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# OPTIMAL OPERATION OF BUILDING MICROGRIDS - COMPARISON FOR MIXED-INTERGER LINEAR AND CONTINUOUS NON-LINEAR

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**Abstract.** This paper presents two mathematical models for the load demand to the Energy Management (EM) problem of a Micro-Grid (MG), by means of deterministic Mixed Integer Linear Programming (MILP) and Non-Linear Programming (NLP) approaches. A general architecture of a microgrids is proposed, involving Energy Storage Systems (ESS), Distributed Generation (DG) and a thermal reduced-model of the grid-connected dwelling. It focuses on the modelling process and the optimization performances for both approaches regarding optimal operation of near zero energy buildings related to electric microgrid within a time horizon of 24 hours.

**Keywords.** optimal operation, zero energy buildings, micro-grid, MILP, SQP

## 1. Introduction

### 1.1 Background and Motivation

Building operation is an important topic in order to contribute to consumptions reduction and energy grid interaction. Demand response in electricity market has already been addressed by the research community [1]. According to local constraints of energy delivery, the optimal control of grid-connected and standalone nearly/net zero energy buildings must be addressed [2]. In this paper, we are investigating the optimal operation of the smart building regarding weather forecast, local renewable energy production and grid prices. The Energy Management Problem (EMP), *i.e.* optimization algorithm associated with predictive models and criteria, may have different nature. We will focus on the influence of linear or non-linear formulation and optimization performances.

### 1.2 Literature Review

#### 1.2.1 Microgrid Energy Management

The demand response and demand side management concepts are a trend that is currently in progress in the modern electric energy industry [1, 3–5]. The controllable loads might reduce fossil fuel consumption, load peak shaving, as well as postpone investments in new transmission and distribution lines if successfully implemented. Also, in the modern electric energy industry, the microgrids (MGs) are emerging as an additional element to maintain the growth and sustainability [6]. Microgrid's EMP, also known as scheduling problem, aims to minimize, the operation costs of DERs, as well as the power exchange with the main grid [6, 7]. In the paper, we choose a centralized approach to ensure global optimality and a smart management of electrical and thermal storages.

#### 1.2.2 Thermal Load Management

Demand side management (DSM) in the electricity grid usually involves energy efficiency and demand response (DR). Numerous studies has been devoted to promote load shifting, efficient energy technologies or even energy awareness. In [8], Palensky et al. propose an overview of DSM, including Demand Response (DR). For thermal demand side management, several studies reach significant savings using multi-agent based [9] or integrated centralized approaches coupled with thermal storage [10, 11]. In microgrids' literature, thermal demand side management is not usually introduced in the electrical modelling. As a new approach, we consider the smart-building as a MG unit, subjected to comfort and air quality criteria. A Direct Load Control on the forced-air heating system of the Smart-Building is considered as the thermal mass of the building could be used for storage.

### 40 1.2.3 MILP vs. NLP formulation

41 The difference between linear and non-linear formulation is related to the nature of constraints and objec-  
 42 tive functions. In the case of MILP formulations, mixed continuous-discrete are usually artificially intro-  
 43 duced in the modelling stage to overcome non-linearities. In addition, it often introduces approximations  
 44 and may leads to different modelling, which make the comparison difficult [12]. This approximation stage  
 45 is tedious and error prone. One possible solution is to automate this transformation from a physical model  
 46 into MILP suited model using Model Driven Engineering (MDE). It usually leads to different nature of  
 47 decision variables which are kept continues in the case of non-linear and are mixed continuous-discrete in  
 48 the case of MILP formulations. One of the main techniques are based on discretization, piecewise approx-  
 49 imation or Taylor series expansions in order to be compatible with MILP algorithm. The main drawbacks  
 50 of NLP are the continuous definition set and convergence properties, indeed, the global convergence is  
 51 guarantee only for convex cases (convex objective function and inequality constraints).

### 52 1.3 Contribution

53 Novel contributions of this work include:

- 54 • Introduction and formulation of the Thermostatically Controlled Loads (TCL) within the microgrid.  
 55 The resulting Linear Program handles the mutli-objective trade-off between discomfort and cost of  
 56 use taking into account air quality criterion.
- 57 • Linearisation and formulation of the ventilation system behaviour. Which is generally non-linear  
 58 and non-convex equality constraints, involving air quality model, heat transfer, and ventilation  
 59 power.
- 60 • Comparison of both MILP and NLP methods on a general use case which provides a solution that  
 61 can be interpreted for implementation.

### 62 1.4 Paper Outline

63 This paper is organized as follows. Section 2. describes the general optimization problem and presented  
 64 both, NLP and MILP, formulations. It describes the models and the linearisation techniques for each  
 65 involved physic (e.g. electrical, thermal and ventilation). Section 3. provides a computational example  
 66 where the building structure and electrical units are described. Results' analysis are performed in section  
 67 4. Finally, section 5. summarizes keys results.

## 68 2. General Formulation and Linearisation

69 In the paper, the EM optimization is related to the physical and economical framework in which the  
 70 MG is inserted. The smart building involves a thermal envelope, an ideal heating system and a general  
 71 ventilation system (with or without heat recovery). It is connected to the MG which consists of a battery  
 72 pack, photovoltaic panels and a connection to the main grid. The system is subjected to forecasts such  
 73 as occupancy, solar irradiation, external temperature and energy prices.

74 This section is organized by field: electric, thermal and ventilation models will be detailed. For each  
 75 physic, both NLP and MILP are formulation and differences will be highlighted. Finally, the full EMP  
 76 will be presented in section 2.4.

### 77 2.1 Electrical

#### 78 2.1.1 Main grid connection

In the paper, the MG can buy or sell energy from or to the main grid. Buying and selling time-dependent  
 costs are considered. Lets note,  $p_{in}$ ,  $p_{out}$  the input and output powers and the related instantaneous  
 costs  $c_{in}$ ,  $c_{out} \in \mathbb{R}$ . The main grid can be formulated as follow:

$$\begin{aligned}
 (1) \quad & p_{in}(t), p_{out}(t) \in \mathbb{R}^+ && p_{mg}(t) \in \mathbb{R} \\
 (2) \quad & p_{mg}(t) = p_{in}(t) - p_{out}(t) \\
 (3) \quad & J_{mg} = \sum_t [c_{in}(t) \cdot p_{in}(t) + c_{out}(t) \cdot p_{out}(t)] \cdot \delta t
 \end{aligned}$$

If the instantaneous cost function is convex with respect to the power  $p_{mg}$ , *i.e.* when  $c_{in}(t) \geq -c_{out}(t)$ ,  
 selling and buying in the same time is impossible since the cost is to be minimize. Otherwise, to ensure  
 not selling and buying energy in the same time, one can introduce a binary variable and additional

constraints such as

$$\begin{aligned}
 (4) \quad & u_{mg} \in \{0 ; 1\} & p_{mg}^{max}, p_{mg}^{min} \in \mathbb{R} \\
 (5) \quad & p_{in}(t) - u_{mg}(t) \cdot p_{mg}^{max} \leq 0 \\
 (6) \quad & p_{out}(t) + u_{mg}(t) \cdot p_{mg}^{min} \leq p_{mg}^{min}
 \end{aligned}$$

### 2.1.2 Battery modelling

For an energetic context, the battery can be modelled by a constant charging and discharging maximal power with respect to the state of charge (SOC) [13,14]. If we consider charging and discharging efficiency  $\eta_c$  and  $\eta_d$ , one should consider a binary formulation of the following form:

$$\begin{aligned}
 (7) \quad & e(t) \geq e_{min} \in \mathbb{R}^+ & e(t) \leq e_{max} \\
 (8) \quad & e(0) = e_0 & e(t_f) = e_f \\
 (9) \quad & p_c, p_d \in \mathbb{R}^+ & u_b \in \{0 ; 1\} \\
 (10) \quad & p_c(t) - u_b(t) \cdot p_c^{max} \leq 0 & p_c^{max} \in \mathbb{R}^+ \\
 (11) \quad & p_d(t) + u_b(t) \cdot p_d^{max} \leq p_d^{max} & p_d^{max} \in \mathbb{R}^+ \\
 (12) \quad & \frac{\partial e(t)}{\partial t} = p_c(t) \cdot \eta_c - \frac{p_d(t)}{\eta_d}
 \end{aligned}$$

This model is drastically simplified considering ideal efficiencies, and eqs. (9)–(12) can be replaced by the linear and continuous eqs. (13) and (14).

$$\begin{aligned}
 (13) \quad & p_{bat}(t) \geq -p_d^{max} & p_{bat}(t) \leq p_c^{max} \\
 (14) \quad & \frac{\partial e(t)}{\partial t} = p_{bat}(t)
 \end{aligned}$$

In MG literature, we usually consider a battery reserve in order to prevent a possible disconnection of the main grid. For simplicity, and because the non-linear formulation does not consider it, we will not include it. It is also possible to add battery cost, depending on the charging and discharging powers or on SOC.

### 2.1.3 Generalities

The electrical sub-optimization problem can be seen as a classical unit commitment problem, where loads, DES and the grid connexion are the main components involved. In the MILP case, the unit commitment problem can be written as follows :

$$\begin{aligned}
 (15) \quad & p_{pv}(t) + p_{mg}(t) + p_{de}(t) = p_{ex}(t) + p_h(t) + p_{bat}(t) + \tilde{p}_v(t) + p_e(t) \\
 (16) \quad & p_h(t) \in [0 ; p_h^{max}] & p_{de}(t) \in \mathbb{R}^+ & p_{ex}(t) \in \mathbb{R}^+
 \end{aligned}$$

where  $p_h$  is the heating electrical power,  $p_{pv}$  is the forecast power of the PV panels,  $\tilde{p}_v$  is the linear approximation of the ventilation power, and  $p_e$  is the electrical load demand.

Two positive decision variables are added to ensure the feasibility of eq. (15) in every configurations, the deficit power  $p_{de}$  and the excess power  $p_{ex}$ . Those variables are also included in the electric objective function, eq. (17) to penalise excess and deficit power. The full electrical objective is thus describes as:

$$(17) \quad J_{elec} = J_{mg} + \sum_t [M_{de} \cdot p_{de}(t) + m_{ex} \cdot p_{ex}(t)] \cdot \delta t$$

The positive weight are usually chosen such as  $M_{de} \gg c_{in}$  and  $m_{ex} \approx 0$ .

Note that the continuous formulation shows some limits when it comes to different selling/buying costs or charging/discharging efficiency in eqs. (3) and (12).

## 2.2 Thermal

### 2.2.1 Thermal envelop

Both formulation, NLP and MILP are based on a linear model of the building structure and derives from a resistance-capacity network. It is usually described by a linear state-space system [15, 16]. It consist on a set of equality constraint to ensure heat flow and energy conservation. In the general case, a  $n \in \mathbb{N}$  thermal-zone building involves  $n$  states  $T_j(t)$  and  $n$  ideal heating inputs  $p_h^j(t)$ . We note  $p_h(t) = \sum_j p_h^j(t)$  the total instantaneous heating power, where  $j \in \{0, \dots, n\}$  denotes the thermal zone index. The thermal model also considers heat gain, such as occupancy, solar or electrical gains.

A general the temperature variation within the thermal-zones can be model by the following state-space system.

$$(18) \quad \dot{T}(t) = A.T(t) + B.U(t)$$

where  $A$  and  $B$  are fixed matrix, taking into account, resistance, capacity values and thermal network topology.  $U(t)$  usually represents boundary conditions, such as a fixed external temperature, and the indoor heat gains and forced-air heater power.

### 2.2.2 Comfort assumptions

Comfort modelling is usually non-linear and discrete, between winter and summer period or between days and night. The simple incomfort cost expression has the same form than the electricity cost. In general, it can be formulated as follows:

$$(19) \quad J_{th} = \sum_t c_i(t).y(t).\delta t$$

$$(20) \quad y(t) \in S$$

where  $c(t)$  and  $y(t)$  represent a time-dependent discomfort cost and the discomfort variable.  $S$  describes a bounded convex set.

As an example, for discomfort modelling in winter period, one could constraint the discomfort to be positive for temperature under the reference temperature profile  $T_r^j(t)$ , where  $j$  denotes the thermal zone index, and null above it as follows:

$$(21) \quad y^j(t) \in \mathbb{R}^+$$

$$(22) \quad y^j(t) \geq T_r^j(t) - T_{int}^j(t)$$

$$(23) \quad J_{th} = \sum_t \sum_j c_i(t).y^j(t).\delta t$$

## 2.3 Ventilation and Air Quality

In the paper, we consider ventilation system of the building. It is actually the key point of the NLP/MILP comparison and has an important impact on both thermal and air-quality phenomenons. As we consider a trade off objective function between cost-of-use and thermal comfort, ventilation control will also have a important impact on the electrical grid and thus, plays a central role in the system. Moreover, it presents non-linearity and non-convexity.

### 2.3.1 Linearisation of the ventilation power

For the non-linear continuous formulation, the ventilation power  $p_v$  is identified as a second order polynomial, eq. (24).

$$(24) \quad p_v(t) = f(Q_v) = \alpha_2.Q_v(t)^2 + \alpha_1.Q_v(t) + \alpha_0$$

where  $Q_v$  si the ventilation air flow ( $m^3.h^{-1}$ ), and  $\alpha_0, \alpha_1 \in \mathbb{R}$  and  $\alpha_2 \in \mathbb{R}^+$  are constant values. Constructor or experimental data could be easily used to identify such behaviour. We propose here a piecewise linear approximation for the MILP formulation.

It is important to note that  $p_v$  is directly linked to the term  $p_{mg}$  by eq. (15). In other words, minimizing  $p_v$  is always equivalent to minimize the global objective function. Then, a linear approximation of  $p_v(t)$ , noted  $\tilde{p}_v(t)$  can be done by adding a set of inequalities.

$$(25) \quad n_{d2} \in \mathbb{N} \qquad k \in \{0 ; 1 ; \dots ; n_{d2} - 1\}$$

$$(26) \quad \tilde{p}_v(t) \leq \nu_1^k \cdot Q_v(t) + \nu_0^k, \quad \forall k \qquad \tilde{p}_v \in \mathbb{R} \qquad Q_v \in \mathbb{R}$$

where the coefficients  $\nu_1^k$  and  $\nu_0^k$  are computed as follow :

$$(27) \quad \nu_1^k = f'(C^k) \qquad \nu_0^k = f(C^k)$$

136 The term  $C^k$  represents a set of linearisation points. This is a really light formulation. Note that it is  
 137 possible with all non-linear convex function, differentiable on the set  $C^k$ . Eq. (26) can be improved in  
 138 particular case to minimize the mean error of this approximation, but a general formulation is difficult.  
 139 For non-convex function, one could use a SOS2 formulation as in eq. (31).

### 140 2.3.2 Approximation of the products $x.y$

141 In this paragraph, we propose a general linear approximation of the product  $z = x.y$ , where  $x, y \in \mathbb{R}$  are  
 142 continuous optimization variables. First we decouple the two variables by setting the substitution as in  
 143 eq. (29).

$$(28) \quad x, y, z \in [0 ; 1]$$

$$(29) \quad a = \frac{1}{2}(x + y) \qquad b = \frac{1}{2}(x - y)$$

144 Note that  $z = x.y = a^2 - b^2$ . This formulation is still good for other decision variables limits. In order to  
 145 lower the matrix range, and so the convergence quality, one could normalize the definition range of the  
 146 variables.

147 For both terms,  $a^2$  and  $b^2$  we define a special ordered set of type 2 (SOS2) using the real valued decision  
 148 variables  $w_a^i$  and  $w_b^i$  and the quantity break points  $A^i$  and  $B^i$ . For clarity, we explicit a binary formulation  
 149 of the SOS2 for the linear approximation of the term  $a^2$  in eqs. (30) and (31).

$$(30) \quad n_d \in \mathbb{N} \qquad i \in \{0 ; \dots ; n_{d1}\} \qquad j \in \{0 ; \dots ; n_{d1} - 1\}$$

$$(31) \quad \left\{ \begin{array}{l} u_a^j, \in \{0 ; 1\} \\ w_a^i, \in [0 ; 1] \\ \sum_j u_a^j = 1 \\ u_a^j \leq w_a^j + w_a^{j+1} \\ a = \sum_i w_a^i \cdot A^i \end{array} \right.$$

150 The piecewise approximation of both terms  $a^2$  and  $b^2$  is then straight forward knowing the weight  $w_a^i$ ,  
 151  $w_b^i$ .

$$(32) \quad a^2 \approx \tilde{z}_a = w_a^i \cdot (A^i)^2 \qquad b^2 \approx \tilde{z}_b = w_b^i \cdot (B^i)^2$$

Finally, approximation of  $z$ , noted  $\tilde{z}$ , can be express as the difference between both approximations  $\tilde{z}_a$   
 and  $\tilde{z}_b$  as:

$$(33) \quad z \approx \tilde{z} = \tilde{z}_a - \tilde{z}_b$$

### 152 2.3.3 Linearisation of heat flow

153 Thermal behaviour of ventilation system can vary from on technology to another. However, it is generally  
 154 modelled as a non-linear heat flow input, with respect to the air flow  $Q_v$ , the inner and outer temperatures  
 155  $T_{ext}$ ,  $T_{int}$ , an exchanger efficiency (for heat recovering ventilation) and air properties. The NL expressing  
 156 of  $q_v(t)$  is described in eq. (34).

$$(34) \quad q_v(t) = \rho \cdot c_p \cdot (1 - \eta_v) \cdot Q_v(t) \cdot (T_{int}(t) - T_{ext}(t))$$

157 The product  $Q_v(t) \cdot T_{int}(t)$  is approximated using the method explained in 2.3.2. This linearisation leads  
158 to the introduction of the approximated heat flow noted  $\tilde{q}_v$ .

#### 159 2.3.4 The air quality model

160 Control of ventilation is usually a trade-off between air quality constraints and thermal comfort. Thus,  
161 an air quality model is needed. In this article, we only consider  $CO_2$  concentration in air quality model.  
162 Thermal discomfort is to be minimized whereas air quality, *i.e.*  $CO_2$  concentration is constrained to a low  
163 level since the link between air quality and comfort is not straight forward, see eq. (35). The non-linear  
164 conservation equation of the gas takes into account, air flow rate  $Q_v$  and the occupancy  $N_p$ , see eq. (37).

$$(35) \quad C_{co2}(t) \in [C_{co2}^{air}; C_{co2}^{max}]$$

$$(36) \quad Q_v(t) \in [Q_v^{min}; Q_v^{max}]$$

$$(37) \quad V \cdot \frac{\partial C_{co2}(t)}{\partial t} = N_p(t) \cdot Q_p \cdot C_{co2}^p - Q_v(t) \cdot (C_{co2}(t) - C_{co2}^{air})$$

165 where  $V$ ,  $Q_p$ ,  $C_{co2}^p$  and  $C_{co2}^{air}$  denote respectively the volume of the room, the expired air flow by occupant,  
166 the mean expired  $CO_2$  concentration by occupant and the outside  $CO_2$  concentration. The product  
167  $Q_v(t) \cdot C_{co2}(t)$  in eq. (37) is linearised using the method explained in 2.3.2 and leads to the linear approx-  
168 imation of the  $CO_2$  concentration noted  $\widetilde{C}_{co2}(t)$ .

### 169 2.4 Full Optimization Problem

170 Both continuous non-linear, and mixed integer linear formulations consider the same objective function,  
171 *i.e.* no approximation or linearisation are needed. It sums the electricity cost and the thermal discomfort  
172 cost (*c.f.* eqs. (17) and (19)).

#### 173 2.4.1 Non-linear formulation

174 The continuous non-linear formulation includes all the constraints related to the main-grid, batteries, the  
175 thermal envelop and the ventilation system (*c.f.* eqs. (1)–(3), (7), (8), (13)–(20), (24) and (34)–(37)).

#### 176 2.4.2 Mixed-Integer Linear Formulation

177 The mixed-integer linear programming includes all the constraints related to the main-grid, batteries, the  
178 thermal envelop and the piecewise approximations of the ventilation system (*c.f.* eqs. (1)–(12), (15)–(20)  
179 and (25)–(27)). Note that approximations are not explicitly develop every time for consistency matters.  
180 Piecewise linear approximations of  $p_v$ ,  $q_v$  and  $C_{co2}$  are all based on the method developed in section 2.3.2  
181 and eqs. (28)–(32).

## 182 3. Computational Experiment

183 In this section we propose a simple test case for an actual comparison between NLP and MILP formu-  
184 lations. We thus propose a fixed architecture of the microgrid, a thermal envelop, common boundary  
185 conditions and sizing.

186 For the MILP approach, simulation of the system and EM optimization are solved in the same optimisa-  
187 tion loop, using Gurobi<sup>1</sup> and a step time  $dt_L$ . In the other hand, simulation is considered as an internal  
188 loop in the EM optimisation for the NLP formulation. As a result, two time steps are used: one for the  
189 simulation (usually 10min) and one for the EM optimization,  $dt_{NL} = 1h$ .

### 190 3.1 Boundary conditions

191 The boundary condition consist of fixed time-dependent profiles :

- 192 • weather forecasts *i.e.* the outdoor temperature and the radiance which correspond to a winter day  
193 in Europe,
- 194 • the electricity price, which is assumed to be piecewise constant (see fig. 2a),

<sup>1</sup><http://www.gurobi.com>

- the PV power generation, based on weather forecasts,
- occupancy profiles, *i.e.* presence, electricity use, thermal reference and  $CO_2$  concentration upper limit ( $CO_2^{max} = 1000 \text{ ppm}$ ).

### 3.2 MG Configuration

The electrical configuration of the microgrid involves a connexion to the main grid without selling option (*i.e.*  $c_{out} = 0$ ),  $24m^2$  PV panel, one battery pack with a capacity of  $10kWh$  and the heating and ventilation system. Initial values and parameters' description can be found in table 3.

### 3.3 Single-zone thermal envelop

In this numerical experimentation, the thermal envelop involves one zone and is modelled by a RC network of the 5th order. It incorporates 4 walls, each described by one capacity and two resistance, one inner capacity  $C_{air}$ , two resistances representing infiltrations and ventilation and one heat source representing the heating power, solar, electric and occupancy inputs, noted  $p_h, p_e, p_{sol}$  and  $p_{occ}$ . The identification of the parameters has been done using EnergyPlus as a reference [17]. Network topology and identified values are respectively available in fig. 1 and table 1.

Capacity	Value ( $\times 10^6$ )	Resistance	Value
$C_{air}$	3.7473	$R_1$	$1.0400 \times 10^{-2}$
$C_1$	3.8832	$R_2$	$7.8945 \times 10^{-3}$
$C_2$	2.3333	$R_3$	$4.1700 \times 10^{-3}$
$C_3$	3.0388	$R_4$	$1.7195 \times 10^{-1}$
$C_4$	8.1109	$R_5$	$1.6600 \times 10^{-1}$
		$R_6$	$6.7131 \times 10^{-2}$
		$R_7$	$1.3500 \times 10^{-2}$
		$R_8$	$1.2774 \times 10^{-1}$
		$R_9$	$2.2000 \times 10^{-3}$

Table 1. Capacities and thermal resistances values identified using EnergyPlus

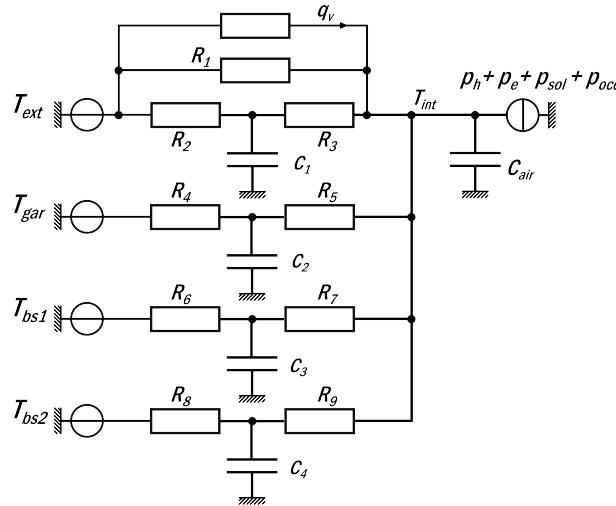


Figure 1. Thermal Envelop

## 4. Results

This section consists of a comparison of both NLP, MILP approaches applied to the use case presented in 3.. It includes a modelling and optimization performances comparison. Main figures can be found in table 2 and the following paragraphs will refer to it.

### 4.1 Modelling comparison

NLP formulation is developed within the CADES framework <sup>2</sup>, providing automatic differentiation of the model. This ensure an easy modelling stage using a combination of SML, *i.e.* the CADES language and

<sup>2</sup><http://www.vesta-system.fr/fr/produits/cades/veda-cades.html>



**Table 2. Performances comparison**

formulation	time step	lines of code	continuous/binary variables	nbr. constraints / matrix size*	computation time (s)	elec. cost (€)
NLP	(10 min, 1 h)	547	72/0	72	2.00	5.212
MILP	1 h	3 859	554/111	660/665/2256	0.73	5.456
	30 min	7 607	1 083/231	1358/1314/4615	8.91	4.876
	20 min	11 441	1 617/346	2032/1963/6912	43.71	4.737

\*nbr. of rows/nbr. of columns/nbr. of nonzeros

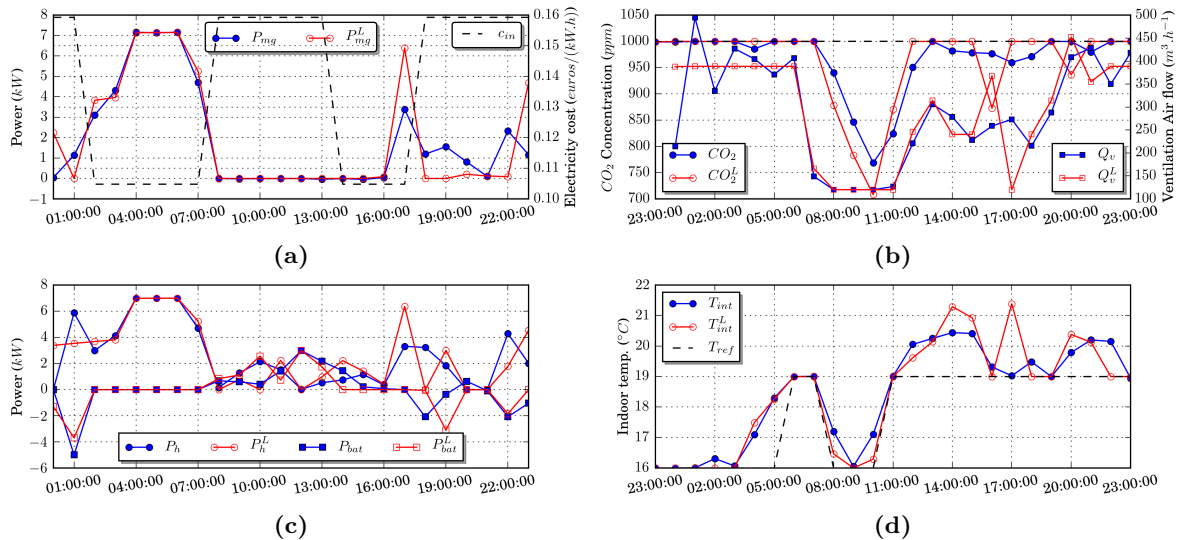
216 C++ functions. About 547 lines of codes are dedicated to the modelling (the solver is not included) for  
 217 about 72 optimization variables and 72 constraints, see table 2. Indeed, dynamic constraints, *e.g.* the  
 218 thermal state system eq. (18), are solved during an internal loop and are not considered by the global  
 219 NLP solver.

220 For a one-hour time step, the MILP formulation consists of 3 859 lines in a LP language, which can  
 221 be drastically minimized using a high level modelling language, such as AMPL, GAMS or Pyomo. It  
 222 includes 660 constraints, 554 continuous and 111 binary variables, see table 2.

## 223 4.2 Optimization results and performances

224 This paragraph aims to compare optimization results, *i.e.* convergence proprieties and speed. Computation  
 225 time and optimal electrical costs are detailed in table 2. NLP is soled using a sequential quadratic  
 226 programming (SQP) solver [18] within the CADES framework, whereas the MILP is solved using bunch  
 227 and bound and dual simplex algorithms within Gurobi.

228 Most of time-dependant quantities are presented in fig. 2. Results are plotted for a discomfort cost related  
 229 to the cost of electricity where  $\frac{c_i(t)}{c_{in}(t)} = 0.8$ , and leads to a null discomfort cost for each case, see fig. 2d.



**Figure 2.** Optimal energy management results - a comparative study between NLP and MILP (noted  $X^L$ ) formulations. (a) Main grid power and buying price, (b)  $CO_2$  concentration (ppm) and ventilation air-flow ( $m^3 \cdot h^{-1}$ ), (c) heating and battery pack power (kW) and (d) Indoor and reference temperatures ( $^{\circ}C$ ).

230 Note that several optimal solutions are likely because of the multiple control variables, thermal and  
 231 electrical storage capacities. One can see the global agreement between both results, especially from 23h  
 232 to 15h. The indoor temperature profiles gently follows the references, taking into account dynamics of  
 233 the model. Between 14 and 15h, thermal storage of the building is used during the low-cost period. We  
 234 then note small differences between 16h and 17h for the air quality and ventilation control (see fig. 2b)  
 235 and at 17h for the thermal control (see figs. 2b and 2c). Those differences could be explains, by several  
 236 optimal solutions or by small differences between NLP formulation and linear approximations.

237 In terms of convergence quality, we note a better accuracy for the MILP formulation. Between 8 and  
 238 15h,  $P_{mg}^L$  is lower than  $1 \times 10^{-12}$ , whereas  $P_{mg}$  vary between  $-40$  and  $-3W$  which is irrelevant in this  
 239 context. Nevertheless, optimal electrical costs are coherent and tend to show that both formulations are  
 240 relevant for the EM optimization.

241 Computation time are about the same for one hour time step, with an advantage for the MILP formulation

242 as expected. However, Results for smaller time-step tends to show some scalability issues due to the high  
 243 number of binary variables (table 2).

## 244 5. Conclusion and Perspective

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### 250 Notations

251 Notation, short description and units of all quantities are available in table 3.

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Not.	Description	Unit
$\circ p_{in}$	MG input power	kW
$\circ p_{out}$	MG output power	kW
$\Delta c_{in}$	input instantaneous cost	€.h <sup>-1</sup>
$\Delta c_{out}$	input instantaneous cost	€.h <sup>-1</sup>
$\circ p_{mg}$	MG power	kW
$\square p_{mg}^{max}$	maximal MG power	25 kW
$\square p_{mg}^{min}$	minimal MG power	25 kW
$\circ J_{mg}$	total cost of the MG	€
$\diamond u_{mg}$	on/off binary variable	-
$\circ e$	battery SOC	kW.h
$\square e_{min}$	minimal SOC	0.5 kW.h
$\square e_{max}$	maximal SOC	10 kW.h
$\square e_0$	initial SOC	5 kW.h
$\square e_f$	final SOC	50 kW.h
$\circ p_{bat}$	battery power	kW
$\bullet p_c$	battery charging power	kW
$\square p_c^{max}$	maximal charging power	5 kW
$\bullet p_d$	battery discharging power	kW
$\square p_d^{max}$	maximal discharging power	5 kW
$\diamond u_b$	on/off binary variable	-
$\Delta p_{pv}$	photovoltaic generated power	kW
$\circ p_{de}$	deficit power	kW
$\circ p_{ex}$	excess power	kW
$\bullet p_h$	heating power	kW
$\square p_h^{max}$	maximal heating power	7 kW
$\circ p_v$	ventilation electric power	kW
$\circ \tilde{p}_v$	approximation of $p_v$	kW
$\Delta p_e$	elec. load demand	kW
$\Delta p_{pv}$	PV generation	kW
$\circ s^+$	positive discomfort	°C
$\circ s^-$	negative discomfort	°C
$\circ q_v$	ventilation heat flow	m <sup>3</sup> .s <sup>-1</sup>
$\Delta T_r$	internal reference	°C
$\square \eta_v$	heat recovering efficiency	0 -
$\square rho$	air density	1.225 kg.m <sup>-3</sup>
$\circ T_{int}$	internal temperature	°C
$\square c_p$	air heat capacity	2.775e <sup>-3</sup> kW.h/kg/K
$\Delta c_i^+$	positive deviation cost	€.h <sup>-1</sup>
$\Delta c_i^-$	negative deviation cost	€.h <sup>-1</sup>
$\circ J_{th}$	thermal discomfort cost	€
$\square Q_p$	respiration air flow	4e <sup>4</sup> m <sup>3</sup> .h <sup>-1</sup>
$\circ Q_v$	ventilation air flow	m <sup>3</sup> .h <sup>-1</sup>
$\square Q_v^{min}$	min. ventilation air flow	120 m <sup>3</sup> .h <sup>-1</sup>
$\square Q_v^{max}$	max. ventilation air flow	1000 m <sup>3</sup> .h <sup>-1</sup>
$\circ C_{co2}$	CO <sub>2</sub> concentration	ppm
$\square C_{co2}^{max}$	max. CO <sub>2</sub> concentration	1e <sup>4</sup> ppm
$\square C_{co2}^{air}$	external CO <sub>2</sub> concentration	400 ppm
$\square C_{co2}^p$	expired CO <sub>2</sub> concentration	4e <sup>5</sup> ppm
$\Delta N_p$	occupancy	-
$\square V$	volume of the room	650.45 m <sup>3</sup>

$\circ$  : optimisation variable,  $\square$  : fixed scalar,  $\Delta$  : given input profile,  $\bullet$  : decision variable

Table 3. Notations

- 289       problem of microgrids,” *Electric Power Systems Research*, vol. 122, pp. 19–28, 2015.
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