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# Oracle inequalities for the stochastic differential equations \*

Pchelintsev E.A.,<sup>†</sup> Pergamenschikov S.M.<sup>‡</sup>

## Abstract

This paper is a survey of recent results on the adaptive robust non parametric methods for the continuous time regression model with the semi - martingale noises with jumps. The noises are modeled by the Lévy processes, the Ornstein – Uhlenbeck processes and semi-Markov processes. We represent the general model selection method and the sharp oracle inequalities methods which provide the robust efficient estimation in the adaptive setting. Moreover, we present the recent results on the improved model selection methods for the nonparametric estimation problems.

**Key words:** Non-parametric regression, Weighted least squares estimates, Improved non-asymptotic estimation, Robust quadratic risk, Lévy process, Ornstein – Uhlenbeck process, semi-Markov process, Model selection, Sharp oracle inequality, Adaptive estimation, Asymptotic efficiency

**AMS (2010) Subject Classification :** primary 62G08; secondary 62G05

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# 1 Introduction

This paper is a survey on the adaptive non parametric estimation methods for the general semi-martingale regression model in continuous time defined as

$$dy_t = S(t)dt + d\xi_t, \quad 0 \leq t \leq n, \quad (1.1)$$

where  $S(\cdot)$  is an unknown 1 - periodic function,  $(\xi_t)_{0 \leq t \leq n}$  is an unobservable noise defined by semimartingale with the values in the Skorokhod space  $\mathcal{D}[0, n]$  such that, for any function  $f$  from  $\mathcal{L}_2[0, n]$ , the stochastic integral

$$I_n(f) = \int_0^n f(s)d\xi_s$$

is well defined and has the following properties

$$\mathbf{E}_Q I_n(f) = 0 \quad \text{and} \quad \mathbf{E}_Q I_n^2(f) \leq \kappa_Q \int_0^n f^2(s)ds. \quad (1.2)$$

We use  $\mathbf{E}_Q$  for the expectation with respect to the distribution  $Q$  in  $\mathcal{D}[0, n]$  of the process  $(\xi_t)_{0 \leq t \leq n}$ , which is assumed to belong to some probability family  $\mathcal{Q}_n$  and  $\kappa_Q$  is some positive constant depending on the distribution  $Q$ . The problem consists to estimate the function  $S$  on the observations  $(y_t)_{0 \leq t \leq n}$ . Note that if  $(\xi_t)_{0 \leq t \leq n}$  is a brownian motion, then we obtain the well known "signal+white noise" model which is very popular in statistical radio-physics (see, for example, [17, 31, 32, 39]). In this paper we assume that in addition to the intrinsic noise in the radio-electronic system, approximated usually by the Gaussian white or color noise, the useful signal  $S$  is distorted by the impulse flow described by the processes with jumps. The cause of the appearance of a pulse stream in the radio-electronic systems can be, for example, either external unintended (atmospheric) or intentional impulse noise and the errors in the demodulation and the channel decoding for the binary information symbols. Note that, for the first time the impulse noises for the detection signal problems have been introduced on the basis of the compound Poisson processes was introduced by Kassam in [23]. Later, such processes was used in [28, 29, 30, 37] for the parametric and nonparametric signal estimation problems. However, the compound Poisson process can describe only the large impulses influence of fixed single frequency. Taking into account that in the telecommunication systems, the impulses are without limitations on frequencies one needs to extend the framework of the observation model by making use the Lévy processes (2.1) which is a particular case of the general semimartingale regression model introduced in [25]. Generally, we consider nonparametric estimation problems for the function  $S$  from  $\mathcal{L}_2$  under the

condition that the distribution of the noise  $(\xi_t)_{0 \leq t \leq n}$  is unknown. We know only that this distribution belongs to some distribution family  $\mathcal{Q}_n$ . In this case we use the robust estimation approach proposed in [13, 28, 29] for the nonparametric estimation. According to this approach we have to construct an estimator  $\widehat{S}_n$  (any function of  $(y_t)_{0 \leq t \leq n}$ ) for  $S$  to minimize the robust risk defined as

$$\mathcal{R}_n^*(\widehat{S}_n, S) = \sup_{Q \in \mathcal{Q}_n} \mathcal{R}_Q(\widehat{S}_n, S), \quad (1.3)$$

where  $\mathcal{R}_Q(\cdot, \cdot)$  is the usual quadratic risk of the form

$$\mathcal{R}_Q(\widehat{S}_n, S) := \mathbf{E}_Q \|\widehat{S}_n - S\|^2 \quad \text{and} \quad \|S\|^2 = \int_0^1 S^2(t) dt. \quad (1.4)$$

It is clear that if we don't know the distribution of the observation one needs to find an estimator which will be optimal for all possible observation distributions. Moreover in this paper we consider the estimation problem in the adaptive setting, i.e. when the regularity of  $S$  is unknown. To this end we use the adaptive method based on the model selection approach. The interest to such statistical procedures is explained by the fact that they provide adaptive solutions for the nonparametric estimation through oracle inequalities which give the non-asymptotic upper bound for the quadratic risk including the minimal risk over chosen family of estimators. It should be noted that for the first time the model selection methods were proposed by Akaike [1] and Mallows [34] for parametric models. Then, these methods had been developed for the nonparametric estimation and the oracle inequalities for the quadratic risks was obtained by Barron, Birgé and Massart [3], by Fourdrinier and Pergamenshchikov [12] for the regression models in discrete time and [27] in continuous time. Unfortunately, the oracle inequalities obtained in these papers can not provide the efficient estimation in the adaptive setting, since the upper bounds in these inequalities have some fixed coefficients in the main terms which are more than one. To obtain the efficiency property for estimation procedures one has to obtain the sharp oracle inequalities, i.e. in which the factor at the principal term on the right-hand side of the inequality is close to unity. The first result on sharp inequalities is most likely due to Kneip [22] who studied a Gaussian regression model in the discrete time. It will be observed that the derivation of oracle inequalities usually rests upon the fact that the initial model, by applying the Fourier transformation, can be reduced to the Gaussian independent observations. However, such transformation is possible only for Gaussian models with independent homogeneous observations or for inhomogeneous ones with known correlation characteristics. For the general non Gaussian observations one needs to

use the approach proposed by Galtchouk and Pergamenschikov [14, 15] for the heteroscedastic regression models in discrete time and developed then by Konev and Pergamenschikov in [25, 26, 28, 29] for semimartingale models in continuous time. In general the model selection is an adaptive rule  $\hat{\alpha}$  which choses an estimator  $S^* = \hat{S}_{\hat{\alpha}}$  from an estimate family  $(\hat{S}_{\alpha})_{\alpha \in \mathcal{A}}$ . The goal of this selection is to prove the following nonasymptotic oracle inequality: for any sufficient small  $\delta > 0$  and any observation duration  $n \geq 1$

$$\mathcal{R}(S^*, S) \leq (1 + \delta) \min_{\alpha \in \mathcal{A}} \mathcal{R}(\hat{S}_{\alpha}, S) + \delta^{-1} \mathcal{B}_n, \quad (1.5)$$

where the rest term  $\mathcal{B}_n$  is sufficiently small with respect to the minimax convergence rate. Such oracle inequalities are called *sharp*, since the coefficient in the main term  $1 + \delta$  is close to one for sufficiently small  $\delta > 0$ . Moreover, in this paper we represent the new results on the improved estimation methods for the nonparametric models (1.1). Usually, the model selection procedures are based on the least squares estimators. But in [38] it is propose to use the improved least square estimators which enable to improve considerably the non asymptotic estimation accuracy. At the first time such idea was proposed in [12] for the regression in discrete time and in [27] for the Gaussian regression model in continuous time. In [38] these methods are developed for the non - Gaussian regression models in continuous time. It should be noted that generally for the conditionally Gaussian regression models one can not use the well known improved estimators proposed in [19, 11] for Gaussian or spherically symmetric observations. To apply the improved estimation methods to the non Gaussian regression models in continuous time one needs to modify the well known James–Stein procedure in the way proposed in [37, 30]. For the improved model selection procedures the oracle inequality (1.5) is shown also. We note that this inequality allows us to provide the asymptotic efficiency without knowing the regularity of the function being estimated. The efficacy property for a nonparametric estimate  $S^*$  means

$$\lim_{n \rightarrow \infty} v_n \sup_{S \in W_r^k} \mathcal{R}(S^*, S) = \lim_{n \rightarrow \infty} v_n \inf_{\hat{S}} \sup_{S \in W_r^k} \mathcal{R}(\hat{S}, S) = l_*(\mathbf{r}),$$

where

$$l_*(\mathbf{r}) = ((1 + 2k)\mathbf{r})^{1/(2k+1)} \left( \frac{k}{\pi(k+1)} \right)^{2k/(2k+1)}, \quad (1.6)$$

$v_n$  is a normalizing coefficient (convergence rate),  $W_r^k$  is the Sobolev ball of a radius  $r > 0$  and the regularity  $k \geq 1$ . The limit (1.6) is called the Pinsker constant which is calculated by Pinsker in [39].

The rest of the paper is organized as follows. In the next section 2, we describe the Lévy, Ornstein–Uhlenbeck and semi-Markov processes as the examples of a semimartingale impulse noise in the model (1.1). In Section 3 we construct the model selection procedure based on the least square estimators and show the sharp oracle inequalities. In Section 4 we give the improved least squares estimators and we study the improvement effect for the semimartingale model (1.1). In Section 5 we construct the improved model selection procedure and show the sharp oracle inequalities. The asymptotic efficiency is studied in Section 6.

## 2 Examples

### 2.1 Lévy model

First we consider the model (1.1) with the Lévy noise process, i.e. we assume that the noise process  $(\xi_t)_{0 \leq t \leq n}$  is defined as

$$\xi_t = \varrho_1 w_t + \varrho_2 z_t \quad \text{and} \quad z_t = x * (\mu - \tilde{\mu})_t, \quad (2.1)$$

where  $\varrho_1$  and  $\varrho_2$  are some unknown constants,  $(w_t)_{t \geq 0}$  is a standard brownian motion,  $\mu(ds dx)$  is a jump measure with deterministic compensator  $\tilde{\mu}(ds dx) = ds\Pi(dx)$ ,  $\Pi(\cdot)$  is a Lévy measure, i.e. some positive measure on  $\mathbb{R}_* = \mathbb{R} \setminus \{0\}$ , (see, for example, [18, 9] for details) such that

$$\Pi(x^2) = 1 \quad \text{and} \quad \Pi(x^6) < \infty.$$

Here we use the notation  $\Pi(|x|^m) = \int_{\mathbb{R}_*} |z|^m \Pi(dz)$ . Note that the Lévy measure  $\Pi(\mathbb{R}_*)$  could be equal to  $+\infty$ . One can check directly that for the process (2.1) the condition (1.2) holds with  $\kappa_Q = \sigma_Q = \varrho_1^2 + \varrho_2^2$ . We assume that the nuisance parameters  $\varrho_1$  and  $\varrho_2$  of the process  $(\xi_t)_{0 \leq t \leq n}$  satisfy the conditions

$$0 < \underline{\varrho} \leq \varrho_1^2 \quad \text{and} \quad \sigma_Q \leq \zeta^*, \quad (2.2)$$

where the bounds  $\underline{\varrho}$  and  $\zeta^*$  are functions of  $n$ , i.e.  $\underline{\varrho} = \underline{\varrho}_n$  and  $\zeta^* = \zeta_n^*$  such that for any  $\delta > 0$

$$\liminf_{n \rightarrow \infty} n^\delta \underline{\varrho}_n > 0 \quad \text{and} \quad \lim_{n \rightarrow \infty} n^{-\delta} \zeta_n^* = 0. \quad (2.3)$$

For this example  $\mathcal{Q}_n$  is the family of all distributions of process (1.1) – (2.1) on the Skorokhod space  $\mathbf{D}[0, n]$  satisfying the conditions (2.2) – (2.3).

The models (1.1) with the Lévy’s type noise are used in different applied problems (see [7], for details). Such models naturally arise in the nonparametric functional statistics problems (see, for example, [8]).

## 2.2 Ornstein – Uhlenbeck model

Now we consider the noise process  $(\xi_t)_{t \geq 0}$  in (1.1) difened by a non-Gaussian Ornstein-Uhlenbeck process with the Lévy subordinator. Let the noise process in (1.1) obey the equation

$$d\xi_t = a\xi_t dt + du_t, \quad \xi_0 = 0, \quad (2.4)$$

where  $u_t = \varrho_1 w_t + \varrho_2 z_t$  and the process  $z_t$  is defied in (2.1). Here  $a \leq 0$ ,  $\varrho_1$  and  $\varrho_2$  are unknown parameters. We assume that the parameters  $\varrho_1$  and  $\varrho_2$  satisfy the conditions (2.2) and the parameter

$$-a_{max} \leq a \leq 0, \quad (2.5)$$

where the bound  $a_{max} > 0$  is the function of  $n$ , i.e.  $a_{max} = a_{max}(n)$ , such that for any  $\check{\delta} > 0$

$$\lim_{n \rightarrow \infty} \frac{a_{max}}{n^{\check{\delta}}} = 0. \quad (2.6)$$

In this case  $\mathcal{Q}_n$  is the family of all distributions of process (2.4) on the Skorokhod space  $\mathbf{D}[0, n]$  satisfying the conditions (2.2), (2.5) and (2.6). Note also that the processes (2.1) and (2.4) are  $\mathcal{G}$  - conditionally Gaussian square integrated semimartingales, where  $\mathcal{G} = \sigma\{z_t, t \geq 0\}$ .

Such processes are used in the financial Black-Scholes type markets with jumps (see, for example, [2, 10] and the references therein). Note also that in the case when  $\varrho_2 = 0$  for the parametric estimation problem such models are considered in [20, 21, 24].

## 2.3 Semi – Markov model

In [4, 5] it is introduced the regression model (1.1) in which the noise process describes by the equation

$$\xi_t = \varrho_1 L_t + \varrho_2 X_t, \quad (2.7)$$

and the Lévy process  $L_t$  is defined as

$$L_t = \check{\varrho} w_t + \sqrt{1 - \check{\varrho}^2} z_t,$$

where  $0 \leq \check{\varrho} \leq 1$  is an unknown constant. Moreover, we assume that the pure jump process  $(X_t)_{t \geq 0}$  in (2.7) is a semi-Markov process with the following form

$$X_t = \sum_{i=1}^{N_t} Y_i, \quad (2.8)$$

where  $(Y_i)_{i \geq 1}$  is an i.i.d. sequence of random variables with

$$\mathbf{E} Y_i = 0, \quad \mathbf{E} Y_i^2 = 1 \quad \text{and} \quad \mathbf{E} Y_i^4 < \infty.$$

Here  $N_t$  is a general counting process (see, for example, [35]) defined as

$$N_t = \sum_{k=1}^{\infty} \mathbf{1}_{\{T_k \leq t\}} \quad \text{and} \quad T_k = \sum_{l=1}^k \tau_l,$$

where  $(\tau_l)_{l \geq 1}$  is an i.i.d. sequence of positive integrated random variables with distribution  $\eta$  and mean  $\tilde{\tau} = \mathbf{E} \tau_1 > 0$ . We assume that the processes  $(N_t)_{t \geq 0}$  and  $(Y_i)_{i \geq 1}$  are independent between them and are also independent of  $(L_t)_{t \geq 0}$ . Here, the family  $\mathcal{Q}_n$  is defined by of all distributions of process (2.7) on the Skorokhod space  $\mathbf{D}[0, n]$  with the parameters  $\varrho_1$  and  $\varrho_2$  satisfying the conditions (2.2) and  $0 \leq \check{\varrho} \leq 1$ .

Note that the process  $(X_t)_{t \geq 0}$  is a special case of a semi-Markov process (see, e.g., [6] and [33]). It should be noted that if  $\tau_j$  are exponential random variables, then  $(N_t)_{t \geq 0}$  is a Poisson process and, in this case,  $(\xi_t)_{t \geq 0}$  is a Lévy process. But, in the general case when the process (2.8) is not a Lévy process, this process has a memory and cannot be treated in the framework of semi-martingales with independent increments. In this case, we need to develop new tools based on renewal theory arguments from [16].

It should be noted that for  $\check{\varrho} > 0$  the process (2.7) is  $\mathcal{G}$  - conditionally Gaussian also. In this case  $\mathcal{G} = \sigma\{z_t, X_t, t \geq 0\}$ .

### 3 Model selection

Let  $(\phi_j)_{j \geq 1}$  be an orthonormal uniformly bounded basis in  $\mathcal{L}_2[0, 1]$ , i.e. for some constant  $\phi_* \geq 1$ , which may be depend on  $n$ ,

$$\sup_{0 \leq j \leq n} \sup_{0 \leq t \leq 1} |\phi_j(t)| \leq \phi_* < \infty.$$

For example, we can take the trigonometric basis defined as  $\text{Tr}_1 \equiv 1$  and, for  $j \geq 2$ ,

$$\text{Tr}_j(x) = \sqrt{2} \begin{cases} \cos(2\pi[j/2]x) & \text{for even } j; \\ \sin(2\pi[j/2]x) & \text{for odd } j, \end{cases} \quad (3.1)$$

where  $[x]$  denotes the integer part of  $x$ .



To estimate the function  $S$  we use here the model selection procedure for continuous time regression models from [28] based on the Fourier expansion. We recall that for any function  $S$  from  $\mathcal{L}_2[0, 1]$  we can write

$$S(t) = \sum_{j=1}^{\infty} \theta_j \phi_j(t) \quad \text{and} \quad \theta_j = (S, \phi_j) = \int_0^1 S(t) \phi_j(t) dt.$$

So, to estimate the function  $S$  it suffices to estimate the coefficients  $\theta_j$  and to replace them in this representation by their estimators. Using the fact that the function  $S$  and  $\phi_j$  are 1 - periodic we can write that

$$\theta_j = \frac{1}{n} \int_0^n \phi_j(t) S(t) dt.$$

If we replace here the differential  $S(t)dt$  by the stochastic observed differential  $dy_t$  then we obtain the natural estimate for  $\theta_j$  on the time interval  $[0, n]$

$$\hat{\theta}_{j,n} = \frac{1}{n} \int_0^n \phi_j(t) dy_t,$$

which can be represented, in view of the model (1.1), as

$$\hat{\theta}_{j,n} = \theta_j + \frac{1}{\sqrt{n}} \xi_{j,n} \quad \text{and} \quad \xi_{j,n} = \frac{1}{\sqrt{n}} I_n(\phi_j).$$

We need to impose some stability conditions for the noise Fourier transform sequence  $(\xi_{j,n})_{1 \leq j \leq n}$ . To this end for some parameter  $\sigma_Q > 0$  we set the following function

$$\mathbf{L}_{1,n}(Q) = \sum_{j=1}^n \left| \mathbf{E}_Q \xi_{j,n}^2 - \sigma_Q \right|. \quad (3.2)$$

In [28] the parameter  $\sigma_Q$  is called proxy variance .

**C<sub>1</sub>)** *There exists a variance proxy  $\sigma_Q > 0$  such that for any  $\epsilon > 0$*

$$\lim_{n \rightarrow \infty} \frac{\mathbf{L}_{1,n}(Q)}{n^\epsilon} = 0.$$

Moreover, we define

$$\mathbf{L}_{2,n}(Q) = \sup_{|x| \leq 1} \mathbf{E}_Q \left( \sum_{j=1}^n x_j \tilde{\xi}_{j,n} \right)^2 \quad \text{and} \quad \tilde{\xi}_{j,n} = \xi_{j,n}^2 - \mathbf{E}_Q \xi_{j,n}^2.$$

$\mathbf{C}_2$ ) Assume that for any  $\epsilon > 0$

$$\lim_{n \rightarrow \infty} \frac{\mathbf{L}_{2,n}(Q)}{n^\epsilon} = 0.$$

**Remark 3.1.** Note that the condition  $\mathbf{C}_2$ ) means the boundedness in  $\mathcal{L}_2$  of all partial sum of the sequence  $(\tilde{\xi}_{j,n})_{1 \leq j \leq n}$ . In particular, if we set  $x_j = 1/\sqrt{n}$  for  $1 \leq j \leq n$  we obtain that the normalized sum

$$\frac{1}{\sqrt{n}} \sum_{j=1}^n \tilde{\xi}_{j,n}$$

has to be bounded in  $\mathcal{L}_2$ .

Now (see, for example, [17]) we can estimate the function  $S$  by the projection estimators, i.e.

$$\widehat{S}_m(t) = \sum_{j=1}^m \widehat{\theta}_{j,n} \phi_j(t), \quad 0 \leq t \leq 1, \quad (3.3)$$

for some number  $m \rightarrow \infty$  as  $n \rightarrow \infty$ . It should be noted that Pinsker in [39] shows that the projection estimators of the form (3.3) are not efficient. For obtaining efficient estimation one needs to use weighted least square estimators defined as

$$\widehat{S}_\lambda(t) = \sum_{j=1}^n \lambda(j) \widehat{\theta}_{j,n} \phi_j(t), \quad (3.4)$$

where the coefficients  $\lambda = (\lambda(j))_{1 \leq j \leq n}$  belong to some finite set  $\Lambda$  from  $[0, 1]^n$ . As it is shown in [39], in order to obtain efficient estimators, the coefficients  $\lambda(j)$  in (3.4) need to be chosen depending on the regularity of the unknown function  $S$ . Since we consider the adaptive case, i.e. we assume that the regularity of the function  $S$  is unknown, then we chose the weight coefficients on the basis of the model selection procedure proposed in [28] for the general semi-martingale regression model in continuous time. To the end, first we set

$$\nu = \#(\Lambda) \quad \text{and} \quad |\Lambda|_* = 1 + \max_{\lambda \in \Lambda} \sum_{j=1}^n \lambda(j),$$

where  $\#(\Lambda)$  is the cardinal number of  $\Lambda$ . Now, to choose a weight sequence  $\lambda$  in the set  $\Lambda$  we use the empirical quadratic risk, defined as

$$\text{Err}_n(\lambda) = \| \widehat{S}_\lambda - S \|^2,$$

which in our case is equal to

$$\text{Err}_n(\lambda) = \sum_{j=1}^n \lambda^2(j) \widehat{\theta}_{j,n}^2 - 2 \sum_{j=1}^n \lambda(j) \widehat{\theta}_{j,n} \theta_j + \sum_{j=1}^{\infty} \theta_j^2.$$

Since the Fourier coefficients  $(\theta_j)_{j \geq 1}$  are unknown, we replace the terms  $\widehat{\theta}_{j,n} \theta_j$  by

$$\widetilde{\theta}_{j,n} = \widehat{\theta}_{j,n}^2 - \frac{\widehat{\sigma}_n}{n},$$

where  $\widehat{\sigma}_n$  is an estimate for the variance proxy  $\sigma_Q$  defined in (3.2). If it is known, we take  $\widehat{\sigma}_n = \sigma_Q$ , otherwise, we can choose it, for example, as in [28], i.e.

$$\widehat{\sigma}_n = \sum_{j=[\sqrt{n}]+1}^n \widehat{t}_{j,n}^2, \quad (3.5)$$

where  $\widehat{t}_{j,n}$  are the estimators for the Fourier coefficients with respect to the trigonometric basis (3.1), i.e.

$$\widehat{t}_{j,n} = \frac{1}{n} \int_0^n \text{Tr}_j(t) dy_t.$$

Finally, in order to choose the weights, we will minimize the following cost function

$$J_n(\lambda) = \sum_{j=1}^n \lambda^2(j) \widehat{\theta}_{j,n}^2 - 2 \sum_{j=1}^n \lambda(j) \widetilde{\theta}_{j,n} + \delta P_n(\lambda),$$

where  $\delta > 0$  is some threshold which will be specified later and the penalty term is

$$P_n(\lambda) = \frac{\widehat{\sigma}_n |\lambda|^2}{n}. \quad (3.6)$$

We define the model selection procedure as

$$\widehat{S}_* = \widehat{S}_{\widehat{\lambda}} \quad \text{with} \quad \widehat{\lambda} = \operatorname{argmin}_{\lambda \in \Lambda} J_n(\lambda). \quad (3.7)$$

We recall that the set  $\Lambda$  is finite so  $\widehat{\lambda}$  exists. In the case when  $\widehat{\lambda}$  is not unique, we take one of them.

As is shown in [4, 28, 38] both Conditions  $\mathbf{C}_1$ ) and  $\mathbf{C}_2$ ) hold for the processes (2.1), (2.4) and (2.7).

**Proposition 3.1.** *If the conditions  $\mathbf{C}_1$ ) and  $\mathbf{C}_2$ ) hold for the distribution  $Q$  of the process  $\xi$  in (1.1), then, for any  $n \geq 1$  and  $0 < \delta < 1/3$ , the risk (1.4) of estimate (3.7) for  $S$  satisfies the oracle inequality*

$$\mathcal{R}_Q(\widehat{S}_*, S) \leq \frac{1 + 3\delta}{1 - 3\delta} \min_{\lambda \in \Lambda} \mathcal{R}_Q(\widehat{S}_\lambda, S) + \frac{\mathbf{B}_n(Q)}{\delta n}, \quad (3.8)$$

where  $\mathbf{B}_n(Q) = \mathbf{U}_n(Q) (1 + |\Lambda|_* \mathbf{E}_Q |\hat{\sigma}_n - \sigma_Q|)$  and the coefficient  $\mathbf{U}_n(Q)$  is such that for any  $\epsilon > 0$

$$\lim_{n \rightarrow \infty} \frac{\mathbf{U}_n(Q)}{n^\epsilon} = 0. \quad (3.9)$$

In the case, when the value of  $\sigma_Q$  is known, one can take  $\hat{\sigma}_n = \sigma_Q$  and

$$P_n(\lambda) = \frac{\sigma_Q |\lambda|_n^2}{n},$$

then we can rewrite the oracle inequality (3.8) with  $\mathbf{B}_n(Q) = \mathbf{U}_n(Q)$ . Also we study the accuracy properties for the estimator (3.5).

**Proposition 3.2.** *Let in the model (1.1) the function  $S(\cdot)$  is continuously differentiable. Then, for any  $n \geq 2$ ,*

$$\mathbf{E}_Q |\hat{\sigma}_n - \sigma_Q| \leq \frac{\varkappa_n(Q) (1 + \|\dot{S}\|^2)}{\sqrt{n}},$$

where the term  $\varkappa_n(Q)$  possesses the property (3.9) and  $\dot{S}$  is the derivative of the function  $S$ .

To obtain the oracle inequality for the robust risk (1.3) we need some additional condition on the distribution family  $\mathcal{Q}_n$ . We set

$$\varsigma^* = \varsigma_n^* = \sup_{Q \in \mathcal{Q}_n} \sigma_Q \quad \text{and} \quad \mathbf{L}_n^* = \sup_{Q \in \mathcal{Q}_n} (\mathbf{L}_{1,n}(Q) + \mathbf{L}_{2,n}(Q)). \quad (3.10)$$

$\mathbf{C}_1^*$ ) Assume that the conditions  $\mathbf{C}_1)$ – $\mathbf{C}_2)$  hold and for any  $\epsilon > 0$

$$\lim_{n \rightarrow \infty} \frac{\mathbf{L}_n^* + \varsigma_n^*}{n^\epsilon} = 0.$$

Now we impose the conditions on the set of the weight coefficients  $\Lambda$ .

$\mathbf{C}_2^*$ ) Assume that the set  $\Lambda$  is such that for any  $\epsilon > 0$

$$\lim_{n \rightarrow \infty} \frac{\nu}{n^\epsilon} = 0 \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{|\Lambda|_*}{n^{1/2+\epsilon}} = 0.$$

**Theorem 3.3.** *Assume that the conditions  $\mathbf{C}_1^*)$ – $\mathbf{C}_2^*)$  hold. Then the robust risk (1.3) of the estimate (3.7) for continuously differentiable function  $S(t)$  satisfies for any  $n \geq 2$  and  $0 < \delta < 1/3$  the oracle inequality*

$$\mathcal{R}_n^*(\hat{S}_*, S) \leq \frac{1 + 3\delta}{1 - 3\delta} \min_{\lambda \in \Lambda} \mathcal{R}_n^*(\hat{S}_\lambda, S) + \frac{1}{\delta n} \mathbf{B}_n^* (1 + \|\dot{S}\|^2),$$

where the term  $\mathbf{B}_n^*$  satisfies the property (3.9).

It should be noted that Propositions 3.1 - 3.2 and Theorem 3.3 are shown in [4] and [27].

Now we specify the weight coefficients  $(\lambda(j))_{j \geq 1}$  in the way proposed in [14] for a heteroscedastic regression model in discrete time. First we define the normalizing coefficient which defined the minimax convergence rate

$$v_n = \frac{n}{\zeta^*}, \quad (3.11)$$

where the upper proxy variance is  $\zeta^*$  is defined in (3.10). Consider a numerical grid of the form

$$\mathcal{A}_n = \{1, \dots, k^*\} \times \{r_1, \dots, r_m\},$$

where  $r_i = i\varepsilon$  and  $m = \lceil 1/\varepsilon^2 \rceil$ . Both parameters  $k^* \geq 1$  and  $0 < \varepsilon \leq 1$  are assumed to be functions of  $n$ , i.e.  $k^* = k^*(n)$  and  $\varepsilon = \varepsilon(n)$ , such that for any  $\delta > 0$

$$\left\{ \begin{array}{l} \lim_{n \rightarrow \infty} k^*(n) = +\infty, \quad \lim_{n \rightarrow \infty} \frac{k^*(n)}{\ln n} = 0, \\ \lim_{n \rightarrow \infty} \varepsilon(n) = 0 \quad \text{and} \quad \lim_{n \rightarrow \infty} n^\delta \varepsilon(n) = +\infty. \end{array} \right.$$

One can take, for example,

$$\varepsilon(n) = \frac{1}{\ln(n+1)} \quad \text{and} \quad k^*(n) = \sqrt{\ln(n+1)}.$$

For each  $\alpha = (\beta, r) \in \mathcal{A}_n$  we introduce the weight sequence  $\lambda_\alpha = (\lambda_\alpha(j))_{j \geq 1}$  as

$$\lambda_\alpha(j) = \mathbf{1}_{\{1 \leq j \leq d\}} + (1 - (j/\omega_\alpha)^\beta) \mathbf{1}_{\{d < j \leq \omega_\alpha\}} \quad (3.12)$$

where  $d = d(\alpha) = \lfloor \omega_\alpha / \ln(n+1) \rfloor$ ,  $\omega_\alpha = (\tau_\beta r v_n)^{1/(2\beta+1)}$  and

$$\tau_\beta = \frac{(\beta+1)(2\beta+1)}{\pi^{2\beta}\beta}.$$

We set

$$\Lambda = \{\lambda_\alpha, \alpha \in \mathcal{A}_n\}. \quad (3.13)$$

It will be noted that in this case the cardinal of the set  $\Lambda$  is  $\nu = k^*m$ . Moreover, taking into account that  $\tau_\beta < 1$  for  $\beta \geq 1$  we obtain for the set (3.13)

$$|\Lambda|_* \leq 1 + \sup_{\alpha \in \mathcal{A}} \omega_\alpha \leq 1 + (v_n/\varepsilon)^{1/3}.$$

Note that the form (3.12) for the weight coefficients was proposed by Pinsker in [39] for the efficient estimation in the nonadaptive case, i.e. when the regularity parameters of the function  $S$  are known. In the adaptive case these weight coefficients are used in [28, 29] to show the asymptotic efficiency for model selection procedures.

## 4 Improved estimation

In this Section we consider the improved estimation method for the model (1.1). We impose the following additional condition on the noise distribution.

**D<sub>1</sub>)** There exists  $n_0 \geq 1$  such that for any  $n \geq n_0$  there exists a  $\sigma$ -field  $\mathcal{G}_n$  for which the random vector  $\tilde{\xi}_{d,n} = (\xi_{j,n})_{1 \leq j \leq d}$  is the  $\mathcal{G}_n$  conditionally Gaussian in  $\mathbb{R}^d$  with the covariance matrix

$$\mathbf{G}_n = \left( \mathbf{E} \xi_{i,n} \xi_{j,n} | \mathcal{G}_n \right)_{1 \leq i, j \leq d}$$

and for some nonrandom constant  $l_n^* > 0$

$$\inf_{Q \in \mathcal{Q}_n} (\text{tr} \mathbf{G}_n - \lambda_{\max}(\mathbf{G}_n)) \geq l_n^*,$$

where  $\lambda_{\max}(A)$  is the maximal eigenvalue of the matrix  $A$ .

**Proposition 4.1.** *Let in the model (1.1) the noise process describes by the Lévy process (2.1). Then the condition **D<sub>1</sub>**) holds with  $l_n^* = (d-1)\underline{\rho}$  for any  $n \geq 1$ .*

**Proposition 4.2.** *Let in the model (1.1) the noise process describes by the Ornstein–Uhlenbeck process (2.4). Then the condition **D<sub>1</sub>**) holds with  $l_n^* = (d-6)\underline{\rho}/2$  for any  $n \geq n_0$  and  $d \geq d_0 = \inf\{d \geq 7 : 5 + \ln d \leq \check{a}d\}$ ,  $\check{a} = (1 - e^{-a_{\max}})/(4a_{\max})$ .*

Now, for the first  $d$  Fourier coefficients we use the improved estimation method proposed for parametric models in [37]. To this end we set  $\tilde{\theta}_n = (\hat{\theta}_{j,n})_{1 \leq j \leq d}$ . In the sequel we will use the norm  $|x|_d^2 = \sum_{j=1}^d x_j^2$  for any vector  $x = (x_j)_{1 \leq j \leq d}$  from  $\mathbb{R}^d$ . Now we define the shrinkage estimators as

$$\theta_{j,n}^* = (1 - g(j)) \hat{\theta}_{j,n},$$

where  $g(j) = (\mathbf{c}_n / |\tilde{\theta}_n|_d) \mathbf{1}_{\{1 \leq j \leq d\}}$  and

$$\mathbf{c}_n = \frac{l_n^*}{\left( r_n^* + \sqrt{d/v_n} \right) n}. \quad (4.1)$$

The positive parameter  $r_n^*$  is such that  $\lim_{n \rightarrow \infty} r_n^* = \infty$  and for any  $\epsilon > 0$

$$\lim_{n \rightarrow \infty} \frac{r_n^*}{n^\epsilon} = 0 \quad (4.2)$$

and  $v_n$  defined in (3.11). Now we introduce a class of shrinkage weighted least squares estimates for  $S$  as

$$S_\lambda^* = \sum_{j=1}^n \lambda(j) \theta_{j,n}^* \phi_j. \quad (4.3)$$

We denote the difference of quadratic risks of the estimates (3.4) and (4.3) as

$$\Delta_Q(S) := \mathcal{R}_Q(S_\lambda^*, S) - \mathcal{R}_Q(\widehat{S}_\lambda, S).$$

**Theorem 4.3.** *Let the observed process  $(y_t)_{0 \leq t \leq n}$  describes by the equation (1.1) and the condition  $\mathbf{D}_1$ ) holds. Then for any  $n \geq 1$*

$$\sup_{Q \in \mathcal{Q}_n} \sup_{\|S\| \leq r_n^*} \Delta_Q(S) \leq -\mathbf{c}_n^2. \quad (4.4)$$

Propositions 4.1 - 4.2 and Theorem 4.3 are shown in [37] and [38].

**Remark 4.1.** *The inequality (4.4) means that non asymptotically, i.e. for any  $n \geq 1$ , the estimate (4.3) outperforms in mean square accuracy the estimate (3.4). Moreover in the efficient weight coefficients  $d \approx n^{\check{\delta}}$  as  $n \rightarrow \infty$  for some  $\check{\delta} > 0$ . Therefore, in view of the definition (4.1) and the conditions (2.3) and (4.2)  $n\mathbf{c}_n \rightarrow \infty$  as  $n \rightarrow \infty$ . This means that improvement is considerably may better than for the parametric regression when the parameter dimension  $d$  is fixed [37].*

## 5 Improved model selection

This Section gives the construction of a model selection procedure for estimating a function  $S$  in (1.1) on the basis of improved weighted least square estimates  $(S_\lambda^*)_{\lambda \in \Lambda}$  and states the sharp oracle inequality for the robust risk of proposed procedure.

As in Section 3, the performance of any estimate  $S_\lambda^*$  will be measured by the empirical squared error

$$\text{Err}_n(\lambda) = \|S_\lambda^* - S\|^2.$$

In order to obtain a good estimate, we have to write a rule to choose a weight vector  $\lambda \in \Lambda$  in (4.3). It is obvious, that the best way is to minimise the

empirical squared error with respect to  $\lambda$ . Making use the estimate definition (4.3) and the Fourier transformation of  $S$  implies

$$\text{Err}_n(\lambda) = \sum_{j=1}^n \lambda^2(j)(\theta_{j,n}^*)^2 - 2 \sum_{j=1}^n \lambda(j)\theta_{j,n}^* \theta_j + \sum_{j=1}^n \theta_j^2.$$

Here one needs to replace the terms  $\theta_{j,n}^* \theta_j$  by their estimators  $\bar{\theta}_{j,n}$ . We set

$$\bar{\theta}_{j,n} = \theta_{j,n}^* \hat{\theta}_{j,n} - \frac{\hat{\sigma}_n}{n},$$

where  $\hat{\sigma}_n$  is defined in (3.5). For this change in the empirical squared error, one has to pay some penalty. Thus, one comes to the cost function of the form

$$J_n^*(\lambda) = \sum_{j=1}^n \lambda^2(j)(\theta_{j,n}^*)^2 - 2 \sum_{j=1}^n \lambda(j)\bar{\theta}_{j,n} + \delta P_n(\lambda),$$

where  $\delta$  is some positive constant and the penalty term  $P_n(\lambda)$  is defined in (3.6). Substituting the weight coefficients, minimizing the cost function

$$\lambda^* = \operatorname{argmin}_{\lambda \in \Lambda} J_n^*(\lambda), \quad (5.1)$$

in (4.3) leads to the improved model selection procedure

$$S^* = S_{\lambda^*}^*. \quad (5.2)$$

It will be noted that  $\lambda^*$  exists because  $\Lambda$  is a finite set and also if the minimizing sequence in (5.1)  $\lambda^*$  is not unique, one can take any minimizer. For this procedure in [38] it is obtained the following oracle inequality.

**Theorem 5.1.** *If the conditions  $\mathbf{C}_1$ ) and  $\mathbf{C}_2$ ) hold for the distribution  $Q$  of the process  $\xi$  in (1.1), then, for any  $n \geq 1$  and  $0 < \delta < 1/3$ , the risk (1.4) of estimate (5.2) for  $S$  satisfies the oracle inequality*

$$\mathcal{R}_Q(S_{\lambda^*}^*, S) \leq \frac{1 + 3\delta}{1 - 3\delta} \min_{\lambda \in \Lambda} \mathcal{R}_Q(S_\lambda^*, S) + \frac{\check{\mathbf{B}}_n(Q)}{n\delta},$$

where  $\check{\mathbf{B}}_n(Q) = \check{\mathbf{U}}_n(Q) (1 + |\Lambda|_* \mathbf{E}_Q |\hat{\sigma}_n - \sigma_Q|)$  and the coefficient  $\check{\mathbf{U}}_n(Q)$  satisfies the property (3.9).

Now Theorem 5.1 and Proposition 3.2 directly imply the following inequality for the robust risk (1.3) of the procedure (5.2).

**Theorem 5.2.** *Assume that the conditions  $\mathbf{C}_1^*$ ) and  $\mathbf{C}_2^*$ ) hold and the function  $S$  is continuously differentiable. Then for any  $n \geq 2$  and  $0 < \delta < 1/3$*

$$\mathcal{R}_n^*(S_{\lambda^*}^*, S) \leq \frac{1 + 3\delta}{1 - 3\delta} \min_{\lambda \in \Lambda} \mathcal{R}_n^*(S_\lambda^*, S) + \frac{\check{\mathbf{U}}_n^*(1 + \|\dot{S}\|^2)}{n\delta},$$

where the coefficient  $\check{\mathbf{U}}_n^*$  satisfies the property (3.9).



## 6 Asymptotic efficiency

In this section we present the results on the robust asymptotic efficiency obtained in [26] and [27] for the semimartingale regression model (1.1). In order to study the asymptotic efficiency we define the following functional Sobolev ball

$$W_{k,\mathbf{r}} = \{f \in \mathbf{C}_p^k[0, 1] : \sum_{j=0}^k \|f^{(j)}\|^2 \leq \mathbf{r}\},$$

where  $\mathbf{r} > 0$  and  $k \geq 1$  are some unknown parameters,  $\mathbf{C}_p^k[0, 1]$  is the space of  $k$  times differentiable 1 - periodic  $\mathbb{R} \rightarrow \mathbb{R}$  functions such that  $f^{(i)}(0) = f^{(i)}(1)$  for any  $0 \leq i \leq k-1$ . It is well known that for any  $S \in W_{k,\mathbf{r}}$  the optimal rate of convergence is  $n^{-2k/(2k+1)}$  (see, for example, [39, 36]). On the basis of the model selection procedure we construct the adaptive procedure  $S_*$  for which we obtain the following asymptotic upper bound for the quadratic risk, i.e. we show that the parameter (1.6) gives a lower bound for the asymptotic normalized risks. To this end we denote by  $\Sigma_n$  of all estimators  $\widehat{S}_n$  of  $S$  measurable with respect to the process (1.1), i.e.  $\sigma\{y_t, 0 \leq t \leq n\}$ .

**Theorem 6.1.** *The robust risk (1.3) admits the following asymptotic lower bound*

$$\liminf_{n \rightarrow \infty} \inf_{\widehat{S}_n \in \Sigma_n} v_n^{2k/(2k+1)} \sup_{S \in W_{k,\mathbf{r}}} \mathcal{R}_n^*(\widehat{S}_n, S) \geq l_*(\mathbf{r}).$$

We show that this lower bound is sharp in the following sense.

**Theorem 6.2.** *The quadratic risk (1.3) for the estimating procedure  $S^*$  has the following asymptotic upper bound*

$$\limsup_{n \rightarrow \infty} v_n^{2k/(2k+1)} \sup_{S \in W_{k,\mathbf{r}}} \mathcal{R}_n^*(S^*, S) \leq l_*(\mathbf{r}).$$

It is clear that Theorem 6.2 and Theorem 6.1 imply

**Corollary 6.3.** *The model selection procedure  $S^*$  is efficient, i.e.*

$$\lim_{n \rightarrow \infty} (v_n)^{\frac{2k}{2k+1}} \sup_{S \in W_{k,\mathbf{r}}} \mathcal{R}_n^*(S^*, S) = l_*(\mathbf{r}). \quad (6.1)$$

**Remark 6.1.** *Note that the equality (6.1) implies that the parameter (1.6) is the Pinsker constant in this case (cf. [39]). Moreover, it means that the robust efficiency holds with the convergence rate  $(v_n)^{\frac{2k}{2k+1}}$ . It is well known that for the simple risks the optimal (minimax) estimation convergence rate*

for the functions from the set  $W_{k,r}$  is  $n^{2k/(2k+1)}$  (see, for example, [17, 36, 39]). So, if the distribution upper bound  $\zeta^* \rightarrow 0$  as  $n \rightarrow \infty$  we obtain the more rapid rate, and if  $\zeta^* \rightarrow \infty$  as  $n \rightarrow \infty$  we obtain the more slow rate. In the case when  $\zeta^*$  is constant the robust rate is the same as the classical non robust convergence rate.

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