



HAL
open science

Comportements asymptotiques d'écoulements tourbillonnaires diphasiques

Roger Prud'Homme, Louis Gottesdiener

► **To cite this version:**

Roger Prud'Homme, Louis Gottesdiener. Comportements asymptotiques d'écoulements tourbillonnaires diphasiques. 1996. hal-02334658

HAL Id: hal-02334658

<https://hal.science/hal-02334658>

Submitted on 27 Oct 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

COMPORTEMENTS ASYMPTOTIQUES D'ÉCOULEMENTS TOURBILLONNAIRES DIPHASIQUES

par R. Prud'homme & L. Gottesdiener - Laboratoire de modélisation en mécanique,
Université P. et M. Curie, 4 place Jussieu 75252 Paris Cedex 05

Résumé - L'objectif est d'étudier les champs de vitesses et de concentrations en écoulement tourbillonnaire dyphasique. Les méthodes analytiques sont utilisées dans la mesure du possible. Pour chaque tourbillon- irrotationnel ou de Rankine - on étudie d'abord le mouvement d'une particule, par une méthode asymptotique à échelles multiples et on compare les résultats à ceux de l'analyse numérique. On étudie ensuite le continuum de particules, d'abord en coordonnées de Lagrange, puis en variables d'Euler, en supposant que les solutions -instationnaires ou stationnaires- sont invariantes par rotation¹. La méthode est étendue à des particules en cours d'évaporation.

¹ ASYMPTOTIC ANALYSIS OF VORTEX FLOWS IN SUSPENSIONS by R. Prud'homme, L.Gottesdiener, E. Dodemand, P. Kuentzmann, soumis à IJMF, 1996

EQUATION DE LA QUANTITE DE MOUVEMENT

$$\frac{4}{3} \pi r_p^3 \rho_{ps} \frac{d_p \vec{v}_p}{dt} = 6\pi \mu r_p f_p (\vec{v}_g - \vec{v}_p) + \frac{2}{3} \pi r_p^3 \rho_{gs} \left(\frac{d_g \vec{v}_g}{dt} - \frac{d_p \vec{v}_p}{dt} \right) - \frac{4}{3} \pi r_p^3 \rho_{ps} \vec{grad}(p) + \frac{4}{3} \pi r_p^3 \rho_{ps} \vec{g} + \vec{F}_l + \vec{F}_b$$

\vec{F}_l est la force de portance et \vec{F}_b est la force de Basset.

En écrivant cette relation pour le fluide pur et en soustrayant membre à membre, on obtient:

$$\frac{4}{3} \pi r_p^3 (\rho_{ps} + 0.5 \rho_{gs}) \frac{d_p \vec{v}_p}{dt} = 6\pi \mu r_p f_p (\vec{v}_g - \vec{v}_p) + 2\pi r_p^3 \rho_{gs} \frac{d_g \vec{v}_g}{dt}$$

Posons:

$$\tau_v = \frac{2\rho_{ps} r_p^2}{9\mu f_p (1 - 0.5E)}, \quad E = \frac{\rho_{gs}}{\rho_{ps} + 0.5\rho_{gs}}$$

EFFETS DU GRADIENT DE PRESSION

Si le mouvement du fluide est celui d'un tourbillon à vorticit  concentr e (dit irrotationnel):

$$\frac{d_p \vec{v}_p}{dt} = \frac{1}{r} \ddot{\theta} \vec{e}_\theta - \dot{v}_p \vec{e}_r - \frac{3E}{2r^3} \vec{e}_r$$

En coordonn es cylindriques:

$$\frac{1}{r} \frac{d(r^2 \dot{\theta})}{dt} = \frac{1}{r} - r \dot{\theta}$$

$$\ddot{r} - r \dot{\theta}^2 = -\dot{r} - \frac{3E}{2r^3}$$

soit:

```

*****
r^2 \dot{\theta} = 1 + (\alpha^2 \beta - 1) e^{-t}      *
*
\ddot{r} + \dot{r} = \frac{(1 + (\alpha^2 \beta - 1) e^{-t})^2 - 1.5E}{r^3}      *
*****
    
```

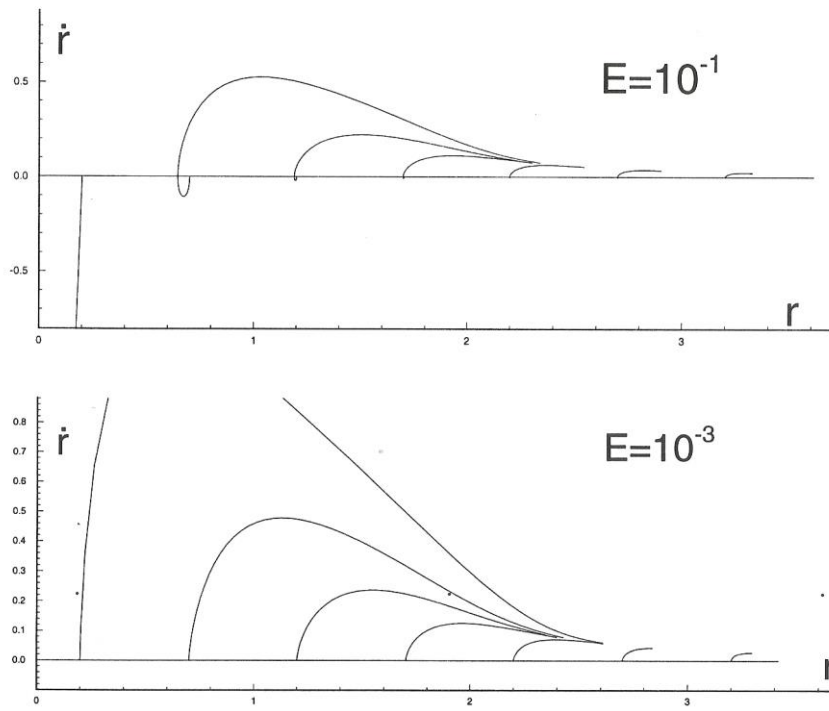


Figure 19 - Particles at rest at the initial time, $b(a)=0$, $\beta(a)=0$ (or $K=0$).
 Influence of the gas/particles density ratio $E=\rho_{gs}/(\rho_{ps}+0.5\rho_{gs})$.

TOURBILLON DIPHASIQUE

$$\frac{d_p \vec{v}_p}{dt} = \frac{\vec{v}_g - \vec{v}_p}{\tau_v} \quad v_{gr} = 0, \quad v_{g\theta} = \Gamma/2\pi r$$

$$t_{ref} = \tau_v, l_{ref} = r_v = \sqrt{\Gamma\tau_v/2\pi}, v_{ref} = \sqrt{\Gamma/2\pi\tau_v}$$

$$\frac{d_p \vec{v}_p}{dt} = \frac{1}{r} \vec{e}_\theta - \vec{v}_p$$

$$\frac{1}{r} \frac{d(r^2 \dot{\theta})}{dt} = \frac{1}{r} - r \dot{\theta}$$

$$\ddot{r} - r \dot{\theta}^2 = -\dot{r}$$

$\dot{r}, \ddot{r}, \dot{\theta}$ dérivées temporelles. Les conditions à $t = 0$, sont les suivantes:

$$r(0) = a, \theta(0) = \alpha, \dot{r}(0) = b, \dot{\theta}(0) = \beta$$

Après une première integration (Soo 1990):

```

*****
*      r2  $\dot{\theta} = 1 + (a^2 \beta - 1)e^{-t}$       *
*       $\ddot{r} + \dot{r} = \frac{(1 + (a^2 \beta - 1)e^{-t})^2}{r^3}$       *
*****

```

DEVELOPPEMENTS ASYMPTOTIQUES - ECHELLES MULTIPLES

$$r = \varepsilon^p \bar{r}, t = \varepsilon^q \bar{t}, v_{pr} = \varepsilon^n \bar{v}_{pr}, v_{p\theta} = \varepsilon^m \bar{v}_{p\theta}$$

$\bar{r}, \bar{t}, \bar{v}_{pr}, \bar{v}_{p\theta}$ sont $O(1)$ par définition. $d\bar{r}/d\bar{t}, d\bar{v}_{pr}/d\bar{t}$ etc. sont $O(1)$ par definition.

$v_{pr} = dr/dt$ entraîne:

$$n=p-q$$

On obtient:

$$a^2 \beta = \varepsilon^{m+p} \bar{a}^2 \bar{\beta}$$

On pose:

$$\theta = \alpha + \varepsilon^l \bar{\theta}, l = m + q - p$$

D'où:

$$* \quad \varepsilon^{m+p} \bar{r}^2 \frac{d\bar{\theta}}{d\bar{t}} = 1 + \left(\varepsilon^{m+p} \bar{a}^2 \bar{\beta} - 1 \right) e^{-\varepsilon^q \bar{t}} \quad *$$

* * *

$$* \quad \varepsilon^{4p-2q} \frac{d^2 \bar{r}}{d\bar{t}^2} + \varepsilon^{4p-q} \frac{d\bar{r}}{d\bar{t}} = \frac{1}{\bar{r}^3} \left(1 + \left(\varepsilon^{m+p} \bar{a}^2 \bar{\beta} - 1 \right) e^{-\varepsilon^q \bar{t}} \right)^2 \quad *$$

Case	$m+p$	Case	q	results along θ	Case	results along r	results in vector format	particle movement
I	> 0	1	> $m+p$	$\frac{1}{r} \frac{d}{dt}(r^2 \dot{\theta}) = 0$ $r^2 \dot{\theta} = a^2 \beta$	a	$m+q=p$ $\ddot{r} - r\dot{\theta}^2 = 0, \ddot{r} = \frac{a^4 \beta^2}{r^3}$	$\frac{d}{dt} \bar{v}_p = \bar{0}$	uniform movement in a straight line
					b	$m+q > p$ $\ddot{r} = 0$		$r = a + bt, \theta = \alpha - \frac{a^2 \beta}{b(a+bt)} + \frac{a\beta}{b}$
		2	= $m+p$	$\frac{1}{r} \frac{d}{dt}(r^2 \dot{\theta}) = \frac{1}{r}$ $r^2 \dot{\theta} = a^2 \beta + t$	a	$q > p$ $\ddot{r} = 0$		$r = a + bt, \theta = \alpha + \frac{1}{b^2} \text{Log} \frac{a+bt}{a} - \frac{(ab\beta-1)t}{b(a+bt)}$
					b	$q = p$ $\ddot{r} - r\dot{\theta}^2 = 0, \ddot{r} = \frac{(a^2 \beta + t)^2}{r^3}$	$\frac{d}{dt} \bar{v}_p = \bar{v}_g$	particle acceleration equal to the gas velocity
II	= 0	1	> 0	$\frac{1}{r} \frac{d}{dt}(r^2 \dot{\theta}) = 0$ $r^2 \dot{\theta} = a^2 \beta$	a	$2p=q$ $\ddot{r} - r\dot{\theta}^2 = 0, \ddot{r} = \frac{a^4 \beta^2}{r^3}$	$\frac{d}{dt} \bar{v}_p = \bar{0}$	uniform movement in a straight line
					b	$2p < q$ $\ddot{r} = 0$		same as 11b
		2	= 0	$\frac{1}{r} \frac{d}{dt}(r^2 \dot{\theta}) = \frac{1}{r} - r\dot{\theta}$ $r^2 \dot{\theta} = 1 + (a^2 \beta - 1)e^{-t}$	a	$p < 0$ $\ddot{r} = -\dot{r}$		$r = a + b(1 - e^{-t}), \theta = \theta(t)$
					b	$p = 0$ $\ddot{r} - r\dot{\theta}^2 = -\dot{r}$	$\frac{d}{dt} \bar{v}_p = \bar{v}_g - \bar{v}_p$	general undegenerated case
		3	< 0	$0 = \frac{1}{r} - r\dot{\theta}$ $r^2 \dot{\theta} = 1$	a	$4p=q$ $-r\dot{\theta}^2 = -\dot{r}, \dot{r} = \frac{1}{r^3}$		$r^4 = 4t + a^4, \theta = \alpha - \frac{a^2}{2} + \sqrt{t + \frac{a^4}{4}}$
					b	$4p < q$ $0 = -\dot{r}$	$\bar{0} = \bar{v}_g - \bar{v}_p$	the particle follows the gas
III	< 0	1	> 0	$\frac{1}{r} \frac{d}{dt}(r^2 \dot{\theta}) = 0$ $r^2 \dot{\theta} = a^2 \beta$	a	$m=p$ $\ddot{r} - r\dot{\theta}^2 = 0, \ddot{r} = \frac{a^4 \beta^2}{r^3}$	$\frac{d}{dt} \bar{v}_p = \bar{0}$	uniform movement in a straight line
					b	$m > p$ $\ddot{r} = 0$		same as 11b
		2	= 0	$\frac{1}{r} \frac{d}{dt}(r^2 \dot{\theta}) = -r\dot{\theta}$ $r^2 \dot{\theta} = a^2 \beta e^{-t}$	a	$m=p$ $\ddot{r} - r\dot{\theta}^2 = -\dot{r}, \ddot{r} + \dot{r} = \frac{a^4 \beta^2 e^{-2t}}{r^3}$	$\frac{d}{dt} \bar{v}_p = -\bar{v}_p$	damped movement in a straight line
					b	$m > p$ $\ddot{r} = -\dot{r}$		$r = a + b(1 - e^{-t}), \theta = \theta(t)$
		3	< 0	$0 = -r\dot{\theta}$ $\dot{\theta} = 0$		$0 = -\dot{r}$	$\bar{0} = -\bar{v}_p$	unmoving

Table 1

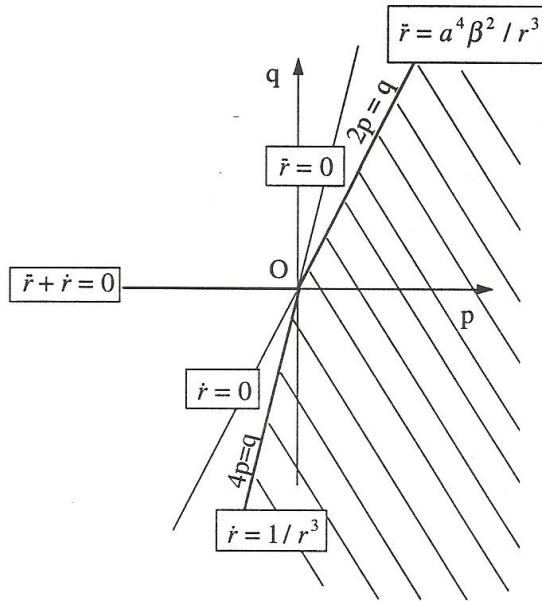


Figure 1

Position in plane (p, q)

(orders of magnitude: ε^p of the distance to the center of the vortex and ε^q of the time)
of the solutions of case II for which v_{p0} and v_{g0} are assumed to be of the same order of magnitude.

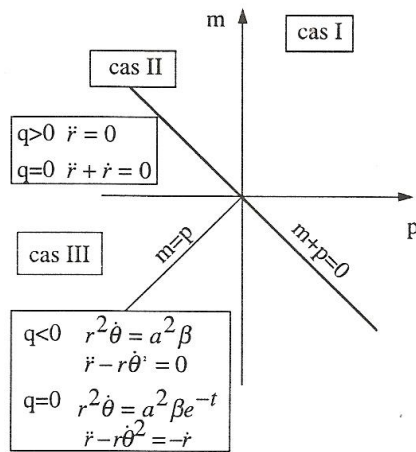
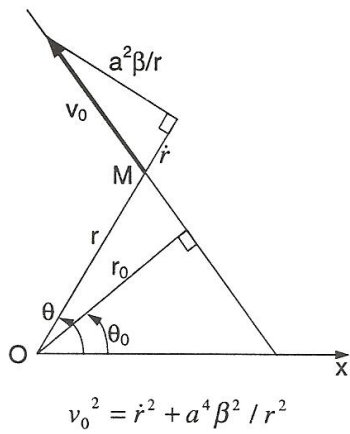


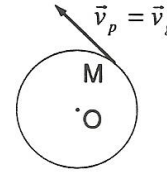
Figure 2

Position in plane (p, m)

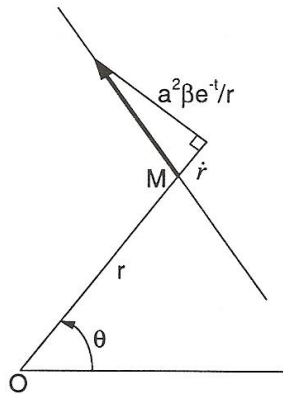
characterizing the orders of magnitude of r and v_{p0} respectively for the different cases, in particular the solutions of case III where $v_{p0} \gg v_{g0}$.



a - Case of uniform motion in a straight line
The particle does not see the gas



b - Case of circular motion
The particle follows the gas, its inertia is negligible



c - Case of damped motion in a straight line
The particle is slowed down by the gas whose motion it does not see



d - Case at rest

Figure 3 Representation in the physical plan for three elementary movements

I1a (see figure 3a) - Uniform motion in a straight line

$$v_{pr} = v_0 \sqrt{1 - \frac{r_0^2}{r^2}}, \quad v_0^2 = b^2 + a^2 \beta^2, \quad r_0^2 = \frac{a^4 \beta^2}{b^2 + a^2 \beta^2}$$

$$v_{p\theta} = \frac{a^2 \beta}{r}$$

I1b

$$v_{pr} = b, \quad r = a + bt, \quad \theta = \alpha + \frac{a\beta}{b} - \frac{a^2 \beta}{b(a+bt)}$$

$$v_{p\theta} = \frac{a^2 \beta}{r}, \quad \theta = \alpha + \frac{a\beta}{b} \left(1 - \frac{a}{r}\right)$$

I2a

$$v_{pr} = b, \quad r = a + bt, \quad \theta = \alpha + \frac{1}{b^2} \text{Log} \frac{r}{a} - \frac{1}{b} \left(a\beta - \frac{1}{b}\right) \left(1 - \frac{a}{r}\right)$$

$$v_{p\theta} = \frac{1}{b} + \frac{a(ab\beta - 1)}{br}$$

I2b A Taylor series expansion is carried out around $t = 0$ giving in the third order:

$$r = a + bt + \frac{a\beta^2}{2} t^2, \quad \theta = \alpha + \beta t + (1 - 2ab\beta)$$

$$v_{pr} = \sqrt{b^2 a \beta^2 (r - a)}, \quad v_{p\theta} = b \left(\frac{\beta}{b} - \frac{1 - 2ab\beta}{a^3 \beta^2} \right) r + \frac{1 - 2ab\beta}{a^2 \beta} r \sqrt{\frac{b^2}{a^2 \beta} - 2 + \frac{2r}{a}}$$

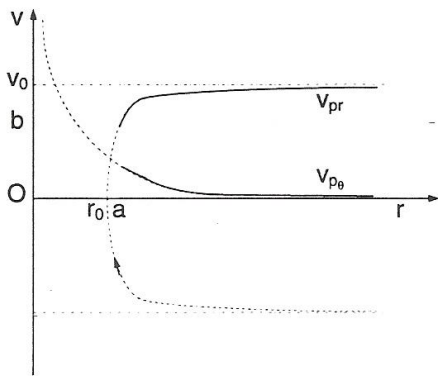
Table 2 - Solutions of case I $v_{p0} \ll v_{g0}$

II1a	Uniform motion in a straight line (same as I1a)
II1b	Same as I1b
II2a	$r = a + b(1 - e^{-t}), \quad \theta = \alpha + \frac{1}{b} \left(a\beta - \frac{1}{a+b} \right) \left(1 - \frac{a}{a+b(1-e^{-t})} \right) + \frac{t + \frac{\text{Log}(a+b(1-e^{-t}))}{a}}{(a+b)^2}$ $v_r = a + b - r, \quad v_\theta = \frac{1 - a^2\beta}{b} + \left(1 + (a^2\beta - 1) \left(\frac{a}{b} + 1 \right) \right) \frac{1}{r}$
II2b	Numerical solution, undegenerated case
II3a	$v_r = \frac{1}{r^3}, \quad v_\theta = \frac{1}{r}, \quad \theta = \alpha - \frac{a^2}{2} + \frac{r^2}{2}$
II3b	Uniform rotation
	$r = a, \quad v_r = 0, \quad v_\theta = \frac{1}{a}$

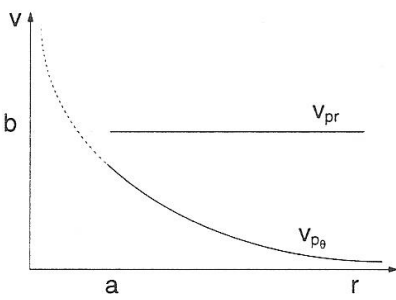
Table 3 - Solutions of case II $v_{p0} = O(v_{g0})$

III1a	Uniform motion in a straight line (same as I1a)
III1b	Same as I1b
III2a	(see figure 3c) - Damped motion in a straight line
III2b	Same as II2a taking $a^2\beta \rightarrow \infty$
	$r = a + b(1 - e^{-t}), \quad \theta = \alpha + \frac{a\beta}{b} \left(1 - \frac{a}{a+b(1-e^{-t})} \right) + \frac{t + \frac{\text{Log}(a+b(1-e^{-t}))}{a}}{(a+b)^2}$ $v_r = a + b - r, \quad v_\theta = -\frac{a^2\beta}{b} + \left(1 + a^2\beta \left(\frac{a}{b} + 1 \right) \right) \frac{1}{r}$
III3	$r = a, \quad \theta = \alpha$

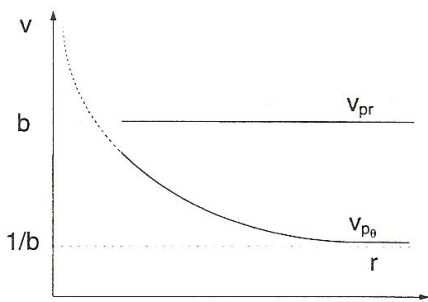
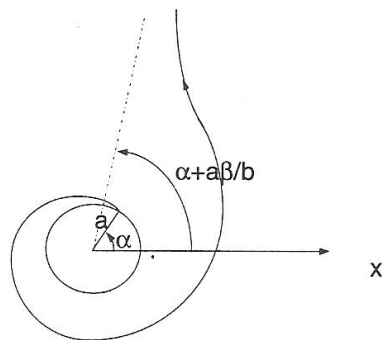
Table 4 - Solutions of case III $v_{p0} \gg v_{g0}$



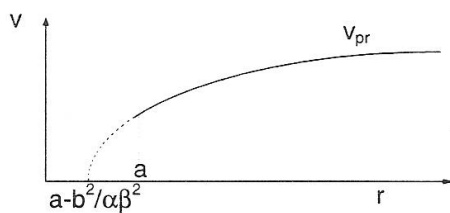
case I 1a



case I 1b



case I 2a



case I 2b

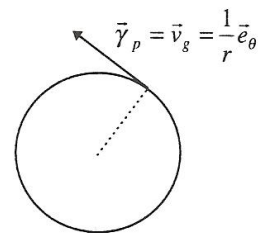


Figure 4 Case I ($v_{p0} \ll v_{g0}$) Path representations in plane (r, v) and in the physical plane.

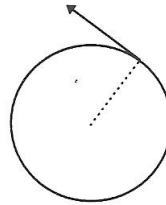
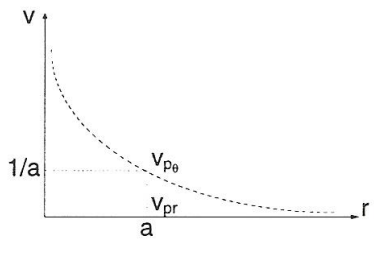
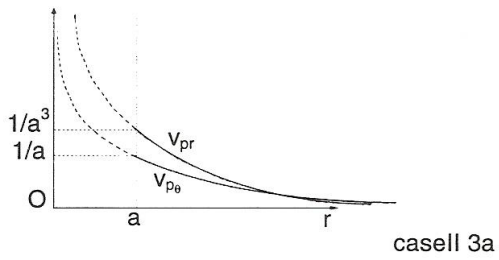
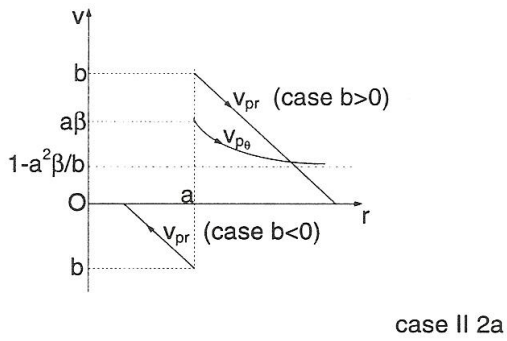
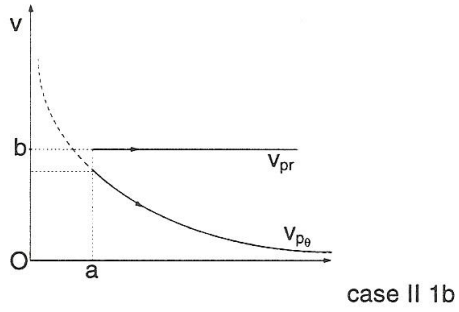
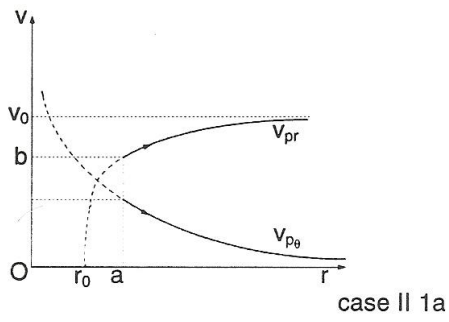


Figure 5 - Case II ($v_{p0} = 0 (v_{g0})$) - Path representations in plane (r, v) and in the physical plane.

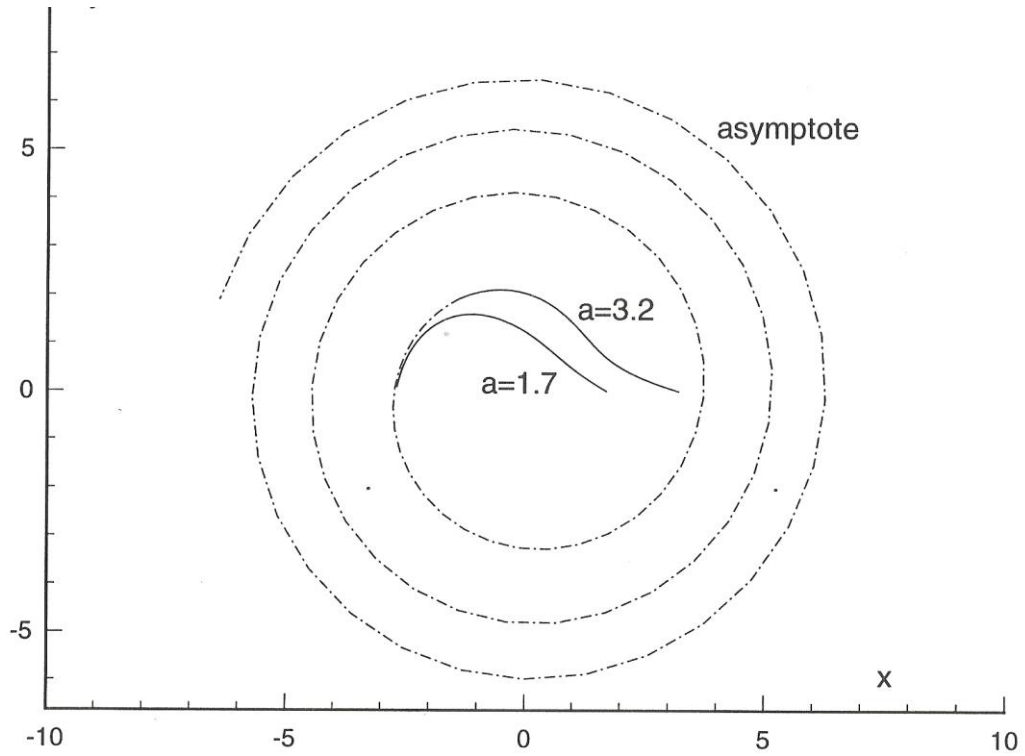


Figure 6. Example of numerically computed path ($a^2\beta=2, b=-2$).

ETUDE D'UN CONTINUUM DE PARTICULES

Coordonnées de Lagrange:

Invariance par rotation:

$$\begin{aligned} r &= r(a,t), & \dot{r} &= \frac{\partial r}{\partial t} & \theta &= \alpha + \theta_0(a,t), & \dot{\theta} &= \frac{\partial \theta}{\partial t} \\ a &= r(a,0), & b &= \dot{r}(a,0) & 0 &= \theta_0(a,0), & \beta &= \dot{\theta}(a,0) \end{aligned}$$

On se donne donc:

$$b=b(a) \qquad \beta=\beta(a)$$

$$r = r(a, \tau), \quad t = \tau, \quad a = r(a, 0)$$

$$dr = \frac{\partial r}{\partial a} da + \frac{\partial r}{\partial \tau} d\tau, \quad dt = d\tau$$

Stationnarité. On montre que:

$$\frac{\partial \dot{\theta} / \partial \tau}{\partial \dot{\theta} / \partial a} = \frac{\partial r / \partial \tau}{\partial r / \partial a} = b(a)$$

EXEMPLE

Cas IIa (or IIIa or IIIa). Chaque particule a une trajectoire rectiligne:

$$r^2 = r_0^2 + \left(v_0 t + \frac{ab}{v_0} \right)^2, \quad v_0^2 = b^2 + a^2 \beta^2$$

$$v_{pr} = v_0 \sqrt{1 - \frac{r_0^2}{r^2}}, \quad r_0^2 = \frac{a^4 \beta^2}{b^2 + a^2 \beta^2}$$

$$v_{p\theta} = \frac{a^2 \beta}{r}$$

Considérons r_0 and v_0 constants et donnons nous les fonctions de a suivantes pour β et b :

$$\beta = \frac{K}{a^2}, \quad b^2 = K^2 \left(\frac{1}{r_0^2} - \frac{1}{a^2} \right)$$

on obtient:

$$v_{pr}^2 = K^2 \left(\frac{1}{r_0^2} - \frac{1}{r^2} \right), \quad v_{p\theta} = \frac{K}{r}, \quad r \geq a \geq r_0$$

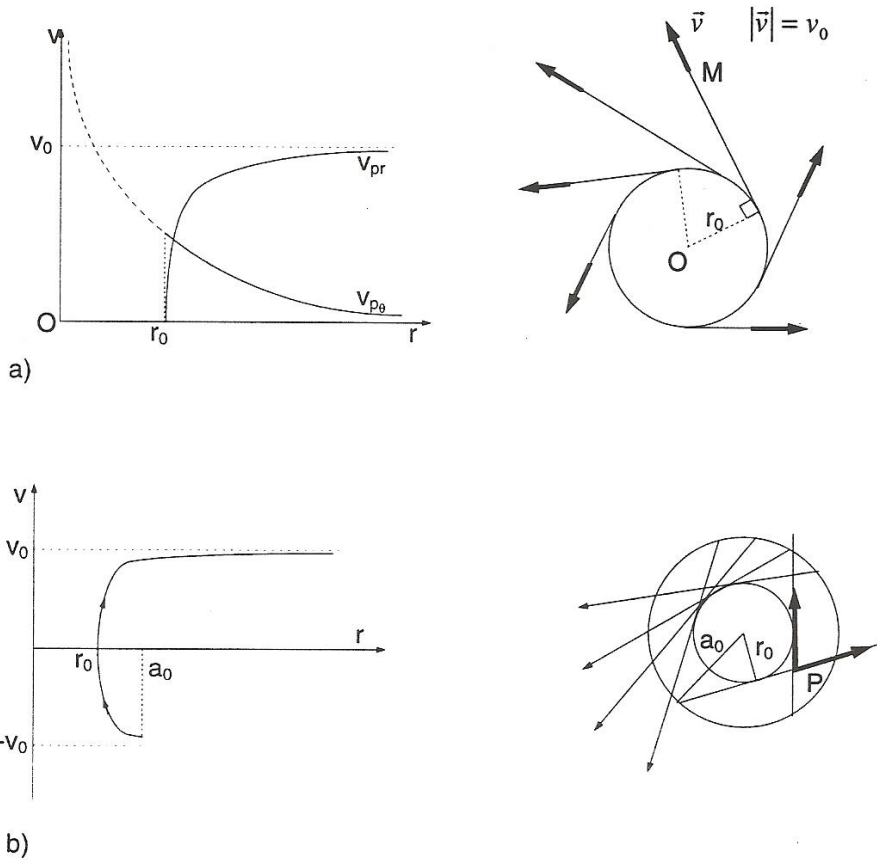


Figure 7. Dispersed phase motions for given values of r_0 and v_0 .

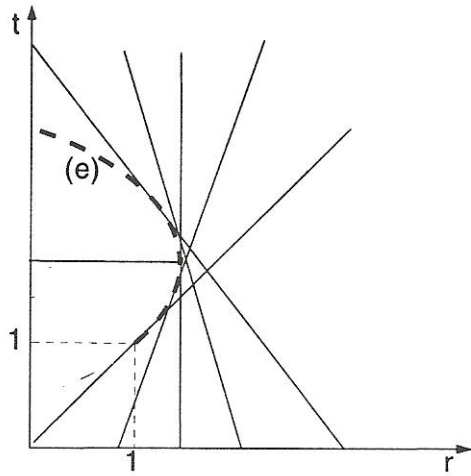


Figure 8. Possible formation of a discontinuity wave.

Dimensionless velocity

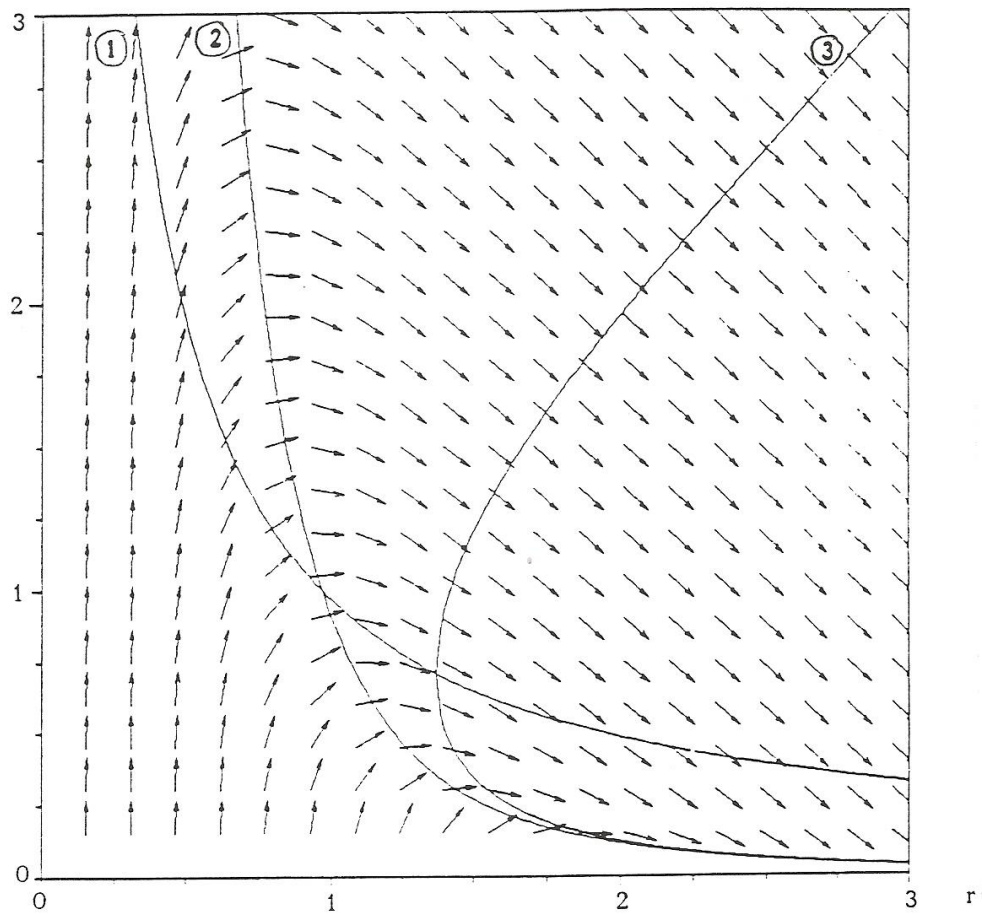


Figure 9: Field of vectors tangent to the curves $v_{pr} = v_{pr}(r)$

Curve 1: $v_{p\theta} = 1/r$

Curve 2: $v_{pr} = 1/r^3$

Curve 3: Locus of points where the product (rv_{pr}) is a maximum (the concentration is minimum) .

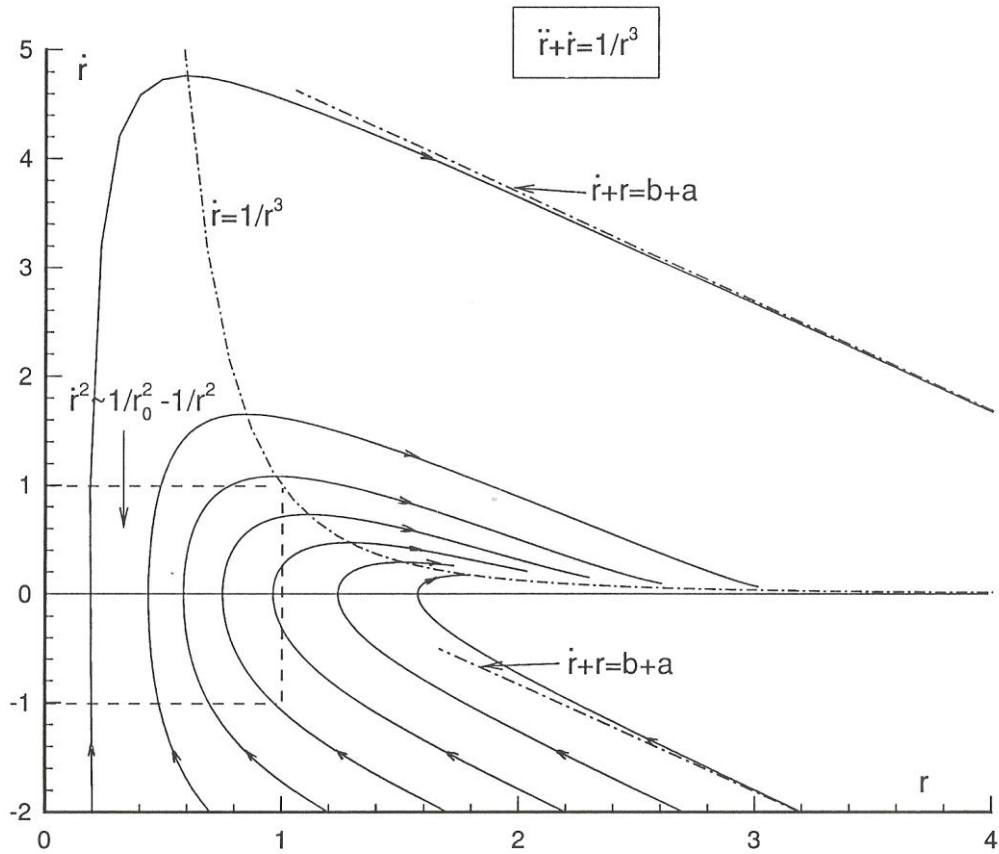


Figure 10: For $a^2\beta=K=1$, a single system of curves; here $b(a)=-2, r_0=r(\dot{r}=0)$

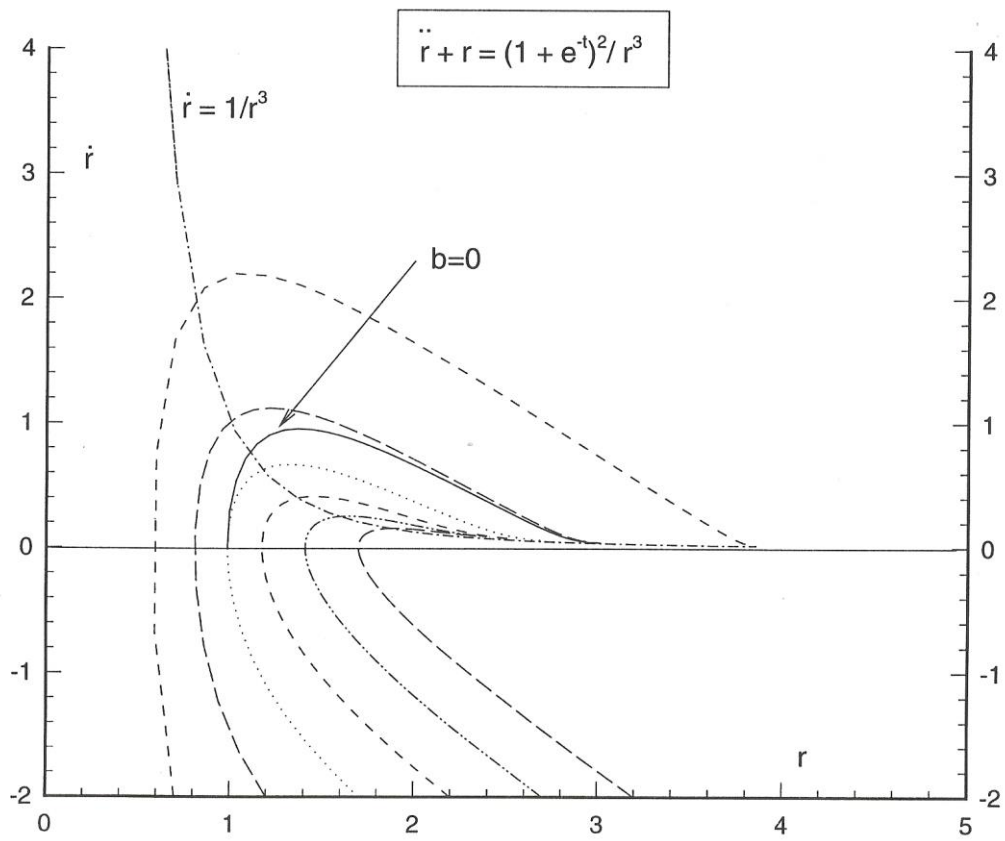


Figure 11: Paths ($a^2\beta=K=2$, $b(a)=-2$); influence of b on one of the paths.

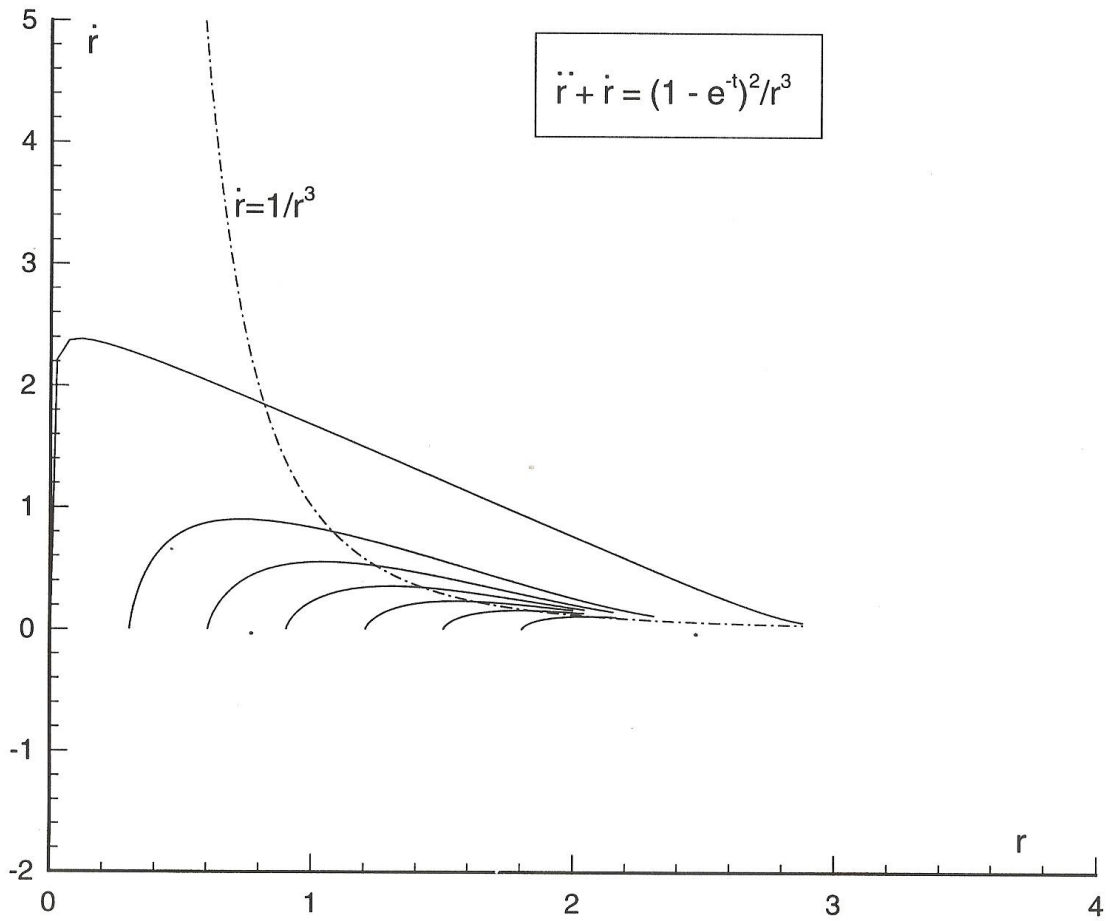


Fig.14 Particles at rest at the initial time, $b(a)=0$, $\beta(a)=0$

MASSE VOLUMIQUE

En variables d'Euler:

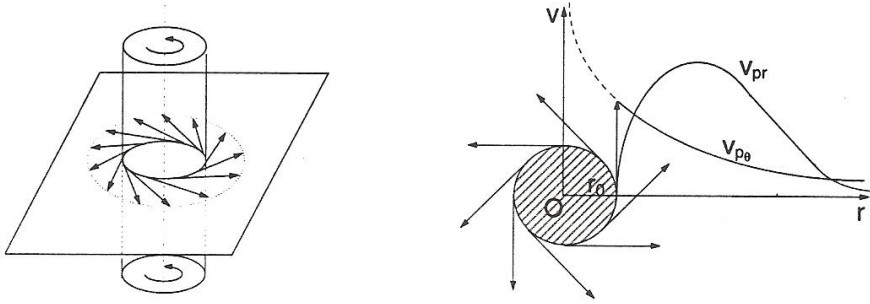
$$\frac{\partial(r\rho_p)}{\partial t} + \frac{(v_{pr} r\rho_p)}{\partial r} = 0$$

En variables de Lagrange:

$$\frac{\partial}{\partial \tau} \left(r\rho_p \frac{\partial r}{\partial a} \right) = 0$$

soit:

$$\rho_p(a, \tau) = \frac{a\rho_p(a, 0)}{r(\partial r / \partial a)}$$



Approximations

II1a t and r small

$$v_{pr}^2 = \frac{1}{r_0^2} - \frac{1}{r^2}, \quad v_{p\theta} = \frac{1}{r}, \quad r \geq r_0, \quad r = \left(r_0^2 + \left(\pm \frac{t}{r_0} + \sqrt{a^2 - r_0^2} \right)^2 \right)^{1/2}$$

Particle mass-flow rate: \dot{m}_p constant

$$\rho_p(a,0) = \frac{\dot{m}_p}{2\pi\sqrt{a^2 - r_0^2}}, \quad a \gg r_0$$

Concentrations:

$$\rho_p = \frac{\dot{m}_p}{2\pi\sqrt{r^2 - r_0^2}}, \quad r \gg r_0$$

II3a r large

$$v_{pr} = \frac{1}{r^3}, \quad v_{p\theta} = \frac{1}{r}, \quad r = (4t + a^4)^{1/4}$$

$$\rho_p(a,0) = \frac{\dot{m}_p a^2}{2\pi}$$

Concentrations:

$$\rho_p = \frac{\dot{m}_p r^2}{2\pi}$$

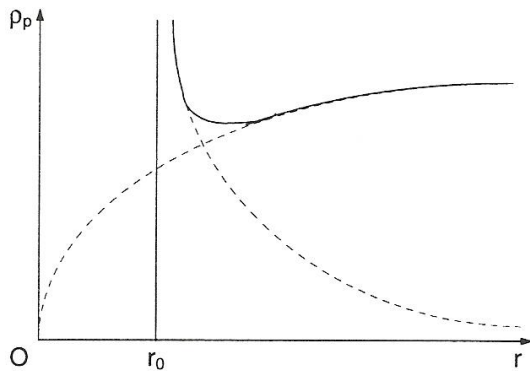
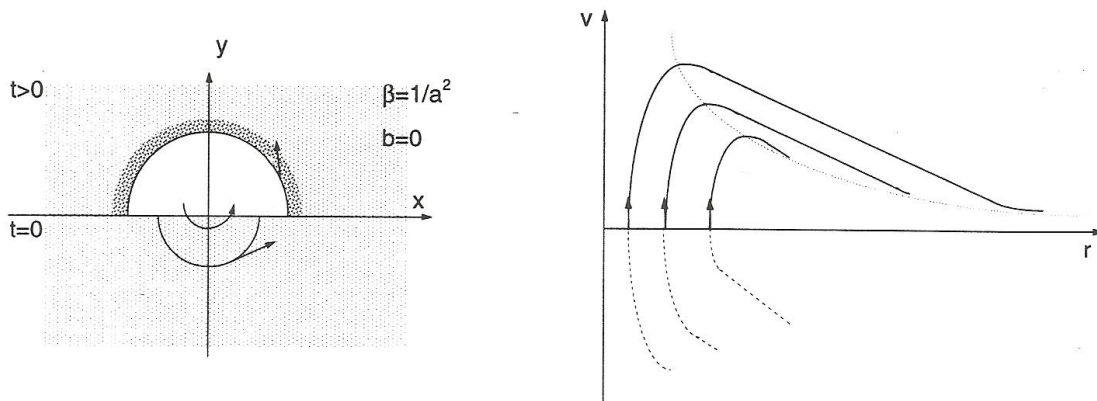


Figure 12: Example of steady flow.



Approximations:

II1a t and r small

$$v_{pr}^2 = \frac{1}{a^2} - \frac{1}{r^2}, \quad v_{p\theta} = \frac{1}{r}, \quad r \geq a, \quad r^2 = a^2 + \frac{t^2}{a^2}$$

Eulerian variables: [28] with $K = 1$

Concentrations: $\rho_p(a,0) = 1, \quad \rho_p(a,t) = \frac{a^4}{a^4 - t^2}$

$$\rho_p = 1 + \frac{2t^2}{r^4 - 4t^2 + r^2\sqrt{r^4 - 4t^2}}, \quad r^2 \geq 2t$$

II3a r large

$$v_{pr} = \frac{1}{r^3}, \quad v_{p\theta} = \frac{1}{r}, \quad r = (4t + a^4)^{1/4}$$

Concentrations: $\rho_p(a,0) = 1, \quad \rho_p(a,t) = \frac{(4t + a^4)^{1/2}}{a^2}$

$$\rho_p = \frac{r^2}{\sqrt{r^4 - 4t}}, \quad r^4 \geq 4t$$

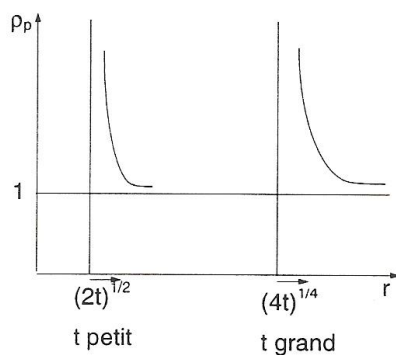


Figure 13: Exemple of time dependent flow

TOURBILLON DE RANKINE

$$\begin{aligned} r \leq R, \quad v_{g\theta} &= \Omega r, \quad v_{gr} = 0 \\ r \geq R, \quad v_{g\theta} &= \Gamma/2\pi r, \quad v_{gr} = 0, \quad \Omega R = \Gamma/2\pi r \end{aligned}$$

Tourbillon de Rankine modifié:

$$v_{g\theta} = \Omega \frac{R^2 r}{r^2 + R^2}, \quad v_{gr} = 0$$

Grandeurs de référence: $t_{ref} = \tau_v, \quad l_{ref} = L, \quad v_{ref} = L/\tau_v$

$$\frac{d_p \vec{v}_p}{dt} = \varpi r \vec{e}_\theta - \vec{v}_p, \quad \varpi = \Omega \tau_v, r \leq R, \quad r_v/R = \sqrt{\varpi}$$

$$\begin{aligned} * \quad \frac{1}{r} \frac{d(r^2 \dot{\theta})}{dt} &= \varpi r - r \dot{\theta}, \quad r \leq R \quad * \\ * \quad \ddot{r} - r \dot{\theta}^2 &= -\dot{r} \quad * \end{aligned}$$

Développements asymptotiques:

$$\varepsilon^{-q} \frac{d(\bar{r}^2 \dot{\bar{\theta}})}{d\bar{t}} + \bar{r}^2 \dot{\bar{\theta}} = \bar{\omega} \varepsilon^{2s+p-m} \bar{r}^2, \quad \bar{\omega} = \varepsilon^{2s} \bar{\omega}$$

$$\varepsilon^{-q} \frac{d^2 \bar{r}}{d\bar{t}^2} + \frac{d\bar{r}}{d\bar{t}} = \varepsilon^{2m-2p+q} \bar{r} \dot{\bar{\theta}}^2$$

Si $q < 0, 2s+p-m=0, 2m-2p+q=0$, on obtient $2s = m-p = -q/2 > 0$, c-à-d. $\bar{\omega} = \Omega \tau_v \ll 1$. On trouve, avec $l_{ref} = L = r_v$:

$$\dot{\theta} = \bar{\omega}, \quad \dot{r} = \bar{\omega}^2 r, \quad r \leq R$$

$$\dot{\theta} = \frac{1}{r^2}, \quad \dot{r} = \frac{1}{r^3}, \quad r \geq R$$

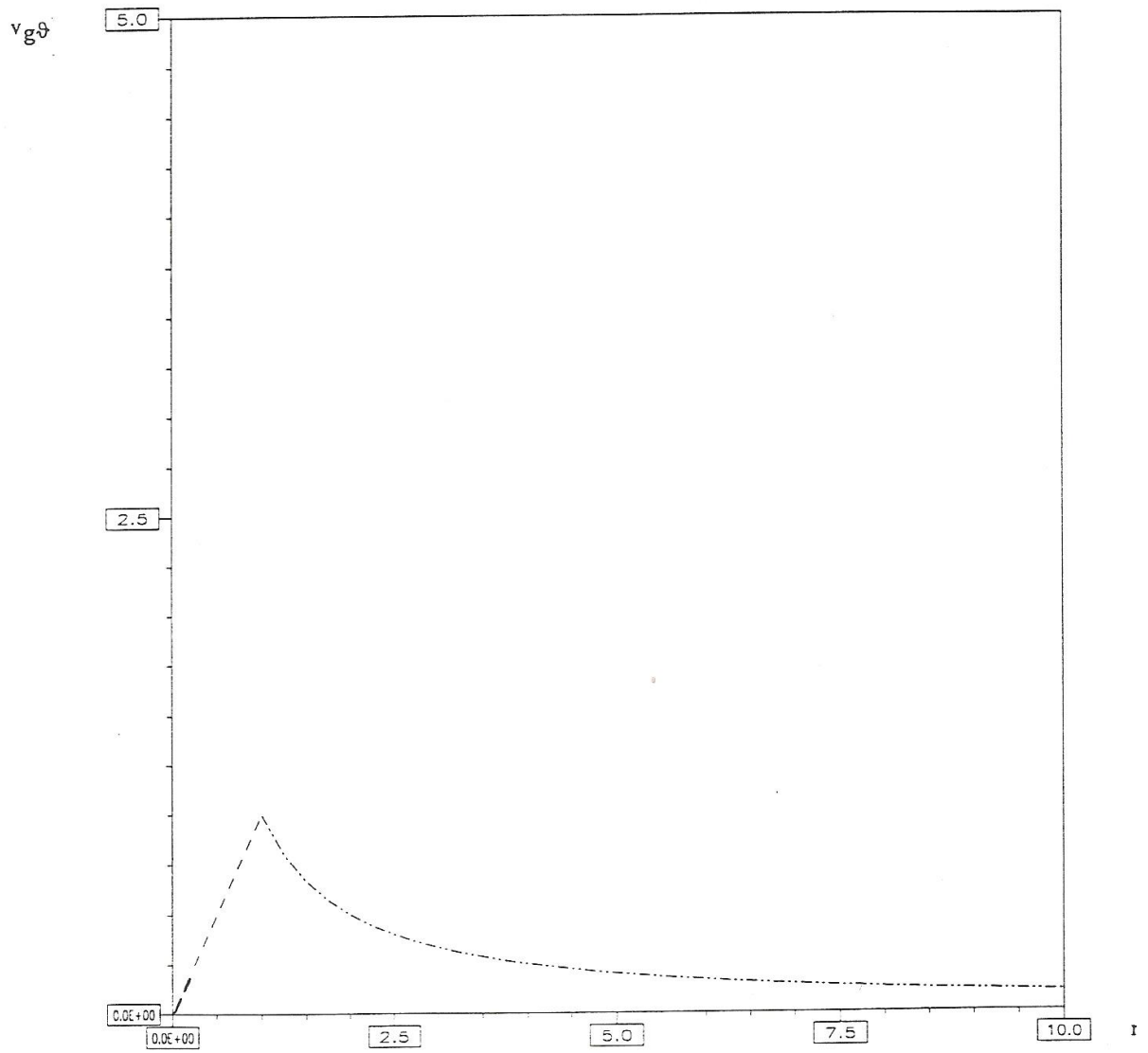


Figure 15 - Tangential velocity versus radius

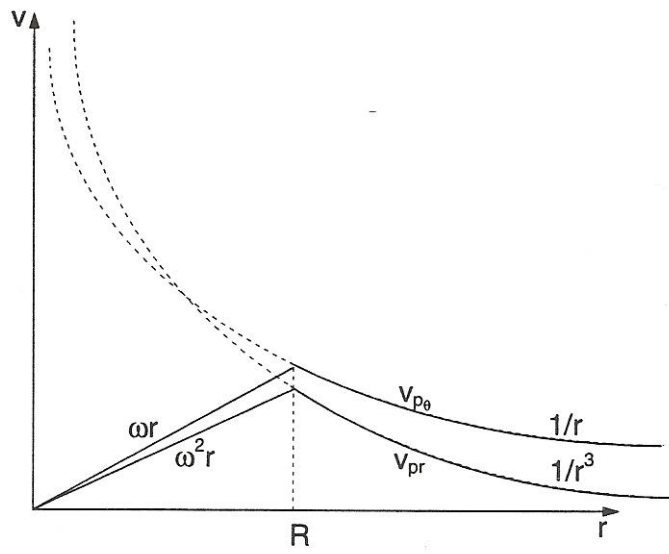


Figure 16 - Particle velocity profiles

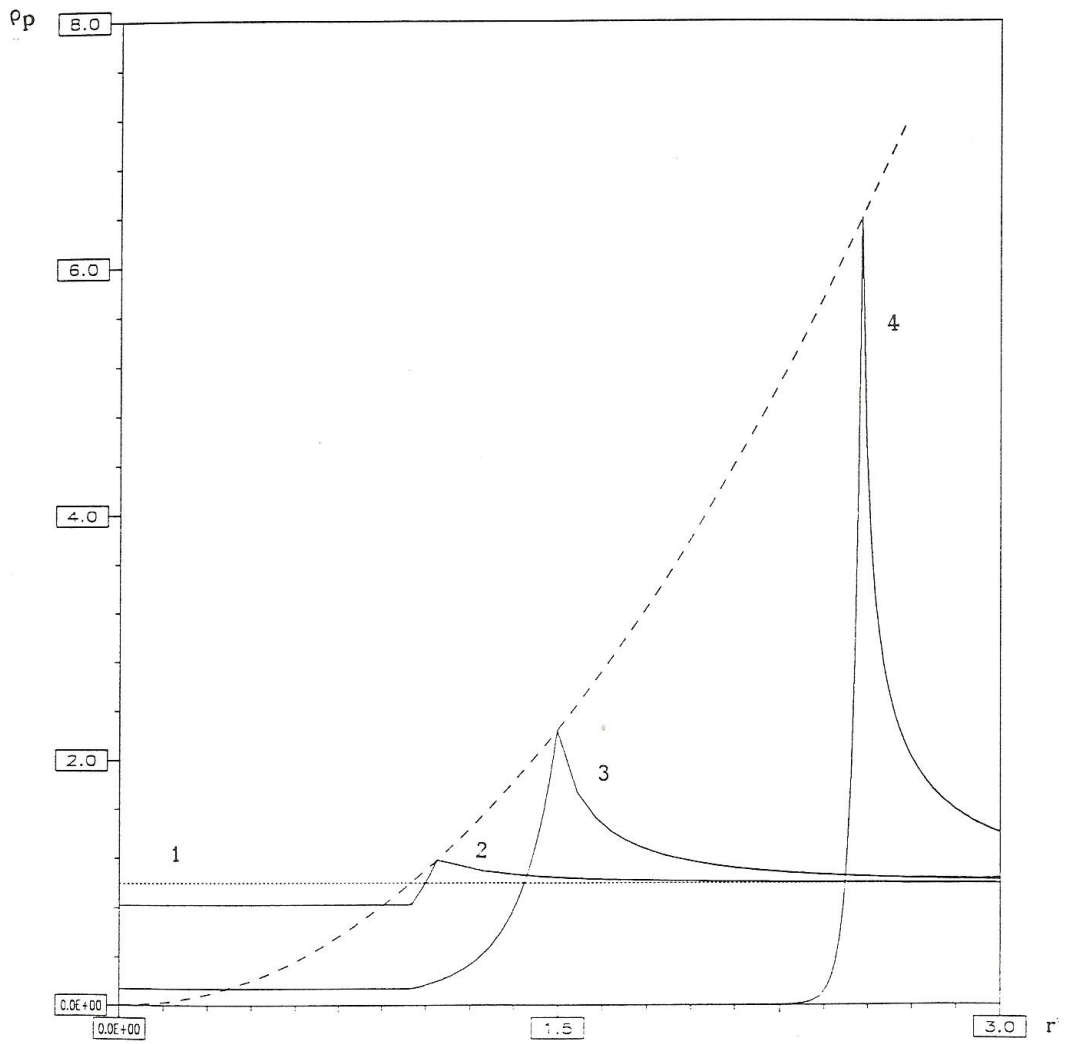


Figure 17 - Particle density versus vector radius. $\rho_p(a,0) = 1, \varpi = 0.1 \ll 1$
 1 : $t=0$, 2 : $t=1$, 3 : $t=10$, 4 : $t=100$

PARTICULES EN COURS D'EVAPORATION

Hypothèses:

- Tourbillon irrotationnel.
- Loi « en D^2 »:

$$r_p^2 = r_{p0}^2 (1 - k't)$$

Grandeurs de référence:

$$t_{eéf} = \tau_v, l_{ref} = r_v = \sqrt{\Gamma \tau_v / 2\pi}, v_{ref} = \sqrt{\Gamma / 2\pi \tau_v}$$

d'où: $k = k' \tau_{v0}$

et l'équation obtenue pour E négligeable:

$$\frac{d_p v_p}{dt} = \frac{\frac{1}{r} \bar{e}_\theta - \bar{v}_p}{1 - kt}$$

Finalement:

$$r^2 \dot{\theta} = 1 + (a^2 \beta - 1)(1 - kt)^{\frac{1}{k}}$$

*

$$\ddot{r} + \frac{\dot{r}}{1 - kt} = \frac{\left[1 + (a^2 \beta - 1)(1 - kt)^{\frac{1}{k}} \right]}{r^3}$$

