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COLLAPSE AND EVAPORATION OF A CANONICAL SELF-GRAVITATING GAS

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We review the out-of-equilibrium properties of a self-gravitating gas of particles in the presence of a strong friction and a random force (canonical gas). We assume a bare diffusion coefficient of the form $D(\rho) = T\rho^{1/n}$, where ρ is the local particle density, so that the equation of state is $P(\rho) = D(\rho)\rho$. Depending on the spatial dimension d , the index n , the temperature T , and whether the system is confined to a finite box or not, the system can reach an equilibrium state, collapse or evaporate. This article focuses on the latter cases, presenting a complete dynamical phase diagram of the system.

Keywords: Gravitational collapse; stochastic processes; dynamical phase transition.

1. Introduction and background

There is currently a renewed interest in the study of long-range interacting systems, particularly outside the realm of astrophysics. These systems may exhibit inequivalence of statistical ensembles (*e.g.* canonical *vs* microcanonical) which affects their equilibrium properties (possibility of negative specific heat in the microcanonical ensemble) and even more dramatically, their dynamical properties. In the context of self-gravitating systems, the Newtonian dynamics is too complicated to permit an exhaustive analytical treatment. Hence, in the past few years, we have developed a model of self-gravitating particles in the presence of a strong friction and a random force,[?] for which inertial effects are negligible. The main interest of this model is to be analytically tractable in many situations, while presenting dynamical phases reminiscent of their Newtonian counterparts. In addition, this model is intimately related to the Keller-Segel model of bacterial chemotaxis.[?]

$$= \tag{1}$$

$$= \tag{2}$$

$$= \tag{3}$$

We thus consider particles obeying the equations of motion $\frac{d\mathbf{x}_i}{dt} = -\nabla\Phi + \sqrt{2D}\eta_i$, where η_i is a delta correlated random Gaussian force, and Φ is the gravitational potential. In a proper mean-field limit, which becomes exact for an infinite number of particles, the density obeys a Fokker-Planck (or Smoluchowski) equation coupled to the Poisson equation (the gravitational constant is set equal to $G = 1$):

$$\frac{\partial\rho}{\partial t} = \nabla \cdot (\nabla P + \rho\nabla\Phi), \quad \Delta\Phi = \rho. \tag{4}$$

The pressure is related to the diffusion coefficient by $P(\rho) = D(\rho)\rho$, and the isothermal case corresponds to $D = T$, while we will consider here the more general poly-

tropic case $D = T\rho^{1/n}$.[?] In addition, the system of total mass M can be placed in a spherical bounded domain of radius R or in an unbounded space.

In the context of chemotaxis,[?] ρ is the density of bacteria, and $c = -\Phi$ is the density of a chemical that they secrete. The bacteria are attracted by the regions of high density of this chemical which generates an effective long-range interaction between them. This interaction exactly takes the form of gravity when neglecting the diffusion and the degradation of the chemical.

Although we shall see that the actual phase diagram depends crucially on the value of the index n , the general physics is controlled by the parameter T which we assimilate to the temperature. When T is small, the kinetic pressure is not strong enough to compensate gravity and the system may collapse. For large T , the system is at equilibrium in a bounded domain or evaporates in an unbounded domain. In the next section, we present the main results that we have obtained in^{?,?,?,?,?,?,?} concerning these different dynamical phases.

2. Collapse and post-collapse dynamics and the critical index n_*

Defining the critical index $n_* = \frac{d}{d-2}$, the system can be shown to always have a polytropic equilibrium state for $0 < n < n_*$, whether the system is confined or not. Hence we shall concentrate on dynamical properties of the case $n > n_*$ and $n < 0$ (including the isothermal case $n = \infty$ ($d > 2$)), and on the interesting particular case $n = n_*$ (including the isothermal case $n = \infty$ ($d = 2$)).

2.1. $n > n_*$ and the isothermal case $n = \infty$ ($d > 2$)

We have shown^{?,?,?,?} that in a bounded domain, and for $T < T_c$, the system displays a finite time singularity at which the central density diverges as $\rho_0(t) \sim (t_{\text{coll}} - t)^{-1}$. The density profile takes a scale-invariant form $\rho(r, t) = \rho_0(t)f[r/r_0(t)]$, where the core radius $r_0(t)$ is related to the central density by $r_0^2(t) = T/\rho_0^{1-1/n}(t)$. The density scaling function decays as $f(x) \sim x^{-\alpha}$, with $\alpha = \frac{2n}{n-1}$, and can be analytically obtained in the case $n = \infty$ ($d > 2$).[?] In this important case, t_{coll} was also computed exactly close to T_c .[?]

For $t > t_{\text{coll}}$, during what we dubbed the post-collapse regime,[?] a delta peak condensate grows at the center as $M_0 \sim (t - t_{\text{coll}})^{d(n-1)/(2n)-1}$, and ultimately saturates exponentially to the total mass M , with a rate which was computed analytically for low T , by using semi-classical quantum methods.[?] Meanwhile, the residual density obeys a reverse scaling of the form $\rho_{\text{res.}}(r, t) = \rho_r(t)f_r[r/r_r(t)]$, where $\rho_r(t) \sim (t - t_{\text{coll}})^{-1}$, and f_r decays with the same exponent $\alpha = \frac{2n}{n-1}$ as f .

In an unbounded domain,[?] the gas collapses below a *non universal* critical temperature T_c depending on the initial state, or evaporates for $T > T_c$. In this evaporation regime, gravity becomes asymptotically negligible and free diffusion is observed, with sub-corrections due to gravity which have been obtained analytically.^{?,?}

2.2. $n < 0$

The case of negative n ,^{?,?,?} including the logotropic case $n = -1$, is similar to the case treated above for $n < -d/2$, with a collapse scaling exponent $\alpha = \frac{2n}{n-1}$. However, for $-d/2 < n < 0$, the collapse is controlled by the $T = 0$ fixed point^{?,?,?} with $\alpha = \frac{2d}{d+2}$.

2.3. $n = n_*$ and the isothermal case $n = \infty$ ($d = 2$)

In this critical case,^{?,?,?} T_c is independent of the radius R of the confining box ($T_c = M/4$, in $d = 2$). For $T < T_c$, there is still a finite time singularity but a delta peak condensate of mass $M_c = (T/T_c)^{d/2}M$ forms at the center, at $t = t_{\text{coll}}$. Moreover, the residual density takes a pseudo scale-invariant form with an effective scaling exponent $\alpha(t)$ very slowly saturating to $\alpha = d$, as t goes to t_{coll} . In an unbounded domain, the gas collapses below the *universal* critical temperature T_c , or evaporates for $T > T_c$. In this evaporation regime, the scaling density profile is strongly affected by gravity and the gas expansion is such that $\langle r^d(t) \rangle \sim (T - T_c)t$.^{?,?}

3. Summary

In Table ??, we summarize the static and dynamic phase diagram of a self-gravitating gas of particles with a bare diffusion coefficient $D = T\rho^{1/n}$, where T is the temperature and d the spatial dimension. This table illustrates the crucial role played by the critical index $n_* = \frac{d}{d-2}$.

Table 1. Static and dynamic phase diagram of a self-gravitating gas.

Index n	Temperature	Bounded domain	Unbounded domain
$n = \infty$ ($d > 2$)	$T > T_c$	Metastable equilibrium state (local minimum of free energy): box-confined isothermal sphere	<ul style="list-style-type: none"> Evaporation : asymptotically free normal diffusion (gravity negligible)
	$T < T_c$	Self-similar collapse with $\alpha = 2$ and self-similar post-collapse leading to a Dirac peak of mass M	<ul style="list-style-type: none"> Collapse: pre-collapse and post-collapse as in a bounded domain
$0 < n < n_*$	$T > T_c$	Equilibrium state: box-confined (incomplete) polytrope	Equilibrium state: complete polytrope (compact support)
	$T < T_c$	Equilibrium state: complete polytrope (compact support)	
$n_* < n < \infty$ $n < 0$	$T > T_c$	Metastable equilibrium state (local minimum of free energy): box-confined polytropic sphere	<ul style="list-style-type: none"> Evaporation: asymptotically free anomalous diffusion (gravity negligible)
	$T < T_c$	Self-similar collapse with $\alpha = 2n/(n-1)$ or $\alpha = 2d/(d+2)$ ($-d/2 < n < 0$) and post-collapse leading to a Dirac peak	<ul style="list-style-type: none"> Collapse: pre-collapse and post-collapse as in a bounded domain
$n = n_*$	$T > T_c$	Equilibrium state: box-confined (incomplete) polytrope	Self-similar evaporation modified by self-gravity
	$T < T_c$	Pseudo self-similar collapse leading to a Dirac peak of mass $(T/T_c)^{d/2}M$ + halo. This is followed by a post-collapse leading to a Dirac peak of mass M	Collapse as in a bounded domain
	$T = T_c$	Infinite family of steady states	As in a bounded domain

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