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Application of Whitney Elements in Current Density Reconstruction of Electric Arcs in Low-voltage Circuit Breakers

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Abstract. This paper presents the formulation of the inverse problem of current density reconstruction in low-voltage circuit breakers. Considering the free-divergence condition for the current density, the inverse problem can be solved in Whitney space W^2 with the divergence condition as a constraint or solved in Whitney space W^1 where the free-divergence condition naturally holds. Tikhonov regularization is used to address the ill-posedness of the inverse problem.

Keywords: Electric arc, magnetostatics, Tikhonov regularization, Whitney elements.

INTRODUCTION

Current interruption in traditional circuit breakers is characterized by electric arcs. Current density distribution in electric arcs is an essential parameter that determines the distribution of heat flux and temperature, the erosion ratio of electrodes, and the interaction of arcs with surroundings. A better knowledge of it helps to understand the complex physical behaviour of arc plasmas. However, no practical non-intrusive methods can be found to get the current density distribution. This work focuses on a non-invasive diagnostic technique able to reconstruct the arc current density from its magnetic field by solving a magnetostatic inverse problem. A key point for this approach is how to guarantee that the divergence of the reconstructed current is zero. To this scope, we use Whitney elements in the formulation of the inverse problem.

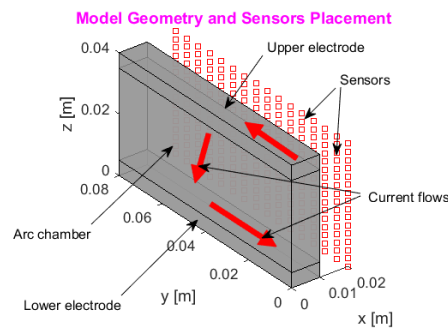


Figure 1: Model geometry and setup of the magnetic sensor array.

FORWARD AND INVERSE PROBLEM FORMULATIONS

A simplified arc chamber with two parallel electrodes is considered, as in Fig 1 [1]. A planar sensor array is used to perform the magnetic measurements. We assume magnetostatic conditions over the whole domain Ω , with no ferromagnetic regions included. The shape function spaces are defined using Whitney elements, where W^0 , W^1 , W^2 , and W^3 are the nodal, edge, face, and volume function spaces, respectively [2]. Representing the current density \mathbf{J} in W^2 , the forward problem reads $[B_m] = [L][I]$, where $[B_m]$ is the vector of magnetic measurements, $[I]$

is the vector of degrees of freedom in W^2 , $[L]$ is the matrix of discrete Biot-Savart operator whose elements corresponding to the i -th sensor and j -th face function w_{f_j} are given by

$$(1) \quad [L]_{ij} = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\mathbf{w}_{f_j}(\mathbf{r}_s) \times (\mathbf{r}_i - \mathbf{r}_s)}{|\mathbf{r}_i - \mathbf{r}_s|^3} d\Omega,$$

where \mathbf{r}_i is the position of i -th sensor, and \mathbf{r}_s is the integration point.

In a magneto-quasi-static approximation, current density is divergence free, i.e., $\text{div } \mathbf{J} = 0$, or in the matrix form $[D][I] = \mathbf{0}$, where $[D]$ is the faces-to-volumes incidence matrix. To address the ill-posedness of the inverse problem, we use the Tikhonov regularization to get a least-squares solution by solving for $[I]$ with the free-divergence condition as a constraint

$$(2) \quad \min_{[I]} \left\{ \|[L][I] - [B_m]\|_2^2 + \lambda^2 \|[I]\|_2^2 \right\}, \quad \text{s.t. } [D][I] = \mathbf{0}.$$

Alternatively, we can introduce the current vector potential \mathbf{T} with $\mathbf{J} = \text{curl } \mathbf{T}$, where \mathbf{T} can be appropriately represented in W^1 . We denote the vector of degrees of freedom in W^1 as $[u]$ for \mathbf{T} , so that $[I] = [C][u]$, where $[C]$ is the edges-to-faces incidence matrix. We solve for $[u]$ where the free-divergence condition naturally holds

$$(3) \quad \min_{[u]} \left\{ \|[L][C][u] - [B_m]\|_2^2 + \lambda^2 \|[u]\|_2^2 \right\}.$$

RESULTS AND CONCLUSIONS

The proposed approaches are numerically tested with 256 simulated magnetic field values of a 16-by-16 sensor array. The forward problem is solved with φ - A formulation using a commercial FEM software [3] avoiding the inverse crime. As can be noted in Fig. 2, both proposed approaches give rise to a good appropriate reconstruction.

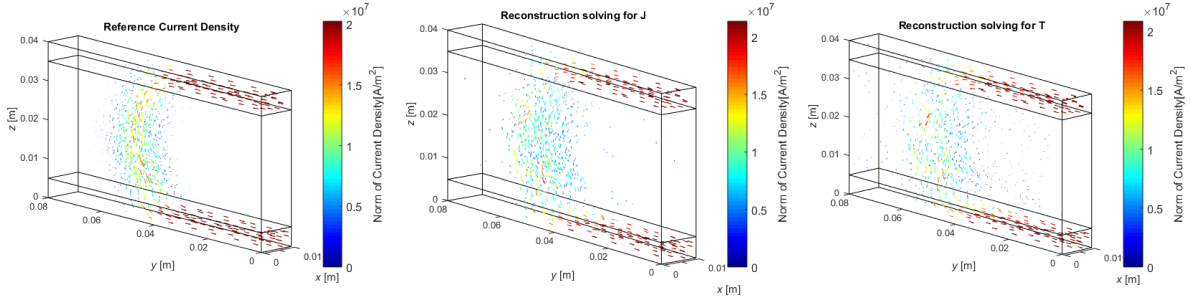


Figure 2: Numerical results: reference current density (left), reconstruction of \mathbf{J} by solving for $[I]$ (middle), and \mathbf{J} by solving for $[u]$ (right).

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