The failure of stabilization policy: balanced-budget fiscal rules in the presence of incompressible public expenditures

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Abstract: *We consider an infinite horizon neoclassical model with a government that (i) balances its budget at each point in time, (ii) needs to finance unavoidable (incompressible) public expenditures, and (iii) further uses a fiscal rule for the share of variable government spending in output with the purpose of stabilizing the economy. We show that insulating this economy from belief driven fluctuations is not possible if the government needs to finance (with distortionary taxes) incompressible public expenditures. In this case, we always have steady state multiplicity (two steady states) and global indeterminacy, while local indeterminacy is also possible. More precisely, even if a sufficiently procyclical share of the variable government spending component in output is still able to eliminate local indeterminacy, two saddle steady states prevail, so that, depending on expectations, the economy may either converge to the low steady state or to the high steady state. This implies that a regime switching rational expectation equilibrium, where the economy switches between paths converging to the two different steady states, easily arises. As expectations influence long run outcomes, our model is able to generate large and sudden expansions and contractions in response to expectation shocks.*
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1   Introduction

In recent years we have observed a revival of interest of macroeconomics in fiscal policy. For example, Feldstein (2009) discusses the recent rise of fiscal activism and Taylor (2011) assesses the size of the fiscal multipliers associated with the US stimulus packages of the period 2001-2009. While fiscal multipliers measure the impact of discretionary fiscal policy on output levels, another related strand of the literature studies instead the stabilization role of fiscal policy. Among those see Mckay and Reis (2016) who revisit the role of macroeconomic stabilizers using modern macrodynamic models.[1] Fiscal rules have also emerged as a key response to the fiscal legacy of the financial crisis. In particular the balanced-budget rule has been advocated, by policy institutions such as the IMF and the OECD,[2] in order to ensure fiscal responsibility and sustainability.

In this paper we discuss the stabilization role of fiscal policy under a balanced-budget rule in the presence of incompressible public expenditures. More concretely, we consider a non-monetary general equilibrium dynamic model where, each period, the government must raise a fixed minimum amount of tax revenues in order to finance unavoidable (incompressible) public expenditures, which should remain constant along business cycles.[3] This implies a countercyclical income tax, which tends to magnify economic fluctuations by creating incentives for spending in good times, and may lead to the emergence of local indeterminacy, generating an additional source of instability associated with the occurrence of local self fulfilling volatile expectations. Moreover, it creates steady state multiplicity and, thereby, may lead to the emergence of global indeterminacy as well.[4] We then discuss whether, in this context of a balanced-budget rule and incompressible public expenditures, the introduction of a *sufficiently procyclical share of additional variable government expenditures in output* is able to stabilize, both locally and globally, endogenous business cycle fluctuations driven by volatile self fulfilling expectations.

Conventional wisdom states that procyclical/progressive tax rates (or equivalently, under a balanced budget, *a procyclical share of government expenditures in output*) have stabilizing effects, which help smooth out business cycle fluctuations due either to exogenous shocks on fundamentals or to volatile expectations (sunspots).[5] Friedman

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[1] They consider a model with a unique determinate equilibrium and focus on the impact of stabilizers on the volatility of endogenous variables, due to exogenous shocks in fundamentals.

[2] See Schaechter (2012) and Fall et al. (2015).

[3] They reflect the views of society on the appropriate size of expenditures associated with the basic functions of government.

[4] Schmitt-Grohé and Uribe (1997) were the first to analize the implications for local indeterminacy of a balanced-budget rule, where the government finances a fixed amount of expenditures using distortionary income taxation. However they did not adress global (in)stability issues.

[5] See, for example, Kletzer (2006) and Moldovan (2010) for the case with exogenous shocks to funda-

(1948) was one of the first to advocate a progressive tax system, which places primary reliance on the income tax, in order to attain both long run goals and short run stability.[6] Here, we show that in the presence of incompressible public expenditures, the stabilization ability of fiscal policy is lost.

But is the presence of incompressible public expenditures an empirically relevant issue? The answer is a resounding yes. They correspond to expenditures associated with the basic functions of government (public safety, defense and general public services) and have been remarkably constant for many developed countries, for very long time horizons.
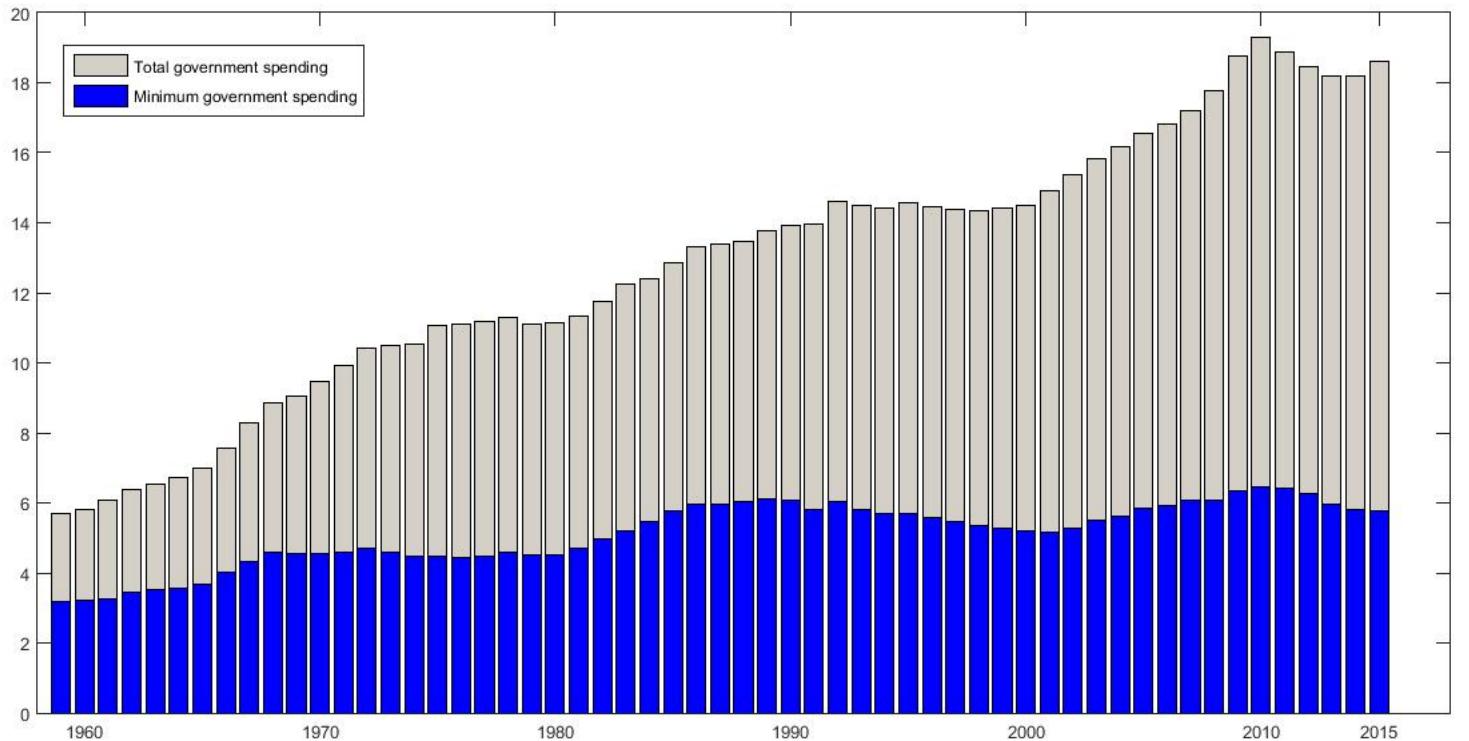
Figure 1: minimum government spending (blue) vs total government spending per capita in the U.S. (in thousands of dollars)

In figure 1 we present the evolution of these public expenditures and total public

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mentals and Guo and Lansing (1998), Dromel and Pintus (2008) for the case of shocks to expectations.

[6]Friedman (1948) also defended that government spending should be stable and determined by the needs of society, which already suggests incompressible public expenditures. Moreover in his proposal the budget should on average be balanced and the government should not issue interest-bearing securities to the public. Note that in our non monetary setup this implies that the government should balance its budget at all periods.
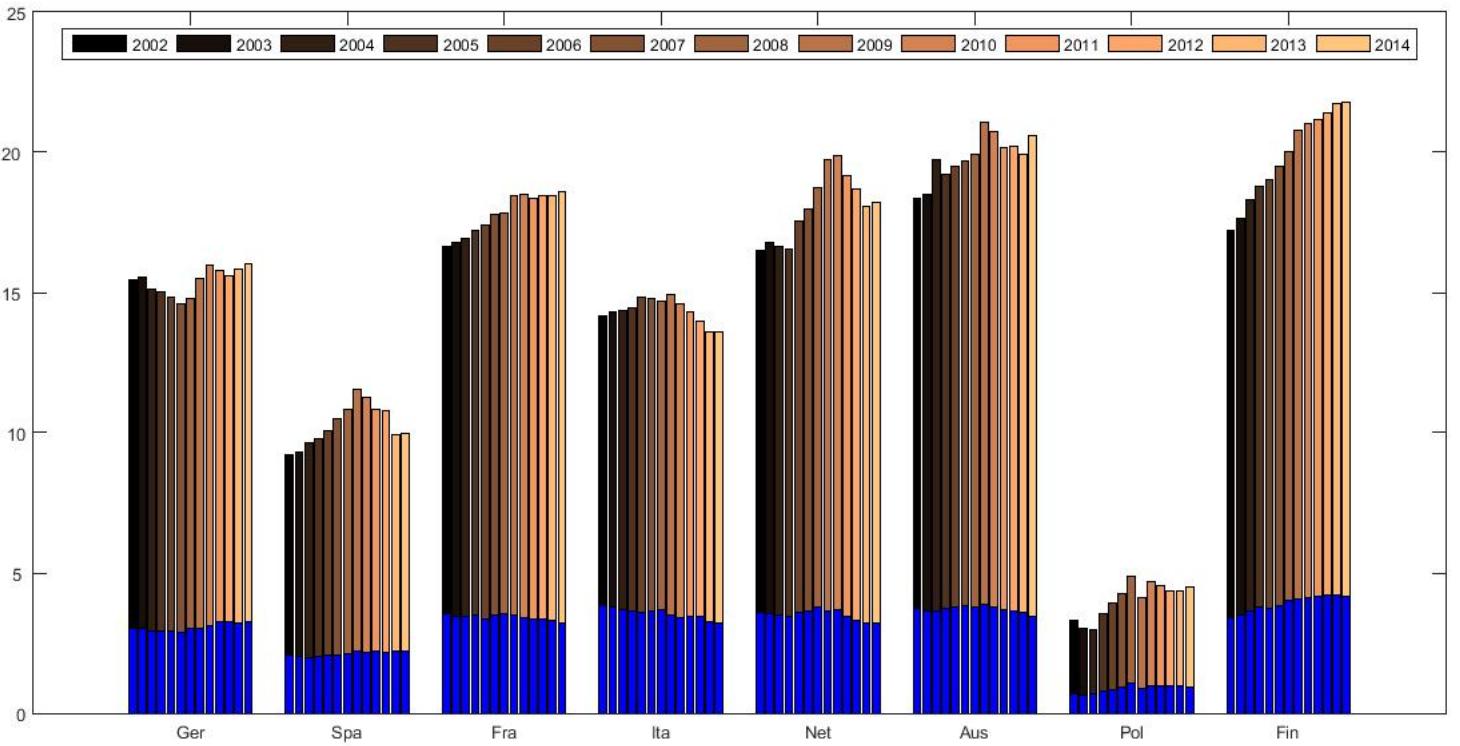
Figure 2: Minimum government spending (blue) vs total government spending per capita in others countries (in thousands of euros)

expenditures for the USA, in per capita terms, from 1959 to 2015 in constant prices of 2015. We can see that while total per capita government spending increased steadily in time, the level of per capita expenditures associated with the basic functions of government did not change much over such a long period, which implies that they do not follow business cycles. To confirm this visual inspection we HP-filtered the data removing the trend and regressed the cyclical component of each type of government spending on the cyclical component of output. The results obtained (see Appendix 9.1) show unequivocally that while per capita expenditures associated with the basic functions of government are constant, i.e. they are not correlated with output, the difference between total government expenditures and incompressible public expenditures is significantly correlated with income, following therefore the business cycle.

Note that the per capita level of incompressible public expenditures may vary across countries, reflecting the views of each society on how much should be spent on the basic functions of government.[7] However, we expect them to be relatively stable in

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[7]Nevertheless, there is no great variation among OECD countries with respect to these basic government expenditures. A developed country needs security and public administration, and it is difficult

time for each country, regardless of the time evolution of total government spending. This is indeed the pattern we find across several European countries from 2002 to 2014, a particularly turbulent period in macroeconomic terms. See figure 2.

Until now the literature considered either fully flexible government expenditures or a totally constant public spending. In this paper, in line with empirical evidence, we address simultaneously the existence of a variable government spending component and one fixed, incompressible, component. We find that, under a balanced-budget rule, when the government needs to finance incompressible public expenditures, two steady states emerge, one being always a saddle. Focusing on local dynamics is therefore not enough for stabilization purposes. Indeed we find that, although a sufficiently *procyclical share of the variable government spending component in output* is able to stabilize locally the indeterminate steady state, it will not eliminate steady state multiplicity.

Our results contrast with those obtained in frameworks where government spending is fully flexible and where there exists a (sufficiently) procyclical share of government expenditures in output. Under a balanced budget, this is equivalent to a (sufficiently) procyclical tax rate for all levels of output, which ensures the existence of a unique steady state and of its local determinacy (saddle).[8] In our case, due to the existence of a countercyclical share of the incompressible government spending component in output, there will always be values of output for which the tax rate is countercyclical. Hence, the tax rate cannot be globally procyclical (or acyclical), which leads to multiple steady states and global indeterminacy, as the economy may switch from one equilibrium path to the other. This means that under a balanced-budget rule, in the presence of incompressible public expenditures, fiscal policy is no longer able to insulate the economy from belief driven fluctuations. In this context the management of expectations is crucial to guarantee that the economy remains on the path converging to the high output equilibrium.

However, if expectations can not be controlled, the existence of multiple equilibria, associated with different expectations about the state of the economy, implies that a regime switching rational expectation equilibrium easily arises. In this equilibrium the economy switches between paths converging to the two different steady states, according to a sunspot variable. This implies that in our framework expectations are able to influence the long run outcomes of the economy, and not just the choice of the convergence path to one steady state. Therefore, in addition to small fluctuations around a

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to compress this kind of State spending. In contrast, with respect to "government social expenditures", the differences between countries are significantly bigger.

[8]See for example Guo and Lansing (1998), where a sufficiently progressive tax rate is able to eliminate indeterminacy caused by increasing returns to scale. Note that, in macro models, progressive and procyclical tax rates are similar in terms of the stability properties of the equilibrium.

locally indeterminate steady state, we are able to account for large fluctuations generated by a regime switching sunspot process. Indeed our model is able to generate large and sudden expansions and contractions in response to expectation shocks. We conclude that the widespread existence of incompressible public expenditures in developed countries, not only implies the failure of traditional fiscal stabilization policies, but may also be responsible for sharp and sudden recessions.

Our paper is related to several recent strands of the literature: global indeterminacy, multiplicity of equilibria and regime switching sunspot equilibria. Benhabib et al. (2001a, b) were the first to show that under monetary Taylor rules the targeted-inflation steady state coexists with a second deflation steady state, and to construct equilibria in which the economy transitions from one steady state to the other. More recent contributions are Aruoba et al. (2017) and Mertens and Ravn (2014), where a Markov-switching sunspot shock can move the economy between the targeted inflation regime and the deflation regime.[9] In this paper, we provide a novel mechanism of multiple equilibria and nonfundamental fluctuations associated now with fiscal policy.[10] We show that fiscal rules, in particular the existence of a balanced-budget rule and incompressible public expenditures, may also be responsible for the emergence of a similar configuration, where the economy may find itself trapped in the low output steady state, which coexists with the high output steady state, and where a regime switching equilibrium easily emerges.

The idea that a economy may switch between alternative equilibrium paths that converge to different steady states has recently been receiving more and more attention in different setups. Like us, Kaplan and Menzio (2016) and Benhabib et al. (2018) consider non monetary models and construct regime switching sunspots equilibria. The mechanisms responsible for equilibria multiplicity and sunspots fluctuations are however totally different.[11] Our mechanism emphasizes the role of fiscal policy and has the merits of being simple and plausible.[12]

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[9]See also Piazza (2016) and Schmitt-Grohe and Uribe (2017).

[10]Note that, although in earlier papers (Benhabib and Farmer (1994), Farmer and Guo (1994) and Christiano and Harrison (1999)) indeterminacy, equilibria multiplicity and nonfundamental shocks emerged because of increasing returns to scale in production, in our paper we do not need increasing returns, incompressible expenditures are sufficient.

[11]Kaplan and Menzio (2016) propose a model where multiplicity is obtained from differences in the shopping behavior of employed and unemployed buyers and where economic fluctuations can be caused by self-fulfilling changes in the agents' expectations about unemployment. Benhabib et al. (2018) show that an adverse selection problem for financial intermediaries generates multiple steady states and both local and global indeterminacy, and can give rise to regime switching sunspots equilibria.

[12]Note that the OECD in a recent economic policy paper (see Fall et al. (2015)) advocates the combined use of a balanced budget rule and an expenditure rule, whose existence is at the basis of our

The rest of the paper is organized as follows. In the next section, we present the model considered and obtain the perfect foresight equilibria. In section 3, we study steady state existence and multiplicity. Section 4 is devoted to the study of local dynamics, while section 5 examines global dynamics. In section 6 we consider more general functional forms and fiscal rules and show that our results are robust. In section 7 we develop an augmented version of the model in which agent's expectations about future economic activity (output) follow a Markov switching process and provide a numerical illustration of the effects of expectation shocks. Finally, in section 8 we provide some concluding remarks. Mathematical proofs are relegated to the Appendix.

2 The model

We consider an infinite-horizon Ramsey model where public spending has two components: a fixed incompressible component, and a variable component which the government uses to stabilize the economy against belief driven cycles. To keep the budget balanced, the government issues distortionary income taxes. Households are infinitely-lived and have a logarithmic utility function in consumption and a perfectly elastic labor supply. Firms have access to a Cobb-Douglas technology, which may exhibit increasing returns to scale, and use labor and capital to produce a single good which is consumed or invested. This section describes such an economy.

2.1 Government

Total government spending, G_t, includes one incompressible component, $\overline{G} \geq 0$, and a variable component, i.e., we assume that

$$G_t = \overline{G} + g(y_t) \tag{1}$$

where $g(y_t) \geq 0$ is the time-varying component of government spending, which is an exogenously given function of aggregate income, y_t, and which the government uses for stabilization purposes.

Distortionary (proportional) income taxation finances wasteful public expenditures G_t and the government budget is balanced at each point in time, i.e. we have that:

$$G_t = (1 - z_t)y_t \tag{2}$$

where $z_t \in (0, 1]$ denotes the fiscal wedge,[13] and $(1 - z_t)$ is the endogenously determined total income tax rate, which is obviously identical to the share of government spending

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mechanism.

[13]The fiscal wedge is the ratio between income net of taxes and gross income.

in output. We could have considered the existence of a stock of debt at the time of entry of the balanced budget rule, $D > 0$ (which obviously remained constant thereafter) and the corresponding interest payments. In Appendix 9.3 we show that our results would either not change or would be reinforced with the consideration of public debt. Therefore, for the sake of simplicity, we assume that $D = 0$.

Using (1) and (2) we obtain

$$z_t = z(y_t) \equiv 1 - \frac{\overline{G}}{y_t} - \tau(y_t) \tag{3}$$

where $\tau(y_t) \equiv \frac{g(y_t)}{y_t}$ is the *share of the variable component of government expenditures in output*. From (3) we observe that the total tax rate on income, $1 - z(y_t)$, has two distinct components. The first component, $\overline{G}/y_t$, which is due to the existence of incompressible public expenditures, is countercyclical, i.e. it decreases when output increases. The second component, $\tau(y_t)$, represents the fiscal rule which the government uses with stabilization purposes. We assume it is a monotonic continuous function of aggregate income, with either $\tau'(y) < 0$ for all $y \in (0, +\infty)$, or $\tau'(y) > 0$ for all $y \in (0, +\infty)$, or $\tau'(y) = 0$ for all $y \in (0, +\infty)$. We also assume that $\tau(y)$ is convex or not too concave.[14] We denote by $\phi(y_t) \equiv \frac{\tau'(y_t) y_t}{\tau(y_t)} \in \Re$ the elasticity of $\tau(y_t)$. Accordingly, this fiscal rule may be procyclical ($\phi(y_t) > 0$), countercyclical ($\phi(y_t) < 0$), or acyclical ($\phi(y_t) = 0$).

We further denote by $\varepsilon_z(y_t) \equiv \frac{z'(y_t) y_t}{z(y_t)}$ the elasticity of $z(y_t)$. Using (3) we obtain

$$\varepsilon_z(y_t) = \frac{\overline{G} - \phi(y_t)\tau(y_t) y_t}{y_t - \overline{G} - \tau(y_t) y_t} > -1. \tag{4}$$

The last inequality is needed to guarantee that after tax income, $z(y_t)y_t$, is increasing in income. It follows that the cyclicality of the total tax rate, $(1 - z(y_t))$, measured by its elasticity $\varepsilon_{1-z}(y_t) = \frac{-z'(y_t) y_t}{1 - z(y_t)}$ is given by:

$$\varepsilon_{1-z}(y_t) = \frac{\phi(y_t)\tau(y_t) - \frac{\bar{G}}{y_t}}{\frac{\bar{G}}{y_t} + \tau(y_t)}. \tag{5}$$

As the denominator is positive, if $\overline{G} > 0$ two configurations occur: either $\phi(y_t) \leq 0$ and the tax rate is countercyclical, or $\phi(y_t) > 0$ and the tax rate is procyclical for y high enough and countercyclical otherwise. Therefore, we can not ensure the existence of a procyclical (or acyclical) tax rate for all values of y when $\overline{G} > 0$.

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[14]Although this assumption is not needed for most of our results (see Appendix 9.2.) it simplifies considerably the exposition.

2.2 Households' behavior

We consider an economy populated by a large number of identical infinitely-lived agents. We assume without loss of generality that the total population is constant and normalized to one. At each period an agent has a perfectly elastic labor supply l_t with $l_t \in [0, \bar{l}]$ and $\bar{l} > 1$ his time endowment. She derives utility from consumption, c_t, and disutility from labor, l_t, according to the instantaneous utility function $U(c_t, l_t)$:

$$U(c_t, l_t) = ln(c_t) - \frac{l_t}{B} \tag{6}$$

where $B > 0$ is a scaling parameter.

Households, when choosing c_t and l_t, face the following budget constraint:

$$\dot{k}_t + c_t = z(y_t)[w_t l_t + r_t k_t] - \delta k_t, \tag{7}$$

where k_t is the capital stock at time t, w_t the wage rate, r_t the rental rate of capital and $\delta > 0$ the depreciation rate of capital.

The intertemporal maximization problem of the representative household is given below:

$$\max_{c_t, k_t, l_t} \quad \int_{t=0}^{+\infty} e^{-\rho t} U\left(c_t, l_t\right) dt \tag{8}$$
$$s.t. \quad (7)$$

where $\rho > 0$ is the discount factor. Note that households take as given the fiscal wedge, $z(y_t)$, when maximizing utility.

Denoting by $\lambda(t)$ the shadow price of capital, the current-value Hamiltonian writes:

$$U(c_t, l_t) + \lambda_t \left[z(y_t)[w_t l_t + r_t k_t] - \delta k_t - c_t\right] \tag{9}$$

The first-order conditions are:

$$c_t^{-1} = \lambda_t \tag{10}$$
$$1 = B\lambda_t z(y_t) w_t \tag{11}$$
$$\frac{\dot{\lambda}_t}{\lambda_t} = -[z(y_t) r_t - (\rho + \delta)] \tag{12}$$

Any solution needs also to satisfy the transversality condition:

$$\lim_{t \to +\infty} e^{-\rho t} \lambda_t k_t = 0 \tag{13}$$

2.3 The production structure

We consider a competitive environment in which a continuum of measure one of identical firms produce a single good y_t using capital k_t and labor l_t. The firms' technology displays constant returns to scale at the private level according to a Cobb-Douglas specification $y_t = F(k_t, l_t, \bar{k}_t, \bar{l}_t) = e(\bar{k}_t, \bar{l}_t)k_t^s l_t^{1-s}$ with $e(\bar{k}_t, \bar{l}_t) \equiv (\bar{k}_t^s \bar{l}_t^{1-s})^\gamma$, $\gamma \geq 0$, a learning-by-doing externality, $\bar{k}_t$, $\bar{l}_t$ being respectively the average-wide stock of capital and hours worked, which are taken as given by individual firms. Since at the aggregate level we have $\bar{k}_t = k_t$ and $\bar{l}_t = l_t$, at equilibrium the technology displays increasing returns to scale when $\gamma > 0$, i.e. $y_t = k_t^\alpha l_t^\beta$ with $\alpha = s(1 + \gamma)$, $\beta = (1 - s)(1 + \gamma)$.

From the profit maximization of the firm, we obtain the real wage rate w_t and the real rental rate of capital r_t as:

$$r_t = se(\bar{k}_t \bar{l}_t)\left(\frac{k_t}{l_t}\right)^{s-1} = \frac{sy_t}{k_t} \equiv r(k_t, y_t) \tag{14}$$

$$w_t = (1 - s)e(\bar{k}_t, \bar{l}_t)\left(\frac{k_t}{l_t}\right)^s = \frac{(1-s)y_t}{l_t} \equiv w(l_t, y_t) \tag{15}$$

Hence, profits are zero and $y_t = w_t l_t + r_t k_t$.

In what follows, we assume that the capital share of output, s, is small, i.e., $s < 0.5$, as usually done in the literature. Moreover, in order to avoid endogenous growth, we consider not too strong productive externalities, i.e. we assume that $\gamma < \frac{1-s}{s}$, so that $\alpha < 1$. Together, these two assumptions imply that $\beta > \max\{\alpha, \gamma\}$. All these assumptions are summarized below in Assumption 1 and we consider them satisfied in the rest of the paper.

Assumption 1. $s < 0.5$ and $0 \leq \gamma < \frac{1-s}{s}$ so that $\alpha < 1$ and $\beta > \max\{\alpha, \gamma\}$.

2.4 Intertemporal equilibrium

In this section, we define the intertemporal perfect foresight equilibrium of this economy. From the aggregate production function we can write $l_t = l(k_t, y_t) \equiv y_t^{1/\beta} k_t^{-\alpha/\beta}$ which implies that, using (15), we can express the wage as a function of k_t and $l(k_t, y_t)$ so that:

$$w_t = (1 - s)\, k_t^{\alpha/\beta} y_t^{(\beta-1)/\beta} \equiv w(k_t, y_t) \tag{16}$$

Substituting (10) and (16) in (11), we solve this equation with respect to c_t and obtain:

$$c_t = B(1 - s)z(y_t)k_t^{\alpha/\beta} y_t^{(\beta-1)/\beta} \equiv c(k_t, y_t) \tag{17}$$

Below, we provide the elasticities of the latter expression with respect to y and k:

$$\varepsilon_{cy} = \frac{[\beta(1+\varepsilon_z(y_t)-1]}{\beta} \quad \varepsilon_{ck} = \frac{\alpha}{\beta} \tag{18}$$

with $\varepsilon_z(y_t)$ given by (4).

Differentiating equation (17) with respect to time, we obtain:

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha}{\beta}\frac{\dot{k}_t}{k_t} + \frac{\beta(1+\varepsilon_z(y_t))-1}{\beta}\frac{\dot{y}_t}{y_t}. \tag{19}$$

Substituting (10) and (14) in (12) we have

$$\frac{\dot{c}_t}{c_t} = \frac{sz(y_t)y_t}{k_t} - (\rho+\delta). \tag{20}$$

Equating now (19) and (20) and rearranging terms we finally obtain:

$$\frac{\dot{y}_t}{y_t} = \frac{s\beta z(y_t)y_t - (\rho+\delta)\beta k_t - \alpha\dot{k}_t}{k_t\left[\beta(1+\varepsilon_z(y_t))-1\right]} \tag{21}$$

with $z(y_t) > 0$, and $\varepsilon_z(y_t)$ given respectively by (3) and (4). Note that if $\beta(1+\varepsilon_z(y_t))-1 = 0$, at some point in time t we say that we have a singularity, and equation (21) is not properly defined. We will discuss later the implications of the existence of singularities on the study of the dynamics of the model.

Substituting now (17) in the households' budget constraint (7), we obtain the law of motion of the capital stock:

$$\dot{k}_t = z(y_t)y_t - \delta k_t - c(k_t, y_t) \tag{22}$$

with $c(k_t, y_t)$ given in (17).

Definition 1. *An intertemporal perfect foresight equilibrium is a path $\{k_t, y_t\}_{t\geq 0}$ satisfying equations (21)-(22) for a given $k_0 > 0$ and the transversality condition (13), with $z(y_t) \in (0,1]$, given in (3), $\varepsilon_z(y_t)$ given in (4) and $c(k_t, y_t)$ given in (17).*

Note that k is a predetermined variable, its value being given from the past, while y is non predetermined. Hence, it is a priori possible that there may exist several different values of y_0 compatible with perfect foresight equilibria trajectories for the same k_0, as it occurs in the presence of (local or global) indeterminacy. In this case there are multiple self-fulfilling expectations-driven equilibria.

3 Steady state existence and multiplicity

A steady state is a positive 4-tuple (k, l, c, y), with $y \in (\underline{y}, \bar{y})$, satisfying:

$$y = k^\alpha l^\beta \tag{23}$$

$$cl = B(1-s)z(y)y \tag{24}$$

$$sz(y)y = (\rho+\delta)k \tag{25}$$

$$c = z(y)y - \delta k \tag{26}$$

$$\text{with } z(y) \equiv 1 - \frac{\overline{G}}{y} - \tau(y) > 0 \tag{27}$$

Solving this system of equations we obtain:

$$
\begin{aligned}
k &= \frac{sz(y)y}{(\rho+\delta)} \\
c &= \frac{[\rho+(1-s)\delta]z(y)y}{(\rho+\delta)} \\
l &= \left[\frac{[B(1-s)(\rho+\delta)]}{[\rho+(1-s)\delta]} \right] \\
\underbrace{(z(y)y)^\alpha y^{-1}}_{\equiv H(y)} &= \underbrace{\left(\frac{s}{(\rho+\delta)} \right)^{-\alpha} \left(\frac{(1-s)(\rho+\delta)B}{\rho+(1-s)\delta} \right)^{-\beta}}_{\equiv \bar{H}} \\
\text{with } z(y) &\equiv 1 - \frac{\overline{G}}{y} - \tau(y) > 0
\end{aligned}
\tag{28}
$$

Steady state existence and multiplicity are determined by the solutions of $H(y) = \bar{H}$. Note that we restrict $y \in (\underline{y}, \bar{y})$ with $\underline{y} > 0$ and $\bar{y} \in (\underline{y}, +\infty)$ to ensure that $z(y) > 0$. See Appendix 9.2. We use the scaling parameter $B > 0$ to ensure the existence of a normalized steady state (NSS), $y = 1$. Hence,

Proposition 1. *The solution $(k_{nss}, l_{nss}, c_{nss}, 1)$ of system (28) is a NSS if and only if $B = B^*$ with:*

$$B^* = \frac{\frac{[\rho+(1-s)\delta]}{(\rho+\delta)}}{\left(\frac{s}{(\rho+\delta)} \right)^{\frac{\alpha}{\beta}} (1-s)z(1)} \tag{29}$$

$$z(1) = 1 - \overline{G} - \tau(1) > 0 \tag{30}$$

Steady sate multiplicity requires that $H(y)$ is a non monotonic function, crossing at least twice the value $\bar{H}$. To study steady state multiplicity, we must then characterize the sign of $\varepsilon_H(y) \equiv \frac{H'(y)y}{H(y)}$. From (28) we have that

$$\varepsilon_H(y) = \alpha(1 + \varepsilon_z(y)) - 1 \tag{31}$$

where $\varepsilon_z(y)$ is given in (4). In Appendix 9.2 we show that:

Proposition 2. *Under Assumption 1 and Proposition 1, when $\overline{G} > 0$, $H(y) > 0$ is always single-peaked in $y \in (\underline{y}, \bar{y})$ and there are exactly two steady states. Moreover, when $\overline{G} = 0$, $H(y) > 0$ is single peaked and there are exactly two steady states if and only if $\phi(y) \equiv \tau'(y)y/\tau(y) < 0$ for all y. In contrast, when $\overline{G} = 0$ and $\phi(y) \geq 0$ for all y, the steady state is unique.*

Proof. See Appendix 9.2. □

Note that if $\varepsilon_z(y) \leq 0$ for all $y \in (\underline{y}, \bar{y})$ then, given that $\alpha < 1$ under Assumption 1, from (31) we can see that the function $H(y)$ is always decreasing for all $y \in (\underline{y}, \bar{y})$, so there can only be one steady state. Using (4) we see that this happens when $\overline{G} = 0$ and $\phi(y) \geq 0$, ensuring that the total tax rate is procyclical (see (5)).

In contrast, as proved in Appendix 9.2, the existence of any countercyclical component in the steady state tax rate, $1 - z(y) = \frac{\overline{G}}{y} + \tau(y)$, is sufficient (regardless of its degree of countercyclicality) for the appearance of two steady states, as in this case the function $H(y) > 0$ is always single-peaked. Therefore:

Corollary 1. *Under the conditions of Proposition 2, a strictly positive $\overline{G}$ is sufficient for steady state multiplicity. Denoting the low output steady state by y_l and the high output steady state by y_h, we have $(1 + \varepsilon_z(y_l)) > \frac{1}{\alpha}$ and $(1 + \varepsilon_z(y_h)) < \frac{1}{\alpha}$.*

Proposition 2 and Corollary 1 tell us that, if the government needs to finance a fixed minimum amount of spending ($\overline{G} > 0$), steady state multiplicity can not be eliminated even if $\phi(y) > 0$, although that would be possible if $\overline{G} = 0$. Note that, when $\overline{G} > 0$, the elasticity of the tax rate at the low output steady state is always negative, i.e., the tax rate at the low output steady state is countercyclical even when $\tau(y_t)$ is procyclical.[15] This implies that, even when $\phi(y)$ is sufficiently positive, we can not ensure the existence of a procyclical (or acyclical) tax rate for all $y \in (\underline{y}, \bar{y})$ when $\overline{G} > 0$, i.e., at equilibrium, the tax rate cannot be made globally procyclical (acyclical). Therefore, in the presence of incompressible public expenditures, the function $\varepsilon_H(y)$ always changes sign for $y \in (\underline{y}, \bar{y})$, so that, in contrast to the case of $\overline{G} = 0$, a procyclical $\tau(y_t)$ is not able to eliminate steady state multiplicity.

4   Local Analysis

We now characterize the local stability properties of our dynamic system around a steady state. We start by linearizing system (21)-(22) around a steady state solution y

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[15]From Corollary 1, when $\overline{G} > 0$ two steady states emerge and at the low output steady state we have $\varepsilon_z(y_l) > \frac{1-\alpha}{\alpha}$. Using (4) and (5) we conclude that at the low output steady state $\varepsilon_{(1-z)}(y_l) < \frac{\alpha-1}{\alpha} \frac{z(y)}{1-z(y)} < 0$, i.e. the elasticity of the tax rate at the low output steady state is always negative.

and k, obtaining:

$$\begin{pmatrix} d\dot{y}(t) \\ d\dot{k}(t) \end{pmatrix} = J \begin{pmatrix} dy(t) \\ dk(t) \end{pmatrix}. \tag{32}$$

The local stability properties of the model are determined by the eigenvalues of the Jacobian matrix J (given in Appendix 9.3) or, equivalently, by its trace, Tr, and determinant, D, which correspond respectively to the product and sum of the two roots (eigenvalues) of the associated characteristic polynomial $Q(\lambda) = \lambda^2 - Tr\lambda + Det$ with:

$$Tr = \rho + \frac{(\rho + \delta)\left[1 - (1 + \varepsilon_z)(\alpha + \beta)\right]}{\{\beta(1 + \varepsilon_z) - 1\}} \tag{33}$$

$$D = \frac{(\rho + \delta)^2\left[1 - \alpha(1 + \varepsilon_z)\right]}{s\{\beta(1 + \varepsilon_z) - 1\}} \tag{34}$$

where $\varepsilon_z \equiv \varepsilon_z(y)$. Necessary and sufficient conditions to obtain local indeterminacy (a sink) are $D > 0$ and $Tr < 0$, while the necessary and sufficient condition to get local saddle-path stability is $D < 0$. Finally, the steady state is locally a source if and only if $D > 0$ and $Tr > 0$.

Proposition 3. *Under Assumption 1 and Propositions 1 and 2, the high output steady state is locally indeterminate (a sink) if and only if $(1 + \varepsilon_z(y_h)) \in (\frac{1}{\beta}, \frac{1}{\alpha})$ and is locally determinate (a saddle) if and only if $(1 + \varepsilon_z(y_h)) < \frac{1}{\beta}$. Furthermore, the low output steady state is always locally determinate (a saddle).*

Proof. Note that the numerator of the determinant at the high output steady state y_h is positive as the condition $1 + \varepsilon_z(y_h) < \frac{1}{\alpha}$ holds. Local indeterminacy requires therefore a positive denominator of both trace and determinant which implies $1 + \varepsilon_z(y_h) < \frac{1}{\beta}$. Since $\alpha < \beta$ under Assumption 1, the latter condition also leads to a negative trace, so that the necessary and sufficient conditions to get local indeterminacy around the high output steady state are $1 + \varepsilon_z(1) \in (\frac{1}{\beta}, \frac{1}{\alpha})$. The rest of the proposition follows since at the low output steady state $(1 + \varepsilon_z(y_l)) > \frac{1}{\alpha} > \frac{1}{\beta}$, so that the determinant is always negative. □

Our local indeterminacy mechanism is once again related with the labor market "wrong slopes" condition.[16] Noting that the slope of the MPL (marginal productivity of labor) curve is $-(1 - \beta)$, while the slope of the inverse labor supply curve is $-\beta\varepsilon_z$,[17]

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[16]Other works where this condition is required are for example Schmitt-Grohe and Uribe (1997), Benhabib and Farmer (1994) or Dufourt et al. (2008).

[17]At the general equilibrium level where $\bar{l} = l$ and $\bar{k} = k$ we can rewrite the MPL schedule (15) as $d\log w_t = \alpha d\log k_t - (1-\beta)d\log l_t$. In what concerns the inverse of the labor supply schedule from (11), considering a constant λ, $z(y)$ given by (3) with $y_t = k_t^\alpha l_t^\beta$, we obtain $d\log w_t = -\beta\varepsilon_z d\log l_t - \alpha\varepsilon_z \log k_t$.

it is easy to see that our indeterminacy condition $\beta(1 + \varepsilon_z(y_h)) > 1$, requires (i) either a negatively sloped inverse labor supply schedule ($\varepsilon_z > 0$) steeper than the (also negatively sloped) MPL curve ($\beta < 1$), or (ii) a positively sloped MPL curve ($\beta > 1$), steeper than the (also positively sloped) inverse labor supply schedule ($\varepsilon_z < 0$), or (iii) a positively sloped MPL curve and a negatively sloped inverse labor supply schedule.[18]

4.1 Stabilizing locally

We consider now Proposition 1 satisfied, so that the normalized steady state exists. Denoting $\phi \equiv \phi(1) = \frac{\tau'(1)}{\tau(1)}$, we choose a parameterization such that the NSS is locally indeterminate in the absence of any cyclical fiscal policy rule ($\phi = 0$).[19] In Proposition 4 below we state the necessary and sufficient conditions for this to happen:

Proposition 4. *Under Assumption 1 and Propositions 1 and 3, consider that the government does not pursue any cyclical fiscal policy ($\phi = 0$). Then the NSS is locally indeterminate if and only if $(1 - \beta)(1 - \tau(1)) < \overline{G} < (1 - \alpha)(1 - \tau(1))$.*

Proof. Note that, since $\varepsilon_z(1) = \frac{\bar{G} - \phi\tau(1)}{1 - \bar{G} - \tau(1)}$, when $\phi = 0$, from Proposition 3, we obtain immediately the condition above. $\qquad\square$

As $\alpha = (1+\gamma)s$ and $\beta = (1+\gamma)(1-s)$, we conclude that in the absence of production externalities, $\gamma = 0$, indeterminacy requires a sufficiently positive $\overline{G}$. Remark that the presence of increasing returns is not needed for any of our main results. See Propositions 2 and 4. However, its consideration allows us to have a more general framework, encompassing several works who considered increasing returns to scale, but not $\overline{G}$, and that are here obtained as particular cases of our model for $\overline{G} = 0$.[20]

Assume now that the government wants to insulate the economy from local belief driven fluctuations around the NSS. This is done by eliminating local indeterminacy. In Proposition 5 below we state how the government can ensure local determinacy of the NSS using fiscal policy:[21]

Proposition 5. *Under Assumption 1 and Propositions 1 and 3, assume that $(1 - \beta)(1 - \tau(1)) < \overline{G} < (1 - \alpha)(1 - \tau(1))$. Then, local indeterminacy of the NSS*

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[18]Note that in the absence of productive externalities $\gamma = 0$, and with $\phi = 0$, so that the share of variable government expenditures in output is constant, our indeterminacy condition collapses into the Schmitt-Grohe and Uribe's (1997) indeterminacy condition $s < \frac{\overline{G}}{y} < 1 - s$, whereas in the absence of government we recover Benhabib and Farmer's (1994) indeterminacy condition $\alpha < 1 < \beta$.

[19]Under Proposition 3 this implies that the NSS is the high output steady-state.

[20]See for example Benhabib and Farmer (1994) and Guo and Lansing (1998).

[21] Remark that ϕ is exogenously made to vary by considering continuos changes in $\tau'(1)$. Hence, existence of the normalized steady state is persistent and always ensured since B^* only depends on $\tau(1)$ and does not depend on $\tau'(1)$. See Proposition 1.

is eliminated, and local saddle path stability of the normalized steady state is achieved with a sufficiently procyclical share of variable government spending in output such that $\phi > \frac{\overline{G}-(1-\beta)(1-\tau(1))}{\tau(1)\beta} > 0.$

Proof. From Proposition 3, it is easy to see that the government can eliminate local indeterminacy, obtaining saddle path stability of the NSS, by increasing ϕ, so that $\varepsilon_z(1) = \frac{\bar{G}-\phi\tau(1)}{1-\bar{G}-\tau(1)}$ decreases, satisfying the inequality $(1 + \varepsilon_z(1)) < \frac{1}{\beta}$, that we can rewrite as $\phi > \frac{\bar{G}-(1-\beta)(1-\tau(1))}{\tau(1)\beta} > 0.$ $\qquad\square$

We conclude that if ϕ is sufficiently positive, the NSS becomes a saddle, which eliminates the existence of local sunspot fluctuations. However, this policy is not able to globally stabilize the economy against endogenous fluctuations if $\overline{G} > 0$. Indeed, we know from our previous analysis, that with $\overline{G} > 0$, another steady state with a lower level of output also exists. Both steady states are saddles and therefore locally determinate. Nevertheless, as will be shown in the next section there is global indeterminacy. Indeed, in the presence of multiple steady states, ensuring that all of them are locally determinate does not eliminate global indeterminacy and sunspots. To address these issues we must analyze the global dynamics of the model.[22]

5 Global Analysis

5.1 Phase diagram

Substituting (22) in (21) our dynamic system (21)-(22) can be rewritten in the following way:

$$\begin{bmatrix} \dot{k}_t \\ \dot{y}_t \end{bmatrix} = \begin{bmatrix} f_1(k_t, y_t) \\ \frac{(1+\gamma)f_2(k_t,y_t)}{f_3(y_t)} \frac{y_t}{k_t} \end{bmatrix} \tag{35}$$

where the vector in the RHS is the vector field of system (35) and:

$$
\begin{aligned}
f_1(k_t, y_t) &\equiv z(y_t)y_t - \delta k_t - c(k_t, y_t) \\
f_2(k_t, y_t) &\equiv s\left[c(k_t, y_t) - sz(y_t)y_t\right] - [\rho(1-s) + \delta(1-2s)]k_t \\
f_3(y_t) &\equiv \beta(1 + \varepsilon_z(y_t)) - 1
\end{aligned}
$$

with $c(k_t, y_t)$, $z(y_t)$ and $\varepsilon_z(y_t)$ given respectively by (17), (3) and (4).

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[22]Guo and Lansing (1998) show that, in an economy with no incompressible expenditures ($\overline{G} = 0$), a policy equivalent to a sufficiently positive ϕ is able to stabilize the economy, if (local) indeterminacy is caused by sufficiently high productive externalities. Here we show that if local indeterminacy is caused by the the existence of incompressible government spending, then such policy will not be able to eliminate global indeterminacy, even if it eliminates local indeterminacy.

In order to analyze global dynamics, in figures 3 and 4, we depict in the space (y, k) the k and the y nullclines and the arrows that represent the vector field. The $k-$nullcline satisfies $f_1(k_t, y_t) = 0$, and the $y-$nullcline satisfies $f_2(k, y) = 0$. Of course these two schedules cross twice, respectively at the low and high output steady states. In Appendix 9.4 we show that along the $k-$nullcline we have $dk/dy > 0$, and that the slope of the $y-$nullcline will change sign at most two times. Moreover, for $k = 0$, i.e., at the intersection between the $y-$nullcline and the line $k = 0$, the slope of the $y-$nullcline is positive and above unity.[23] As both k and y increase, the slope of the $y-$nullcline decreases, and the nullcline reaches a maximum when its slope becomes zero. As y further increases its slope becomes negative, reaching $-\infty$, so that the $y-$nullcline becomes vertical. In Appendix 9.4 we show that for reasonable values of the parameters we obtain a correspondence, i.e., after becoming vertical the nullcline bends inwards, as depicted in figures 3 and 4. It is also easy to show[24] that when $k = 0$ the $y-$nullcline is located on the right of the $k-$nullcline, as represented in figures 3 and 4.

In Appendix 9.4 we also show that above the $k-$nullcline we have $\dot{k} < 0$, i.e., above (below) the $\dot{k} = 0$ line the vertical arrows that represent the vector field of $\dot{k}_t$ point downwards (upwards). Before determining the directions of the horizontal arrows that represent the vector field of $\dot{y}_t$ it is important to note that our model exhibits a singularity for $y = y^s$ such that $f_3(y^s) = \beta(1 + \varepsilon_z(y^s)) - 1 = 0$. In the space (y, k), $y = y^s$ defines a vertical line. This line partitions the space (y, k) into two subsets of regular points: one set, which we denote by Ω_+, where $f_3(y) > 0$, i.e., where the necessary condition for indeterminacy is satisfied, and another, denoted by Ω_-, where this condition is not satisfied, i.e. $f_3(y) < 0$. Of course, on different sides of the vertical line $y = y_s$ horizontal arrows point in opposite directions. The full determination of the direction of the horizontal arrows, depicted in Figures 3 and 4, is also provided in Appendix 9.4.

In the following, we will restrict our analysis to equilibrium regular paths that converge to a steady state.[25]

5.2 Global Dynamics when the NSS is a sink

We will start by addressing the situation where the NSS is a sink, that is depicted

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[23]It is equal to $1/\alpha$.

[24]See Appendix 9.4..

[25] We define equilibrium regular paths as solutions of (35) that do not collide with $y = y_s$ and verify the initial and transversality conditions. For an analysis of singular dynamics paths see Brito et al. (2017).

in figure 3. As explained above the NSS is the high output steady-state, which coexists with a lower output steady state which is a saddle. Since at both steady states $f_3(y) = \beta(1 + \varepsilon_z(y)) - 1 > 0$, they are both located on the same side of the singularity so that $y_s > 1$. All deterministic trajectories starting on the left of the saddle path diverge to either $k = 0$ or to $y = \underline{y}$ and cannot be equilibrium paths. Otherwise, all other deterministic trajectories converge to the higher output sink steady state, with the exception of those starting precisely on the stable arm of the saddle, which converge to the lower output steady state. This means that, for the same initial given value of the predetermined variable, the capital stock, there are several different equilibrium trajectories that converge to different steady states,[26] i.e., we have global indeterminacy.[27] The equilibrium trajectory obtained depends on the value of the non predetermined variable chosen, which is expected output. Also, since the NSS is a sink, it is locally indeterminate and there exist infinitely many stochastic endogenous fluctuations (sunspots), driven by expectations, arbitrarily close to it. We can therefore state the following:

Proposition 6. *Under Assumption 1 and Propositions 1 and 3, when $(1 + \varepsilon_z(1)) \in (\frac{1}{\beta}, \frac{1}{\alpha})$ there are exactly two steady states: the NSS, which is the high output steady and a sink, coexists with the low output steady state which is a saddle. In this case there is global indeterminacy. Furthermore the NSS is locally indeterminate and there exist local stochastic endogenous fluctuations (sunspots).*

5.3   Global Dynamics with two saddles

Now, if the government decides to stabilize locally the NSS, it can, as described above, make it saddle-stable by increasing ϕ so that $f_3(1) = \beta(1 + \varepsilon_z(1)) - 1 < 0$. Both steady states are now locally saddle-stable, but in the low-output steady state $f_3(y_l) = \beta(1 + \varepsilon_z(y_l)) - 1 > 0$. This means that we have $y_l < y_s < 1$ so that the situation depicted in figure 4 emerges. Remark that stabilizing locally implies moving the position of the singularity line from the right to the left of the NSS.

However, although in this case the share of the variable government spending component in output is procyclical ($\phi > 0$), and local indeterminacy and sunspots no longer

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[26]Note however, that since all equilibrium trajectories, with the exception of the one that converges to the low output steady state, end up at the high output steady state, the likelihood of reaching the low output steady state is low.

[27]See Raurich (2000) for a definition and a clear cut discussion about global indeterminacy issues.
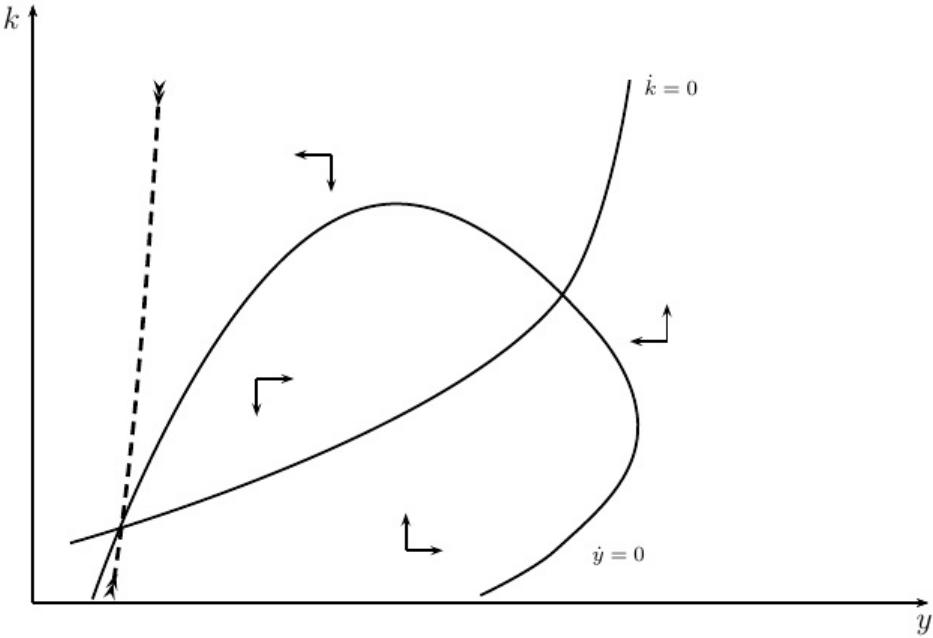
Figure 3: Sink-saddle configuration

exist, the total tax rate, as noted before, is not procyclical (or acyclical) for all y and the problem of global indeterminacy remains. Again, for a given initial value of the capital stock, the model admits equilibria that converge either to the lower steady state or to the NSS.[28] These equilibria differ with respect to the agents' expectations about future output. This implies that expectations about future output, determine the long-run outcomes of the economy, i.e. we also have global indeterminacy.

Proposition 7.   *Under Assumption 1 and Propositions 1 and 3, when $1 + \varepsilon_z(1) < \frac{1}{\beta} < \frac{1}{\alpha}$ there are exactly two steady states: the NSS, which is the high output steady and a saddle, coexists with the low output steady state which is a saddle. In this case there exist two distinct saddle paths and hence there is global indeterminacy.*

We conclude that in our model, and in contrast to previous results, a procyclical share in output of the variable government spending component , i.e. a procyclical $\tau(y)$, is not able to insulate the economy from belief driven fluctuations. Furthermore, since these fluctuations are due to the existence of global (and not local) indeterminacy, the current indeterminacy mechanism is able to generate (or account for) sharp fluctuations in output as, if agents' expectations are revised downwards, the economy is displaced from the upper to the lower stable arm, making the economy converge to the lower

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[28]For all other values of y we obtain divergent trajectories.
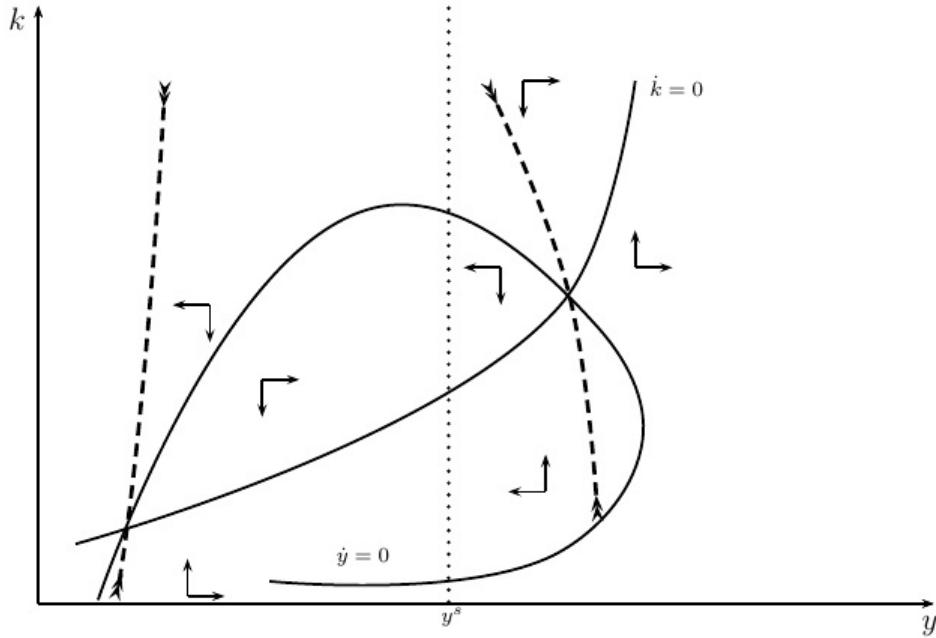
Figure 4: Saddle-saddle configuration

output steady state.[29] Hence, a fiscal policy that locally stabilizes, eliminating small fluctuations, is not able to prevent (big) fluctuations caused by changes in agents' expectations. In this context the management of expectations is crucial to guarantee that the economy remains on the right path, avoiding sharp belief driven fluctuations.

It is clear from the above discussion that steady state multiplicity is responsible for these results. Also, as explained above, in our model steady state multiplicity is pervasive, due to the existence of a positive fixed amount of minimum government expenditures. It follows, that when the government is not able to manage private agents' expectations, abandoning the view that government expenditures, even in recessions, can not fall below a fixed minimum level, is the only way to avoid the perils of stabilization. Indeed, making government spending fully flexible (and procyclical), thus eliminating $\overline{G}$, is the only way to obtain simultaneously saddle path stability and steady state uniqueness, and hence global determinacy, fully restoring the ability to stabilize of a sufficiently procyclical government spending/tax rate policy.

However, if the government is not able to eliminate $\overline{G}$ or to manage expectations we obtain multiple equilibrium trajectories associated with different expectations about future output. Note however that in each equilibrium agent's expectations are correct. Indeed, when agents' confidence falls and the economy lands on the low output

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[29]Furthermore, such major crisis is potentielly long lasting if the revision in agents' expectations is persistent.

trajectory, the output that materializes is the one expected by the agents. Similarly, when confidence is restored and agents are optimistic, the economy switches to the high output trajectory. Again the output that materializes is the one expected by agents.

6 Robustness

In this section, we assess through numerical exercises the robustness of our conclusions by relaxing the assumption of an identical tax rate for labor and capital income and by considering a more general functional form for utility.

6.1 Considering different tax rates rules for labor and capital income

We maintain the assumption that there is a fixed minimum level of government expenditures, $\overline{G} > 0$, but we now assume that, in order to stabilize the economy, the government controls the cyclicality of $g(y) = \tau(y)y$ using two different tools: a policy rule for the labor income tax, $\tau_l(y)$, and a policy rule for the capital income tax, $\tau_k(y)$, such that $\tau(y) = s\tau_k(y) + (1-s)\tau_l(y)$. These two instruments may differ in level and in their response to aggregate output. For numerical purposes, we consider a functional form for the tax rates *à la* Lloyd-Braga *et al.* (2008):

$$\tau_j(y) = \mu_j y^{\phi_j}, \quad j = k, l \tag{36}$$

where the elasticity of the tax rate is denoted by ϕ_j, and the level of the tax rate is governed by the parameter μ_j. The disposable income of households is therefore given now by $z_l(y)wl + z_k(y)rk$ with

$$z_j(y) = 1 - \tau_j(y) - \frac{\overline{G}}{y}, \qquad j = k, l. \tag{37}$$

and we still have $G_t = [1 - z(y_t)]y_t$ where $z(y_t) = [1 - \frac{\overline{G}}{y} - (1-s)\mu_l y_t^{\phi_l} - s\mu_k y_t^{\phi_k}]$.

It is obvious that our conclusions on the existence and the multiplicity of steady states remain, since they rely only on the presence of $\overline{G}$ in the after-tax labor and capital income. We focus therefore on the local and global dynamic properties of the extended model.

We consider calibrated values of the parameters for a quarterly frequency. In particular, we set $(\rho, \delta, s) = (0.01, 0.025, 0.3)$. The first two values are widely used in quarterly calibrations, while the value considered for the share of capital income in national income is standard in the macroeconomic literature. In addition, we set the

Parameters	ρ	δ	s	$\bar{G}$	μ_l	μ_k	γ
Values	0.01	0.025	0.3	0.1	0.35	0.23	0.38

Table 1: Calibration of main parameters

learning-by-doing externality at $\gamma = 0.38$.[30] With these values for the parameters, in the absence of incompressible government expenditures, $\bar{G} = 0$, or any taxation policy, $\phi_l = \phi_k = 0$, the NSS would be locally determinate (a saddle). Since we want to start from a situation where the government desires to stabilize the economy around the NSS, we set $\bar{G} = 0.1$ in order to guarantee that the NSS is a sink.[31] Regarding the tax rates on labor and capital income, we set $(\mu_k, \mu_l) = (0.23, 0.35)$ which can be considered mild values according to the estimates provided by Trabandt and Uhlig (2011). Note that this implies a labor tax rate of 45% at the NSS and capital tax rate of 33% also at the NSS, in line with empirical estimates. The parameter values considered are summarized in Table 1.

In the following numerical exercises different sets of values for ϕ_l and ϕ_k were chosen to allow for different choices of fiscal policy. We observe that the stabilizing power of the two tools is dramatically different. While the labor income tax rate can be used to successfully stabilize locally the economy, the same is not true for the capital income tax rate. Indeed, a value of $\phi_l > 0.231$, fully prevents the economy from stationary expectation-driven fluctuations around the NSS, regardless of the value chosen for ϕ_k. Such conclusions confirm and complement the contribution of Guo (1999) that only a progressive labor income tax is stabilizing.

We now study global dynamics by choosing values of ϕ_l and ϕ_k corresponding to either a locally destabilizing or stabilizing fiscal policy. In particular, in order to significantly deviate from the baseline model with a unique income tax rate, we consider pairs of values of ϕ_l and ϕ_k with opposite signs. Figure 5 illustrates a case where the government sets $\phi_l = -0.5$ and $\phi_k = 1$, which leads to a sink-saddle configuration.

The first conclusion is that this figure is quite similar to figure 3, which supports the robustness of our results. The solid lines represent the $k-$nullcline and the $y-$nullcline, respectively in red and black. As in figure 3, the upper steady state (NSS) is locally indeterminate and therefore, for a given k_0, there are an infinite number of initial values

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[30]This value is significantly below the benchmark value of $\gamma = 0.5$ considered in Benhabib and Farmer (1994), and quite close to the upper limits of the obtained estimation intervals in Basu and Fernald (1997) and Burnside *et al.* (1995).

[31]This value is in accordance with our own computations, which report country shares of incompressible government expenditures in GDP, $\bar{G}/y$, between 0.05 and 0.12.
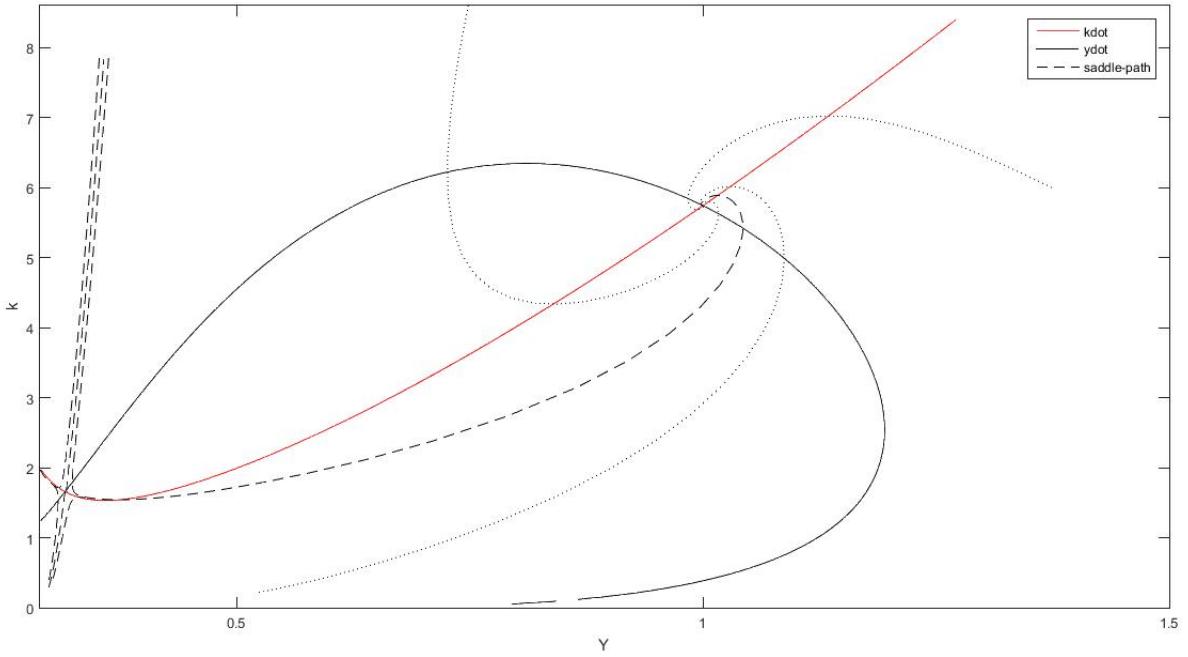
Figure 5: Sink-saddle configuration. $\phi_l = -0.5$, $\phi_k = 1$.

of output y that converge to this steady state. The dotted lines depict this kind of trajectories. We can observe that these equilibrium paths converge in a non-monotonic way, which implies therefore an endogenous propagation mechanism. We also plot two trajectories, in dashed lines, surrounding the nonlinear saddle-path that converges to the lower steady state. In contrast, this equilibrium path, for a given k_0, admits a unique initial value for y for which the economy converges to the lower steady state. However, as in the case with just a unique income tax rate we have global indeterminacy. For initial values of k there are different values of y compatible with convergence to the lower or to higher steady state.

In figure 6, we illustrate the case where the government sets $\phi_l = 0.5$ and $\phi_k = -1$. In this case, with a sufficiently procyclical labor income tax rate, and as in figure 4, both steady states are locally determinate (saddle-path stable). As above, the nullclines are represented by the solid lines,[32] while the location of the two nonlinear saddle-paths is given by dashed lines. We also plots the singularity that occurs at $y = 0.768$. One easily observes that, as in the case with just a unique income tax rate depicted in figure 4, for a given initial value of the capital stock k_0, there are two initial values of output y, each located on a different equilibrium trajectory on different sides of the singularity,

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[32]In this figure we do not fully represent the RHS of the y-nullcline, which, as in the previous figures is backward-bending.
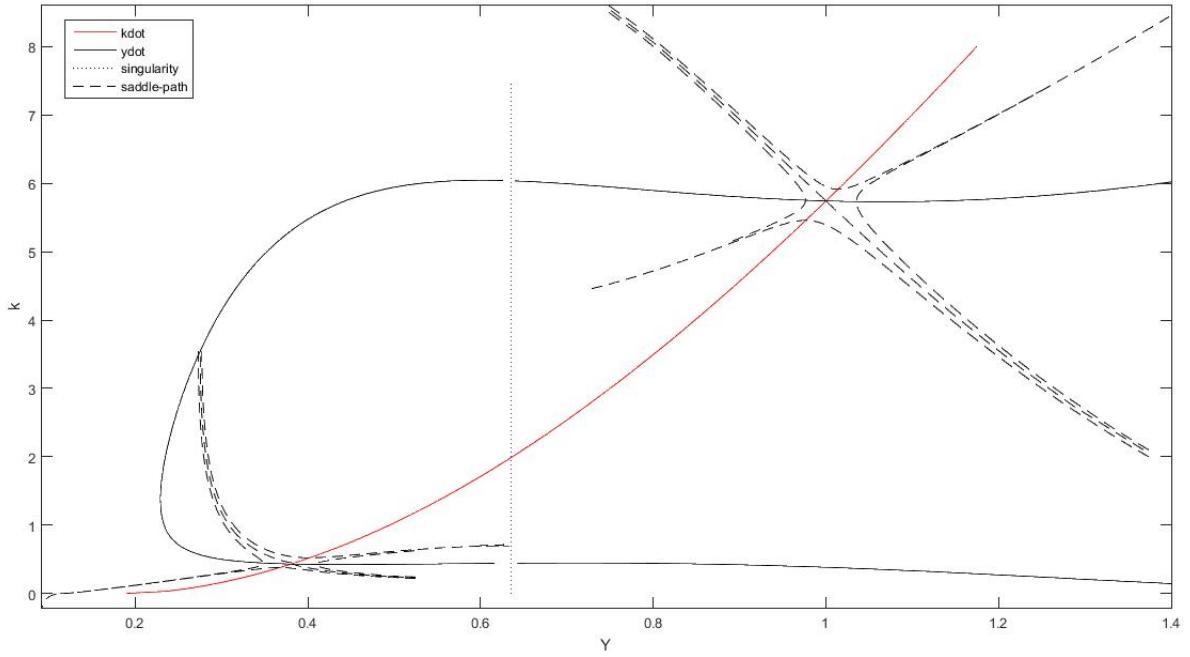
Figure 6: Saddle-saddle configuration. $\phi_l = 0.5$, $\phi_k = -1$.

i.e. we have global indeterminacy. It follows that, also in this case one may construct deterministic cycles and/or regime switching sunspot equilibria between the two saddle paths, which validates our previous results.

6.2   Considering more general preferences

In this section we introduce a more general functional form for the instantaneous utility function $U(c_t, l_t)$:

$$U(c_t, l_t) = \frac{c_t^{(1-\sigma)}}{(1-\sigma)} - \frac{l_t^{(1+\chi)}}{B(1+\chi)} \tag{38}$$

where $\sigma \neq 1$ is the inverse of the elasticity of intertemporal substitution (EIS) and $\chi > 0$ is the inverse of the elasticity of labor supply. Through numerical exercises, we examine the robustness of our conclusions on steady state multiplicity and indeterminacy for empirically accepted values of σ and χ. Using our baseline calibration with an identical (procyclical) income tax rate for labor and capital income,[33] our simulation results show that, keeping $\chi = 0$, we still obtain steady state multiplicity (again two steady states)

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[33]We considered $\tau(y) = \mu y^\phi$ with $\mu = 0.3$ and $\phi = 0.33$. The values considered for the other parameters are the ones in Table 1.

for any $\sigma \in (0.4, 1.18)$.[34] Note that this interval, according with recent works by Vissing-Jorgensen and Attanasio (2003), and Gruber (2013), includes all the empirically relevant values for σ.[35] Similarly, considering $\sigma = 1$, we still obtain two steady states for values of $\chi \in (0, 0.6)$.[36] Moreover, regarding local stability, we find that for $\sigma \in (0.4, 1.18)$ and $\chi \in (0, 66)$, both steady states are saddle-path stable. We conclude therefore that our previous results are robust.

7 Expectations Shocks

In this section, we discuss and illustrate the effects of expectations shocks. We start by providing a version of the model in which agents' expectations about long-run output follow a simple two state Markov switching process, allowing y_t to jump between trajectories. We then use the augmented model to illustrate the effect of shocks to the agents' expectations about long-run output. Finally, we discuss the economic mechanism responsible for the emergence of global indeterminacy and regime switching sunspots.

Note that, in order to explore the quantitative effects of expectations shocks, in this section we use a generalized version of our baseline model, with different capital and labor income tax rates and where the intertemporal elasticity of substitution is no longer equal to unity.

7.1 Modelling Expectation Shocks

In this section we follow Kaplan and Menzio (2016),[37] borrowing their definition of a Markov switching rational expectation equilibrium. We introduce a sunspots variable, S_t, which takes two values, 0 or 1. We assume that $S_t = 1$ is associated with the belief that the economy is in a trajectory converging to the high output steady state (conditional on remaining in the same optimistic state), whereas $S_t = 0$ is associated with the belief that the economy is on a trajectory converging to the low output steady state (conditional on remaining in the same pessimistic state). Agents' expectations switch from optimistic to pessimistic with probability p, i.e., the probability that S_t

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[34]Note however the presence of a transcritical bifurcation around $\sigma = 0.46$.

[35]Both Vissing-Jorgensen and Attanasio (2003) and Gruber (2013) report estimates for σ between 0.5 and 1. See also Hansen and Singleton (1982) and Bansal and Yarow (2004) and Hansen et al. (2007). Note that some earlier papers used to find much higher values for $\sigma \in (1.25, 5)$. See for example Kocherlakota (1996) and Campbell (1999).

[36]Note that Rogerson and Wallenius (2009), besides the consideration of an infinitely elastic labor supply, recommend using values for χ around 0.333. Accordingly, we considered $\chi \in (0, 0.6)$.

[37]See also Benhabib et al. (2018) and Kamihigashi (2017).

changes from unity to zero in a short interval is p. In this case output falls by $D_{1,0}(k, y)$. Similarly the agents' expectations switch from pessimistic to optimistic with probability q, in which case output increases by $D_{0,1}(k, y)$.

In the optimistic state, the evolution of the economy is described by the two equations below:

$$\frac{\dot{y}_t}{y_t} = -\frac{\alpha}{[\beta(1 + \varepsilon_{z_l}(y_t)) - 1]}\frac{\dot{k}_t}{k_t} + \beta\frac{sz_l(y_t)y_t - (\rho + \delta)k_t}{k_t[\beta(1 + \varepsilon_{z_l}(y_t)) - 1]} + pD_{1,0}(k_t, y_t) \quad (39)$$

$$\dot{k}_t = [(1 - s)z_l(y_t) + sz_k(y_t)]y_t - \delta k_t - c(k_t, y_t) \quad (40)$$

where $c(k_t, y_t) = [B(1 - s)z_l(y_t)k_t^{\alpha/\beta}y_t^{(\beta-1)/\beta}]^{\frac{1}{\sigma}}$. The term in the LHS of (39) represents the change in output conditional on the economy remaining in the optimistic state. The first two terms on the RHS correspond to the generalization of the RHS of (21) for the case where $z_l(y_t) \neq z_k(y_t)$ and $\sigma \neq 1$, while the last term is the probability that the economy switches to the pessimistic state, p, times the resulting change in output conditional on the economy switching states, $D_{1,0}(k, y)$. The second equation describes the evolution of capital. The term in the RHS corresponds to the generalization of the RHS of (22) for the case where $z_l(y_t) \neq z_k(y_t)$ and $\sigma \neq 1$.

Similarly, in the pessimistic state the behavior of the economy is described by the following two equations:

$$\frac{\dot{y}_t}{y_t} = -\frac{\alpha}{[\beta(1 + \varepsilon_{z_l}(y_t)) - 1]}\frac{\dot{k}_t}{k_t} + \beta\frac{sz_l(y_t)y_t - (\rho + \delta)k_t}{k_t[\beta(1 + \varepsilon_{z_l}(y_t)) - 1]} + qD_{0,1}(k_t, y_t) \quad (41)$$

$$\dot{k}_t = [(1 - s)z_l(y_t) + sz_k(y_t)]y_t - \delta k_t - c(k_t, y_t) \quad (42)$$

where, as above, $c(k_t, y_t) = [B(1 - s)z_l(y_t)k_t^{\alpha/\beta}y_t^{(\beta-1)/\beta}]^{\frac{1}{\sigma}}$. The term in the LHS of (41) represents the change in output conditional on the economy remaining in the pessimistic state. The first two terms on the RHS correspond, as in the optimistic case, to the generalization of the RHS of (21) for the case where $z_l(y_t) \neq z_k(y_t)$ and $\sigma \neq 1$, while the last term is the probability that the economy switches to the optimistic state, q, times the resulting change in output conditional on the economy switching states, $D_{0,1}(k, y)$. The second equation describes the evolution of capital, which is the same as in the optimistic state.

Since expectations must be rational we need to impose the following conditions. First, when the economy switches from the optimistic to the pessimistic state, the value of output must land on y_l^S, where y_l^S denotes the stable manifold associated with the low-output steady state. This guarantees that, if the economy then remains in the pessimistic state forever, it will converge to the low output steady state. Second, when the economy switches from the pessimistic to the optimistic state, the value of output must fall on y_h^S the stable manifold associated with the high-output steady state if

this steady state is a saddle, or its basin of attraction if it is a sink. This guarantees that, if the economy then remains in the optimistic state forever, it will converge to the high output steady state. Finally, if the initial state of the economy is optimistic, the initial value of output must be on the stable manifold associated with the high-output steady state or in its basin of attraction, while if the initial state of the economy is pessimistic, the initial value of output must be on the stable manifold associated with the low output steady state.

Let S denote a history of realizations of S_t and $t_n(S)$ the nth time at which the state of the process switches in history S. Then, following Kaplan and Menzio (2016) we define:

Definition 2. *A Markov switching rational expectation equilibrium is a history-dependent path $\{k_t(S), y_t(S)\}$ such that, for every S, the following conditions are satisfied: (i) For all $t \in [t_n, t_{n+1})$ with $S_{t_n} = 1$, $\{k_t, y_t\}$ satisfies (39) and (40). (ii) For all $t \in [t_n, t_{n+1})$ with $S_{t_n} = 0$, $\{k_t, y_t\}$ satisfies (41) and (42). (iii) For any k and any $y \in y_h^S(k)$, $y + D_{1,0}(k, y) \in y_l^S(k)$. For any k and any $y \in y_l^S(k)$, $y + D_{0,1}(k, y) \in y_h^S(k)$. (iv) $y_0 \in y_h^S(k_0)$ if $S_0 = 1$, and $y_0 \in y_l^S(k_0)$ if $S_0 = 0$.*

Note that when $p = q = 0$ the solution of (39) and (40)) is any equilibrium path which converges to the high output steady state, depicted in figure 5, if this steady state is a sink, or the saddle path towards the high output steady state depicted in figure 6 if this steady state is a saddle. Similarly, when $p = q = 0$ the solution of (41) and (42) is the saddle path towards the low output steady state, depicted in figures 5 and 6. By continuity these functions exist for small values of p and q and solve respectively (39) and (40), and (41) and (42). In Appendix 9.5 we discuss the approximation used.

7.2 Illustrating the effects of an expectation shock

We now illustrate the behavior of the model economy under global indeterminacy and sunspot shocks. We consider the case where the two steady states are both saddles, so that $y_l < y_s < 1$, and where the economy starts in the optimistic state, being therefore described by equations (39) and (40). However, our economy can be hit by a severe and persistent crisis triggered by a sudden loss in confidence, which brings it to the lower equilibrium trajectory. The next figure depicts such a numerical exercise where we assume $p = 0.02$ and $q = 0.04$. These values imply that the model economy will remain in the optimistic state with probability 0.666. The values chosen for some of the other parameters are given in Table 1. Moreover, we considered $\phi_l = 0.25$ and $\phi_k = -0.95$, so that we have two saddles, and set $\sigma = 0.59$ to ensure that the difference of the two steady states is empirically plausible. Indeed with this calibration we obtain

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$y_l = 0.85$, i.e., a loss in confidence implies a 15 percent decrease in long run output, a value which represents half of the fall in output observed during the Great Depression in the USA. See Ohanian (2001). Moreover, according to Ball (2014), the long-term effects of the global recession of 2008-2009 on output in 23 countries were quite large.[38] He finds that the average long term output loss for the 23 countries, weighted by the sizes of their economies, was 8.4%, and reports losses over 30% in Greece, Hungary, and Ireland. Also half of the countries considered registered long term output losses above 9.57%.[39]

Note also that with this calibration the values obtained for the labor and capital income tax rates at the lower steady state are quite close to the ones corresponding to the high output steady-state. Indeed the labor tax rate at the low output steady state is 0.4527 (vs. 0.45 at the high output steady state) while the capital income tax rate at the low output steady state is 0.3721 (vs. 0.33 at the high output steady state). This is reassuring as it means that our simulation results are not driven by empirically implausible increases in taxation.

In figure 7 the economy starts in the optimistic state and remains there for 32 quarters, reaching the high output steady-state. Then, in period 33, as S_t drops from 1 to 0, agents' expectations about future output become pessimistic and the economy jumps to the trajectory converging to the low output steady-state. We observe an immediate drop in output and investment. However, as expected, the fall in consumption is slower,[40] which is a nice feature of our model, although more persistent. Remark that in the Great Recession, we observed a similar pattern. Indeed, consumption was still below pre-recession levels 8 quarters after the official end of the recession. See Petev et al. (2011). Interestingly, according to these authors this behavior was explained by expectations, i.e. by "a decline in consumer confidence which may have reduced spending through accumulation of precautionary savings (or reduction of debt) as well as deferment of spending, most notably on durables."[41] The model economy then stays in recession for 24 periods, reaching the low output steady state. Remark that when

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[38]Ball measures these effects by comparing current estimates of potential output from the OECD and IMF to the path that potential output was following in 2007, according to estimates at the time. Note that this concept is closer to our difference between two long term solutions than actual output losses.

[39]For the United States he reports a loss of potential output of 5.3%, somewhat smaller than in most countries.

[40]This behaviour of consumption is due to the choice of the intertemporal elasticity of substitution.

[41]They remark that a year and a half after the end of the recession, consumer confidence was still below pre-recession levels.
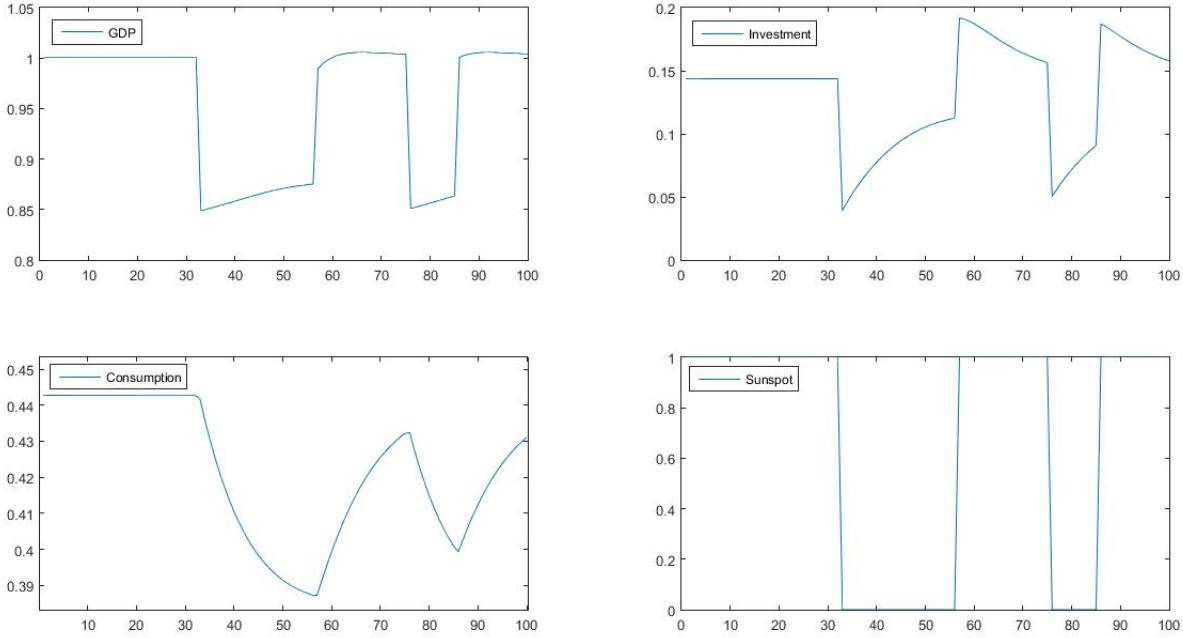
Figure 7: Simulation of a path-switching sunspot process, $\phi_l = 0.25$, $\phi_k = -0.95$, $\sigma = 0.59$.

falling from the upper to the lower saddle-path output always overshoots,[42] so that along the lower saddle path output (and investment) increases. Then, as S_t jumps from 0 to 1 in period 57, agents become optimistic again, and the economy jumps to the saddle path converging to the high output steady state. The same pattern repeats itself one more time, with jumping events at period 76 (recession) and 86 (boom). As all these movements are generated by switches between trajectories converging to quite different long run output levels, the ups and downs we observe are considerably larger than fluctuations around one single trajectory, like the ones generated by exogenous productivity shocks or local sunspots in the case of an indeterminate steady state.

7.3   The economic mechanism behind regime switching sunspots

Below, we describe the economic mechanism behind the emergence of regime switching sunspots in the case with two saddles. For the sake of simplicity we revert back to the

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[42]This is due to the relative slopes of the lower and the upper saddle-paths: at the switching point both are negatively slopped.

case with identical labor and capital income tax rates. Note first that in the absence of distortions, i.e., without productive externalities ($\gamma = 0$) and without government ($\overline{G} = 0$ and $\phi(y) = 0$), we recover the results of the classical Ramsey model: the steady state is unique and saddle-stable. Neither local nor global indeterminacy are possible, so that endogenous fluctuations are ruled out. Furthermore, consumption is a decreasing function of output.[43] With sufficiently strong productive externalities, but no taxes, the steady state is still unique but indeterminate (a sink). Local sunspots fluctuations are therefore possible. Also in this case consumption is increasing in income.[44] When we consider taxation ($\overline{G} > 0$ and $\phi(y) \neq 0$), and even without externalities, we always have steady state multiplicity.[45] A low output steady state, where $[\alpha(1 + \varepsilon_z(y_l)) - 1] > 0$, appears and coexists with a high output steady state where $[\alpha(1 + \varepsilon_z(y_l)) - 1] < 0$. When the two steady states are saddles, we have $\varepsilon_z(y_h) < \frac{1-\beta}{\beta} < \frac{1-\alpha}{\alpha}$, while at the low output steady state $\varepsilon_z(y_l) > \frac{1-\alpha}{\alpha} > \frac{1-\beta}{\beta}$. This means that the function $[\beta(1 + \varepsilon_z(y)) - 1] = \frac{\partial C_t}{\partial y_t}$ (see (18)) is positive for values of $y < y_s$, is zero at $y = y_s$ and becomes negative when $y > y_s$. We conclude, that for a given value of capital, consumption is a single peaked function of income. Therefore, for a given value of capital, there are two values of output on different sides of the singularity, $y_1 < y_s$ and $y_2 > y_s$ that sustain the same level of consumption, i.e., from (17) we have $z(y_1)y_1^{(\beta-1)/\beta} = z(y_2)y_2^{(\beta-1)/\beta}$.

Consider now the following. Departing from a situation where expectations are optimistic, so that the economy is on the saddle path converging to the high output steady state, we observe a sudden drop in confidence. Agents become pessimistic about the future of the economy and expect a simultaneous fall in consumption, capital and output. As along the saddle path converging to the high output steady state we have a negative relation between capital and income, it is easy to see that for these expectations to be self-fulfilling, the economy has to switch to the saddle path on the left of the singularity. Indeed, for the same value of capital, the economy jumps from point A to point B, where consumption is identical, and starts moving downwards along the new saddle path, in the direction of the low output steady state. We observe therefore a simultaneous decrease in output and capital. As consumption increases with capital and, on this side of the singularity, increases with output, consumption also falls

---

[43]Note that with an infinitely elastic labor supply the income effect is constant and equal to 1 and there is no substitution effect. Indeed using (10) and (11) we obtain $c_t = Bw_t$. Substituting now (16) in the previous expression we have $c_t = c(k_t, y_t) \equiv B(1-s)k_t^{\alpha/\beta} y_t^{(\beta-1)/\beta}$ so that equilibrium consumption is decreasing in y and increasing in k since $\beta = (1-s) < 1$.

[44]As before $\frac{\partial C_t}{\partial y_t} = \frac{(\beta-1)}{\beta} y_t^{-\frac{1}{\beta}}$, but now $\beta = (1-s)(1+\gamma) > 1$.

[45]The function $\varepsilon_H(y) = [\alpha(1 + \varepsilon_z(y)) - 1]$, which without any form of countercyclical taxation is always negative, now changes sign once. See Proposition 2.
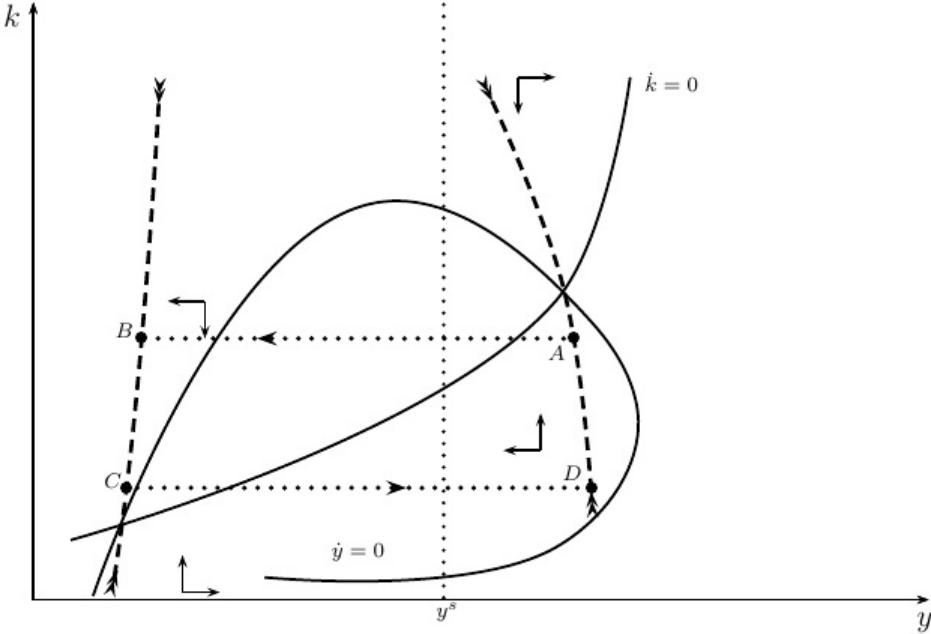
Figure 8: Regime-Switching Expectations.From point A to B: pessimistic expectation. From point C to D: optimistic expectation

unambiguously. Expectations are therefore self-fulfilling. Consider now the situation where agents, while on the path converging to the low output steady state, become optimistic, expecting an increase in output, capital and consumption. Again, in order for the expectations to be self fulfilling, the economy must jump to the saddle path converging to the high output steady state. For the same level of capital the economy jumps from point C to point D, where consumption is identical. The economy then starts moving upwards along the (negatively sloped) saddle path on the right of the singularity, converging to the high output steady state. Moreover, as along the high output saddle path, capital increases and income decreases, since on this side of the singularity consumption decreases with y, we obtain an unambiguous increase in consumption. Therefore again expectations are self fulfilling.

8 Concluding Comments

The balanced-budget rule has been advocated in order to ensure fiscal discipline, sustainability and government solvency, avoiding the perils associated with explosive public debt trajectories and sovereign default. In this paper, we show that under a balanced-budget rule conventional stabilization policy recommendations are no longer valid in

the presence of incompressible public expenditures, such as public safety, defense and general public services. Without such expenditures, a sufficiently procyclical share of government spending in output (or sufficiently procyclical tax rates) is able to guarantee both local and global uniqueness of equilibrium, preventing expectation-driven fluctuations. In contrast, the need to raise a fixed amount of tax revenues in order to finance incompressible public expenditures, always generates global indeterminacy, associated with the emergence of two steady states. We show that the low activity steady state is always saddle-path stable while the high activity one may be either a sink (locally indeterminate) or a saddle (locally determinate). A government, willing to eliminate local expectation-driven fluctuations around the high steady state, can do so by setting a sufficiently procyclical share of the variable government spending in output. But, as global indeterminacy persists, the economy remains exposed to large and persistent fluctuations based on a regime-switching sunspots process.

In this context, a government faces several trade-offs. The first is a "*welfare vs. stabilization*" trade-off. The only way to completely eradicate global indeterminacy and regime-switching fluctuations is to eliminate the incompressible property of expenditures associated with the basic functions of a State. In particular, these expenditures will have to follow the business cycle: increasing in a boom and decreasing in a recession. Of course, this option has severe political and social costs, especially in a recession, being therefore difficult to implement. The second trade-off has to do with the magnitude of the fluctuations. A government who wishes to maintain incompressible expenditures may chose to disregard and to endure "*small*" fluctuations around the high output (sink) steady state. Note that in this context regime-switching fluctuations are unlikely since this steady state acts as a global attractor. However, in the simulations performed, the existing multiple trajectories converging to the high output steady state were non-monotonic and of long duration, which suggests non-negligible fluctuation costs. Finally, a potential solution to simultaneously keep the incompressible expenditures while minimizing expectation-driven fluctuations is to successfully convince economic agents that the economy will remain in the high activity state. This requires a careful expectations' management which is uncertain and very difficult to implement.

We conclude that under a balanced-budget rule the existence of incompressible expenditures can severely undermine the stabilization role of fiscal policy. A government may therefore feel tempted to abandon the fiscal discipline of a balanced-budget rule. The emission of public debt, breaking the link between incompressible expenditures and countercyclical tax rates, could attenuate some of the implications of incompressible public spending. However, issuing more government bonds may not be an option for

those countries that have reached extremely high levels of public debt. Moreover, recent works (see Aires et al. (2018) and Lorenzoni and Werning (2018)) have shown that debt itself may be a source of equilibria multiplicity triggered by volatile expectations, where a good and a bad equilibria coexist. As this suggests that abandoning the balanced-budget rule will not solve the multiplicity problem and associated fluctuations, we conclude that governments must seriously consider whether they can afford to maintain incompressible public expenditures.

9 Appendix

9.1 Regressing the cyclical components of public spending on cyclical output

The cyclical components of all variables were obtained using the Hodrik-Prescott (HP) filter and are expressed in percentage deviations from trend. Below we present the results obtained when regressing by OLS the cyclical component of the difference between total per capita U.S. government spending in constant prices of 2015 and incompressible U.S. public expenditures in constant prices of 2015, $(G_t - \overline{G}_t)^*$, on the cyclical component of per capita U.S. output in constant prices of 2015, y_t^*.

$$(G_t - \overline{G}_t)^* = \alpha_1 + \alpha_2 y_t^* + e_t$$

	Estimate	S.E.	T-stat	p.value
α_1	-0.001	0.019	-0.075	0.94
α_2	-0.078	0.027	-2.915	0.01

$n = 56, \quad R^2 = 0.136$

We can see that the cyclical difference between total government spending and incompressible public expenditures is significantly correlated with cyclical income.

A similar regression was also performed for the cyclical component of incompressible public expenditures, $\overline{G}_t^*$.

$$\overline{G}_t^* = \beta_1 + \beta_2 y_t^* + u_t$$

	Estimate	S.E.	T-stat	p.value
β_1	-0.001	0.011	-0.060	0.95
β_2	0.026	0.016	1.630	0.11

$n = 56, \quad R^2 = 0.047$

In this case we conclude the opposite. The correlation between $\overline{G}_t^*$ and y_t^* is not significantly different from zero, i.e. the hypothesis that $\overline{G}_t^*$ is constant is not rejected by the data at the usual levels of significance.

9.2   Steady State Existence and Multiplicity

Existence and multiplicity are determined by the solutions of $H(y) = \bar{H}$. Assume first that $\bar{G} > 0$. Consider that $z(y) = 1 - \bar{G}/y - \tau(y)$ where $\tau(y) \in [0, +\infty)$ for $y \in (0, +\infty)$ so that $z(y) \leq 1$. Consider also that whenever $\tau(y)$ is positively valued, it is a monotonic continuous function, with either $\tau'(y) < 0$ (countercyclical) for all $y \in (0, +\infty)$ or $\tau'(y) > 0$ (procyclical) for all $y \in (0, +\infty)$ or $\tau'(y) = 0$ (constant) for all $y \in (0, +\infty)$. We do not specify second-order conditions for the moment. Of course, at equilibrium, if $\tau(y) > 0$ we must consider only values for y such that $\tau(y) \in (0, 1)$, and more precisely such that $z(y) \in (0, 1)$. Note that if $\tau(y) = 0$ for every y, then $z(y) = 1 - \bar{G}/y$ and therefore for $y > \bar{G}$ we have that $z > 0$. Let us now consider that $\tau(y) \neq 0$ and that there is some $y > \bar{G}$ such that $\tau(y) < 1 - \bar{G}/y$. Further let $\underline{y}$ be such that $\tau(\underline{y}) = 1 - \bar{G}/\underline{y}$. Similarly, there exists a $\bar{y} \in (\underline{y}, +\infty)$ such that $z(y) > 0$ for any $y \in (\underline{y}, \bar{y})$. Note that if $\tau'(y) < 0$ or $\tau'(y) = 0$ then $\bar{y} = +\infty$. Since $z'(y) = \frac{\bar{G}}{y^2} - \tau'(y)$, we conclude that $z(y)$ is strictly increasing if $\tau'(y) < 0$ or $\tau'(y) = 0$. In contrast, if $\tau'(y) > 0$, then $z(y)$ is single-peaked since $z'(y)$ may change sign only once and that $z(y)$ takes the value zero at $\underline{y}$ and $\bar{y}$.

Rewrite the equation $H(y) = \bar{H}$ as $Q(y) \equiv z(y)^\alpha = \bar{H} y^{1-\alpha}$. The right-hand side (RHS) is strictly increasing and concave. Furthermore, for $y \in (\underline{y}, \bar{y})$, we have that $\bar{H} y^{1-\alpha} \in (\underline{M}, \bar{M})$ with $\bar{M} = +\infty$ if $\bar{y} = +\infty$. From the left-hand side (LHS), given by the function $Q(y)$, we derive:

$$
\begin{aligned}
Q(\underline{y}) &= 0 \\
Q(\bar{y}) &= \begin{cases} < 1 & if \quad \tau'(y) < 0 \ or \ \tau'(y) = 0 \\ = 0 & if \qquad \quad \tau'(y) > 0 \end{cases}
\end{aligned}
\tag{43}
$$

Hence, the left-hand side always starts and ends below the right-hand side. It follows that existence of a steady state implies generically multiplicity. We will have exactly two steady state if $Q(y)$ is concave. The second-order derivative of $Q(y)$ is such that:

$$
-\alpha z(y)^{\alpha-1} \left\{ (1-\alpha) \frac{\left[\frac{\bar{G}}{y^2} - \tau'(y)\right]^2}{z(y)} + \underbrace{\left[\frac{2\bar{G}}{y^3} + \tau''(y)\right]}_{= -z''(y)} \right\}
\tag{44}
$$

A sufficient condition for concavity is that $z(y)$ is concave, which is the case if and only if $\bar{G} > -\frac{y^3 \tau''(y)}{2}$. This last condition is satisfied if the tax function $\tau(y)$ is convex or not

too concave.[46]

We conclude that for any monotonic continuous function $\tau(y) \in [0, +\infty)$ for $y \in (0, +\infty)$, with either $\tau'(y) < 0$ for all $y \in (0, +\infty)$, or $\tau'(y) > 0$ for all $y \in (0, +\infty)$, or $\tau'(y) = 0$ for all $y \in (0, +\infty)$, which is convex or not too concave, the equation $H(y) = \bar{H}$ has exactly two solutions in $y \in (\underline{y}, \bar{y})$. Furthermore, differentiating $H(y)$, one can show that the lowest solution (i.e. the low output steady state y_l) is characterized by $1 + \varepsilon_z(y_l) > \frac{1}{\alpha}$ while the high output steady state $y_h = 1$ satisfies $1 + \varepsilon_z(y_h) < \frac{1}{\alpha}$. See (31). As $H(\underline{y}) = H(\bar{y}) = 0 < \bar{H}$, we conclude that $H(y)$ is single-peaked.

When $\bar{G} = 0$, so that $z(y) = 1 - \tau(y)$, if $\tau'(y) < 0$, it is easy to see that the previous results still apply. However, when $\tau'(y) > 0$, we have $z'(y) < 0$, $\underline{y} = 0$ and $z(0) = 1$. The LHS of the equality $Q(y) = \bar{H}y^{1-\alpha}$ is now strictly decreasing from 1 to 0. As the RHS strictly increases from 0 to $\bar{M}$, these two functions cross only once. Therefore, we have steady state unicity. Note also that we also have steady state unicity when $\bar{G} = 0$ and $\tau'(y) = 0$. In this case the LHS of the equality $Q(y) = \bar{H}y^{1-\alpha}$ is constant and the RHS strictly increases from 0 to $\bar{M}$, so that again these two functions cross only once. $\square$

9.3 Public Debt

Here we allow for the existence of a stock of public debt at the time of entry of the balanced budget rule, $D > 0$, which obviously remained constant thereafter. This introduces a new component into government expenditures: interest payments on public debt. If D corresponds to perpetual debt, i.e. to fixed-income securities with interest rates and payments constant overtime, the balanced budget rule becomes $\bar{G} + g(y_t) + r^*D = (1 - z_t)y_t$, where r^* is the interest rate that was fixed when the debt was issued.[47] Therefore we have

$$z_t = z(y_t) \equiv 1 - \frac{(\bar{G} + r^*D)}{y_t} - \tau(y_t) \tag{45}$$

---

[46]Note that this is only a sufficient condition, which means that two steady states may be obtained even when the degree of concavity of $\tau(y)$ is high. For example, for the functional form considered in Lloyd-Braga et al. (2008), $\tau(y) = \mu y_t^\phi$, we always have two steady states in the presence of any form of countercyclical tax rates.

[47]This constitutes a reasonable assumption as a large part of public debt is in the form of medium and long term fixed-income securities. Moreover this share has been increasing in recent years. For example Chen et al. (2018) report that the overall weighted average maturity for advanced economies issuances during 1995 to 2014 was 8.1 years and that for emerging markets and developing economies was 8.5 years. Furthermore in advanced economies the median short-term debt ratio declined from 22 percent in 1995 to 14 percent in 2011, while in emerging markets and developing economies it peaked in 1997 at 29 percent, dropping almost continuously to 4 percent in 2013.

i.e., the consideration of public debt merely increases the size and changes our definition of incompressible government expenditures, that now also include fixed interest rate payments. If public debt is totally held by foreigners, there will be no other changes in the model and in our results. However, if residents hold public debt, these interest payments enter the budget constraint of households so that (7) becomes

$$\dot{k}_t + c_t = z(y_t)[w_t l_t + r_t k_t] + r^* D - \delta k_t. \tag{46}$$

It is easy to check that at the steady state (23) to (25) still hold and that (26) becomes

$$c = z(y)y + r^* D - \delta k$$

where $z(y) \equiv 1 - \frac{(\overline{G} + r^* D)}{y} - \tau(y)$. Therefore a steady state satisfies the following system of equations:[48]

$$k = \frac{s z(y) y}{(\rho + \delta)}$$

$$c = \frac{[\rho + (1-s)\delta] z(y) y}{(\rho + \delta)} + r^* D$$

$$l = \frac{B(1-s)(\rho + \delta) z(y) y}{[\rho + (1-s)\delta] z(y) y + (\rho + \delta) r^* D} \tag{47}$$

$$H(y) = \bar{H}$$

$$with\ H(y) = \frac{(z(y) y)^{\alpha + \beta}}{y \{[\rho + (1-s)\delta] z(y) y + (\rho + \delta) r^* D\}^{\beta}}$$

$$\bar{H} = \left(\frac{s}{(\rho + \delta)} \right)^{-\alpha} [B(1-s)(\rho + \delta)]^{-\beta}$$

$$and\ z(y) \equiv 1 - \frac{(\overline{G} + r^* D)}{y} - \tau(y) > 0$$

Steady state existence and multiplicity are determined by the solutions of $H(y) = \bar{H}$.[49] To study steady state multiplicity, we must characterize the sign of $\varepsilon_H(y) \equiv \frac{H'(y) y}{H(y)}$. From (47) we have that

$$\varepsilon_H(y) = (\alpha + \beta h(y))(1 + \varepsilon_z(y)) - 1 \tag{48}$$

$$where\ h(y) \equiv \frac{r^* D(\rho + \delta)}{[\rho + (1-s)\delta] z(y) y + r^* D(\rho + \delta)}$$

$$and\ \varepsilon_z(y_t) \equiv \frac{z'(y_t) y_t}{z(y_t)} = \frac{(\overline{G} + r^* D) - \phi(y_t) \tau(y_t) y_t}{y_t - (\overline{G} + r^* D) - \tau(y_t) y_t}. \tag{49}$$

---

[48] Note that we restrict $y \in (\underline{y}, \bar{y})$ with $\underline{y} > 0$ and $\bar{y} \in (\underline{y}, +\infty)$ to ensure that $z(y) > 0$.

[49] Again we use the scaling parameter $B > 0$ to ensure the existence of a normalized steady state (NSS), $y = 1$.

Comparing (48) with (31),[50] we conclude that now, in contrast with our previous results, even if $\varepsilon_z(y) < 0$ for all $y \in (\underline{y}, \bar{y})$, we can not guarantee the existence of only one steady. This happens because $\alpha + \beta h(y) > \alpha$, the term that now multiplies $(1 + \varepsilon_z(y))$ in (48), will be greater than one for some values of y, implying that the function $H(y)$ may not be always decreasing for all $y \in (\underline{y}, \bar{y})$ when $\varepsilon_z(y) < 0$. Moreover, from (49) it is easy to see that, when $D > 0$, it is no longer true that $\varepsilon_z(y) < 0$ for all $y \in (\underline{y}, \bar{y})$ when $\overline{G} = 0$ and $\phi(y) \equiv \tau'(y)y/y \geq 0$, so that steady state multiplicity becomes possible when $\overline{G} = 0$ and $\phi(y) \equiv \tau'(y)y/y \geq 0$. We conclude that the consideration of a positive stock of public debt held by residents renders the emergence of steady state multiplicity more likely, which reinforces our global indeterminacy argument.

9.4   Matrix J

$$
J = \begin{pmatrix} \frac{(\rho+\delta)\beta(1+\varepsilon_z)\frac{k}{y} - \alpha\left[(1+\varepsilon_z)z - \frac{\partial c(k,y)}{\partial y}\right]}{\beta(1+\varepsilon_z)-1} & \frac{-(\rho+\delta)\beta + \alpha\left[\delta + \frac{\partial c(k,y)}{\partial k}\right]}{\beta(1+\varepsilon_z)-1} \\[2em] (1+\varepsilon_z)z - \frac{\partial c(k,y)}{\partial y} & -\left(\delta + \frac{\partial c(k,y)}{\partial k}\right) \end{pmatrix}
$$

9.5   Derivation of the phase diagram

Consider equations (35). The $k-$nullcline satisfies $f_1(k_t, y_t) = 0$. The implicit solution $k = k_1(y)$ of the above relationship also satisfies the following relation (along the nullcline):

$$
\frac{dk_1/k_1}{dy/y} = \frac{[\beta(1 + \varepsilon_z(y))\delta k + c(k,y)]}{[\beta \delta k + \alpha]} > 0 \tag{50}
$$

Furthermore, we find that $f_1(0, y) > 0$, $f_1(+\infty, y) < 0$. It follows that for a fixed y, $\dot{k} > 0$ for any given $k \in (0, k_1)$. See figures 3 and 4 where the we depict the $k-$ nullcline and the arrows that represent the vector field.

The $y-$nullcline satisfies $f_2(k, y) = 0$. Let us first show that the relation between k and y derived from this expression, $k = k_2(y)$, may be multi-valued, i.e. for a fixed $y \in (\underline{y}, \bar{y})$, we may have zero, one or two values of k satisfying $f_2(k, y) = 0$. Note that for a fixed $y \in (\underline{y}, \bar{y})$, we have $\widetilde{f}_2(0) \equiv f_2(0, y) < 0$ and $\widetilde{f}_2(+\infty) \equiv f_2(+\infty, y) < 0$, since $c(k, y)$ is a concave function in k while the last term is linear in k. We also have:

$$
\frac{\partial f_2(k, y)}{\partial k} = \frac{s^2}{(1-s)} \frac{c(k,y)}{k} - [(1-s)\rho + \delta(1-2s)].
$$

---

[50]Remark that when $D = 0$, so that $h(y) = 0$, (48) collapses into (31).

Moreover $\frac{c(k,y)}{k} = (1-s)Bz(y_t)y_t^{\frac{\beta-1}{\beta}}k_t^{\frac{\alpha-\beta}{\beta}}$ is strictly decreasing in k. Then, for a fixed $y \in (\underline{y}, \bar{y})$, there is a critical value $\tilde{k}(y)$ such that $\frac{\partial f_2(k,y)}{\partial k} > (<)0$ if $k < (>)\tilde{k}(y)$, i.e. the function $\tilde{f_2}(k)$ is first increasing and them decreasing in k. This implies that, for a given value of y, $\tilde{f_2}(k) = 0$ may have zero, one or two solutions in k satisfying $f_2(k,y) = 0$. It follows that $k_2(y)$ is a two-valued function with an upper (lower) solution satisfying $\frac{\partial f_2(k,y)}{\partial k} < (> 0)$. Notice now that, as $\frac{c}{k}$ evaluated at any steady state is independent of y, see (28), the sign of this derivative is identical for both steady states and is given by:

$$s[\rho + \delta(1-s)] - (1-s)[(1-s)\rho + \delta(1-2s)] \tag{51}$$

This means that both steady states are either on the upper or the lower solution $k_2(y)$. To simplify the exposition and without loss of generality, we will assume for the rest of this section that this expression is negative[51] which implies that both steady states are on the upper branch of $k_2(y)$.

We can now study the shape of the $y-$nullcline. We have that:

$$\begin{aligned} \frac{dk_2/k_2}{dy/y} &= \frac{sc(k,y) - \beta(1 + \varepsilon_z(y))[\rho(1-s) + \delta(1-2s)]k}{\alpha\left[sc(k,y) - \frac{(1-s)}{s}[\rho(1-s) + \delta(1-2s)]k\right]} \\ &= \frac{s^2 z(y)y - [\beta(1 + \varepsilon_z(y)) - 1][\rho(1-s) + \delta(1-2s)]k}{\alpha\left\{s^2 z(y)y - \frac{(1-2s)}{s}[\rho(1-s) + \delta(1-2s)]k\right\}} \end{aligned} \tag{52}$$

It is easy to see that for $k = 0$, i.e., at the intersection between the $y-$nullcline and the horizontal axis, the slope of the $y-$nullcline is equal to $\frac{1}{\alpha} > 1$. Also, from (52), we can see that the slope of the $y-$nullcline will change sign at most two times and that the numerator is positive on the right-hand side of the singularity point y^s. Furthermore, when evaluated at a steady state, the slope of the nullcline is given by:

$$\frac{[s[\rho + \delta(1-s)] - (1-s)[(1-s)\rho + \delta(1-2s)]\alpha(1 + \varepsilon_z(y_j))]}{\alpha[s[\rho + \delta(1-s)] - (1-s)[(1-s)\rho + \delta(1-2s)]]}, j = h, l \tag{53}$$

where by assumption the denominator is negative, see (51), so that the two steady states are located on the upper branch of the nullcline. Therefore, when the NSS is a saddle, i.e. located at the RHS of y^s, the slope of the $y-$nullcline is negative. This implies that the $y-$nullcline admits a maximum at a point $y^* < y^s$, as it must change sign before the singularity. We still need to characterize the slope around the lower steady state and around the upper steady state when it is a sink. Around the lower (upper) steady state, we have $(1 + \varepsilon_z(yl)) > (<)\frac{1}{\alpha}$. Since by assumption $\frac{s[\rho+\delta(1-s]}{(1-s)[(1-s)\rho+\delta(1-2s)]} < 1$, it follows that $\frac{dk}{dy} > 0$ around the lower steady state. In contrast, the sign of the numerator is left undetermined for the upper steady state when it is a sink i.e. it can be located on the

---

[51]For a standard parameterization $(\rho, \delta) = (0.01, 0.025)$, this is satisfied fo any $s \in (0, 0.39)$.

increasing or decreasing part of the upper-solution of $k_2(y)$. Obviously, on the lower branch of the y-nullcline, the derivative has an opposite sign. As a result, the y-nullcline is first increasing in y and then decreases until $\frac{s^2}{(1-s)}c(k,y) = (1-s)\rho + \delta(1-2s)]k$. It bends therefore backwards and goes back to the origin (without attaining it).

We now determine the directions of the arrows that represent the vector field of $\dot{y}_t$. Remember that our model exhibits a singularity when $g(y^s) = \beta(1+\varepsilon_z(y^s)) - 1 = 0$, which in the space (y,k), defines a vertical line $y = y_s$. Of course, on different sides of the vertical line $y = y_s$ horizontal arrows point in opposite directions. Now consider a point (y_1, k_1) on the LHS of y_s and above the $y-$ nullcline. As we know that $\frac{\partial f_2(k,y)}{\partial k} = (1+\gamma)\{s^3 z(y)y - [\rho(1-s) + \delta(1-2s)](1-2s)k\}$ we know that moving from y_1 on the zero motion line, i.e. on $f_2(y_1, k) = 0$, vertically to (y_1, k_1), $f_2(y, k)$ is decreasing. Hence $\dot{y} > 0$ at (y_1, k_1), changing sign whenever, for the same k_1, we cross the $y-$nullcline or the $y = y_s$ line. The same reasoning applies to any fixed k on the LHS of y_s and above the $y-$nullcline. It follows that for any k on the LHS of y_s but below the the $y-$nullcline $f_2(y, k)$ is decreasing, i.e., $\dot{y} < 0$, changing again sign whenever, for the same k, we cross the $y-$nullcline or the $y = y_s$ line. See figures 3 and 4.

It is also easy to show that when $k = 0$ the $y-$nullcline is located on the right of the $k-$nullcline as depicted in figures 3 and 4. Indeed, although $\lim_{k\to 0} c(k,y) = 0$, it is easy to see that k will tend to zero faster than $C(k,y)$. Therefore, rewriting $f_1(k_t, y_t) = 0$ *and* $f_2(k_t, y_t) = 0$ *respectively as:*

$$z(y_t)y_t = c(k_t, y_t) + \delta k_t$$
$$z(y_t)y_t = \frac{c(k_t, y_t)}{s} - \frac{[\rho(1-s) + \delta(1-2s)]k(t)}{s^2}$$

when $k \to 0$, on the $\dot{k} = 0$ nullcline we have that $z(y)y = \lim_{k\to 0} c(k,y)$, while on the $\dot{y} = 0$ nullcline $z(y)y = \lim_{k\to 0}\frac{c(k,y)}{s} > \lim_{k\to 0} c(k,y)$. As $z(y)y$ is an increasing function of y, on the horizontal axis, the $y-$nullcline starts on the right hand side of the $k-$nullcline.

9.6 Obtaining the stable manifolds in the Markov switching rational expectation equilibrium with two saddles

We first assume p and q arbitrarily small (i.e. $p = q = 0$). Then, using the reverse shooting method, we obtain two numerical solution paths for our deterministic non linear dynamic system, converging respectively to the high and the low output steady states. We then apply non linear polynomial fitting procedures to obtain the two stable arms $y^j(k_t)$ for $j = h, l$.

We are now ready to simulate our regime switching economy. Indeed, for a given capital, observed output is obtained by randomizing across both stable-arms using the sunspot variables S_t:

$$y_t(k_t, S_t) = S_t y^h(k_t) + (1 - S_t) y^l(k_t) \tag{54}$$

where $S_t = (1 - p)S_{t-1} + q(1 - S_{t-1})$. The realization of observed output in turn determines which saddle-path we are in. Hence, substituting $y_t(k_t, S_t)$ in (40) the next equilibrium value of the capital stock conditional on the sunspot, $k_{t+1}(S_t)$ is determined. From there, we compute the next values of both $y_{t+1}^h(k_{t+1})$ and $y_{t+1}^l(k_{t+1})$ and given a realization of S_{t+1}, obtain a new observed output $y_{t+1}(k_{t+1}, S_{t+1})$ and so on. $\qquad\square$

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