_i||_{\mathcal{L}_2} \leq ||P_i||_{\mathcal{H}_\infty} ||w||_{\mathcal{L}_2} \;\forall\; i = 1, ..., n.$$

The string stability sensitivity function corresponding to the controlled input is

$$\Gamma(s) := \frac{\frac{u_i(s)}{w(s)}}{\frac{u_{i-1}(s)}{w(s)}} = P_i(s)(P_{i-1}(s))^{-1}. \tag{17}$$

If we derive the above string stability function assuming heterogeneity in the parameters of vehicles in the platoon, we would obtain

$$\Gamma(s, \bar{p}, \hat{p}_i, \hat{p}_{i-1}) = \frac{(K^{\text{ff}} \mathrm{e}^{-\phi_{b(i-1)}s} + G_{i-1} K^{\text{fb}} \mathrm{e}^{-\phi_{ci}s})}{(1 + K^{\text{fb}}(h_i s + 1) G_i \mathrm{e}^{-\phi_{ci}s})}, \tag{18}$$

$\forall\; i = 2, ..., n$, G_i contains the plant dynamics as in (4),

$$\begin{aligned} K(s) &= C_c(sI - A_c)^{-1} B_c + D_c \\ &= \begin{bmatrix} K^{\text{fb1}}(s) & K^{\text{fb2}}(s) & K^{\text{ff}}(s) \end{bmatrix} \end{aligned} \tag{19}$$

is the stabilising controller ($u_i(s) = K(s)y_i(s)$, $u_i \in \mathbb{R}$, $y_i \in \mathbb{R}^3$). For simplicity in representation, we use $K^{\text{fb}}(s) = K^{\text{fb1}}(s) + sK^{\text{fb2}}(s)$ to denote the feedback part of controller corresponding to the signal e_i. The function Γ in (18) contains the plant and the controller parameters of vehicle i, and some plant parameters of vehicle $i-1$. Recall that we denote the controller parameters and the plant parameters of vehicle i using $\bar{p}$ and $\hat{p}_i$ respectively. The CACC configuration that arises from (9) for heterogeneous (parameter) vehicles is given in Fig. 4.

Note that the authors in [1] consider (18) for strict string stability. This is sufficient for their case, as the string stability sensitivity function corresponding to the controlled input and the acceleration are the same for homogeneous vehicular platoons. However, this is not valid for the case of heterogeneous vehicular platoons. Therefore, we rewrite the string stability sensitivity function in terms of acceleration

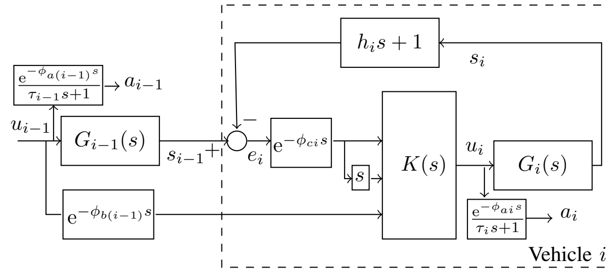

Fig. 4. The one vehicle look-ahead configuration of CACC.

(observing the accelerations for the exogenous input w). The new string stability sensitivity function (corresponding to acceleration) becomes

$$\begin{aligned} \Psi(s, \bar{p}, \hat{p}_i, \hat{p}_{i-1}) &= \frac{\frac{a_i(s)}{w(s)}}{\frac{a_{i-1}(s)}{w(s)}} = \frac{\frac{\mathrm{e}^{-\phi_{ai}s}}{\tau_i s+1} \cdot \frac{u_i(s)}{w(s)}}{\frac{\mathrm{e}^{-\phi_{a(i-1)}s}}{\tau_{i-1}s+1} \cdot \frac{u_{i-1}(s)}{w(s)}} \\ &= \frac{(K^{\text{ff}} \mathrm{e}^{-\phi_{b(i-1)}s} + G_{i-1} K^{\text{fb}} \mathrm{e}^{-\phi_{ci}s})(\tau_{i-1}s+1)\mathrm{e}^{-\phi_{ai}s}}{(1 + K^{\text{fb}}(h_i s+1) G_i \mathrm{e}^{-\phi_{ci}s})(\tau_i s+1)\mathrm{e}^{-\phi_{a(i-1)}s}}, \end{aligned} \tag{20}$$

for all $i = 2, ..., n$. Note that Ψ is the string stability sensitivity function corresponding to both acceleration and velocity ($a_i(s) = s \cdot v_i(s)$). Based on the assumptions mentioned above, for $\mathcal{L}_2$ *strict* string stability of heterogeneous vehicular platoons, we define the following (similar to [1]).

Definition 1: We consider the interconnected system (9) to be $\mathcal{L}_2$ *strict* string stable if $c(\bar{p}) < 0$ and

$$\sup_{\omega \in \mathbb{R}} |\Psi(j\omega, \bar{p}, \hat{p}_i, \hat{p}_{i-1})| \leq 1 \;\forall\; i = 2, ..., n.$$

Finally, we optimise (using frequency-gridding approach) the function

$$\min_{\bar{p}} \left\{ \max_{\hat{p}_i, \hat{p}_{i-1}} \left(\sup_{\omega \in \mathbb{R}} |\Psi(j\omega, \bar{p}, \hat{p}_i, \hat{p}_{i-1})| \right) \right\} : \; i \in \{2, ..., n\},$$

while $c(\bar{p}) < 0$, to obtain a stabilising controller that achieves strict $\mathcal{L}_2$ string stability.

D. Investigating a robust string stability achieving controller

The platoon string stability problem in the previous subsection can be extended to find a controller for an uncertain string stability sensitivity function (with uncertainties in vehicle parameters confined to real intervals) to increase scalability to multiple vehicles. Now we consider the Ψ function to be *uncertain* with perturbations at all the vehicle/plant parameters (including time-delays). We define the vectors $\hat{p}_\delta$ (corresponding to the parameters of vehicle i) and $\breve{p}_\delta$ (corresponding to the parameters of vehicle ahead) as the uncertainty belonging to some region $\mathcal{R} \in \mathbb{R}^5$ respectively. We define the new robust induced-$\mathcal{L}_2$ norm (as the worst case $\mathcal{L}_2$ gain for all perturbations) while the corresponding $\alpha < 0$ (exponential stability),

$$\chi_\infty := \max_{\hat{p}_\delta, \breve{p}_\delta \in \mathcal{R}} \left\{ \sup_{\omega \in \mathbb{R}} |\Psi(j\omega, \bar{p}, \hat{p}_\delta, \breve{p}_\delta)| \right\}. \tag{21}$$

If the corresponding $\alpha > 0$, then $\chi_\infty = \infty$. By intuitively minimising the worst case scenarios using the approach in Section V-C, the robust induced-$\mathcal{L}_2$ norm may be brought to a desirable value ($\chi_\infty \leq 1$). This provides robust performance for all the bounded perturbations/uncertainties in terms of maximum induced-$\mathcal{L}_2$ norm. In this paper, we determined χ_∞ by maximising induced-$\mathcal{L}_2$ norm for all the possible combinations of vehicle parameters confined in their respective regions in the real coordinate space.

VI. Simulation Based Studies

Let us consider a case of heterogeneous platoon with three vehicles ($n = 3$), whose parameters are given in Table II. We consider only three vehicles in the platoon for simplicity in presentation. Our aim is to guarantee (exponential) stability

TABLE II
THE VEHICLE PARAMETERS USED FOR SIMULATIONS.

i	τ_i	h_i	ϕ_{ai}	ϕ_{bi}	ϕ_{ci}
1	0.07	0.7	0.18	0.018	0.18
2	0.1	0.8	0.2	0.02	0.2
3	0.01	0.6	0.15	0.015	0.15

and (string stability) performance for the platoon[2]. The string stability sensitivity function considered is

$$\Psi(s, \bar{p}, \hat{p}_l, \hat{p}_k) = \dots$$
$$\dots \frac{(K^{\mathrm{ff}} \mathrm{e}^{-\phi_{bk}s} + G_k K^{\mathrm{fb}} \mathrm{e}^{-\phi_{cl}s})(\tau_k s + 1)\mathrm{e}^{-\phi_{al}s}}{(1 + K^{\mathrm{fb}}(h_l s + 1)G_l \mathrm{e}^{-\phi_{cl}s})(\tau_l s + 1)\mathrm{e}^{-\phi_{ak}s}}, \tag{22}$$

for all $k, l = 1, 2, 3$, where vehicle l is following vehicle k. Note that (20) and (22) are the same function, however, we abuse the subscript notations (k, l) to show that we include all possible combinations of the heterogeneous vehicles from Table II in the platoon.

Considering that the vehicles (with parameters in Table II) are to be controlled by identical controllers, we minimise the objective function (while $c(\bar{p}) < 0$)

$$\min_{\bar{p}} \left\{ \max_{\hat{p}_l, \hat{p}_k} \left(\sup_{\omega \in \mathbb{R}} |\Psi(j\omega, \bar{p}, \hat{p}_l, \hat{p}_k)| \right) \right\} \leq 1 : \; k, l \in \{1, 2, 3\}, \tag{23}$$

to obtain the controller

$$K = \begin{cases} \dot{x}_{ci}(t) &= \begin{bmatrix} -1.4999 & 1.5909 \\ 0.5346 & -3.8166 \end{bmatrix} x_{ci}(t) \\ &\quad + \begin{bmatrix} 1.9677 & -1.2820 & -1.7317 \\ -0.4932 & 1.1862 & 0.7864 \end{bmatrix} y_i(t) \\ u_i(t) &= \begin{bmatrix} -1.0527 & 0.3931 \end{bmatrix} x_{ci}(t) \\ &\quad + \begin{bmatrix} 1.7204 & 0.0702 & 0.0178 \end{bmatrix} y_i(t), \\ & \qquad \forall\ i = 1, 2, 3. \end{cases} \tag{24}$$

A preliminary minimisation of $c(\bar{p})$ was performed to ensure that the starting values for $\bar{p}$ in (23) had $c(\bar{p}) < 0$ (exponential stability). The frequency responses are plotted in Fig. 5 for the function $\Psi(s, \bar{p}, \hat{p}_l, \hat{p}_k)$ with K given in (24) for all $k, l = 1, 2, 3$. The time responses of accelerations and velocities for a reference signal ($V_{ref,0}$, $F_{a0} = \dot{V}_{ref,0}$) are shown in Figs. 6-7 for a combination of the three vehicles from Table II (in the platoon) simulated using MATLAB. Similarly, the time responses of position errors for vehicles in the same arrangement are given in Fig. 8. The robust induced-$\mathcal{L}_2$ norm for the one vehicle look-ahead vehicular platoon using K given in (24) was also investigated. We found $\chi_\infty \leq 1$ and $\alpha = -0.1485$ for

$$\begin{aligned} &\tau_i \in [0.01, 0.1], h_i \in [0.6, 0.8], \phi_{ai} \in [0.15, 0.2], \\ &\phi_{bi} \in [0.015, 0.02], \text{ and } \phi_{ci} \in [0.15, 0.2] \end{aligned} \tag{25}$$

$\forall\ i = 1, ..., n$, for any natural number n. In summary, the controller K given in (24) is guaranteed to maintain *exponential* stability and *strict* $\mathcal{L}_2$ string stability for any number of vehicles in system (9) and for any combination within the platoon, given that the vehicle parameters are confined to the intervals in (25).

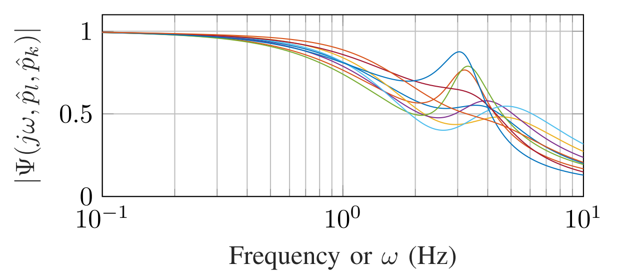

Fig. 5. Frequency response of the function Ψ for all the nine possible combinations of the three heterogeneous vehicles in the platoon.

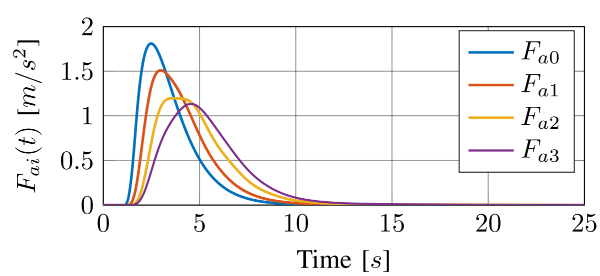

Fig. 6. Acceleration response for the reference signal $V_{ref,0}$ (platoon of the 3 vehicles controlled using the controller K).

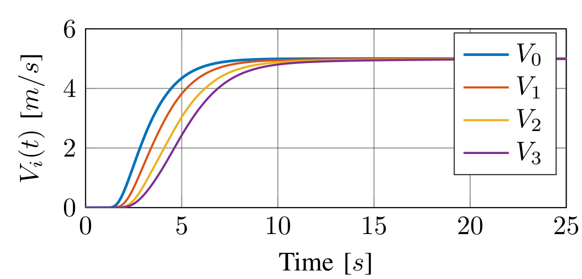

Fig. 7. Velocity response for the reference signal $V_{ref,0}$ (platoon of the 3 vehicles controlled using the controller K).

[2]The software tool and the vehicular platoon example are available in http://twr.cs.kuleuven.be/research/software/delay-control/CACCproblem.zip

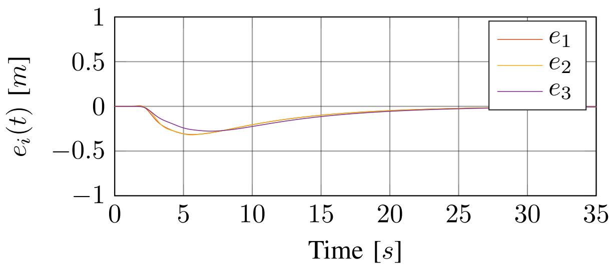

Fig. 8. Position errors for the reference signal $V_{ref,0}$ (platoon of the 3 vehicles controlled using the controller K).

VII. Conclusion

The design problem of stabilising (identical) controllers that achieves strict $\mathcal{L}_2$ string stability for heterogeneous (parameter) vehicular platoons in the one vehicle look-ahead topology was considered. We proposed an approach to design the controllers satisfying the stability and performance requirements for linearised third order heterogeneous vehicle plant models. The proposed approach was implemented in MATLAB and the corresponding results were presented.

We solved the controller design problem using direct optimisation techniques in the frequency domain, grounded in necessary and sufficient stability conditions. One of the main limitations for the direct optimisation based approach is the non-convexity of the optimisation problem. Therefore, the algorithm can converge to local optimum. We mitigate this problem by considering sufficiently large number of randomly generated starting points, and choosing the most optimal solution from them.

A scalable design approach to obtain (identical) decentralised controllers was also proposed for the case of heterogeneous (parameter) one vehicle look-ahead platoon, wherein the computational complexity is independent of the platoon size. We ensure that the achieved exponential stability and string stability properties are independent of the number of vehicles in the platoon, given that their parameters are confined to some real intervals. This improves the computational efficiency and scalability.

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