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Abstract	<p><i>Rule-based argumentation systems</i> are developed for reasoning about defeasible information. As a major feature, their logical language distinguishes between <i>strict rules</i> (encoding strict information) and <i>defeasible rules</i> (describing general behavior with exceptional cases). They build <i>arguments</i> by chaining such rules, define <i>attacks</i> between them, use a <i>semantics</i> for evaluating the arguments and finally identify the <i>plausible conclusions</i> that follow from the rules. Focusing on the family of inconsistency-based attack relations, this paper presents the first study of the outcomes of such systems under various acceptability semantics, namely naive, stable, semi-stable, preferred, grounded and ideal. It starts by extending the existing list of <i>rationality postulates</i> that any rule-based system should satisfy. Then, it defines the key notion of <i>option</i> of a <i>theory</i> (a theory being a set of facts, a set of strict rules and a set of defeasible rules). For each of the cited semantics, it characterizes the extensions of a rule-based system that satisfies all the</p>	

postulates in terms of options of the theory under which the system is built. It also fully characterizes the set of plausible conclusions of the system. The results show that designing a rule-based argumentation system requires great care.

Keywords (separated by '-') Defeasible reasoning - Rule-based systems - Argumentation

Footnote Information

A formal characterization of the outcomes of rule-based argumentation systems

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Abstract Rule-based argumentation systems are developed for reasoning about defeasible information. As a major feature, their logical language distinguishes between *strict rules* (encoding strict information) and *defeasible rules* (describing general behavior with exceptional cases). They build *arguments* by chaining such rules, define *attacks* between them, use a *semantics* for evaluating the arguments and finally identify the *plausible conclusions* that follow from the rules. Focusing on the family of inconsistency-based attack relations, this paper presents the first study of the outcomes of such systems under various acceptability semantics, namely naive, stable, semi-stable, preferred, grounded and ideal. It starts by extending the existing list of *rationality postulates* that any rule-based system should satisfy. Then, it defines the key notion of *option* of a *theory* (a theory being a set of facts, a set of strict rules and a set of defeasible rules). For each of the cited semantics, it characterizes the extensions of a rule-based system that satisfies all the postulates in terms of options of the theory under which the system is built. It also fully characterizes the set of plausible conclusions of the system. The results show that designing a rule-based argumentation system requires great care.

Keywords Defeasible reasoning · Rule-based systems · Argumentation

1 Introduction

Argumentation is a promising approach for reasoning about inconsistent information. It consists of generating *arguments* that support claims, defining *attacks* between them, evaluating

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the arguments using a given *semantics* and finally identifying the plausible claims on the basis of the strength of their arguments.

Dung proposed in [1] various semantics at an abstract level, i.e., without taking into account the structure of arguments or the nature of attacks. His abstract framework was instantiated by several scholars. The idea is as follows: Start with a *knowledge base* whose elements are encoded in a logical language, generate arguments using the consequence operator attached to the language, identify the attacks and apply Dung's semantics for the evaluation task. There were two major categories of instantiations for this abstract framework. The first category uses *deductive logics* (such as propositional logic [2,3] or any Tarskian logic [4]), whereas the second category uses *rule-based languages*. These languages distinguish between:

- *facts* which are information about particular instances like “My laptop is heavy.”
- *strict rules* which encode general laws about classes of instances like “First generation laptops are heavy.” Such rules do not have exceptions.
- *defeasible rules* which describe general behavior with exceptional cases. Defeasible rules correspond thus to what is called *defaults* in [5] or *conditional assertions* in [6,7].

Examples of rule-based argumentation systems are: ASPIC [8], its extended version ASPIC+ [9], DELP [10] and the systems developed in [11–15]. Despite the popularity of these systems, the results they return have not been characterized yet, except the system discussed in [11]. The following questions are thus still open:

- what are the underpinnings of the extensions under various semantics?
- do the semantics return different results as at the abstract level?
- what is the number of extensions a system may have?
- what are the plausible conclusions with such systems?

In this paper, we answer all the four questions in three steps: We start by defining a rule-based argumentation system over a knowledge base called *theory* (a set of facts, a set of strict rules and a set of defeasible rules). The system uses a notion of *derivation schema* for generating arguments from the theory. For the sake of generality, the attack relation is left *unspecified*. However, it has the property of being *conflict-dependent*, that is, it captures the inconsistency that may be present in the theory. It is worth mentioning that all existing attack relations (like rebuttal and assumption attack) are conflict-dependent. A notable exception is undercutting which aims at blocking the application of defeasible rules [16].

In a second step, we extend the list of postulates (consistency, closure under strict rules) proposed in [17]. The aim of those postulates is to mathematically capture what humans perceive as rationale behavior from the semantics of defeasible theories. They are thus desirable properties that a system should satisfy. We introduce three new postulates. The first one, *strict precedence*, ensures that any claim that follows from the strict part of a theory is a plausible conclusion of the argumentation system. The second postulate, *exhaustiveness*, ensures a form of completeness of the extensions of an argumentation system. The third postulate, *closure under sub-arguments*, states that an argument cannot be accepted if one of its sub-parts is questionable.

Finally, we investigate the outputs of rule-based argumentation systems that satisfy all the postulates. We show that naive extensions return *options* of the theory (an *option* being a sub-theory that gathers a maximal-up to consistency-set of the facts, strict rules, and defeasible rules). Furthermore, the set of plausible conclusions under the naive semantics contains all the conclusions that are drawn from all the options. Stable extensions return *preferred options* but not necessarily all of them; it depends on the attack relation at work. Unlike options, preferred options are options that contain the strict part. Should not all preferred

67 options be picked as stable extensions, defining an attack relation that discards exactly the
 68 spurious ones thus turns out to be tricky. The same results hold under preferred semantics.
 69 We also characterize both ideal and grounded extensions. We show that the ideal extension of
 70 an argumentation system is the set of arguments built from the free part of a theory (i.e., the
 71 sub-theory that contains the strict part as well as all defeasible rules that are not involved in
 72 any minimal conflict). The grounded extension is a subset of the ideal extension. This means
 73 that under grounded semantics, an argumentation system may miss intuitive conclusions.

74 The paper is structured as follows: Section 2 recalls Dung’s semantics. Section 3 introduces
 75 the logical language that will be used in the paper. Section 4 defines rule-based argumen-
 76 tation systems. Section 5 introduces a list of postulates that such systems should satisfy.
 77 Section 6 studies the outcomes of rule-based systems under naive, stable, semi-stable, pre-
 78 ferred, grounded and ideal semantics. Section 7 discusses how our results may apply to
 79 existing systems, and the last section concludes.

81 **2 Abstract argumentation framework**

82 An *argumentation framework* consists of a set of *arguments* and a binary relation express-
 83 ing *attacks* among the arguments. Throughout this section, the structure and the origin of
 84 arguments are left unspecified.

85 **Definition 1** (*Argumentation framework*) An *argumentation framework* is a pair $\mathcal{H} =$
 86 $(\mathcal{A}, \mathcal{R})$ where \mathcal{A} is a non-empty (possibly infinite) set of arguments and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is
 87 an attack relation. A pair $(a, b) \in \mathcal{R}$ means that a attacks b . A set $\mathcal{E} \subseteq \mathcal{A}$ attacks an argument
 88 b iff $\exists a \in \mathcal{E}$ such that $(a, b) \in \mathcal{R}$.

89 **Notation** We sometimes use the infix notation $a\mathcal{R}b$ to denote $(a, b) \in \mathcal{R}$.

90 An argumentation framework $(\mathcal{A}, \mathcal{R})$ is represented as a graph, *argumentation graph*,
 91 whose nodes are the arguments of \mathcal{A} and its edges are the attacks in \mathcal{R} . Arguments are
 92 evaluated using a *semantics*, i.e., a set of criteria that should be satisfied by an argument
 93 in order to be *acceptable*. Throughout this paper, we focus on extension-based semantics
 94 initially introduced by Dung [1]. Such semantics look for acceptable sets of arguments,
 95 called *extensions*. Each extension represents a coherent point of view and satisfies two basic
 96 properties: *conflict-freeness* and *defense*.

97 **Definition 2** (*Conflict-freeness, defense, admissibility*) Let $\mathcal{H} = (\mathcal{A}, \mathcal{R})$ be an argumentation
 98 framework and $\mathcal{E} \subseteq \mathcal{A}$.

- 99 • \mathcal{E} is *conflict-free* iff $\nexists a, b \in \mathcal{E}$ such that $(a, b) \in \mathcal{R}$.
- 100 • \mathcal{E} *defends* an argument a iff $\forall b \in \mathcal{A}$, if $(b, a) \in \mathcal{R}$, then \mathcal{E} attacks b .
- 101 • \mathcal{E} is an *admissible* set iff \mathcal{E} is conflict-free and defends all its elements.

102 The following definition recalls the main semantics that were proposed in [1, 18, 19]. It is
 103 worth noticing that all those semantics are based on the notion of admissibility.

104 **Definition 3** (*Semantics*) Let $\mathcal{H} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework, and $\mathcal{E} \subseteq \mathcal{A}$ be
 105 a conflict-free set.

- 106 • \mathcal{E} is a *naive* extension iff it is a maximal (w.r.t. \subseteq) conflict-free set.
- 107 • \mathcal{E} is a *complete* extension iff \mathcal{E} is an admissible set that contains any argument it defends.

- 108 • \mathcal{E} is a *preferred* extension iff \mathcal{E} is a maximal (w.r.t. \subseteq) complete extension.
- 109 • \mathcal{E} is a *stable* extension iff \mathcal{E} attacks any argument in $\mathcal{A} \setminus \mathcal{E}$.
- 110 • \mathcal{E} is a *semi-stable extension* iff \mathcal{E} is a complete extension and the union of the set \mathcal{E} and
- 111 the set of all arguments attacked by \mathcal{E} is maximal (w.r.t. \subseteq).
- 112 • \mathcal{E} is a *grounded extension* iff \mathcal{E} is a minimal (w.r.t. \subseteq) complete extension.
- 113 • \mathcal{E} is an *ideal extension* iff \mathcal{E} is a maximal (w.r.t. \subseteq) admissible set contained in every
- 114 preferred extension.

115 An argumentation framework has a single grounded (respectively ideal) extension. How-
 116 ever, it may have several naive, admissible, complete, preferred, stable and semi-stable
 117 extensions. It may also have zero stable extensions.

118 **Notations** Let $\mathcal{H} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. We denote by $\text{Ext}_x(\mathcal{H})$ the set
 119 of all extensions of \mathcal{H} under semantics $x \in \{n, p, s, ss\}$, where n (respectively
 120 p, s, ss) stands for naive (respectively preferred, stable and semi-stable). We
 121 denote by $\text{GE}(\mathcal{H})$ (respectively $\text{IE}(\mathcal{H})$) the single grounded (respectively ideal)
 122 extension of \mathcal{H} . When we do not need to refer to a particular semantics, we write
 123 $\text{Ext}(\mathcal{H})$ for short.

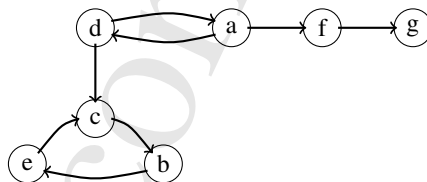
124 The following result recalls some key properties of these semantics.

125 **Property 1** [1, 18, 19] Let $\mathcal{H} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework.

- 126 • $\text{Ext}_s(\mathcal{H}) \subseteq \text{Ext}_n(\mathcal{H})$
- 127 • $\text{Ext}_s(\mathcal{H}) \subseteq \text{Ext}_p(\mathcal{H})$
- 128 • If $|\text{Ext}_s(\mathcal{H})| > 0$, then $\text{Ext}_s(\mathcal{H}) = \text{Ext}_{ss}(\mathcal{H})$
- 129 • \mathcal{H} has one grounded (respectively ideal) extension
- 130 • $\text{GE}(\mathcal{H}) \subseteq \text{IE}(\mathcal{H})$

131 When $\text{Ext}_s(\mathcal{H}) = \text{Ext}_p(\mathcal{H})$, the framework \mathcal{H} is said to be *coherent*. It is also worth
 132 recalling that an argumentation framework that has an infinite set of arguments may have an
 133 infinite number of extensions (under multiple-extensions semantics).

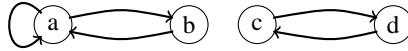
134 Let us now illustrate the different semantics on the argumentation framework \mathcal{H}_1 depicted
 135 below.



136
 137 This framework has eight naive extensions:

- 138 • $\mathcal{E}_1 = \{a, c, g\}$,
- 139 • $\mathcal{E}_2 = \{d, e, f\}$,
- 140 • $\mathcal{E}_3 = \{b, d, f\}$,
- 141 • $\mathcal{E}_4 = \{a, e, g\}$,
- 142 • $\mathcal{E}_5 = \{a, b, g\}$,
- 143 • $\mathcal{E}_6 = \{b, e, g\}$,
- 144 • $\mathcal{E}_7 = \{b, d, g\}$, and
- 145 • $\mathcal{E}_8 = \{c, f\}$.

146 \mathcal{H}_1 has one stable/semi-stable extension \mathcal{E}_3 and two preferred extensions: \mathcal{E}_3 and $\mathcal{E}_6 = \{a, g\}$.
 147 Its grounded and ideal extensions are empty ($\text{GE}(\mathcal{H}_1) = \text{IE}(\mathcal{H}_1) = \emptyset$).
 148 Consider now the following argumentation framework \mathcal{H}_2 borrowed from [19]. It lays
 149 bare some differences between ideal and grounded semantics.



150

151 It can be checked that:

- 152 • $\text{GE}(\mathcal{H}_2) = \emptyset$,
- 153 • $\text{Ext}_p(\mathcal{H}_2) = \{\{b, c\}, \{b, d\}\}$, and
- 154 • $\text{IE}(\mathcal{H}_2) = \{b\}$.

155 Throughout the paper, we will refer to the seven semantics of Definition 3 by the *reviewed*
 156 semantics, and by extension-based semantics to any semantics, which partitions the power
 157 set of the set of arguments into two parts: *extensions* and *non-extensions*. Note that there
 158 are other semantics in the literature like recursive [20] and stage [21] that follow this line of
 159 research. This distinction is important since some of the results in the next sections hold for
 160 any extension-based semantics, while others hold under the reviewed ones.

161 3 Rule-based logical language

162 In what follows, \mathcal{L} is a set of *literals*, i.e., atoms or negation of atoms. The negation of an
 163 atom x from \mathcal{L} is denoted $\neg x$. We consider two additional constants \top and σ such that $\top \notin \mathcal{L}$
 164 and $\sigma \notin \mathcal{L}$. Three kinds of information are distinguished:

- 165 • *Facts*, which are elements of $\mathcal{L} \cup \{\top\}$
- 166 • *Strict rules*, which are of the form $x_1, \dots, x_n \rightarrow x$ (x, x_1, \dots, x_n denoting literals in \mathcal{L})
- 167 • *Defeasible rules*, which are of the form $x_1, \dots, x_n \Rightarrow x$ or of the form $\top \Rightarrow x$
 168 (x, x_1, \dots, x_n denoting literals in \mathcal{L})

169 Facts are information about particular instances. A strict rule expresses general information
 170 that has no exception. It is read as follows: If x_1, \dots, x_n hold, then x always holds. A
 171 defeasible rule of the form $x_1, \dots, x_n \Rightarrow x$ expresses general information that may have
 172 exceptions and is read as follows: If x_1, \dots, x_n hold, then generally x holds as well. A
 173 defeasible rule of the form $\top \Rightarrow x$ expresses that x is a *defeasible fact* and is read as follows:
 174 generally x holds. Unlike existing systems like ASPIC [8] where a strict rule with an empty
 175 body represents a fact, in our formalism we keep general information and factual information
 176 separate.

177 Let \mathcal{L}' be a set of *atoms* used for naming rules with the constraints $\mathcal{L} \cap \mathcal{L}' = \emptyset$, $\top \notin \mathcal{L}'$ and
 178 $\sigma \notin \mathcal{L}'$. Every rule has a unique name and two rules cannot have the same name. Throughout
 179 the paper, rules are named r, r_1, r_2, \dots

180 **Definition 4 (Theory)** A *theory* is a triple $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ where $\mathcal{F} = \{\top\} \cup X$, with $X \subseteq \mathcal{L}$,
 181 is a set of facts, and $\mathcal{S} \subseteq \mathcal{L}'$ (respectively $\mathcal{D} \subseteq \mathcal{L}'$) is a set of strict (respectively defeasible)
 182 rules' names. \mathcal{T} is *finite* iff all three sets \mathcal{F}, \mathcal{S} and \mathcal{D} are finite.

183 Note that \top is a fact in any theory. Note also that the two sets \mathcal{S} and \mathcal{D} contain names of
 184 rules and not the corresponding rules themselves. Throughout the paper, $(\mathcal{F}, \mathcal{S}, \emptyset)$ is referred
 185 to as the *strict part* of a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$.

Notations Let $r \in \mathcal{L}'$, the function $\text{Rule}(r)$ returns the (strict or defeasible) rule whose name is r . For each rule $x_1, \dots, x_n \rightarrow x$ (respectively $x_1, \dots, x_n \Rightarrow x$ or $\top \Rightarrow x$) whose name is r , the *head* of the rule is $\text{Head}(r) = x$ and the *body* of the rule is $\text{Body}(r) = \{x_1, \dots, x_n\}$ or $\text{Body}(r) = \{\top\}$. Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ and $\mathcal{T}' = (\mathcal{F}', \mathcal{S}', \mathcal{D}')$ be two theories. We say that \mathcal{T} is a *sub-theory* of \mathcal{T}' , written $\mathcal{T} \sqsubseteq \mathcal{T}'$, iff $\mathcal{F} \subseteq \mathcal{F}'$ and $\mathcal{S} \subseteq \mathcal{S}'$ and $\mathcal{D} \subseteq \mathcal{D}'$. The relation \sqsubseteq is the strict version of \sqsubseteq (i.e., it is the case that at least one of the three inclusions is strict).

The notion of consistency is defined as follows:

Definition 5 (Consistency) A set $X \subseteq \mathcal{L}$ is *consistent* iff $\nexists x, y \in X$ such that $x = \neg y$. It is *inconsistent* otherwise.

This simple definition of consistency is sufficient since the language \mathcal{L} contains only literals. However, it is not suitable in case of richer languages. Assume that \mathcal{L} is a propositional language. Thus, the set $\{x, y, \neg x \vee \neg y\}$ is consistent with respect to the above definition while it is clearly not the case. Thus, richer languages require a stronger definition of consistency like the one proposed in [22].

Without loss of generality, throughout the paper we make the three following assumptions about rules.

Assumptions The body of every (strict/defeasible) rule is finite and not empty. Moreover, for each rule r , $\text{Body}(r) \cup \{\text{Head}(r)\}$ is consistent. We say that r is consistent.

Note that the fact that rules are consistent does not ensure the consistency of a set of rules. [23] discussed different forms of rule consistency. One of them is illustrated by the example $\{x \Rightarrow y, y \Rightarrow \neg x\}$ where both defeasible rules are consistent whereas together lead to an inconsistent rule $x \Rightarrow \neg x$.

Let us now show how new information (i.e., literal) is produced from a given theory. This is generally the case when (strict and/or defeasible) rules are fired in a *derivation schema*. Below we provide a definition which generalizes derivations as defined by [10, 24] and others.

Definition 6 (Derivation schema) Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory, $x \in \mathcal{L}$. A *derivation schema* for x from \mathcal{T} is a finite sequence $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ such that:

- $x_n = x$
- for $i \in \{1, \dots, n\}$,
 - $x_i \in \mathcal{F}$ and $r_i = \sigma$, or
 - $x_i = \text{Head}(r_i)$, with $r_i \in \mathcal{S} \cup \mathcal{D}$ and $\text{Body}(r_i) \subseteq \{x_1, \dots, x_{i-1}\}$

$\text{Seq}(d) = \{x_1, \dots, x_n\}$.

$\text{Facts}(d) = \{x_i \mid i \in \{1, \dots, n\}, r_i = \sigma\}$.

$\text{Strict}(d) = \{r_i \mid i \in \{1, \dots, n\}, r_i \in \mathcal{S}\}$.

$\text{Def}(d) = \{r_i \mid i \in \{1, \dots, n\}, r_i \in \mathcal{D}\}$.

In order to improve readability, we somehow abuse the notation: we use the rules themselves instead of their names.

Example 1 Consider the theory \mathcal{T}_1 such that $\mathcal{F}_1, \mathcal{S}_1, \mathcal{D}_1$ are as follows.

$$\mathcal{F}_1 \begin{cases} p \\ q \end{cases} \quad \mathcal{S}_1 \begin{cases} p \rightarrow s & (r_1) \\ q \rightarrow \neg s & (r_2) \\ p, s \rightarrow u & (r_3) \end{cases} \quad \mathcal{D}_1 \begin{cases} \neg s \Rightarrow t & (r_4) \\ t, u \Rightarrow v & (r_5) \\ p \Rightarrow q & (r_6) \end{cases}$$

- 226 Each of (1)–(7) below is a derivation schema from theory \mathcal{T}_1
- 227
$$\langle (p, \sigma) \rangle \tag{1}$$
- 228
$$\langle (q, \sigma), (\neg s, r_2) \rangle \tag{2}$$
- 229
$$\langle (p, \sigma), (s, r_1), (u, r_3) \rangle \tag{3}$$
- 230
$$\langle (p, \sigma), (s, r_1), (p, \sigma), (u, r_3) \rangle \tag{4}$$
- 231
$$\langle (p, \sigma), (q, \sigma), (s, r_1), (u, r_3) \rangle \tag{5}$$
- 232
$$\langle (p, \sigma), (q, r_6), (\neg s, r_2) \rangle \tag{6}$$
- 233
$$\langle (p, \sigma), (q, \sigma), (\neg s, r_2), (s, r_1), (u, r_3), (t, r_4), (v, r_5) \rangle \tag{7}$$

234 A derivation schema is not necessarily consistent (such as (7) above), as it may contain
 235 opposite literals in the form $x_i = \neg x_j$ for some i and j . (A derivation d is consistent
 236 iff $\text{Seq}(d)$ is consistent.) Moreover, a derivation schema is not necessarily minimal (for set
 237 inclusion) as shown in Example 1: compare (3) with (4). The former is a proper sub-sequence
 238 of the latter.

239 **Definition 7** (*Minimal derivation schema*) Let \mathcal{T} be a theory and $x \in \mathcal{L}$. A derivation schema
 240 for x from \mathcal{T} is *minimal* iff none of its proper subsequences is a derivation schema for x from
 241 \mathcal{T} .

242 Interestingly enough, there are two ways for a derivation schema not to be minimal for
 243 set inclusion: (i) involving superfluous literals, i.e., literals that do not serve toward inferring
 244 the conclusion as is illustrated by (5) in Example 1 (q is of no use there), (ii) involving
 245 redundancy (hence, repeated literals) as illustrated by (4) in Example 1 (p is repeated twice).

246 **Definition 8** (*Focused derivation schema*) Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory and $x \in \mathcal{L}$. A
 247 derivation schema $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ for x from \mathcal{T} is *focused* iff it can be reduced
 248 to a minimal one by just deleting repeated pairs (x_i, r_i) .

249 **Property 2** Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory and $x \in \mathcal{L}$. A derivation schema $d =$
 250 $\langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ for x from \mathcal{T} is minimal iff d is focused and the literals x_1, \dots, x_n
 251 are pairwise distinct.

252 **Notations** For a theory \mathcal{T} , $\text{CN}(\mathcal{T})$ denotes the set of all literals that have a derivation schema
 253 from \mathcal{T} . We call $\text{CN}(\mathcal{T})$ the *potential consequences* drawn from \mathcal{T} (for short,
 254 *consequences*) but they need not be definitive as they may happen to be dismissed
 255 by opposite conclusions.

256 The following property applies to the consequences drawn from a given theory.

257 **Property 3** Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory.

- 258 • $\text{CN}(\mathcal{T}) \subseteq \mathcal{F} \cup \{\text{Head}(r) \mid r \in \mathcal{S} \cup \mathcal{D}\} \subseteq \mathcal{L}$
- 259 • If \mathcal{T} is finite, then $\text{CN}(\mathcal{T})$ is finite
- 260 • $\mathcal{F} \subseteq \text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{CN}(\mathcal{T})$
- 261 • $\top \in \text{CN}(\mathcal{T})$
- 262 • $\text{CN}(\mathcal{T}) = \{\top\}$ iff $\mathcal{F} = \{\top\}$ and $\nexists r \in \mathcal{D}$ such that $\text{Body}(r) = \{\top\}$.
- 263 • If d is a derivation schema from \mathcal{T} , $\text{Seq}(d) \subseteq \text{CN}(\mathcal{T})$

264 Some rules may not be *activated* (i.e., the literals in their body have no derivation schema).
 265 Let us consider the following example.

266 *Example 2* Let $\mathcal{T}_2 = (\mathcal{F}_2, \mathcal{S}_2, \mathcal{D}_2)$ be a theory such that

$$267 \quad \mathcal{F}_2 \begin{cases} p \\ q \end{cases} \quad \mathcal{S}_2 \begin{cases} p \rightarrow t & (r_1) \\ s \rightarrow u & (r_2) \end{cases} \quad \mathcal{D}_2 \begin{cases} p \Rightarrow q & (r_3) \\ u \Rightarrow v & (r_4) \end{cases}$$

268 There are rules here whose head is not a consequence of \mathcal{T}_2 . In symbols, $\text{CN}(\mathcal{T}_2) = \{p, q, t\} \subset$
 269 $\{p, q, t, u, v\} = \mathcal{F}_2 \cup \{\text{Head}(r) \mid r \in \mathcal{S}_2 \cup \mathcal{D}_2\}$. Namely, the two rules r_2 and r_4 are not
 270 activated.

271 It is also easy to show that CN is monotonic. Note that a similar result was shown in [25]
 272 for the logic underlying the ASPIC system [8].

273 **Property 4** Let \mathcal{T} and \mathcal{T}' be two theories. If $\mathcal{T} \sqsubseteq \mathcal{T}'$ then $\text{CN}(\mathcal{T}) \subseteq \text{CN}(\mathcal{T}')$.

274 Let us now introduce the key notion of *option* which is useful for characterizing the
 275 extensions of argumentation systems under various semantics. An option is a maximal (for
 276 set inclusion) consistent sub-theory of a given theory.

277 **Definition 9** (*Option*) Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory. An *option* of \mathcal{T} is a sub-theory
 278 $\mathcal{T}' = (\mathcal{F}', \mathcal{S}', \mathcal{D}')$ of \mathcal{T} such that:

- 279 • $\mathcal{F}' \subseteq \mathcal{F}$, $\mathcal{S}' \subseteq \mathcal{S}$ and $\mathcal{D}' \subseteq \mathcal{D}$
- 280 • $\text{CN}(\mathcal{T}')$ is consistent
- 281 • $\nexists \mathcal{T}'' \sqsubseteq \mathcal{T}$ such that $\mathcal{T}' \subset \mathcal{T}''$ and $\text{CN}(\mathcal{T}'')$ is consistent.

282 $\text{Opt}(\mathcal{T})$ denotes the set of all options of \mathcal{T} .

283 Let us illustrate this new notion by the following example.

284 *Example 3* Consider \mathcal{T}_3 such that $\mathcal{F}_3, \mathcal{S}_3, \mathcal{D}_3$ are as follows.

$$285 \quad \mathcal{F}_3 \begin{cases} p \\ q \\ \neg s \end{cases} \quad \mathcal{S}_3 \{t, v \rightarrow s \quad (r_1)\} \quad \mathcal{D}_3 \begin{cases} p \Rightarrow t & (r_2) \\ q \Rightarrow u & (r_3) \\ u \Rightarrow v & (r_4) \end{cases}$$

286 The theory \mathcal{T}_3 has seven options:

- 287 • $\mathcal{O}_1 = (\mathcal{F}_3, \mathcal{S}_3, \{r_2, r_3\})$
- 288 • $\mathcal{O}_2 = (\mathcal{F}_3, \mathcal{S}_3, \{r_2, r_4\})$
- 289 • $\mathcal{O}_3 = (\mathcal{F}_3, \mathcal{S}_3, \{r_3, r_4\})$
- 290 • $\mathcal{O}_4 = (\mathcal{F}_3, \emptyset, \mathcal{D}_3)$
- 291 • $\mathcal{O}_5 = (\{p, q\}, \mathcal{S}_3, \mathcal{D}_3)$
- 292 • $\mathcal{O}_6 = (\{p, \neg s\}, \mathcal{S}_3, \mathcal{D}_3)$
- 293 • $\mathcal{O}_7 = (\{q, \neg s\}, \mathcal{S}_3, \mathcal{D}_3)$

294 A theory has at least one option which is the theory itself in case it is consistent. This is
 295 the case in Example 2: $\text{Opt}(\mathcal{T}_2) = \{\mathcal{T}_2\}$.

296 **Property 5** Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory.

- 297 • $\text{Opt}(\mathcal{T}) \neq \emptyset$.
- 298 • $\text{Opt}(\mathcal{T}) = \{\mathcal{T}\}$ iff $\text{CN}(\mathcal{T})$ is consistent.

299 We show next that options are all pairwise distinct.

300 **Proposition 1** For all $\mathcal{O}, \mathcal{O}' \in \text{Opt}(\mathcal{T})$, if $\text{CN}(\mathcal{O}) = \text{CN}(\mathcal{O}')$, then $\mathcal{O} = \mathcal{O}'$.

301 The definition of option does not make any difference between the strict part of a theory
 302 (i.e., its facts and strict rules) and its defeasible part. In rule systems [24], the former
 303 takes precedence over the latter since it represents the “certain” information of a theory. For
 304 instance, in default logic the certain part belongs to every extension of a theory [5]. This
 305 precedence is captured by the following notion of *preferred option*.

306 **Definition 10** (*Preferred option*) Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory. A *preferred option* of \mathcal{T} is
 307 a sub-theory $\mathcal{T}' = (\mathcal{F}', \mathcal{S}', \mathcal{D}')$ of \mathcal{T} such that:

- 308 • $\mathcal{F}' = \mathcal{F}$ and $\mathcal{S}' = \mathcal{S}, \mathcal{D}' \subseteq \mathcal{D}$,
- 309 • $\text{CN}(\mathcal{T}')$ is consistent,
- 310 • $\forall r \in \mathcal{D} \setminus \mathcal{D}', \text{CN}((\mathcal{F}, \mathcal{S}, \mathcal{D}' \cup \{r\}))$ is inconsistent.

311 $\text{POpt}(\mathcal{T})$ denotes the set of all preferred options of \mathcal{T} .

312 Let us illustrate this notion by an example.

313 *Example 3 (Cont)* The theory \mathcal{T}_3 has three preferred options: $\mathcal{O}_1, \mathcal{O}_2$ and \mathcal{O}_3 . □

314 Unlike options, the defeasible rules of a theory do not necessarily belong to at least one
 315 preferred option of the theory as shown by the following example.

316 *Example 4* The theory \mathcal{T}_4 such that

$$317 \quad \mathcal{F}_4 \begin{cases} p \\ q \end{cases} \quad \mathcal{S}_4 \{ p \rightarrow s \quad (r_1) \quad \quad \mathcal{D}_4 \{ p \Rightarrow \neg s \quad (r_2)$$

318 has a single preferred option $\mathcal{O} = (\mathcal{F}_4, \mathcal{S}_4, \emptyset)$ which does not contain the unique defeasible
 319 rule r_2 .

320 Every preferred option is an option. The converse holds only when the theory is consistent
 321 in which case the latter is the only (preferred) option (cf. Property 5).

322 **Property 6** Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory.

- 323 • $\text{POpt}(\mathcal{T}) \subseteq \text{Opt}(\mathcal{T})$.
- 324 • $\text{Opt}(\mathcal{T}) \subseteq \text{POpt}(\mathcal{T})$ iff $\text{CN}(\mathcal{T})$ is consistent.

325 A theory may not have preferred options. This is in particular the case when the strict part
 326 (the set of facts and strict rules) is inconsistent.

327 **Property 7** Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory.

- 328 • $\text{POpt}(\mathcal{T}) = \emptyset$ iff $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is inconsistent.
- 329 • For all $r \in \mathcal{D}$, if $\text{CN}((\mathcal{F}, \mathcal{S}, \{r\}))$ is consistent, then there exists a preferred option \mathcal{O}
 330 such that $(\mathcal{F}, \mathcal{S}, \{r\}) \subseteq \mathcal{O}$.

331 Notice that the set of consequences of an (preferred) option is not necessarily maximal
 332 for set inclusion as shown by Example 3.

333 *Example 3 (Cont)* We have $\text{CN}(\mathcal{O}_1) = \{p, q, \neg s, t, u\}$ and $\text{CN}(\mathcal{O}_2) = \{p, q, \neg s, t\}$. Thus,
 334 $\text{CN}(\mathcal{O}_2) \subseteq \text{CN}(\mathcal{O}_1)$. □

335 **Notations** For a set \mathcal{B} of theories, we denote the set of its maximal elements as $\text{Max}(\mathcal{B}) =$
 336 $\{\mathcal{T} \in \mathcal{B} \mid \nexists \mathcal{T}' \in \mathcal{B} \text{ such that } \text{CN}(\mathcal{T}) \subsetneq \text{CN}(\mathcal{T}')\}$.

337 Note that in general, maximal preferred options may be different from maximal options.
 338 Consider, for instance, the theory in Example 3. Its maximal options are \mathcal{O}_4 and \mathcal{O}_5 , while
 339 its maximal preferred options are \mathcal{O}_1 and \mathcal{O}_3 .

340 Let us now introduce the concept of *free* part of a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. It is the sub-
 342 theory that is made of the set of facts, the set of strict rules and the defeasible rules which
 343 are involved in every preferred option of \mathcal{T} .

344 **Definition 11** (*Free sub-theory*) The *free* sub-theory of a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ is
 345 $\text{Free}(\mathcal{T}) = (\mathcal{F}, \mathcal{S}, \bigcap_{(\mathcal{F}, \mathcal{S}, \mathcal{D}_i) \in \text{POpt}(\mathcal{T})} \mathcal{D}_i)$.

346 The following result summarizes some basic properties of this sub-theory.

347 **Property 8** Let \mathcal{T} be a theory.

- 348 • For any $\mathcal{O} \in \text{POpt}(\mathcal{T})$, $\text{Free}(\mathcal{T}) \sqsubseteq \mathcal{O}$
- 349 • $\text{CN}(\text{Free}(\mathcal{T}))$ is consistent

350 4 Rule-based argumentation systems

351 In this section, we propose an instantiation of Dung's framework that allows reasoning
 352 about defeasible information, i.e., drawing conclusions from a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. The
 353 instantiation is referred to as argumentation system keeping thus the term framework for the
 354 abstract formalism of Dung. The backbone of an argumentation system is naturally the notion
 355 of *argument*. Intuitively, an argument is a justification of a claim, i.e., it provides evidence
 356 that the claim is true. Thus, it should satisfy at least the three following basic properties:
 357 (i) internal coherence, (ii) relevance to the claim it justifies and (iii) truth preserving (i.e.,
 358 it guarantees the truth of the claim). It is true that humans' arguments may be inconsistent,
 359 but they are seen as fallacious by reasonable people. Furthermore, the topic of the paper is
 360 not reasoning about humans' arguments. It is rather reasoning about inconsistent theories by
 361 using arguments as a building block of the proposed logic.

362 **Definition 12** (*Argument*) Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory. An *argument* defined from \mathcal{T} is
 363 a pair (d, x) such that:

- 364 • $x \in \mathcal{L}$
- 365 • d is a derivation schema for x from \mathcal{T} (Truth preserving)
- 366 • $\text{Seq}(d)$ is consistent (Internal coherence)
- 367 • $\nexists T' \sqsubset (\text{Facts}(d), \text{Strict}(d), \text{Def}(d))$ such that $x \in \text{CN}(T')$ (Relevance)

368 An argument (d, x) is *strict* iff $\text{Def}(d) = \emptyset$.

369 *Example 1 (Cont)* Below are the nine arguments that are built from the theory \mathcal{T}_1 .

- 370 • $((p, \sigma), p)$
- 371 • $((q, \sigma), q)$
- 372 • $((p, \sigma), (q, r_6), q)$
- 373 • $((p, \sigma), (s, r_1), s)$
- 374 • $((q, \sigma), (\neg s, r_2), \neg s)$
- 375 • $((p, \sigma), (q, r_6), (\neg s, r_2), \neg s)$
- 376 • $((p, \sigma), (s, r_1), (u, r_3), u)$

- 377 • $((q, \sigma), (\neg s, r_2), (t, r_4)), t)$
- 378 • $((p, \sigma), (q, r_6), (\neg s, r_2), (u, r_3)), t)$

379 Note that there is no argument in favor of v since all derivation schemas for v are inconsistent.
 380 Derivations (4) and (5) do not give birth to arguments since they are not minimal. \square

381 **Notations** Let \mathcal{T} be a theory, $\text{Arg}(\mathcal{T})$ denotes the set of all arguments built from \mathcal{T} in the
 382 sense of Definition 12. If $a = (d, x)$ is an argument, then $\text{Conc}(a) = x$. For a
 383 set \mathcal{E} of arguments, $\text{Concs}(\mathcal{E}) = \{x \mid (d, x) \in \mathcal{E}\}$ and $\text{Th}(\mathcal{E})$ is a theory such
 384 that

$$385 \quad \text{Th}(\mathcal{E}) = \left(\bigcup_{(d,x) \in \mathcal{E}} \text{Facts}(d), \bigcup_{(d,x) \in \mathcal{E}} \text{Strict}(d), \bigcup_{(d,x) \in \mathcal{E}} \text{Def}(d) \right).$$

386 The following result shows that an argument provides a minimal derivation schema for a
 387 conclusion.

388 **Theorem 1** Let \mathcal{T} be a theory. For any consistent sequence $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ from
 389 \mathcal{T} , the following two statements are equivalent:

- 390 • (d, x) is an argument (from \mathcal{T})
- 391 • d is a focused derivation schema from \mathcal{T} such that $x = x_n$

392 An argument may have several sub-parts, each of which may give birth to an argument,
 393 called *sub-argument* of the original argument.

394 **Definition 13** (*Sub-argument*) An argument (d, x) is a *sub-argument* of (d', x') iff
 395 $(\text{Facts}(d), \text{Strict}(d), \text{Def}(d)) \sqsubseteq (\text{Facts}(d'), \text{Strict}(d'), \text{Def}(d'))$.

396 **Notations** The function $\text{Sub}(\cdot)$ returns the set of all sub-arguments of a given argument.

397 *Example 1* (*Cont*) The argument $((q, \sigma), (\neg s, r_2), \neg s)$ has two sub-arguments: $((q, \sigma))$,
 398 q and itself. By contrast, $((q, \sigma), q)$ is not a sub-argument of $((p, \sigma), (q, r_6), q)$. \square

399 **Property 9** If (d, x) is a sub-argument of (d', x') , then $\text{Seq}(d) \subseteq \text{Seq}(d')$.

400 The converse is not true as shown next.

401 *Example 5* Consider the two arguments a and b :

- 402 • $a = ((p, \sigma), (t, p \rightarrow t)), t)$
- 403 • $b = ((p, \sigma), (q, p \rightarrow q), (t, q \Rightarrow t)), t)$

404 Note that $\text{Seq}(a) = \{p, t\} \subseteq \{p, q, t\} = \text{Seq}(b)$ but a is not a sub-argument of b since the
 405 theory $(\{p\}, \{p \rightarrow t\}, \sigma)$ is not a sub-theory of $(\{p\}, \{p \rightarrow q\}, \{q \Rightarrow t\})$.

406 Argumentation systems that use a Tarskian logic such as propositional logic may have
 407 infinite sets of arguments even when the theories (called *knowledge bases*) over which they
 408 are built are themselves finite (cf. [26]). We show that this is not the case for rule-based
 409 argumentation systems. Indeed, the sets of arguments are finite as soon as the theories are
 410 finite.

411 **Proposition 2** If a theory \mathcal{T} is finite, then $\text{Arg}(\mathcal{T})$ is finite.

412 The set of arguments built from a given theory cannot be empty since the set of facts of a
413 theory contains at least \top .

414 **Property 10** For a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$, $\text{Arg}(\mathcal{T}) \neq \emptyset$.

415 The construction of arguments in all existing *structured* argumentation systems is a
416 monotonic process. By structured systems, we mean argumentation systems that build their
417 arguments from knowledge bases encoded in particular logics. These include ASPIC [8],
418 ASPIC+ [9], DeLP [10], ABA [27] and those discussed in [4]. Thus, unlike Dung's frame-
419 work where arguments are abstract entities, in structured systems arguments have a clear
420 origin and a precise structure. Hunter studied in [25] the properties of the logics underlying
421 existing structured systems. The results show that the set of arguments built from a knowl-
422 edge base cannot be shrunk when the base is extended by new information. The following
423 result shows that this property holds also for the kind of logic discussed in this paper.

424 **Proposition 3** Let \mathcal{T} and \mathcal{T}' be two theories. If $\mathcal{T} \sqsubseteq \mathcal{T}'$, then $\text{Arg}(\mathcal{T}) \subseteq \text{Arg}(\mathcal{T}')$.

425 A rule-based instantiation of Dung's abstract framework is defined as follows:

426 **Definition 14** (*Argumentation system*) An *argumentation system* defined over a theory $\mathcal{T} =$
427 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ is a pair $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ where $\text{Arg}(\mathcal{T})$ is the set of arguments built from \mathcal{T}
428 in the sense of Definition 12 and $\mathcal{R} \subseteq \text{Arg}(\mathcal{T}) \times \text{Arg}(\mathcal{T})$ is an attack relation.

429 For the sake of generality, the attack relation of an argumentation system is left *unspecified*
430 in the sequel. Thus, it may be instantiated in different ways. In existing rule-based argumen-
431 tation systems like the ASPIC system as defined in [8, 17] and its extended version ASPIC+
432 [9], three kinds of attack relations are used: (i) *rebut*, initially proposed in [28], which requires
433 that two arguments have opposite conclusions, (ii) *assumption attack*, proposed also in [28],
434 according to which an argument undermines a premise of another argument, and (iii) *under-*
435 *cut*, proposed in [16], which allows an argument to prevent the application of a defeasible
436 rule in another argument. The two first relations are *conflict-dependent*, i.e., they capture
437 the inconsistency of the theory over which an argumentation system is built. Such relations
438 should show no attack from argument a to b unless their derivation schemas contain opposite
439 literals.

440 **Definition 15** (*Conflict-dependency*) Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation sys-
441 tem. The attack relation \mathcal{R} is *conflict-dependent* iff for all $(d, x), (d', x') \in \text{Arg}(\mathcal{T})$, if
442 $(d, x) \mathcal{R} (d', x')$ then $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent.

443 An important feature of conflict-dependent attack relations is that they do not admit self-
444 attacking arguments, mainly since arguments are consistent.

445 **Proposition 4** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system. If \mathcal{R} is *conflict-*
446 *dependent*, then for all $a \in \text{Arg}(\mathcal{T})$ $(a, a) \notin \mathcal{R}$.

447 Conflict-dependency is somehow related to the notion of conflict-freeness of sets of argu-
448 ments. Indeed, when the attack relation is conflict-dependent, the set of arguments built from
449 any consistent theory is conflict-free with respect to this relation.

450 **Proposition 5** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} .
451 For every $\mathcal{T}' \sqsubseteq \mathcal{T}$, if $\text{CN}(\mathcal{T}')$ is consistent and \mathcal{R} is *conflict-dependent*, then $\text{Arg}(\mathcal{T}')$ is
452 *conflict-free*.

453 Another feature of all the attack relations in existing rule-based argumentation systems is
 454 the fact that they privilege strict arguments.

455 **Definition 16** (*Strict argument precedence*) Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation
 456 system built over a theory T . An attack relation \mathcal{R} *privileges strict arguments* iff for all
 457 $a = (d, x), b = (d', x') \in \text{Arg}(T)$, if a is strict and $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent, then
 458 $a\mathcal{R}b$.

459 A consequence of this property is that the set $\text{Arg}(\text{Free}(T))$ is admissible (i.e., it is
 460 conflict-free and defends all its elements). We will show in a subsequent section that this
 461 result is crucial for characterizing ideal extension.

462 **Theorem 2** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory $T =$
 463 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is consistent. If \mathcal{R} is conflict-dependent and privileges
 464 strict arguments, then $\text{Arg}(\text{Free}(T))$ is an admissible set of \mathcal{H} .

465 Unless stated otherwise, in what follows we do not make any assumption about the attack
 466 relation of a rule-based argumentation system. However, the arguments of the latter are
 467 evaluated using any of the semantics recalled in Definition 3. The extensions of a system
 468 are used for defining the *plausible conclusions* to be drawn from the theory over which the
 469 system is built. A literal is a plausible conclusion of a system iff it is a common conclusion
 470 to all the extensions.

471 **Definition 17** (*Plausible conclusions*) Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system
 472 built over a theory T . The set of *plausible conclusions* of \mathcal{H} under semantics x is

473
$$\text{Output}(\mathcal{H}) = \begin{cases} \{z \in \mathcal{L} \mid \forall \mathcal{E} \in \text{Ext}_x(\mathcal{H}), \exists a \in \mathcal{E} \text{ s.t. } \text{Conc}(a) = z\} & \text{if } \text{Ext}_x(\mathcal{H}) \neq \emptyset \\ \emptyset & \text{else} \end{cases}$$

474 The set of plausible conclusions coincides with the set of common conclusions of the
 475 extensions, of course when extensions exist.

476 **Property 11** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory T such
 477 that $\text{Ext}_x(\mathcal{H}) \neq \emptyset$ where x is any of the reviewed semantics. The equality $\text{Output}(\mathcal{H}) =$
 478 $\bigcap_{\mathcal{E}_i \in \text{Ext}_x(\mathcal{H})} \text{Concs}(\mathcal{E}_i)$ holds.

479 Finally, it is obvious that the plausible conclusions of an argumentation system are con-
 480 sequences of the theory over which it is built.

481 **Property 12** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory T . The
 482 inclusion $\text{Output}(\mathcal{H}) \subseteq \text{CN}(T)$ holds under any extension-based semantics.

483 It is worth noticing that under admissible semantics, the set of plausible conclusions of any
 484 argumentation system is empty. This is mainly due to the fact that the empty set is always
 485 an admissible extension. This makes this semantics unsuitable for defeasible reasoning.
 486 Complete semantics suffers from the same problem. Indeed, since under this semantics
 487 extensions are not maximal for set inclusion, the empty set may be an extension leading
 488 thus to an empty set of plausible conclusions. Stable semantics may also be unsuitable for
 489 argumentation systems that do not have extensions. However, we show in a subsequent section
 490 that rule-based systems that satisfy some desirable properties do have stable extensions, in
 491 particular when the attack relation is conflict-dependent.

492 **5 Postulates for rule-based argumentation systems**

493 Like any reasoning model, argumentation systems should enjoy some desirable properties or
 494 *rationality postulates* that ensure their soundness. The first work on postulates in argumen-
 495 tation was done by [17] in the context of rule-based systems. Starting from the observation
 496 that some existing systems like those proposed in [12, 29] suffer from two main problems: (i)
 497 returning inconsistent sets of plausible conclusions and (ii) forgetting intuitive conclusions,
 498 the authors proposed three postulates which prevent the encountered problems. In what fol-
 499 lows, we recall the three postulates and propose three new ones. The first postulate proposed
 500 in [17] concerns the consistency of the set of conclusions supported by every extension.

501 **Postulate 1** (Consistency) *An argumentation system $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ built over a theory*
 502 *$T = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ satisfies consistency under semantics x iff for any $\mathcal{E} \in \text{Ext}_x(\mathcal{H})$, $\text{Concs}(\mathcal{E})$*
 503 *is consistent.*

504 A rule-based system which satisfies this postulate has necessarily a consistent set of
 505 plausible conclusions.

506 **Property 13** [17] *If an argumentation system $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ satisfies consistency under*
 507 *semantics x (x being any extension-based semantics), then $\text{Output}(\mathcal{H})$ is consistent.*

508 The second postulate ensures a form of “completeness” of the outputs of an argumentation
 509 system. It says that if there is an argument with conclusion x in an extension of the system,
 510 and there exists a strict rule $x \rightarrow y$ in the theory over which the system is built, then y
 511 should also be supported by an argument in the same extension. Recall that a strict rule has
 512 no exception. Thus, as soon as x is true, y holds for sure.

513 **Postulate 2** (Closure under strict rules) *An argumentation system $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ built*
 514 *over a theory $T = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ is closed under strict rules under semantics x iff for any*
 515 *$\mathcal{E} \in \text{Ext}_x(\mathcal{H})$, $\text{Concs}(\mathcal{E}) = \text{CN}((\text{Concs}(\mathcal{E}), \mathcal{S}, \emptyset)$.*

516 If an argumentation system is closed under strict rules, then its set of plausible conclusions
 517 is also closed under strict rules.

518 **Property 14** [17] *Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory*
 519 *$T = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. If \mathcal{H} is closed under strict rules under semantics x (x being any extension-*
 520 *based semantics), then $\text{Output}(\mathcal{H}) = \text{CN}((\text{Output}(\mathcal{H}), \mathcal{S}, \emptyset)$.*

521 A third postulate, called *indirect consistency*, was proposed in [17]. It ensures that every
 522 closed (under strict rules) extension should satisfy consistency. It was shown that a system that
 523 satisfies consistency and closure under strict rules satisfies this form of indirect consistency.

524 **Property 15** [17] *Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory*
 525 *$T = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. If \mathcal{H} satisfies consistency and is closed under strict rules under semantics x*
 526 *(x being any extension-based semantics), then for any $\mathcal{E} \in \text{Ext}_x(\mathcal{H})$, $\text{CN}((\text{Concs}(\mathcal{E}), \mathcal{S}, \emptyset)$*
 527 *is consistent.*

528 It is worth mentioning that the three previous results hold for any attack relation and
 529 under any extension-based acceptability semantics, thus under any of the semantics recalled
 530 in Definition 3 and others like recursive semantics [20].

531

532 In any axiomatic approach, the axioms (or postulates) should ideally all be *independent*
 533 from each other, i.e., none is deduced from the others. Thus, in the sequel indirect consistency
 534 is abandoned since it follows from Postulates 1 and 2. We propose next three new postulates
 535 which were already defined in [30] for argumentation systems that use a logic in the sense of
 536 [22]. The first one says that if an argument belongs to an extension, then all its sub-arguments
 537 should be in the extension. Thus, an argument cannot be accepted if one of its sub-parts is
 538 questionable. This is a natural requirement since plausible conclusions inferred from a theory
 539 rely on their supporting arguments which should be unassailable.

540 **Postulate 3** (Closure under sub-arguments) *An argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$*
 541 *built over a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ is closed under sub-arguments under semantics x iff for*
 542 *any $\mathcal{E} \in \text{Ext}_x(\mathcal{H})$, if $a \in \mathcal{E}$ then $\text{Sub}(a) \subseteq \mathcal{E}$.*

543 Argumentation systems that satisfy both consistency and closure under sub-arguments
 544 enjoy a strong version of consistency. Indeed, the set of consequences that follow from the
 545 theory of an extension is consistent.

546 **Proposition 6** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$*
 547 *$(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{Ext}_x(\mathcal{H}) \neq \emptyset$ (x being any extension-based semantics). If \mathcal{H} satisfies*
 548 *consistency and closure under sub-arguments, then for any $\mathcal{E} \in \text{Ext}_x(\mathcal{H})$, $\text{CN}(\text{Th}(\mathcal{E}))$ is*
 549 *consistent.*

550 Let us illustrate this result with an example.

551 *Example 6* Let $\mathcal{T}_5 = (\mathcal{F}_5, \mathcal{S}_5, \mathcal{D}_5)$ be a theory such that $\mathcal{S}_5 = \emptyset$ and

552
$$\mathcal{F}_5 \begin{cases} x \\ \neg x \end{cases} \quad \mathcal{D}_5 \begin{cases} x \Rightarrow y & (r_1) \\ \neg x \Rightarrow z & (r_2) \end{cases}$$

553 Consider the two arguments (d_1, y) , (d_2, z) with $d_1 = \langle (x, \sigma), (y, r_1) \rangle$ and $d_2 =$
 554 $\langle (\neg x, \sigma), (z, r_2) \rangle$. Assume that the set $\mathcal{E} = \{(d_1, y), (d_2, z)\}$ is an extension of $(\text{Arg}(\mathcal{T}_5), \mathcal{R})$
 555 under a given semantics. Clearly, $\text{CN}(\text{Concs}(\mathcal{E}), \mathcal{S}, \emptyset) = \{y, z\}$ is consistent. However,
 556 $\text{Th}(\mathcal{E}) = \mathcal{T}$ and $\text{CN}(\mathcal{T}) = \{x, \neg x, y, z\}$ is inconsistent. Proposition 6 ensures that the argu-
 557 mentation system $(\text{Arg}(\mathcal{T}_5), \mathcal{R})$ violates at least one of the consistency or closure under
 558 sub-arguments postulates.

559 Since facts and strict rules are the certain part in a theory (facts being observable and
 560 strict rules having no exceptions), they should be plausible conclusions of any argumentation
 561 system. It is worth mentioning that this principle is applied, for instance, in default logic
 562 where the non-defeasible information of a default theory is part of all extensions [5]. Of
 563 course this makes sense when the non-defeasible information is consistent.

564 **Postulate 4** (Strict precedence) *An argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ built over a*
 565 *theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ satisfies strict precedence under semantics x iff $\text{CN}(\mathcal{F}, \mathcal{S}, \emptyset) \subseteq$*
 566 *Output(\mathcal{H}).*

567 Notice that argumentation systems that have no extensions violate this postulate. Similarly,
 568 systems that evaluate their arguments using a semantics which considers the empty set as an
 569 extension (like admissible semantics) violate strict precedence. Such systems are thus not
 570 suitable for defeasible reasoning since they may miss intuitive conclusions.

571 **Proposition 7** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory*
 572 *$\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\emptyset \in \text{Ext}_x(\mathcal{H})$. \mathcal{H} violates strict precedence under semantics x .*

573 We show next that if an argumentation system satisfies consistency and strict precedence,
574 then the strict part of the theory over which it is built is consistent.

575 **Proposition 8** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
576 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{Ext}_x(\mathcal{H}) \neq \emptyset$. If \mathcal{H} satisfies consistency and strict precedence under
577 semantics x , then $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is consistent.*

578 The last postulate ensures a form of completeness of the extensions of an argumentation sys-
579 tem under any semantics. It says that if the sequence of an argument is part of the conclusions
580 of a given extension, then the argument should belong to the extension. Informally: If each
581 step in the argument is good enough to be in the extension, then so is the argument itself. It
582 is worth pointing out that this postulates holds for both strict and defeasible rules.

583 **Postulate 5** (Exhaustiveness) *An argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ built over a
584 theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ satisfies exhaustiveness under semantics x iff for any $\mathcal{E} \in \text{Ext}_x(\mathcal{H})$,
585 for any $(d, x) \in \text{Arg}(\mathcal{T})$, if $\text{Seq}(d) \subseteq \text{Concs}(\mathcal{E})$, then $(d, x) \in \mathcal{E}$.*

586 Argumentation systems that satisfy exhaustiveness and closure under sub-arguments have
587 complete extensions, i.e., they are closed in terms of arguments.

588 **Proposition 9** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system such that $\text{Ext}_x(\mathcal{H}) \neq \emptyset$
589 (x being any extension-based semantics). If \mathcal{H} is closed under sub-arguments and satisfies
590 exhaustiveness under semantics x , then for all $\mathcal{E} \in \text{Ext}_x(\mathcal{H})$, $\mathcal{E} = \text{Arg}(\text{Th}(\mathcal{E}))$.*

591 When an argumentation system satisfies strict precedence and exhaustiveness, then its
592 strict arguments are part of any extension. This holds under any extension-based semantics.

593 **Proposition 10** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system such that $\text{Ext}(\mathcal{H}) \neq \emptyset$
594 (under an extension-based semantics). If \mathcal{H} satisfies exhaustiveness and strict precedence,
595 then for any $\mathcal{E} \in \text{Ext}(\mathcal{H})$, $\text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \mathcal{E}$.*

596 An axiomatic approach should obey an important feature: The postulates should be *com-*
597 *patible*, i.e., they can be satisfied all together by an argumentation system under a given
598 semantics. Fortunately, this is the case of the five postulates discussed in this section.

599 **Proposition 11** *The five postulates are compatible.*

600 The four postulates (consistency, closure under sub-arguments, closure under strict rules,
601 strict precedence) are independent. None of them follows from a subset of the three others.
602 However, as will be shown in the next section, exhaustiveness follows from consistency and
603 closure under sub-arguments when an argumentation system uses a conflict-dependent attack
604 relation and naive or stable semantics for evaluating arguments.

605 6 Outcomes of rule-based argumentation systems

606 This section analyzes the outputs of rule-based argumentation systems under the reviewed
607 semantics, i.e., those recalled in Definition 3, that are suitable for defeasible reasoning. Recall
608 that complete semantics is not a good candidate for such reasoning since its extensions are
609 not maximal (for set inclusion) and may thus lead to an empty set of plausible conclusions,
610 and missing intuitive conclusions. We analyze the extensions under each semantics. Indeed,

611 we characterize the set $\text{Concs}(\cdot)$ of conclusions and the theory $\text{Th}(\cdot)$ of each extension. We
 612 also characterize the set $\text{Output}(\cdot)$ of plausible conclusions that are drawn by a rule-based
 613 argumentation system from a theory.

614 Note that the argumentation system described in Sect. 4 is not fully specified since its attack
 615 relation is left undefined and may thus be instantiated in different ways. For the purpose of
 616 our study, we do not need to consider a particular attack relation. Since any reasonable
 617 argumentation system should satisfy the discussed postulates, throughout this section we
 618 only focus on systems that satisfy the postulates. Such systems exist and ASPIC, defined
 619 in [17], is one of them. Indeed, it was shown in [17] that ASPIC, which uses *restricted*
 620 *rebut* as attack relation, satisfies consistency and closure under both sub-arguments and strict
 621 rules under *all* Dung’s semantics. Furthermore, the attack relation in ASPIC privileges strict
 622 arguments (by definition) and the strict part of a theory is assumed to be consistent. Thus, the
 623 system satisfies strict precedence under the same semantics. Finally, from our Proposition
 624 15 (respectively Proposition 13), it follows that it also satisfies exhaustiveness under stable
 625 (respectively naive) semantics. The results we provide next hold for any instantiation of the
 626 attack relation \mathcal{R} . This means that whatever the attack relation that is considered, the outcome
 627 will be the same. This shows also that all the reasonable rule-based argumentation systems
 628 that can be built over the same theory are *equivalent* [31,32], in the sense they provide the
 629 same extensions and the set of plausible conclusions under a given semantics.

630 Before presenting the formal results concerning the reviewed semantics, below are some
 631 results that hold under any extension-based semantics, thus under all the reviewed semantics
 632 but also under several other semantics (e.g., recursive semantics [20], the one used in DeLP
 633 system [10], stage semantics [21], ...). The first result characterizes the set of conclusions of
 634 each extension of an argumentation system which is closed under sub-arguments.

635 **Proposition 12** *Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system such that $\text{Ext}(\mathcal{H}) \neq$
 636 \emptyset (under an extension-based semantics). If \mathcal{H} is closed under sub-arguments, then for any
 637 $\mathcal{E} \in \text{Ext}(\mathcal{H})$,*

- 638 • $\text{Concs}(\mathcal{E}) = X \cup \{\text{Head}(r) \mid r \in Y \cup Z\}$ where $\text{Th}(\mathcal{E}) = (X, Y, Z)$
- 639 • $\text{Concs}(\mathcal{E}) = \text{CN}(\text{Th}(\mathcal{E}))$
- 640 • $\forall (d, x) \in \text{Arg}(\text{Th}(\mathcal{E})), \text{Seq}(d) \subseteq \text{Concs}(\mathcal{E})$

641 The next result shows that if an argumentation system over a theory satisfies strict prece-
 642 dence, closure under both sub-arguments and strict rules, then the set of literals deduced
 643 from $\text{Th}(\mathcal{E})$, the theory of an extension \mathcal{E} , is exactly the same set that is obtained from $\text{Th}(\mathcal{E})$
 644 extended by all facts and strict rules which are not in $\text{Th}(\mathcal{E})$.

645 **Theorem 3** *Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
 646 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{Ext}(\mathcal{H}) \neq \emptyset$ (under an extension-based semantics). If \mathcal{H} satisfies strict
 647 precedence and closure under both strict rules and sub-arguments, then for any $\mathcal{E} \in \text{Ext}(\mathcal{H})$,*

648
$$\text{CN}(\text{Th}(\mathcal{E})) = \text{CN} \left(\left(\mathcal{F}, \mathcal{S}, \bigcup_{(d,x) \in \mathcal{E}} \text{Def}(d) \right) \right).$$

649 We also show that the theory of an extension can be extended into a sub-theory (of the
 650 argumentation system) which infers, using the notion of derivation, all the conclusions that
 651 are supported by arguments of the extension. This (i.e., Theorem 4) will be useful in proving
 652 various results in the next sections.

653 **Theorem 4** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
 654 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{Ext}(\mathcal{H}) \neq \emptyset$ (under an extension-based semantics). If \mathcal{H} satisfies strict
 655 precedence, and closure under both strict rules and sub-arguments, then for any $\mathcal{E} \in \text{Ext}(\mathcal{H})$,
 656 $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$ for $\mathcal{O} = (\mathcal{F}, \mathcal{S}, \zeta)$ such that

$$657 \quad \zeta = \left(\bigcup_{(d,x) \in \mathcal{E}} \text{Def}(d) \right) \cup \{r \mid r \in \mathcal{D} \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\text{Th}(\mathcal{E}))\}.$$

658 6.1 Naive semantics

659 Before characterizing the extensions as well as the plausible conclusions of a rule-based
 660 argumentation system, let us first show some additional links between the postulates in the
 661 particular case of naive semantics. The first result shows that exhaustiveness follows from
 662 consistency and closure under sub-arguments. This is the case when the attack relation is
 663 conflict-dependent.

664 **Proposition 13** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T}
 665 such that \mathcal{R} is conflict-dependent. If \mathcal{H} satisfies consistency and closure under sub-arguments
 666 under naive semantics, \mathcal{H} satisfies exhaustiveness under naive semantics.

667 The second result shows that when an argumentation system is closed under sub-arguments
 668 and satisfies the consistency postulate under naive semantics, then every naive extension of
 669 the system is closed in terms of arguments.

670 **Proposition 14** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 671 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 672 arguments under naive semantics. For any $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$, $\mathcal{E} = \text{Arg}(\text{Th}(\mathcal{E}))$.

673 Strict precedence is problematic in case of naive semantics since it may be violated by a
 674 rule-based argumentation system. This is mainly due to the fact that the orientation of attacks
 675 is not taken into account when computing naive extensions; thus, there is no way to enforce
 676 the postulate. We show next that strict arguments are part of any naive extension only when
 677 they neither are attacked nor attack any argument.

678 **Theorem 5** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
 679 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent. For any $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$, $\text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \mathcal{E}$ iff
 680 for any $a \in \text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$, $\nexists b \in \text{Arg}(T)$ such that $a\mathcal{R}b$ or $b\mathcal{R}a$.

681 We have previously shown that the five postulates are compatible in the general case.
 682 Indeed, under stable and preferred semantics, it was shown that the ASPIC system satisfies
 683 all the postulates. In case of naive semantics, this is not always true. Strict precedence is
 684 not compatible with consistency when the strict part is inconsistent, or it is consistent but
 685 in conflict with the defeasible part. For instance, any argumentation system built over the
 686 theory of Example 3 will violate at least one of the two postulates under naive semantics.

687 **Theorem 6** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
 688 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent. If $\exists a, b \in \text{Arg}(T)$ such that $a \in \text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$
 689 and $\text{Conc}(a) = \neg \text{Conc}(b)$, then \mathcal{H} cannot satisfy both strict precedence and consistency
 690 under naive semantics.

691 In case of the ASPIC system, the argument b cannot be strict since the strict part (i.e.,
 692 $\text{CN}(\mathcal{F}, \mathcal{S}, \emptyset)$) is assumed to be consistent. Moreover, there is only one conflict between a and
 693 b and which emanates from a since strict arguments cannot be attacked by defeasible ones.
 694 Thus, strict precedence is violated.

695 We show next that, assuming consistency and closure under sub-arguments, naive exten-
 696 sions are maximal.

697 **Theorem 7** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 698 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-arguments
 699 under naive semantics. For all $\mathcal{E}, \mathcal{E}' \in \text{Ext}_n(\mathcal{H})$, if $\text{Concs}(\mathcal{E}') \subseteq \text{Concs}(\mathcal{E})$ then $\mathcal{E} = \mathcal{E}'$.*

700 The following theorem characterizes naive extensions. It says that every naive extension
 701 of an argumentation system which satisfies consistency and closure under sub-arguments has
 702 a unique corresponding maximal option in the theory at hand.

703 **Theorem 8** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T}
 704 such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 705 arguments under naive semantics. For any $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$, there exists a unique option
 706 $\mathcal{O} \in \text{Max}(\text{Opt}(\mathcal{T}))$ such that $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$ and $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$.*

707 Note that the inclusion $\text{Th}(\mathcal{E}) \subseteq \mathcal{O}$ is due to the fact that a theory $\text{Th}(\mathcal{E})$ of an extension
 708 \mathcal{E} contains only activated (strict and defeasible) rules, while maximal options may contain
 709 non-activated ones. Thus, the elements which in \mathcal{O} but not in $\text{Th}(\mathcal{E})$ are non-activated rules.

710 **Notations** For \mathcal{E} any extension of \mathcal{H} such that $\mathcal{O} \in \text{Max}(\text{Opt}(\mathcal{T}))$ satisfies $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$ and
 711 $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$, let

$$\text{Option}(\mathcal{E}) \stackrel{\text{def}}{=} \mathcal{O}.$$

713 We prove that no two naive extensions return the same option. Moreover, every extension is
 714 exactly the set of all arguments that can be built from its corresponding option.

715 **Theorem 9** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 716 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-arguments
 717 under naive semantics.*

- 718 • For all $\mathcal{E}, \mathcal{E}' \in \text{Ext}_n(\mathcal{H})$, if $\text{Option}(\mathcal{E}) = \text{Option}(\mathcal{E}')$, then $\mathcal{E} = \mathcal{E}'$
- 719 • For any $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$, $\mathcal{E} = \text{Arg}(\text{Option}(\mathcal{E}))$

720 We have shown that each naive extension captures exactly one maximal option and it
 721 supports all, and only, the consequences of that option. Theorem 10 states that every maximal
 722 option has a corresponding naive extension. So, there is a bijection from the set of naive
 723 extensions to the set of maximal options.

724 **Theorem 10** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 725 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-arguments
 726 under naive semantics.*

- 727 • For any $\mathcal{O} \in \text{Max}(\text{Opt}(\mathcal{T}))$, $\text{Arg}(\mathcal{O}) \in \text{Ext}_n(\mathcal{H})$
- 728 • For any $\mathcal{O} \in \text{Max}(\text{Opt}(\mathcal{T}))$, $\mathcal{O} = \text{Option}(\text{Arg}(\mathcal{O}))$
- 729 • For all $\mathcal{O}, \mathcal{O}' \in \text{Max}(\text{Opt}(\mathcal{T}))$, if $\text{Arg}(\mathcal{O}) = \text{Arg}(\mathcal{O}')$ then $\mathcal{O} = \mathcal{O}'$

730 *Example 3 (Cont)* The theory \mathcal{T}_3 has seven options, of which only two are maximal:
 731 $\text{Max}(\text{Opt}(\mathcal{T}_3)) = \{\mathcal{O}_4, \mathcal{O}_5\}$. For all argumentation system \mathcal{H} built over \mathcal{T}_3 , if the attack
 732 relation of \mathcal{H} is to be conflict-dependent and consistency and closure under sub-arguments
 733 satisfied, then $\text{Ext}_n(\mathcal{H}) = \{\text{Arg}(\mathcal{O}_4), \text{Arg}(\mathcal{O}_5)\}$.

734 From the previous results, it follows that there is a bijection between the set of naive
 735 extensions of an argumentation system and the maximal options of the theory over which the
 736 system is built.

737 **Corollary 1** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 738 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-arguments
 739 under naive semantics. There is a bijection between $\text{Ext}_n(\mathcal{H})$ and $\text{Max}(\text{Opt}(T))$.

740 The previous results require only the satisfaction of two postulates: consistency and closure
 741 under sub-arguments. We show next that when a rule-based argumentation system satisfies
 742 all the five postulates, there is a bijection between the set of naive extensions of the system
 743 and the maximal preferred options of the theory over which it is built. The reason is that in
 744 such a case, the maximal options of the theory coincide with the maximal preferred ones.
 745 Recall that in general, maximal preferred options may be different from maximal options.

746 **Theorem 11** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 747 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency, strict precedence and closure under
 748 both strict rules and sub-arguments under naive semantics. The equality $\text{Max}(\text{POpt}(T)) =$
 749 $\text{Max}(\text{Opt}(T))$ holds.

750 **Corollary 2** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 751 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency, strict precedence and closure under
 752 both strict rules and sub-arguments under naive semantics. There is a bijection between
 753 $\text{Ext}_n(\mathcal{H})$ and $\text{Max}(\text{POpt}(T))$.

754 It is possible to delimit the number of naive extensions of any argumentation system that
 755 satisfies consistency and closure under sub-arguments. It is exactly the number of maximal
 756 options of the theory at hand.

757 **Corollary 3** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 758 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-arguments
 759 under naive semantics. The equality $|\text{Ext}_n(\mathcal{H})| = |\text{Max}(\text{Opt}(T))|$ holds.

760 It follows also that when a theory is finite, then any system built over it has a finite number
 761 of naive extensions.

762 **Corollary 4** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 763 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-arguments
 764 under naive semantics. If \mathcal{T} is finite, then \mathcal{H} has a finite number of naive extensions.

765 What about the plausible conclusions that are drawn from a theory using an argumentation
 766 system that satisfies the postulates? From the previous results, it is easy to show that they are
 767 the literals that follow from all the maximal options.

768 **Theorem 12** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 769 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-arguments
 770 under naive semantics.

$$771 \quad \text{Output}(\mathcal{H}) = \bigcap_{\mathcal{O}_i \in \text{Max}(\text{Opt}(T))} \text{CN}(\mathcal{O}_i)$$

772 *Example 3 (Cont)* Any argumentation system \mathcal{H} that can be built over the theory \mathcal{T}_3 and has a
 773 conflict-dependent attack relation and satisfies consistency and closure under sub-arguments
 774 will have as output the set $\text{Output}(\mathcal{H}) = \text{CN}(\mathcal{O}_4) \cap \text{CN}(\mathcal{O}_5) = \{p, q, t, u, v\}$.

775 Let us summarize the main results: under naive semantics, any rule-based argumentation
 776 system may violate strict precedence. However, the other postulates can be satisfied. In such
 777 a case, if the attack relation is conflict-dependent, then any argumentation system will infer
 778 exactly the literals that follow from all the maximal options of the theory over which the
 779 system is built. This is due to the bijection that holds between the set of naive extensions and
 780 the set of maximal options. In case the system satisfies also strict precedence and closure
 781 under strict rules, then the maximal options of the theory coincide with the maximal preferred
 782 options.

783 6.2 Stable semantics

784 As for naive semantics, exhaustiveness follows from consistency and closure under sub-
 785 arguments in case of stable semantics.

786 **Proposition 15** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T}*
 787 *such that \mathcal{R} is conflict-dependent. If \mathcal{H} satisfies consistency and closure under sub-arguments*
 788 *under stable semantics, then the following two properties hold:*

- 789 • \mathcal{H} satisfies exhaustiveness under stable semantics.
- 790 • For any $\mathcal{E} \in \text{Ext}_s(\mathcal{H})$, $\mathcal{E} = \text{Arg}(\text{Th}(\mathcal{E}))$.

791 Stable extensions of rule-based argumentation systems satisfying the five postulates return
 792 maximal preferred options. This means that if one instantiates Dung's framework and does
 793 not get maximal preferred options with stable extensions, then the instantiation certainly
 794 violates at least one of the postulates. Note that strict precedence may be satisfied by an
 795 argumentation system under stable semantics while it is violated by the same system under
 796 naive semantics. This is due to the fact that the orientation of attacks plays an important
 797 role in stable semantics, then strict precedence can be enforced by choosing an appropriate
 798 orientation.

799 **Theorem 13** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system defined over a theory \mathcal{T}*
 800 *such that \mathcal{R} is conflict-dependent. If \mathcal{H} satisfies consistency, strict precedence and closure*
 801 *under both sub-arguments and strict rules under stable semantics, and $\text{Ext}_s(\mathcal{H}) \neq \emptyset$, then*
 802 *for any $\mathcal{E} \in \text{Ext}_s(\mathcal{H})$, there exists a unique option $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that:*

- 803 • $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$
- 804 • $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$
- 805 • $\mathcal{E} = \text{Arg}(\mathcal{O})$

806 Two stable extensions cannot capture the same maximal preferred option.

807 **Theorem 14** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system defined over a theory \mathcal{T}*
 808 *such that \mathcal{R} is conflict-dependent. If \mathcal{H} satisfies consistency, strict precedence and closure*
 809 *under both sub-arguments and strict rules under stable semantics and $\text{Ext}_s(\mathcal{H}) \neq \emptyset$, then*
 810 *for all $\mathcal{E}, \mathcal{E}' \in \text{Ext}_s(\mathcal{H})$, if $\text{Option}(\mathcal{E}) = \text{Option}(\mathcal{E}')$ then $\mathcal{E} = \mathcal{E}'$.*

811 The previous results characterize the stable extensions of rule-based argumentation sys-
 812 tems that satisfy the postulates. However, they do not guarantee that each maximal preferred

option of a theory has a corresponding stable extension. To put it differently, it does not guarantee a bijection between the sets $\text{Ext}_s(\mathcal{H})$ and $\text{Max}(\text{POpt}(T))$ and thus does not ensure the equality $|\text{Ext}_s(\mathcal{H})| = |\text{Max}(\text{POpt}(T))|$. In case of argumentation systems that use a Tarskian logic for representing information and for computing arguments, it was shown in [4] that this equality depends on the attack relation that is chosen. We show next that this is also the case for rule-based systems.

Given T , let \mathfrak{R}_s be the set of *all* attack relations that are conflict-dependent and that ensure the five postulates under stable semantics:

$$\mathfrak{R}_s = \{\mathcal{R} \subseteq \text{Arg}(T) \times \text{Arg}(T) \mid \mathcal{R} \text{ is conflict-dependent and } (\text{Arg}(T), \mathcal{R}) \text{ satisfies the five postulates under stable semantics for any theory } T\}.$$

This set contains three *disjoints* subsets of attack relations, i.e., $\mathfrak{R}_s = \mathfrak{R}_{s_1} \cup \mathfrak{R}_{s_2} \cup \mathfrak{R}_{s_3}$:

- \mathfrak{R}_{s_1} : the set of relations such that $|\text{Ext}_s(\mathcal{H})| = 0$
- \mathfrak{R}_{s_2} : the set of relations such that $|\text{Ext}_s(\mathcal{H})| = |\text{Max}(\text{POpt}(T))|$
- \mathfrak{R}_{s_3} : the set of relations such that $|\text{Ext}_s(\mathcal{H})| < |\text{Max}(\text{POpt}(T))|$

Let us analyze separately each category of attack relations. The following result shows that the set \mathfrak{R}_{s_1} is empty, meaning that there is no attack relation which prevents the existence of stable extensions. To say it differently, any argumentation system which satisfies the postulates has at least one stable extension.

Theorem 15 $\mathfrak{R}_{s_1} = \emptyset$.

A consequence of this postulate is that stable extensions coincide with semi-stable ones. Indeed, it was shown in [18] that when stable extensions exist, they coincide with semi-stable extensions.

Corollary 5 For all argumentation system $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$, if $\mathcal{R} \in \mathfrak{R}_{s_2} \cup \mathfrak{R}_{s_3}$, then $\text{Ext}_s(\mathcal{H}) = \text{Ext}_{s_s}(\mathcal{H})$.

From the previous results, it is possible to delimit the number of stable extensions of rule-based argumentation systems that satisfy the five postulates.

Corollary 6 Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system defined over a theory T such that \mathcal{R} is conflict-dependent. If \mathcal{H} satisfies the five postulates, then

$$1 \leq |\text{Ext}_s(\mathcal{H})| \leq |\text{Max}(\text{POpt}(T))|.$$

It follows that when a theory is finite, any argumentation system built over it has a finite number of stable extensions.

Corollary 7 Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory T such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates. If T is finite, then \mathcal{H} has a finite number of stable extensions.

Attack relations of category \mathfrak{R}_{s_2} induce a bijection between the set of stable extensions of an argumentation system and the set of maximal preferred options of the theory over which the system is built. Indeed, every preferred option gives a stable extension.

Theorem 16 Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system over a theory T such that $\mathcal{R} \in \mathfrak{R}_{s_2}$. For any $O \in \text{Max}(\text{POpt}(T))$, $\text{Arg}(O) \in \text{Ext}_s(\mathcal{H})$.

852 *Example 3 (Cont)* The theory \mathcal{T}_3 has seven options, of which only two are preferred maximal:
 853 $\text{Max}(\text{POpt}(\mathcal{T}_3)) = \{\mathcal{O}_1, \mathcal{O}_3\}$. Thus, for all argumentation system \mathcal{H} built over \mathcal{T}_3 , if the
 854 attack relation of \mathcal{H} is of category \mathfrak{R}_{s_2} , then $\text{Ext}_s(\mathcal{H}) = \{\text{Arg}(\mathcal{O}_1), \text{Arg}(\mathcal{O}_3)\}$. Recall
 855 that under naive semantics, there is no argumentation system over \mathcal{T}_3 that can satisfy strict
 856 precedence. The ones that guarantee consistency and closure under sub-arguments will all
 857 have the following naive extensions: $\text{Ext}_n(\mathcal{H}) = \{\text{Arg}(\mathcal{O}_4), \text{Arg}(\mathcal{O}_5)\}$.

858 Argumentation systems with an attack relation from \mathfrak{R}_{s_2} are *coherent*, meaning that the
 859 preferred extensions exhaust all the stable ones. It follows thus that the three semantics (semi-
 860 stable, stable, preferred) coincide. This means that semi-stable and preferred semantics have
 861 no added value with respect to stable semantics since they guarantee the same results.

862 **Theorem 17** For any argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ such that $\mathcal{R} \in \mathfrak{R}_{s_2}$, it holds
 863 $\text{Ext}_s(\mathcal{H}) = \text{Ext}_{ss}(\mathcal{H}) = \text{Ext}_p(\mathcal{H})$.

864 In case an argumentation system satisfies strict precedence under naive semantics (see
 865 Theorem 5), then its extensions coincide with the stable ones. To put differently, in case
 866 naive semantics can guarantee strict precedence, stable semantics becomes useless since it
 867 provides no added value with respect to naive semantics.

868 **Theorem 18** For all argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ such that $\mathcal{R} \in \mathfrak{R}_{s_2}$, if \mathcal{H}
 869 satisfies the postulates under naive semantics, then

870
$$\text{Ext}_n(\mathcal{H}) = \text{Ext}_s(\mathcal{H}) = \text{Ext}_{ss}(\mathcal{H}) = \text{Ext}_p(\mathcal{H}).$$

871 Plausible conclusions of rule-based argumentation systems that use attack relations in
 872 category \mathfrak{R}_{s_2} are exactly the literals that follow from all the maximal preferred options of the
 873 theory at hand.

874 **Theorem 19** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 875 that $\mathcal{R} \in \mathfrak{R}_{s_2}$.

876
$$\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T}))} \text{CN}(\mathcal{O}_i).$$

877 Systems that use relations in \mathfrak{R}_{s_3} choose a proper subset of the maximal preferred options
 878 of \mathcal{T} and make inferences from them. Their output sets are as follows:

879 **Theorem 20** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 880 that $\mathcal{R} \in \mathfrak{R}_{s_3}$.

881
$$\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{O}_i \in \mathcal{X}} \text{CN}(\mathcal{O}_i)$$

882 with $\mathcal{X} = \{\mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T})) \mid \mathcal{E}_i = \text{Arg}(\mathcal{O}_i) \in \text{Ext}_s(\mathcal{H})\}$.

883 These attack relations introduce a critical discrimination between the maximal preferred
 884 options of a theory. Hence, great care must be exercised when designing rule-based argumen-
 885 tation systems based on stable semantics: The principles governing the interaction between
 886 \Rightarrow and \mathcal{R} must be both rigorously and meticulously specified so as to avoid trouble of which
 887 the following example is an easy case.

888 *Example 7* Consider \mathcal{T}_6 such that $\mathcal{F}_6, \mathcal{S}_6, \mathcal{D}_6$ are as follows:

$$889 \quad \mathcal{F}_6 \begin{cases} p \\ q \end{cases} \quad \mathcal{S}_6 = \emptyset \quad \mathcal{D}_6 \begin{cases} p \Rightarrow s & (r_1) \\ q \Rightarrow \neg s & (r_2) \end{cases}$$

890 The theory \mathcal{T}_6 has two maximal preferred options:

- 891 • $\mathcal{O}_1 = (\mathcal{F}_6, \mathcal{S}_6, \{r_1\})$
- 892 • $\mathcal{O}_2 = (\mathcal{F}_6, \mathcal{S}_6, \{r_2\})$

893 For a system \mathcal{H} whose attack relation is in \mathfrak{N}_{s_3} either (i) $\text{Arg}(\mathcal{O}_1)$ or (ii) $\text{Arg}(\mathcal{O}_2)$ is its
894 unique stable extension. In case (i), $s \in \text{Output}(\mathcal{H})$ and $\neg s \notin \text{Output}(\mathcal{H})$.

895 In case (ii), $\neg s$ is the plausible conclusion. By the obvious symmetry (don't be misled by
896 negation!¹), either choice would be arbitrary, and this is an instance where an attack relation
897 from \mathfrak{N}_{s_2} is alright.

898 To sum up, attack relations satisfying the postulates can be split into two categories: \mathfrak{N}_{s_2}
899 and \mathfrak{N}_{s_3} . Relations from \mathfrak{N}_{s_2} make semi-stable semantics and preferred semantics to collapse
900 into stable semantics. They offer no added value with respect to the latter. Stable semantics
901 may, however, be *more valuable* than naive semantics. Indeed, the theories for which strict
902 precedence cannot be satisfied under naive semantics are handled correctly under stable
903 semantics. This latter can enforce the satisfaction of strict precedence if the attack relation
904 is defined in an appropriate way. For those theories where the postulate is satisfied, stable
905 semantics collapses into naive semantics. With attack relations from category \mathfrak{N}_{s_3} , pitfalls
906 threaten as preferred options are discarded, and a lot of care must be exercised when designing
907 such an argumentation system.

908 6.3 Preferred semantics

909 Preferred semantics was originally proposed in order to overcome the limitation of stable
910 semantics which does not guarantee the existence of extensions. Indeed, any argumentation
911 system has at least one preferred extension which may be empty. We show that in case of
912 rule-based systems the empty set cannot be an extension.

913 **Theorem 21** *Let \mathcal{H} be an argumentation system built over a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such
914 that \mathcal{H} satisfies the strict precedence postulate under preferred semantics. $\text{Ext}_p(\mathcal{H}) \neq \{\emptyset\}$.*

915 Unlike the cases of naive and stable extensions, a preferred extension may capture a proper
916 sub-part of a maximal preferred option. For instance, it is not impossible that a preferred
917 extension captures only the strict part of theory \mathcal{T}_6 in Example 7.

918 **Theorem 22** *Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
919 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates under preferred semantics. For
920 any $\mathcal{E} \in \text{Ext}_p(\mathcal{H})$, $\exists \mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$ and $\text{Concs}(\mathcal{E}) \subseteq \text{CN}(\mathcal{O})$.*

921 Each preferred extension corresponds to exactly one maximal preferred option. It either
922 returns all the consequences of that option, or chooses a subset. The latter contains all the
923 conclusions that follow from the strict part and some conclusions that follow using defeasible
924 rules. We show next that there is at least one maximal preferred option which is captured by
925 a preferred extension. This is mainly due to the fact that stable extensions exist.

¹ There is an apparent asymmetry between s and $\neg s$ but it is meaningless because we can choose an atom t to represent the intuitive statement formalized by $\neg s$ and then the intuitive statement formalized by s gets represented as $\neg t$. As an illustration about numbers, by letting *odd* instead of *even*, or vice versa, to be an atom of \mathcal{L} , asymmetry about negation could be reversed, while in both cases the meaning would be the same.

926 **Theorem 23** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
 927 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates under preferred
 928 semantics. There exists $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\text{Arg}(\mathcal{O}) \in \text{Ext}_p(\mathcal{H})$.

929 *Example 7 (Cont)* At least one of $\text{Arg}(\mathcal{O}_1)$ and $\text{Arg}(\mathcal{O}_2)$ is a preferred extension of an
 930 argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}_6), \mathcal{R})$ which satisfies the five postulates.

931 Two preferred extensions refer to different preferred options.

932 **Theorem 24** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system such that \mathcal{R} is conflict-
 933 dependent and \mathcal{H} satisfies exhaustiveness and closure under sub-arguments under preferred
 934 semantics. Let $\mathcal{E}, \mathcal{E}' \in \text{Ext}_p(\mathcal{H})$ and $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$. If $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$ and $\text{Th}(\mathcal{E}') \sqsubseteq \mathcal{O}$,
 935 then $\mathcal{E} = \mathcal{E}'$.

936 We show next that the *free* part of a theory, i.e., the sub-theory, which consists of the set
 937 of facts, the set of strict rules and the defeasible rules which are involved in every preferred
 938 option, is part of any preferred extension of argumentation systems that satisfy the postulates.
 939 Indeed, the set $\text{Arg}(\text{Free}(\mathcal{T}))$ is part of every preferred extension of any argumentation
 940 system which satisfies consistency, exhaustiveness, strict precedence and closure under sub-
 941 arguments.

942 **Theorem 25** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system over a theory $\mathcal{T} =$
 943 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent and privileges strict arguments (recall Defi-
 944 nition 16), and \mathcal{H} satisfies consistency, exhaustiveness, strict precedence and closure under
 945 sub-arguments under preferred semantics.

946
$$\text{Arg}(\text{Free}(\mathcal{T})) \subseteq \bigcap_{\mathcal{E}_i \in \text{Ext}_p(\mathcal{H})} \mathcal{E}_i.$$

947 From the previous results, it follows that the number of preferred extensions does not
 948 exceed the number of maximal preferred options of the theory over which the system is built.

949 **Theorem 26** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 950 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates under preferred semantics.

951
$$1 \leq |\text{Ext}_p(\mathcal{H})| \leq |\text{Max}(\text{POpt}(\mathcal{T}))|$$

952 When a theory is finite, any argumentation system built over it has a finite number of
 953 preferred extensions.

954 **Corollary 8** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such
 955 that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates under preferred semantics. If
 956 \mathcal{T} is finite, then \mathcal{H} has a finite number of preferred extensions.

957 Let us now characterize the plausible conclusions that are drawn from a theory \mathcal{T} by an
 958 argumentation system \mathcal{H} satisfying the rationality postulates under preferred semantics. Let
 959 \mathfrak{R}_p be the set of *all* attack relations that ensure the postulates under preferred semantics:

960
$$\mathfrak{R}_p = \{\mathcal{R} \subseteq \text{Arg}(\mathcal{T}) \times \text{Arg}(\mathcal{T}) \mid \mathcal{R} \text{ is conflict-dependent and } (\text{Arg}(\mathcal{T}), \mathcal{R})$$

 961
$$\text{satisfies the five postulates under preferred semantics}\}.$$

962 In his seminal paper [1], Dung has shown that the stable extensions of an argumentation
 963 system are also preferred extensions of the system. Consequently, the set \mathfrak{R}_p is a subset of
 964 \mathfrak{R}_s .

965 **Property 16** $\mathfrak{R}_p \subseteq \mathfrak{R}_s$.

966 Then, \mathfrak{R}_p contains three *disjoint* subsets of attack relations: $\mathfrak{R}_p = \mathfrak{R}_{p_1} \cup \mathfrak{R}_{p_2} \cup \mathfrak{R}_{p_3}$:

- 967 • \mathfrak{R}_{p_1} : the relations which are in $\mathfrak{R}_p \cap \mathfrak{R}_{s_1}$.
- 968 • \mathfrak{R}_{p_2} : the relations which are in $\mathfrak{R}_p \cap \mathfrak{R}_{s_2}$.
- 969 • \mathfrak{R}_{p_3} : the relations which are in $\mathfrak{R}_p \cap \mathfrak{R}_{s_3}$.

970 Let us analyze each category of attack relations separately. The first set is empty (i.e.,
971 $\mathfrak{R}_{p_1} = \emptyset$) since we have shown previously that there is no attack relation which prevents an
972 argumentation system from having stable extensions ($\mathfrak{R}_{s_1} = \emptyset$).

973 Attack relations of category \mathfrak{R}_{p_2} lead to coherent argumentation systems (i.e., $\text{Ext}_s(\mathcal{H}) =$
974 $\text{Ext}_p(\mathcal{H})$) as shown in Theorem 17. Thus, preferred semantics does not provide an added
975 value with respect to stable semantics. Moreover, there is a bijection between the two sets:
976 $\text{Ext}_p(\mathcal{H})$ and $\text{Max}(\text{POpt}(T))$.

977 **Corollary 9** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system over a theory T such that
978 $\mathcal{R} \in \mathfrak{R}_{p_2}$.

- 979 • For any $\mathcal{E} \in \text{Ext}_p(\mathcal{H})$, $\exists \mathcal{O} \in \text{Max}(\text{POpt}(T))$ such that $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$ and
980 $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$.
- 981 • For any $\mathcal{O} \in \text{Max}(\text{POpt}(T))$, $\text{Arg}(\mathcal{O}) \in \text{Ext}_p(\mathcal{H})$.
- 982 • For any $\mathcal{O} \in \text{Max}(\text{POpt}(T))$, $\mathcal{O} = \text{Option}(\text{Arg}(\mathcal{O}))$.

983 In the case of attack relations of category \mathfrak{R}_{p_2} , $\text{Arg}(\text{Free}(T))$ is equal to the intersection
984 of all preferred extensions.

985 **Theorem 27** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system over a theory T . If $\mathcal{R} \in$
986 \mathfrak{R}_{p_2} , then

$$987 \quad \text{Arg}(\text{Free}(T)) = \bigcap_{\mathcal{E}_i \in \text{Ext}_x(\mathcal{H})} \mathcal{E}_i$$

988 where $x \in \{p, s, ss\}$.

989 The output of an argumentation system is in this case the same as under stable semantics,
990 i.e., the plausible conclusions given in Theorem 19.

991 Let us now analyze attack relations of category \mathfrak{R}_{p_3} . Remember that in this case stable
992 semantics chooses only some maximal preferred options of the theory at hand. Four situations
993 may be encountered:

- 994 1. The stable extensions and the preferred extensions of an argumentation system coincide.
995 Thus, preferred semantics has no added value with respect to stable semantics. Moreover,
996 it may lead to arbitrary results as discussed in the previous subsection when $\mathcal{R} \in \mathfrak{R}_{s_3}$
997 (see Example 7 where one of the defeasible rules is chosen in an arbitrary way).
- 998 2. The preferred extensions consider additional but *not all* maximal preferred options (other
999 than the ones chosen by stable semantics). This case is similar to the previous one, and the
1000 argumentation system may return arbitrary results. Note that Example 7 is not sufficient
1001 to show this case since stable semantics will return one of \mathcal{O}_1 and \mathcal{O}_2 while preferred
1002 semantics will return the second one, which corresponds more to the case above. In order
1003 to exemplify this case, consider the following theory \mathcal{T}_7 .

1004 *Example 8* Consider \mathcal{T}_7 such that $\mathcal{F}_7, \mathcal{S}_7, \mathcal{D}_7$ are as follows:

$$1005 \quad \mathcal{F}_7 \begin{cases} p \\ t \end{cases} \quad \mathcal{S}_7 = \emptyset \quad \mathcal{D}_7 \begin{cases} p \Rightarrow q & (r_1) \\ q \Rightarrow s & (r_2) \\ t \Rightarrow \neg s & (r_3) \\ t \Rightarrow \neg q & (r_4) \end{cases}$$

1006 The theory \mathcal{T}_7 has three maximal preferred options:

- 1007 • $\mathcal{O}_1 = (\mathcal{F}_7, \mathcal{S}_7, \{r_1, r_2\})$
- 1008 • $\mathcal{O}_2 = (\mathcal{F}_7, \mathcal{S}_7, \{r_1, r_3\})$
- 1009 • $\mathcal{O}_3 = (\mathcal{F}_7, \mathcal{S}_7, \{r_3, r_4\})$

1010 Assume that $\text{Arg}(\mathcal{O}_1)$ is the single stable extension of an argumentation system $\mathcal{H} =$
 1011 $(\text{Arg}(\mathcal{T}_7), \mathcal{R})$ which satisfies the five postulates. Thus, $\text{Arg}(\mathcal{O}_1)$ is also a preferred extension
 1012 of \mathcal{H} . Case 2 suggests either $\text{Arg}(\mathcal{O}_2)$ or $\text{Arg}(\mathcal{O}_3)$ (not both) is another preferred extension.
 1013 Thus, as in Example 7, some rules are discarded in an arbitrary way.

- 1014 3. The preferred extensions return *all* the maximal preferred options of the theory. This
 1015 means that stable semantics chooses *some* maximal preferred options and preferred
 1016 semantics considers the remaining ones. This case coincides exactly with the case of
 1017 attack relations of category \mathfrak{R}_{p_2} (see Theorem 19). Indeed, the argumentation system
 1018 returns all the conclusions that follow from all maximal preferred options of the theory.
 1019 Note that this output is also ensured by stable semantics when $\mathcal{R} \in \mathfrak{R}_{s_2}$.
- 1020 4. Some of the preferred extensions provide proper sub-parts of maximal preferred options.
 1021 In this case, the result of the argumentation system may be arbitrary as can be seen on
 1022 the following example.

1023 *Example 8 (Cont)* Consider an argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}_7), \mathcal{R})$ such that $\mathcal{R} \in$
 1024 \mathfrak{R}_{p_3} . Assume that \mathcal{H} has two preferred extensions: \mathcal{E}_1 and \mathcal{E}_2 . From Theorem 23, one of
 1025 them captures necessarily a maximal preferred option. Let \mathcal{E}_1 be such extension, and let
 1026 $\text{Option}(\mathcal{E}_1) = \mathcal{O}_1$. Case 4 suggests that there is at least another preferred extension, say
 1027 \mathcal{E}_2 such that $\text{Option}(\mathcal{E}_2) \sqsubset \mathcal{O}_i$ ($i = 2, 3$). Assume that $i = 2$ and $\text{Th}(\mathcal{E}_2) = (\mathcal{F}_7, \mathcal{S}_7, \{r_3\})$.
 1028 Note that since preferred extensions are maximal for set inclusion, it cannot be the case that
 1029 $\text{Th}(\mathcal{E}_2) = (\mathcal{F}_7, \mathcal{S}_7, \{r_1\})$ (since \mathcal{E}_2 would be a subset of \mathcal{E}_1). One can notice that among the
 1030 four rules, r_4 is not used, which is unjustified.

1031 To sum up, attack relations of category \mathfrak{R}_{p_3} may lead either to *arbitrary* results or to
 1032 results which can be provided by stable semantics.

1033 6.4 Grounded: ideal semantics

1034 This section analyzes the outcomes of rule-based systems under grounded and ideal seman-
 1035 tics. Recall that both semantics ensure only one extension, which may be empty, for an
 1036 argumentation system. Moreover, the grounded extension $\text{GE}(\mathcal{H})$ of an argumentation sys-
 1037 tem \mathcal{H} is a sub-part of the ideal extension $\text{IE}(\mathcal{H})$ of the same system. Consequently, the
 1038 conclusions supported by the former are also supported by the latter, i.e., $\text{Concs}(\text{GE}(\mathcal{H})) \subseteq$
 1039 $\text{Concs}(\text{IE}(\mathcal{H}))$. Note also that the output set of an argumentation system is exactly
 1040 $\text{Concs}(\text{GE}(\mathcal{H}))$ (respectively $\text{Concs}(\text{IE}(\mathcal{H}))$) in case of grounded (respectively ideal)
 1041 semantics. Before presenting the formal results, it is worth mentioning that an argumentation
 1042 system that satisfies the postulates under preferred semantics does not necessarily satisfy the
 1043 postulates under grounded/ideal semantics. Similarly, a system that satisfies the postulates

under ideal semantics may violate some of the postulates under grounded semantics. That is why in the following we study each semantics separately.

The *ideal* extension, introduced in [19], is a maximal (for set inclusion) admissible set that is a subset of each preferred extension. In case of a rule-based argumentation system which satisfies the postulates, it returns a sub-part of one of the preferred options of the theory over which the system is built. Formally:

Theorem 28 *If an argumentation system \mathcal{H} satisfies the five postulates under ideal semantics, then there exists a preferred option $\mathcal{O} \in \text{POpt}(T)$ such that $\text{Th}(\text{IE}(\mathcal{H})) \subseteq \mathcal{O}$ and $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Concs}(\text{IE}(\mathcal{H})) \subseteq \text{CN}(\mathcal{O})$.*

Note that the outcome under ideal semantics may be arbitrary. This is in particular the case when the semantics selects one preferred option and draws all the conclusions that follow from this option. However, when the attack relation is of category \mathfrak{R}_{p_2} and privileges strict arguments (recall Definition 16), then the ideal extension is exactly the set $\text{Arg}(\text{Free}(T))$.

Theorem 29 *If an argumentation system \mathcal{H} satisfies the five postulates under ideal semantics and $\mathcal{R} \in \mathfrak{R}_{p_2}$ and privileges strict arguments, then $\text{IE}(\mathcal{H}) = \text{Arg}(\text{Free}(T))$.*

The above result shows that ideal semantics allows the inference of literals only from the free part of a theory.

Corollary 10 *If an argumentation system \mathcal{H} satisfies the five postulates under ideal semantics and $\mathcal{R} \in \mathfrak{R}_{p_2}$ and privileges strict arguments, then $\text{Output}(\mathcal{H}) = \text{CN}(\text{Free}(T))$.*

Note that in this case grounded extension may be more cautious than ideal one and may miss intuitive (free) conclusions since $\text{GE}(\mathcal{H}) \subseteq \text{Arg}(\text{Free}(T))$.

The grounded extension of any argumentation system which satisfies the postulates under grounded semantics captures a sub-part of a preferred option, i.e., it behaves exactly like ideal extension.

Theorem 30 *If an argumentation system \mathcal{H} satisfies the five postulates under grounded semantics, then there exists a preferred option $\mathcal{O} \in \text{POpt}(T)$ such that $\text{Th}(\text{GE}(\mathcal{H})) \subseteq \mathcal{O}$ and $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Concs}(\text{GE}(\mathcal{H})) \subseteq \text{CN}(\mathcal{O})$.*

7 Related work

The abstract argumentation framework proposed by Dung [1] was used for reasoning about defeasible information, and more generally for handling inconsistency. It was thus instantiated in different ways, considering different logical languages for representing information. Examples of such languages are propositional language (e.g., [2, 3]) and rule-based ones (e.g., [8, 9, 11, 27, 29, 33]).

All the instantiations are defined in a similar way: define arguments and attacks, then apply Dung's semantics on the defined graph, and infer the formulas that follow from all extensions. Some of these works are incomplete since there is one important step which is missing: *characterizing the set of inferences* that are drawn from a theory/knowledge base, i.e., describing formally how the output relates to the theory.

For filling this gap, in [4], we considered argumentation systems that use Tarskian logics, covering thus the systems studied in [2, 3]. In [11, 33], we focused on rule-based logics. Here we faced two issues: First, the logical languages that are considered in the literature

are different. In ASPIC [8], defeasible rules express defaults and any uncertain information. In ABA [27], uncertain information is encoded by assumptions. In ASPIC+ [9], several types of information are considered (axioms, ordinary premises, issues, assumptions, strict rules and defeasible rules). The differences between all these types are unclear, especially between strict rules and axioms (both represent certain and non-defeasible information), and between ordinary premises, assumptions and defeasible rules (which all represent uncertain and defeasible information). Consequently, we have chosen the logical language used in the ASPIC system [8, 17]. It considers facts and strict rules (for encoding strict information) and defeasible rules (for encoding assumptions, defeasible rules, ordinary premises). Another issue with rule-based argumentation systems is that there are two types of attack relations: inconsistency-based ones and undercut which amounts to blocking the application of defeasible rules. For a better understanding of each type of attack relation, we studied in [11] argumentation systems that use undercut as their sole attack relation, and in this paper we studied the impact of inconsistency-based ones.

Our formalism uses the same logical language as ASPIC and a more general inconsistency-based attack relation. Our results apply thus to ASPIC when its undercut relation is empty. Note that our results and those from [11] should be combined for characterizing the outcomes of the ASPIC system when it uses the two kinds of relations. This is left for future work.

ASPIC+ uses a “richer” logical language since its aim was to unify all existing argumentation systems. It can thus be seen a union of several elementary systems: ABA for dealing with assumptions, ASPIC for dealing with strict/defeasible information, and the systems defined in [4] for dealing with Tarskian logics. In [4], we have characterized this sub-class of ASPIC+. In this paper, we characterized the sub-class capturing ASPIC.

The last well-known argumentation system, called ABA, cannot be compared to our formalism since the two systems use different logical languages. While ABA uses assumptions for capturing the defeasible information in a theory, our formalism uses defeasible rules.

8 Conclusion

The paper provides the first investigation on the outputs of rule-based argumentation systems that use inconsistency-based attack relations. The study is general in the sense that it keeps the attack relation unspecified. Thus, the system can be instantiated with any of the attack relations that are used in existing systems. The results show that under naive semantics, the systems return the literals that follow from all the options of the theory at hand. Stable and preferred semantics either do not provide an added value with respect to naive semantics or the attack relation of a system should be formalized in a very rigorous way in order to avoid arbitrary results. Ideal semantics returns the free part of a theory, whereas the grounded semantics returns a sub-part of the free part meaning that it may miss interesting conclusions.

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Appendix: Proofs

Proof of Property 2 Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory and $x \in \mathcal{L}$. Let $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ be a derivation schema for x from \mathcal{T} .

(\rightarrow) Let us assume that there exist x_i and x_j such that $x_i = x_j$ but $i \neq j$. Clearly, we can further assume $i < j$ without loss of generality. For each (x_k, r_k) in d where $k > j$, $x_j \in \text{Body}(r_k)$ is trivially equivalent to $x_i \in \text{Body}(r_k)$ hence $\text{Body}(r_k) \subseteq \{x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_{k-1}\}$. Therefore, $\langle (x_1, r_1), \dots, (x_{j-1}, r_{j-1}), (x_{j+1}, r_{j+1}), \dots, (x_n, r_n) \rangle$ is also a derivation schema, but it is a proper subsequence of d , a contradiction arises. Now, let us assume that d fails to be focussed. There exists $i \in \{1, \dots, n-1\}$ such that $x_i \notin \text{Body}(r_j)$ for every $j > i$. Consequently, $\langle (x_1, r_1), \dots, (x_{i-1}, r_{i-1}), (x_{i+1}, r_{i+1}), \dots, (x_n, r_n) \rangle$ is also a derivation schema for x in \mathcal{T} , contradicting the minimality of d .

(\leftarrow) Let us assume that d fails to be minimal although d is focussed and the literals x_1, \dots, x_n are pairwise distinct. As d is not minimal, there exists a proper subsequence d' of d which is a derivation schema for x in \mathcal{T} . Let us write $\langle (x_{k+1}, r_{k+1}), \dots, (x_n, r_n) \rangle$ for the largest common final subsequence of d and d' . Now, k exists (and $k > 0$) because d' is a proper subsequence of d . As d' is a derivation schema for x_n and d' is a subsequence of d and x_1, \dots, x_n are pairwise distinct, $k < n$ ensues. Since d is focussed, $x_k \in \text{Body}(r_j)$ for some $j > k$. So, (x_j, r_j) is in $\langle (x_{k+1}, r_{k+1}), \dots, (x_n, r_n) \rangle$. As d' is a derivation schema, $\text{Body}(r_j) \subseteq \{x_1, \dots, x_{k-1}\}$ (remember, d' is a subsequence of $\langle (x_1, r_1), \dots, (x_{k-1}, r_{k-1}), (x_{k+1}, r_{k+1}), \dots, (x_n, r_n) \rangle$). Hence, $x_k \in \{x_1, \dots, x_{k-1}\}$. That is, x_1, \dots, x_n are not pairwise distinct. \square

Proof of Property 3 Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory.

- The inclusions $\text{CN}(\mathcal{T}) \subseteq \mathcal{F} \cup \{\text{Head}(r) \mid r \in \mathcal{S} \cup \mathcal{D}\} \subseteq \mathcal{L}$ follow trivially from Definition 6.
- If \mathcal{T} is finite, then $\mathcal{F}, \mathcal{S}, \mathcal{D}$ are finite. Thus, the set $\mathcal{F} \cup \{\text{Head}(r) \mid r \in \mathcal{S} \cup \mathcal{D}\}$ is finite. From the first item, $\text{CN}(\mathcal{T})$ is finite.
- For any $x \in \mathcal{F}$, the sequence $\langle (x, \sigma) \rangle$ is a derivation schema for x from \mathcal{T} . Thus, $x \in \text{CN}(\mathcal{T})$ and this proves the inclusion $\mathcal{F} \subseteq \text{CN}(\mathcal{T})$.
- $\top \in \text{CN}(\mathcal{T})$ since $\top \in \mathcal{F}$ and $\mathcal{F} \subseteq \text{CN}(\mathcal{T})$.
- Assume that $\mathcal{F} = \{\top\}$ and $\nexists r \in \mathcal{D}$ such that $\text{Body}(r) = \{\top\}$. Thus, since the body of any other rule in \mathcal{T} is assumed to be non-empty, no rule in $\mathcal{S} \cup \mathcal{D}$ can be applied, hence $\text{CN}(\mathcal{T}) = \{\top\}$. Conversely, if $\text{CN}(\mathcal{T}) = \{\top\}$, then $\mathcal{F} = \{\top\}$ (since $\mathcal{F} \subseteq \text{CN}(\mathcal{T})$) and $\nexists r \in \mathcal{D}$ such that $\text{Body}(r) = \{\top\}$ (since each such rule is applicable when it exists).
- Let $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ be a derivation schema for $x \in \mathcal{L}$ from \mathcal{T} . From Definition 6, for each x_i ($i = 1, \dots, n$), there exists a derivation schema from \mathcal{T} for x_i . Thus, $\text{Seq}(d) \subseteq \text{CN}(\mathcal{T})$. \square

Proof of Property 4 Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ and $\mathcal{T}' = (\mathcal{F}', \mathcal{S}', \mathcal{D}')$ be two theories such that $\mathcal{T} \sqsubseteq \mathcal{T}'$. Let $x \in \text{CN}(\mathcal{T})$. So, there exists a derivation schema $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ for x from \mathcal{T} . Since $\mathcal{T} \sqsubseteq \mathcal{T}'$, $\text{Facts}(d) \subseteq \mathcal{F}'$ and $\text{Strict}(d) \subseteq \mathcal{S}'$ and $\text{Def}(d) \subseteq \mathcal{D}'$. Therefore, d is also a derivation schema for x from \mathcal{T}' . \square

Proof of Property 5 The two properties follow trivially from the definition of option. \square

Proof of Property 6 The inclusion $\text{POpt}(\mathcal{T}) \subseteq \text{Opt}(\mathcal{T})$ follows trivially from Definitions 9 and 10.

Assume that $\text{CN}(\mathcal{T})$ is consistent. From Property 5, $\text{Opt}(\mathcal{T}) = \{\mathcal{T}\}$. Since $(\mathcal{F}, \mathcal{S}, \emptyset) \sqsubseteq \mathcal{T}$, $\text{POpt}(\mathcal{T}) = \{\mathcal{T}\}$. Assume now that $\text{Opt}(\mathcal{T}) \subseteq \text{POpt}(\mathcal{T})$. Since $\text{Opt}(\mathcal{T}) \neq \emptyset$, for all $\mathcal{O} \in \text{Opt}(\mathcal{T})$ it holds that $\mathcal{O} \in \text{POpt}(\mathcal{T})$. Thus, for all $\mathcal{O} \in \text{Opt}(\mathcal{T})$, $(\mathcal{F}, \mathcal{S}, \emptyset) \sqsubseteq \mathcal{O}$. It

1169 follows that $(\mathcal{F}, \mathcal{S}, \emptyset) \sqsubseteq \text{Free}(T)$.² Assume that $\text{CN}(T)$ is inconsistent. Then, there exists
 1170 a minimal conflict² $C = (X, Y, Z) \sqsubseteq T$. Since $(\mathcal{F}, \mathcal{S}, \emptyset) \sqsubseteq \text{Free}(T)$, $X = Y = \emptyset$. But,
 1171 by assumption, the body of every defeasible rule is not empty. Thus, $\text{CN}(C) = \emptyset$. This
 1172 contradicts the fact that $\text{CN}(C)$ is inconsistent. \square

1173 *Proof of Property 7* Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory.

1174 Assume that $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is consistent. Thus, there exists a preferred option \mathcal{O} such that
 1175 either (i) for all $r \in \mathcal{D}$, $\text{CN}((\mathcal{F}, \mathcal{S}, \{r\}))$ is inconsistent meaning that $\mathcal{O} = \text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ or
 1176 (ii) there exists $r \in \mathcal{D}$ such that $\text{CN}((\mathcal{F}, \mathcal{S}, \{r\}))$ is consistent thus $(\mathcal{F}, \mathcal{S}, \emptyset) \sqsubset \mathcal{O}$. In both
 1177 cases, $\text{POpt}(\mathcal{T}) \neq \emptyset$. Assume now that $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is inconsistent. Since \mathcal{F} and \mathcal{S} should
 1178 be part of any preferred option and the set of consequences of a preferred option should be
 1179 consistent, then $\text{POpt}(\mathcal{T}) = \emptyset$.

1180 Let $r \in \mathcal{D}$ and assume that $\text{CN}((\mathcal{F}, \mathcal{S}, \{r\}))$ is consistent. From Definition 10, $(\mathcal{F}, \mathcal{S}, \{r\})$ is
 1181 either a preferred option (iff for all $r' \in \mathcal{D}$ such that $r \neq r'$, $\text{CN}((\mathcal{F}, \mathcal{S}, \{r, r'\}))$ is inconsistent).
 1182 Or, there exists a preferred option $\mathcal{O} = (\mathcal{F}, \mathcal{S}, \{r\} \cup \mathcal{D}')$ where $\mathcal{D}' \subseteq \mathcal{D} \setminus \{r\}$. \square

1183 *Proof of Property 8* Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory and $\text{Free}(T) = (\mathcal{F}, \mathcal{S}, \mathcal{D}')$. From
 1184 the definition of $\text{Free}(T)$, $\text{Free}(T) \sqsubseteq \mathcal{O}$ for all $\mathcal{O} \in \text{POpt}(T)$. From Property 4,
 1185 $\text{CN}(\text{Free}(T)) \subseteq \text{CN}(\mathcal{O})$. Since $\text{CN}(\mathcal{O})$ is consistent, then so is for $\text{CN}(\text{Free}(T))$. \square

1186 *Proof of Property 9* Let (d, x) be a sub-argument of (d', x') . Let $x_i \in \text{Seq}(d)$. There are
 1187 two possibilities:

- 1188 • $x_i \in \text{Facts}(d)$, thus $x_i \in \text{Facts}(d')$ since $\text{Facts}(d) \subseteq \text{Facts}(d')$. So, $x_i \in$
 1189 $\text{Seq}(d')$.
- 1190 • $x_i = \text{Head}(r)$ with $r \in \text{Strict}(d) \cup \text{Def}(d)$; thus, $r \in \text{Strict}(d') \cup \text{Def}(d')$ since
 1191 $\text{Strict}(d) \subseteq \text{Strict}(d')$ and $\text{Def}(d) \subseteq \text{Def}(d')$. So, $x_i \in \text{Seq}(d')$.

1192 \square

1193 *Proof of Property 10* Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ be a theory. Since $\top \in \mathcal{F}$ by Definition 4,
 1194 $((\top, \sigma), \top) \in \text{Arg}(T)$ and thus $\text{Arg}(T) \neq \emptyset$. \square

1195 *Proof of Property 11* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system over a theory
 1196 \mathcal{T} and $\text{Ext}(\mathcal{H})$ its set of extensions under any extension-based semantics. Assume that
 1197 $\text{Ext}(\mathcal{H}) \neq \emptyset$.

1198 Let $x \in \text{Output}(\mathcal{H})$. Thus, for all $\mathcal{E} \in \text{Ext}(\mathcal{H})$, $\exists a \in \mathcal{E}$ such that $\text{Conc}(a) = x$. It
 1199 follows that $x \in \text{Concs}(\mathcal{E}_i)$, $\forall \mathcal{E}_i \in \text{Ext}(\mathcal{H})$ and hence $x \in \bigcap_{\mathcal{E}_i \in \text{Ext}(\mathcal{H})} \text{Concs}(\mathcal{E}_i)$.

1200 Assume now that $x \in \bigcap_{\mathcal{E}_i \in \text{Ext}(\mathcal{H})} \text{Concs}(\mathcal{E}_i)$. Thus, $\forall \mathcal{E}_i, \exists a_i \in \mathcal{E}_i$ such that $\text{Conc}(a_i) =$
 1201 x . Consequently, $x \in \text{Output}(\mathcal{H})$. \square

1202 *Proof of Property 12* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1203 \mathcal{T} . Let $x \in \text{Output}(\mathcal{H})$. From Definition 17, $\exists (d, x) \in \text{Arg}(T)$. From Definition 12, d
 1204 is a derivation for x from \mathcal{T} . Thus, $x \in \text{CN}(T)$. \square

1205 *Proof of Property 16* Let $\mathcal{R} \in \mathfrak{R}_p$ and let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be a rule-based argumentation
 1206 system built over a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. Since \mathcal{H} satisfies the five postulates, thus for all
 1207 $\mathcal{E} \in \text{Ext}_p(\mathcal{H})$,

² Let \mathcal{T} be a theory. $\text{Free}(T)$ is a sub-theory (X, Y, Z) of \mathcal{T} such that for all minimal conflict $C = (X', Y', Z')$
 of \mathcal{T} , it holds that $X \cap X' = \emptyset$ and $Y \cap Y' = \emptyset$ and $Z \cap Z' = \emptyset$. A minimal conflict of theory \mathcal{T} is a sub-theory
 C of \mathcal{T} such that $\text{CN}(C)$ is inconsistent and $\nexists C' \sqsubset C$ such that $\text{CN}(C')$ is inconsistent.

- 1208 • $\text{Concs}(\mathcal{E})$ is consistent
- 1209 • $\text{Concs}(\mathcal{E}) = \text{CN}(\text{Concs}(\mathcal{E}), \mathcal{S}, \emptyset)$
- 1210 • For all $a \in \mathcal{E}$, $\text{Sub}(a) \subseteq \mathcal{E}$
- 1211 • for all $(d, x) \in \text{Arg}(\mathcal{T})$, if $\text{Seq}(d) \subseteq \text{Concs}(\mathcal{E})$, then $(d, x) \in \mathcal{E}$.

1212 From Property 1, $\text{Ext}_s(\mathcal{H}) \subseteq \text{Ext}_p(\mathcal{H})$, then for all $\mathcal{E} \in \text{Ext}_s(\mathcal{H})$, \mathcal{E} satisfies the above
 1213 four properties. Thus, \mathcal{H} satisfies consistency, exhaustiveness and closure under both sub-
 1214 arguments and strict rules. Let us now show that it also satisfies strict precedence under
 1215 stable semantics. From Property 11, $\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{E}_i \in \text{Ext}_p(\mathcal{H})} \text{Concs}(\mathcal{E}_i)$. Thus, for
 1216 all $\mathcal{E} \in \text{Ext}_p(\mathcal{H})$, $\text{Output}(\mathcal{H}) \subseteq \text{Concs}(\mathcal{E})$. Since \mathcal{H} satisfies strict precedence under
 1217 preferred semantics, $\text{CN}(\mathcal{F}, \mathcal{S}, \emptyset) \subseteq \text{Concs}(\mathcal{E})$. Thus, the property is satisfied by every
 1218 stable extension. \square

1219 *Proof of Proposition 1* Let $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. Let $\mathcal{O}, \mathcal{O}' \in \text{opt}(\mathcal{T})$ be such that $\text{CN}(\mathcal{O}) =$
 1220 $\text{CN}(\mathcal{O}')$. Let $\mathcal{O} = (X, Y, Z)$ and $\mathcal{O}' = (X', Y', Z')$. For all $x \in X$, $x \in \text{CN}(\mathcal{O})$ and thus
 1221 $x \in X'$. The same holds for all $x' \in X'$. Thus, $X = X'$.

1222 Let $r \in Y \cup Z$. There are two cases: (i) $\text{Body}(r) \not\subseteq \text{CN}(\mathcal{O})$. Consequently, $\text{Body}(r) \not\subseteq$
 1223 $\text{CN}(\mathcal{O}')$. Thus, $\text{CN}(\mathcal{O}' \oplus r)$ is consistent. So, $r \in Y' \cup Z'$ (by definition of an option).

1224 (ii) $\text{Body}(r) \subseteq \text{CN}(\mathcal{O})$. Consequently, $\text{Body}(r) \subseteq \text{CN}(\mathcal{O}')$. Thus, $\text{CN}(\mathcal{O}' \oplus r)$ is consis-
 1225 tent. So, $r \in Y' \cup Z'$ (by definition of an option). \square

1226 *Proof of Proposition 2* If \mathcal{T} is finite, then $\text{CN}(\mathcal{T})$ is finite (apply Property 3). Consequently,
 1227 $\text{Arg}(\mathcal{T})$ is finite. \square

1228 *Proof of Proposition 3* Let \mathcal{T} and \mathcal{T}' be two theories such that $\mathcal{T} \sqsubseteq \mathcal{T}'$. Let (d, x) be an
 1229 argument defined from \mathcal{T} . All items in Definition 12 are independent from \mathcal{T} except for d
 1230 being a derivation schema for x from \mathcal{T} . Hence, for (d, x) to be an argument defined from
 1231 \mathcal{T}' , it is enough that d be a derivation schema for x from \mathcal{T}' . Now, this is equivalent to
 1232 $x \in \text{CN}(\mathcal{T}')$. By Property 4, the latter follows from $x \in \text{CN}(\mathcal{T})$ (which is itself proved from
 1233 the fact that d is a derivation schema for x from \mathcal{T}). Thus, (d, x) is an argument defined from
 1234 \mathcal{T}' . \square

1235 *Proof of Proposition 4* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system such that \mathcal{R} is
 1236 conflict-dependent. Let $a = (d, x) \in \text{Arg}(\mathcal{T})$ be such that $(a, a) \in \mathcal{R}$. Since \mathcal{R} is conflict-
 1237 dependent, $\text{Seq}(d)$ is inconsistent. This is impossible since a is an argument (thus $\text{Seq}(d)$
 1238 should be consistent). \square

1239 *Proof of Proposition 5* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a
 1240 theory \mathcal{T} such that $\text{CN}(\mathcal{T})$ is consistent and \mathcal{R} is conflict-dependent. Assume that $\text{Arg}(\mathcal{T})$
 1241 is not conflict-free. Thus, there exist $(d, x), (d', x') \in \text{Arg}(\mathcal{T})$ such that $(d, x)\mathcal{R}(d', x')$.
 1242 Consequently, $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent. Besides, from Property 3, $\text{Seq}(d) \subseteq \text{CN}(\mathcal{T})$
 1243 and $\text{Seq}(d') \subseteq \text{CN}(\mathcal{T})$. Thus, $\text{CN}(\mathcal{T})$ is inconsistent. Contradiction. \square

1244 *Proof of Proposition 6* Let \mathcal{H} be an argumentation system which satisfies consistency and
 1245 closure under sub-arguments. From Proposition 12, $\forall \mathcal{E} \in \text{Ext}(\mathcal{H}) \text{Concs}(\mathcal{E}) = \text{CN}(\text{Th}(\mathcal{E}))$.
 1246 Since \mathcal{H} satisfies consistency, $\forall \mathcal{E} \in \text{Ext}(\mathcal{H}) \text{Concs}(\mathcal{E})$ is consistent. Thus, so is for
 1247 $\text{CN}(\text{Th}(\mathcal{E}))$. \square

1248 *Proof of Proposition 7* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1249 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\emptyset \in \text{Ext}(\mathcal{H})$. Thus, $\text{Output}(\mathcal{H}) = \emptyset$. Assume that \mathcal{H} satisfies
 1250 strict precedence, then $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Output}(\mathcal{H})$. Since $\top \in \mathcal{F}$ and from Property 3, it
 1251 holds that $\mathcal{F} \subseteq \text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$, then $\top \in \text{Output}(\mathcal{H})$. \square

1252 *Proof of Proposition 8* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a
 1253 theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. Assume that \mathcal{H} satisfies consistency and strict precedence. From
 1254 Property 13, it holds that $\text{Output}(\mathcal{H})$ is consistent. From strict precedence, $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$
 1255 $\subseteq \text{Output}(\mathcal{H})$. Thus, $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is consistent. \square

1256 *Proof of Proposition 9* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system such that
 1257 $\text{Ext}(\mathcal{H}) \neq \emptyset$ (under an extension-based semantics). Assume that \mathcal{H} is closed under
 1258 sub-arguments and satisfies exhaustiveness. Let $\mathcal{E} \in \text{Ext}(\mathcal{H})$. From the monotonicity of
 1259 Arg , it holds that $\mathcal{E} \subseteq \text{Arg}(\text{Th}(\mathcal{E}))$. Let $(d, x) \in \text{Arg}(\text{Th}(\mathcal{E}))$. From Proposition 12,
 1260 $\text{Seq}(d) \subseteq \text{Concs}(\mathcal{E})$. From the exhaustiveness postulate, $(d, x) \in \mathcal{E}$. \square

1261 *Proof of Proposition 10* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system such that
 1262 $\text{Ext}(\mathcal{H}) \neq \emptyset$ (under an extension-based semantics). Assume that \mathcal{H} satisfies exhaustiveness
 1263 and strict precedence. Since \mathcal{H} satisfies strict precedence, $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Output}(\mathcal{H})$.
 1264 From Property 11, for all $\mathcal{E} \in \text{Ext}(\mathcal{H})$, $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Concs}(\mathcal{E})$. Let $(d, x) \in$
 1265 $\text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$. Thus, $\text{Seq}(d) \subseteq \text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$. From exhaustiveness, $(d, x) \in \mathcal{E}$. \square

1266 *Proof of Proposition 11* In order to prove the compatibility of the postulates, it is sufficient
 1267 to give an example of a system which satisfies all the five postulates. This system is ASPIC
 1268 as defined in [17]. Proposition 1 in [17] shows that the system is closed under sub-arguments
 1269 under any Dung's semantics. Proposition 8 in [17] shows that the system is closed under
 1270 strict rules under complete semantics, thus under stable semantics. Property 2 in [17] shows
 1271 that the system satisfies consistency under any Dung's semantics. From Proposition 13, the
 1272 system satisfies exhaustiveness. Let us now show that the system satisfies strict precedence.
 1273 This follows from the definition of attack relation (Definition 16 in [17]) according to which
 1274 a strict argument cannot be attacked. Thus, it belongs to any stable extension. \square

1275 *Proof of Proposition 12* Let \mathcal{H} be an argumentation system such that $\text{Ext}(\mathcal{H}) \neq \emptyset$ where
 1276 $\text{Ext}(\mathcal{H})$ is its set of extensions under an extension-based semantics. Assume that \mathcal{H} is closed
 1277 under sub-arguments and let $\mathcal{E} \in \text{Ext}(\mathcal{H})$ and $\text{Th}(\mathcal{E}) = (X, Y, Z)$.

- 1278 • Let $x \in \text{Concs}(\mathcal{E})$. Thus, $\exists(d, x) \in \mathcal{E}$ where d is a derivation for x from $(\text{Facts}(d),$
 1279 $\text{Strict}(d), \text{Def}(d))$. From Property 3, $x \in \text{Facts}(d)$ (thus $x \in X$), or $x = \text{Head}(r)$
 1280 where $r \in \text{Strict}(d)$ (thus $r \in Y$) or $x \in \text{Def}(d)$ (thus $x \in Z$).
 1281 Assume now that $x \in X$. Thus, $\exists(d, y) \in \mathcal{E}$ such that $x \in \text{Facts}(d)$. Besides,
 1282 $((x, \sigma), x)$ is a sub-argument of (d, y) . Since \mathcal{H} is closed under sub-arguments,
 1283 $((x, \sigma), x) \in \mathcal{E}$, and thus, $x \in \text{Concs}(\mathcal{E})$.
 1284 Let $r \in Y \cup Z$. Thus, $\exists(d, x) \in \mathcal{E}$ such that $r \in \text{Strict}(d) \cup \text{Def}(d)$. Let
 1285 $d = \langle (x_1, r_1), \dots, (x_i, r), (x_{i+1}, r_{i+1}), \dots, (x_n = x, r_n) \rangle$ with $x_i = \text{Head}(r)$. Thus,
 1286 there exists a sub-sequence d' of d which is a derivation for x_i . This derivation is minimal
 1287 (for set inclusion since (d, x) is an argument). Thus, (d', x_i) is an argument. Moreover,
 1288 it is a sub-argument of (d, x) . Since \mathcal{H} is closed under sub-arguments, $(d', x_i) \in \mathcal{E}$.
 1289 Consequently, $x_i \in \text{Concs}(\mathcal{E})$. Thus, $\text{Concs}(\mathcal{E}) = X \cup \{\text{Head}(r) \mid r \in Y \cup Z\}$.
- 1290 • From the definitions of the two functions Concs and Th , it follows that $\text{Concs}(\mathcal{E}) \subseteq$
 1291 $\text{CN}(\text{Th}(\mathcal{E}))$. From Property 3, $\text{CN}(\text{Th}(\mathcal{E})) \subseteq X \cup \{\text{Head}(r) \mid r \in Y \cup Z\}$. From above,
 1292 $\text{CN}(\text{Th}(\mathcal{E})) \subseteq \text{Concs}(\mathcal{E})$.
- 1293 • Assume now that $a = (d, x) \in \text{Arg}(\text{Th}(\mathcal{E}))$. For all $x_i \in \text{Seq}(d)$, $x_i \in \text{CN}(\text{Th}(\mathcal{E}))$.
 1294 Since \mathcal{H} is closed under sub-arguments, $\text{CN}(\text{Th}(\mathcal{E})) = \text{Concs}(\mathcal{E})$. Then, $x_i \in$
 1295 $\text{Concs}(\mathcal{E})$. Thus, $\text{Seq}(d) \subseteq \text{Concs}(\mathcal{E})$.

1296 \square

1297 *Proof of Proposition 13* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a
 1298 theory \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under
 1299 sub-arguments. Assume that \mathcal{H} violates exhaustiveness. Thus, there exists $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$
 1300 and there exists $a = (d, x) \in \text{Arg}(\mathcal{T})$ such that $\text{Seq}(d) \subseteq \text{Concs}(\mathcal{E})$ but $(d, x) \notin \mathcal{E}$. So,
 1301 $\exists b = (d', x') \in \mathcal{E}$ such that $a\mathcal{R}b$ or $b\mathcal{R}a$. Since \mathcal{R} is conflict-dependent, $\text{Seq}(d) \cup \text{Seq}(d')$
 1302 is inconsistent. Thus, $\exists y \in \text{Seq}(d)$ such that $\neg y \in d'$. But, $y, \neg y \in \text{CN}(\text{Th}(\mathcal{E}))$. Since \mathcal{H}
 1303 is closed under sub-arguments, $\text{CN}(\text{Th}(\mathcal{E})) = \text{Concs}(\mathcal{E})$. Thus, $y, \neg y \in \text{Concs}(\mathcal{E})$. This
 1304 contradicts the fact that \mathcal{H} satisfies consistency. \square

1305 *Proof of Proposition 14* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a
 1306 theory \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 1307 arguments. From Proposition 13, \mathcal{H} satisfies exhaustiveness. From Proposition 9, it follows
 1308 that for all $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$, $\mathcal{E} = \text{Arg}(\text{Th}(\mathcal{E}))$. \square

1309 *Proof of Proposition 15* The proof is similar to that of Propositions 13 and 14. \square

1310 *Proof of Theorem 1* Let \mathcal{T} be a theory and $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ a consistent sequence
 1311 from \mathcal{T} .

1312 (\rightarrow) Assume that (d, x) is an argument from \mathcal{T} . $d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$ yields
 1313 $x = x_n$ (Definition 12). Assume that d is not focused. Let d^* be obtained from d by deleting
 1314 all repeated pairs. Since d is not focused, d^* is not minimal. Therefore, there exists (x_k, r_k) in
 1315 d^* (hence in d) such that depriving d^* from (x_k, r_k) still gives a derivation of x from \mathcal{T} . Since
 1316 d^* contains no repeated pair, for every (x_i, r_i) in d^* , if $i \neq k$ then either $x_i \neq x_k$ or $r_i \neq r_k$.
 1317 For $r_k \neq \sigma$, the former implies the latter hence $r_i \neq r_k$ whenever $i \neq k$. Thus, depriving
 1318 d^* from (x_k, r_k) gives a derivation d' of x from \mathcal{T} such that $\text{Facts}(d') \subseteq \text{Facts}(d)$ and
 1319 $\text{Strict}(d') \subseteq \text{Strict}(d)$ and $\text{Def}(d') \subseteq \text{Def}(d)$, with one of the latter two inclusions
 1320 being strict. That is, there exists $T' \sqsubset (\text{Facts}(d), \text{Strict}(d), \text{Def}(d))$ such that $x \in$
 1321 $\text{CN}(T')$, thereby contradicting Definition 12. The remaining case is $r_k = \sigma$. Since d^* contains
 1322 no repeated pair, no (x_i, r_i) in d^* is (x_k, σ) except for $i = k$ and it follows that d^* deprived
 1323 from (x_k, r_k) gives a derivation d' of x from \mathcal{T} such that $\text{Facts}(d') \subset \text{Facts}(d)$ while
 1324 $\text{Strict}(d') \subseteq \text{Strict}(d)$ and $\text{Def}(d') \subseteq \text{Def}(d)$. As above, a contradiction arises.

1325 (\leftarrow) Assume that d is a focused derivation schema from \mathcal{T} such that $x_n = x$. By
 1326 the definitions, $x \in \mathcal{L}$ and d is a derivation schema for x from \mathcal{T} . Due to the hypoth-
 1327 esis in the statement of the theorem, $\text{Seq}(d)$ is consistent. Assume that there exists
 1328 $T' = (\mathcal{F}', \mathcal{S}', \mathcal{D}') \sqsubset (\text{Facts}(d), \text{Strict}(d), \text{Def}(d))$ such that $x \in \text{CN}(T')$. That is, there
 1329 exists a derivation $d' = \langle (x'_1, r'_1), \dots, (x'_m, r'_m) \rangle$ for some $m < n$ such that $\text{Facts}(d') = \mathcal{F}'$
 1330 and $\text{Strict}(d') = \mathcal{S}'$ and $\text{Def}(d') = \mathcal{D}'$. Let d^* be a minimal derivation schema for x from
 1331 \mathcal{T} obtained from d by deleting all repeated pairs. Accordingly, $\text{Facts}(d^*) = \text{Facts}(d)$ and
 1332 $\text{Strict}(d^*) = \text{Strict}(d)$ and $\text{Def}(d^*) = \text{Def}(d)$. Since $\text{Strict}(d') \subseteq \text{Strict}(d)$
 1333 and $\text{Def}(d') \subseteq \text{Def}(d)$, if (x'_i, r'_i) is in d' with $r'_i \neq \sigma$ then there exists k such that (x_k, r_k)
 1334 is in d^* where $r_k = r'_i$ (also, $x_k = x'_i$ because $x'_i = \text{Head}(r'_i) = \text{Head}(r_k) = x_k$). Similarly,
 1335 since $\text{Facts}(d') \subseteq \text{Facts}(d)$, if (x'_i, r'_i) is in d' with $r'_i = \sigma$ then there exists k such that
 1336 (x_k, r_k) is in d^* where $x'_i = x_k$ and $r_k = r'_i = \sigma$. That is, d' is a proper subsequence of a
 1337 reordering of d^* , thereby contradicting the minimality of d^* .

1338 Indeed, let us show that no initial proper fragment of a reordering d_i^* of d^* is a
 1339 minimal derivation of x . Assume a reordering $d_i^* = \langle (x_{i1}^*, r_{i1}^*), \dots, (x_{ip}^*, r_{ip}^*) \rangle$ of $d^* =$
 1340 $\langle (x_1^*, r_1^*), \dots, (x_p^*, r_p^*) \rangle$ such that $d_i = \langle (x_{i1}^*, r_{i1}^*), \dots, (x_{iq}^*, r_{iq}^*) \rangle$ is a minimal derivation of
 1341 x from \mathcal{T} for some $q < p$. Let j be the greatest index from $1 \dots p$ such that x_j^* is in $\text{Seq}(d^*)$
 1342 but not in $\text{Seq}(d_i)$ (clearly, $j < p$). Since d^* is minimal, there must exist $h > j$ such that
 1343 $x_j^* \in \text{Body}(r_h^*)$ (otherwise d^* deprived of (x_j^*, r_j^*) would give a proper subsequence also

1344 being a derivation of x). By Property 2, $x_h^* \neq x_i^*$ for $i \neq h$. Hence, $\text{Head}(r_i^*) \neq x_h^*$ for $i \neq h$.
 1345 Together with $x_j^* \notin \text{Seq}(d_i)$ and $x_j^* \in \text{Body}(r_h^*)$, this entails $x_h^* \notin \text{Seq}(d_i)$. Therefore, j is
 1346 not the greatest index such that x_j^* is in $\text{Seq}(d^*)$ but not in $\text{Seq}(d_i)$. \square

1347 **Lemma 1** Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
 1348 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is consistent and \mathcal{R} is conflict-dependent and privileges
 1349 strict arguments. For all $a \in \text{Arg}(\text{Free}(T))$, $b \in \text{Arg}(T)$, if $a\mathcal{R}b$ or $b\mathcal{R}a$, then $\exists a' \in$
 1350 $\text{Sub}(a)$ such that a' is strict and $a'\mathcal{R}b$.

1351 *Proof* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$
 1352 such that $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is consistent. Assume that \mathcal{R} is conflict-dependent and privileges
 1353 strict arguments.

1354 Assume that $\exists a = (d, x) \in \text{Arg}(\text{Free}(T))$ and $\exists b = (d', x') \in \text{Arg}(T)$ such that
 1355 $b\mathcal{R}a$ or $a\mathcal{R}b$. Since \mathcal{R} is conflict-dependent, $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent and thus
 1356 $\text{CN}(\text{Th}(\{a\})) \cup \text{CN}(\text{Th}(\{b\}))$ is inconsistent (since from Property 3, $\text{Seq}(d) \subseteq \text{CN}(\text{Th}(\{a\}))$
 1357 and $\text{Seq}(d') \subseteq \text{CN}(\text{Th}(\{b\}))$).

1358 Let $\text{Th}(\{a\}) = (X, Y, Z)$ and $\text{Th}(\{b\}) = (X', Y', Z')$. Let us show that $\text{CN}((\mathcal{F}, \mathcal{S}, Z'))$ is
 1359 inconsistent. Assume that $\text{CN}((\mathcal{F}, \mathcal{S}, Z'))$ is consistent. Thus, there exists a preferred option
 1360 $\mathcal{O} \in \text{POpt}(T)$ such that $(\mathcal{F}, \mathcal{S}, Z') \sqsubseteq \mathcal{O}$. Since $(X, Y, Z) \sqsubseteq \text{Free}(T)$ and $\text{Free}(T) \sqsubseteq \mathcal{O}$,
 1361 $(\mathcal{F}, \mathcal{S}, Z \cup Z') \sqsubseteq \mathcal{O}$. From Property 4, $\text{CN}((\mathcal{F}, \mathcal{S}, Z \cup Z')) \subseteq \text{CN}(\mathcal{O})$. Thus, $\text{CN}(\mathcal{O})$ is
 1362 inconsistent. This contradicts the fact that \mathcal{O} is an option.

1363 Let Z^* be the smallest (for set inclusion) subset of Z' such that $\text{CN}((\mathcal{F}, \mathcal{S}, Z^*))$ is in-
 1364 consistent. Thus, for all $r \in Z^*$, $\text{CN}((\mathcal{F}, \mathcal{S}, Z^* \setminus \{r\}))$ is consistent. It follows that for all $r \in Z^*$,
 1365 there exists a preferred option $\mathcal{O} \in \text{POpt}(T)$ such that $(\mathcal{F}, \mathcal{S}, Z \cup Z^* \setminus \{r\}) \sqsubseteq \mathcal{O}$ by Prop-
 1366 erty 8.

1367 Assume that for every strict $a'' \in \text{Sub}(a)$, $\text{Seq}(d'') \cup \text{Seq}(d')$ is consistent. How-
 1368 ever, $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent (since $a\mathcal{R}b$ or $b\mathcal{R}a$ while \mathcal{R} is conflict-dependent).
 1369 Hence, $\text{Head}(\text{Def}(d)) \cup \text{Head}(\text{Def}(d'))$ is inconsistent, say $y \in \text{Head}(\text{Def}(d))$ and
 1370 $\neg y \in \text{Head}(\text{Def}(d'))$. Should no such y be in $\mathcal{F} \cup \text{Head}(\mathcal{S})$, then there would be a preferred
 1371 option $\mathcal{O} = (F, S, D_{\mathcal{O}})$ with $\neg y \in \text{Head}(D_{\mathcal{O}})$. A contradiction arises, because $a = (d, x)$
 1372 being in $\text{Arg}(\text{Free}(T))$ means that $\text{Def}(d)$ is a subset of $D_{\mathcal{O}}$ for every preferred option
 1373 $\mathcal{O} = (F, S, D_{\mathcal{O}})$.

1374 That is, there exists $a'' = (d'', x'') \in \text{Sub}(a)$ such that a'' is strict and $\text{Seq}(d'') \cup \text{Seq}(d')$
 1375 is inconsistent. Since \mathcal{R} privileges strict arguments, $a''\mathcal{R}b$. \square

1376 *Proof of Theorem 2* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1377 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is consistent. Assume that \mathcal{R} is conflict-dependent
 1378 and privileges strict arguments.

1379 We show first that $\text{Arg}(\text{Free}(T))$ is conflict-free. From Property 8, $\text{CN}(\text{Free}(T))$ is
 1380 consistent. Since \mathcal{R} is conflict-dependent, from Proposition 5, $\text{Arg}(\text{Free}(T))$ is conflict-
 1381 free.

1382 Let us now show that $\text{Arg}(\text{Free}(T))$ defends its elements. Assume that $\exists a = (d, x) \in$
 1383 $\text{Arg}(\text{Free}(T))$ and $\exists b = (d', x') \in \text{Arg}(T)$ such that $b\mathcal{R}a$. From Lemma 1, there exists
 1384 $a' = (d'', x'') \in \text{Sub}(a)$ such that a' is strict, hence $a \in \text{Arg}(\text{Free}(T))$, and $a'\mathcal{R}b$. \square

1385 *Proof of Theorem 3* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1386 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that $\text{Ext}(\mathcal{H}) \neq \emptyset$. Assume that \mathcal{H} satisfies strict precedence and
 1387 closure under both strict rules and sub-arguments. Let $\mathcal{E} \in \text{Ext}(\mathcal{H})$ and $\text{Th}(\mathcal{E}) = (X, Y, Z)$.
 1388 Since $X \subseteq \mathcal{F}$ and $Y \subseteq \mathcal{S}$, $(X, Y, Z) \sqsubseteq (\mathcal{F}, \mathcal{S}, Z)$. From Property 4, $\text{CN}((X, Y, Z)) \subseteq$
 1389 $\text{CN}((\mathcal{F}, \mathcal{S}, Z))$.

1390 Let us now show that $\text{CN}((\mathcal{F}, \mathcal{S}, Z)) \subseteq \text{CN}((X, Y, Z))$. Since \mathcal{H} satisfies strict precedence, $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Output}(\mathcal{H})$. From Property 11, $\text{Output}(\mathcal{H}) \subseteq \text{Concs}(\mathcal{E})$.
 1391
 1392 Since \mathcal{H} is closed under sub-arguments, $\text{Concs}(\mathcal{E}) = \text{CN}(\text{Th}(\mathcal{E}))$ by Proposition 12. Hence,
 1393 $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{CN}((X, Y, Z))$. Furthermore, from Property 3, $\mathcal{F} \subseteq \text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$. Thus,
 1394 $X = \mathcal{F}$, i.e., $\text{Th}(\mathcal{E}) = (X, Y, Z)$. Let $x \in \text{CN}((\mathcal{F}, \mathcal{S}, Z))$. Then, there exists a derivation
 1395 schema

$$d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$$

1397 for x from $(\mathcal{F}, \mathcal{S}, Z)$. There are two cases:

- 1398 • For any $i = 1, \dots, n$, $r_i \in \{\sigma\} \cup Y \cup Z$. Hence, d is also a derivation schema for x from
 1399 (\mathcal{F}, Y, Z) . Thus, $x \in \text{CN}((\mathcal{F}, Y, Z))$.
- 1400 • There exists $1 < i \leq n$ such that $r_i \in \mathcal{S} \setminus Y$ (note that the two theories (\mathcal{F}, Y, Z) and
 1401 $(\mathcal{F}, \mathcal{S}, Z)$ differ only on $\mathcal{S} \setminus Y$). Let i be the first step where an element of $\mathcal{S} \setminus Y$ is used
 1402 in the derivation d . Note also that $i > 1$ since strict rules have non-empty bodies. Thus,
 1403 for any $j < i$, $r_j \in \{\sigma\} \cup Y \cup Z$ and $\langle (x_1, r_1), \dots, (x_j, r_j) \rangle$ is a derivation schema of x_j
 1404 from (\mathcal{F}, Y, Z) . Thus, $x_j \in \text{CN}((\mathcal{F}, Y, Z))$. Furthermore, $\text{Body}(r_i) \subseteq \{x_1, \dots, x_{i-1}\}$, so
 1405 $\text{Body}(r_i) \subseteq \text{CN}((\mathcal{F}, Y, Z))$. Since $\text{Concs}(\mathcal{E}) = \text{CN}(\text{Th}(\mathcal{E}))$, $\text{Body}(r_i) \subseteq \text{Concs}(\mathcal{E})$.
 1406 Since \mathcal{H} is closed under strict rules, $\text{Head}(r_i) \in \text{Concs}(\mathcal{E}) = \text{CN}((\mathcal{F}, Y, Z))$, i.e., $x_i \in$
 1407 $\text{CN}((\mathcal{F}, Y, Z))$. We repeat the same reasoning for showing that each $x_i \in \text{CN}((\mathcal{F}, Y, Z))$
 1408 and conclude that $x \in \text{CN}((\mathcal{F}, Y, Z))$.

□

1410 *Proof of Theorem 4* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1411 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{H} satisfies strict precedence, and closure under both strict rules
 1412 and sub-arguments.

1413 Let $\mathcal{E} \in \text{Ext}(\mathcal{H})$ and $\text{Th}(\mathcal{E}) = (X, Y, Z)$. From Theorem 3 and Proposition 12, it holds
 1414 that

$$1415 \quad \text{CN}(\text{Th}(\mathcal{E})) = \text{CN}((\mathcal{F}, \mathcal{S}, Z)) = \text{Concs}(\mathcal{E}). \quad (*)$$

1417 Let $\mathcal{O} = (\mathcal{F}, \mathcal{S}, Z \cup Z')$ where $Z' = \{r \mid r \in \mathcal{D} \setminus Z \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\text{Th}(\mathcal{E}))\}$. Since
 1418 $(\mathcal{F}, \mathcal{S}, Z) \sqsubseteq \mathcal{O}$, then from Property 4 and (*), $\text{Concs}(\mathcal{E}) \subseteq \text{CN}(\mathcal{O})$. Let us now show that
 1419 $\text{CN}(\mathcal{O}) \subseteq \text{Concs}(\mathcal{E})$. Let $x \in \text{CN}((\mathcal{F}, \mathcal{S}, Z \cup Z'))$. Then, there exists a derivation schema

$$d = \langle (x_1, r_1), \dots, (x_n, r_n) \rangle$$

1421 for x from $(\mathcal{F}, \mathcal{S}, Z \cup Z')$. There are two cases:

- 1422 • For any $i = 1, \dots, n$, $r_i \in \{\sigma\} \cup \mathcal{S} \cup Z$. Hence, d is also a derivation schema for x from
 1423 $(\mathcal{F}, \mathcal{S}, Z)$. Thus, $x \in \text{CN}((\mathcal{F}, \mathcal{S}, Z))$, and from (1), $x \in \text{Concs}(\mathcal{E})$.
- 1424 • Assume that there exists $1 < i \leq n$ such that $r_i \in Z'$ (note that the two theories $(\mathcal{F}, \mathcal{S}, Z)$
 1425 and $(\mathcal{F}, \mathcal{S}, Z \cup Z')$ differ only on Z'). Let i be the first step where an element of Z' is
 1426 used in the derivation d . Since the bodies of defeasible rules are not empty, $i > 1$. It
 1427 follows that for any $j < i$, $r_j \in \{\sigma\} \cup \mathcal{S} \cup Z$, thus $\langle (x_1, r_1), \dots, (x_j, r_j) \rangle$ is a derivation
 1428 schema of x_j from $(\mathcal{F}, \mathcal{S}, Z)$. Thus, $x_j \in \text{CN}((\mathcal{F}, \mathcal{S}, Z))$. Furthermore, by Definition 6,
 1429 $\text{Body}(r_i) \subseteq \{x_1, \dots, x_{i-1}\}$. Then, $\text{Body}(r_i) \subseteq \text{CN}((\mathcal{F}, \mathcal{S}, Z))$. This contradicts the fact
 1430 that $r_i \in Z'$ and thus such r_i does not exist.

□

1432 *Proof of Theorem 5* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1433 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent. Assume that for all $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$,
 1434 $\text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \mathcal{E}$. Assume now that $\exists a \in \text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$ and $\exists b \in \text{Arg}(\mathcal{T})$ such
 1435 that $a\mathcal{R}b$ or $b\mathcal{R}a$. Since \mathcal{R} is conflict-dependent, $(b, b) \notin \mathcal{R}$ (cf. Proposition 4). Thus,
 1436 $\exists \mathcal{E} \in \text{Ext}_n(\mathcal{H})$ such that $b \in \mathcal{E}$. Consequently, $a, b \in \mathcal{E}$, this contradicts the fact that \mathcal{E} is
 1437 conflict-free (since it is a naive extension).

1438 Assume now that for all $a \in \text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$, $\nexists b \in \text{Arg}(\mathcal{T})$ such that $a\mathcal{R}b$ or $b\mathcal{R}a$. This
 1439 means that arguments of $\text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$ are not attacked. Thus, they belong to every naive
 1440 extension. □

1441 *Proof of Theorem 6* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1442 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent. Assume that $\exists a, b \in \text{Arg}(\mathcal{T})$ such that
 1443 $a \in \text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$ and $\text{Conc}(a) = \neg \text{Conc}(b)$. Thus, $(b, b) \notin \mathcal{R}$ and $\exists \mathcal{E} \in \text{Ext}_n(\mathcal{H})$
 1444 such that $b \in \mathcal{E}$. If \mathcal{H} satisfies strict precedence, then $\text{Conc}(a) \in \text{Concs}(\mathcal{E})$ meaning that
 1445 $\text{Concs}(\mathcal{E})$ is inconsistent. Thus, \mathcal{H} violates consistency. If \mathcal{H} satisfies consistency, then
 1446 $\text{Conc}(a) \notin \text{Concs}(\mathcal{E})$ meaning that \mathcal{H} violates strict precedence. □

1447 *Proof of Theorem 7* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1448 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 1449 arguments. Let $\mathcal{E}, \mathcal{E}' \in \text{Ext}_n(\mathcal{H})$ such that $\text{Concs}(\mathcal{E}') \subseteq \text{Concs}(\mathcal{E})$.

1450 Assume that $\exists a = (d, x) \in \mathcal{E} \setminus \mathcal{E}'$. Thus, $\exists b = (d', x') \in \mathcal{E}'$ such that $a\mathcal{R}b$ or $b\mathcal{R}a$. Since \mathcal{R}
 1451 is conflict-dependent, $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent. But \mathcal{H} is closed under sub-arguments.
 1452 Thus, Proposition 12 gives $\text{Concs}(\mathcal{E}) = \text{CN}(\text{Th}(\mathcal{E}))$ and $\text{Concs}(\mathcal{E}') = \text{CN}(\text{Th}(\mathcal{E}'))$.
 1453 Besides, $\text{Seq}(d) \subseteq \text{CN}(\text{Th}(\mathcal{E}))$ and $\text{Seq}(d') \subseteq \text{CN}(\text{Th}(\mathcal{E}'))$ using Propositions 12 and 14.
 1454 Since $\text{CN}(\text{Th}(\mathcal{E}')) \subseteq \text{CN}(\text{Th}(\mathcal{E}))$, $\text{Seq}(d) \cup \text{Seq}(d') \subseteq \text{CN}(\text{Th}(\mathcal{E}))$. Thus, $\text{CN}(\text{Th}(\mathcal{E}))$ is
 1455 inconsistent. This contradicts the fact that \mathcal{H} satisfies consistency.

1456 The same reasoning holds for $a = (d', x') \in \mathcal{E}' \setminus \mathcal{E}$. □

1457 *Proof of Theorem 8* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1458 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 1459 arguments. Let $\mathcal{O} \in \text{Ext}_n(\mathcal{H})$.

1460 From Proposition 6, $\text{CN}(\text{Th}(\mathcal{O}))$ is consistent. Thus, $\exists \mathcal{O} \in \text{Opt}(\mathcal{T})$ such that $\text{Th}(\mathcal{O}) \sqsubseteq$
 1461 \emptyset . From Property 4, $\text{CN}(\text{Th}(\mathcal{O})) \subseteq \text{CN}(\emptyset)$. Since \mathcal{H} is closed under sub-arguments,
 1462 $\text{CN}(\text{Th}(\mathcal{O})) = \text{Concs}(\mathcal{O})$ by Proposition 12. Thus, $\text{Concs}(\mathcal{O}) \subseteq \text{CN}(\emptyset)$. Assume now
 1463 that $\exists x \in \text{CN}(\emptyset) \setminus \text{Concs}(\mathcal{O})$. Then, there exists a minimal derivation d for x from \emptyset . From
 1464 Property 3, $\text{Seq}(d) \subseteq \text{CN}(\emptyset)$. Since $\text{CN}(\emptyset)$ is consistent, (d, x) is an argument. In addition
 1465 $(d, x) \notin \mathcal{O}$. Then, $\exists (d', x') \in \mathcal{O}$ such that $(d, x)\mathcal{R}(d', x')$ or $(d', x')\mathcal{R}(d, x)$. Since \mathcal{R} is
 1466 conflict-dependent, then $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent. But, $\text{Seq}(d') \subseteq \text{Concs}(\mathcal{O})$.
 1467 So, $\text{Seq}(d) \cup \text{Seq}(d') \subseteq \text{CN}(\emptyset)$. This contradicts the fact that \emptyset is an option. So,
 1468 $\text{CN}(\emptyset) \subseteq \text{Concs}(\mathcal{O})$.

1469 Since both $\text{Concs}(\mathcal{O}) \subseteq \text{CN}(\emptyset)$ and $\text{CN}(\emptyset) \subseteq \text{Concs}(\mathcal{O})$ have now been proved, the
 1470 required $\text{CN}(\emptyset) = \text{Concs}(\mathcal{O})$ follows.

1471 Let us now show that $\emptyset \in \text{Max}(\text{Opt}(\mathcal{T}))$. Assume that $\exists \mathcal{O}' \in \text{Opt}(\mathcal{T})$ such that
 1472 $\text{CN}(\emptyset) \subseteq \text{CN}(\mathcal{O}')$. Thus, $\exists x \in \text{CN}(\mathcal{O}')$ and $x \notin \text{CN}(\emptyset)$. Thus, there exists a minimal deriva-
 1473 tion d for x from \mathcal{O}' . Since $\text{CN}(\mathcal{O}')$ is consistent and $\text{Seq}(d) \subseteq \text{CN}(\mathcal{O}')$ (from Property 3),
 1474 (d, x) is an argument. In addition $(d, x) \notin \emptyset$ (since $x \notin \text{CN}(\emptyset)$). Then, $\exists (d', x') \in \emptyset$ such
 1475 that $(d, x)\mathcal{R}(d', x')$ or $(d', x')\mathcal{R}(d, x)$. Since \mathcal{R} is conflict-dependent, $\text{Seq}(d) \cup \text{Seq}(d')$ is
 1476 inconsistent. But, $\text{Seq}(d') \subseteq \text{Concs}(\emptyset)$. So, $\text{Seq}(d) \cup \text{Seq}(d') \subseteq \text{CN}(\emptyset)$. This contradicts
 1477 the fact that \emptyset is an option.

1478 From Proposition 1, it follows that for all $\mathcal{O}, \mathcal{O}' \in \text{Max}(\text{Opt}(T))$, if $\text{CN}(\mathcal{O}) = \text{CN}(\mathcal{O}') =$
 1479 $\text{Concs}(\mathcal{E})$, then $\mathcal{O} = \mathcal{O}'$. \square

1480 *Proof of Theorem 9* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1481 T such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 1482 arguments.

- 1483 • Let $\mathcal{E}, \mathcal{E}' \in \text{Ext}_n(\mathcal{H})$. From Theorem 8, $\exists \mathcal{O}, \mathcal{O}' \in \text{Max}(\text{Opt}(T))$ such that $\text{Concs}(\mathcal{E}) =$
 1484 $\text{CN}(\mathcal{O})$ and $\text{Concs}(\mathcal{E}') = \text{CN}(\mathcal{O}')$. If $\mathcal{O} = \mathcal{O}'$, then $\text{Concs}(\mathcal{E}) = \text{Concs}(\mathcal{E}')$. From
 1485 Theorem 7, $\mathcal{E} = \mathcal{E}'$.
- 1486 • Let $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$ and $\mathcal{O} = \text{Option}(\mathcal{E})$. Thus, $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$ and $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$.
 1487 From Proposition 3, $\text{Arg}(\text{Th}(\mathcal{E})) \subseteq \text{Arg}(\mathcal{O})$. From Proposition 14, $\text{Arg}(\text{Th}(\mathcal{E})) = \mathcal{E}$.
 1488 Thus, $\mathcal{E} \subseteq \text{Arg}(\mathcal{O})$. Assume now that $\exists a = (d, x) \in \text{Arg}(\mathcal{O})$ and $a \notin \mathcal{E}$. Thus,
 1489 $\exists b = (d', x') \in \mathcal{E}$ and aRb or bRa . Since \mathcal{R} is conflict-dependent, $\text{Seq}(d) \cup \text{Seq}(d')$ is
 1490 inconsistent. Besides, $\text{Seq}(d) \subseteq \text{CN}(\mathcal{O})$ and $\text{Seq}(d') \subseteq \text{CN}(\text{Th}(\mathcal{E}))$. Since \mathcal{H} is closed
 1491 under sub-arguments, $\text{CN}(\text{Th}(\mathcal{E})) = \text{CN}(\mathcal{O})$. Thus, $\text{Seq}(d) \cup \text{Seq}(d') \subseteq \text{CN}(\mathcal{O})$. This
 1492 contradicts the fact that \mathcal{O} is an option. \square

1493 *Proof of Theorem 10* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1494 T such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 1495 arguments.

- 1496 • Let $\mathcal{O} \in \text{Max}(\text{Opt}(T))$. Thus, $\text{CN}(\mathcal{O})$ is consistent. From Proposition 5, since \mathcal{R} is
 1497 conflict-dependent, $\text{Arg}(\mathcal{O})$ is conflict-free. Assume now that $\text{Arg}(\mathcal{O}) \notin \text{Ext}_n(\mathcal{H})$.
 1498 Thus, $\exists a = (d, x) \in \text{Arg}(T)$ such that $a \notin \text{Arg}(\mathcal{O})$ and $\text{Arg}(\mathcal{O}) \cup \{a\}$ is conflict-
 1499 free. Consequently, $\exists \mathcal{E} \in \text{Ext}_n(\mathcal{H})$ such that $\text{Arg}(\mathcal{O}) \cup \{a\} \subseteq \mathcal{E}$. It follows that
 1500 $\text{Concs}(\text{Arg}(\mathcal{O}) \cup \{a\}) \subseteq \text{Concs}(\mathcal{E})$. Since $\text{CN}(\mathcal{O})$ is consistent, $\text{Concs}(\text{Arg}(\mathcal{O})) =$
 1501 $\text{CN}(\mathcal{O})$. Thus, $\text{CN}(\mathcal{O}) \cup \{x\} \subseteq \text{Concs}(\mathcal{E})$. From Theorem 8, $\exists \mathcal{O}' \in \text{Max}(\text{Opt}(T))$ such
 1502 that $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O}')$. Then, $\text{CN}(\mathcal{O}) \cup \{x\} \subseteq \text{CN}(\mathcal{O}')$. This contradicts the fact that
 1503 \mathcal{O} is a maximal option.
- 1504 • Let $\mathcal{O} \in \text{Max}(\text{Opt}(T))$. By definition of Th , $\text{Th}(\text{Arg}(\mathcal{O})) \sqsubseteq \mathcal{O}$. From Property 4,
 1505 $\text{CN}(\text{Th}(\text{Arg}(\mathcal{O}))) \subseteq \text{CN}(\mathcal{O})$. Besides, from first item, $\text{Arg}(\mathcal{O}) \in \text{Ext}_n(\mathcal{H})$. From The-
 1506 orem 8, $\exists \mathcal{O}' \in \text{Max}(\text{Opt}(T))$ such that $\text{Th}(\text{Arg}(\mathcal{O})) \sqsubseteq \mathcal{O}'$ and $\text{Concs}(\text{Arg}(\mathcal{O})) =$
 1507 $\text{CN}(\mathcal{O}')$. Since \mathcal{H} is closed under sub-arguments, $\text{CN}(\text{Th}(\text{Arg}(\mathcal{O}))) = \text{Concs}(\text{Arg}(\mathcal{O}))$.
 1508 Consequently, $\text{CN}(\mathcal{O}') \subseteq \text{CN}(\mathcal{O})$. From Proposition 1, $\mathcal{O} = \mathcal{O}'$.
- 1509 • Let $\mathcal{O}, \mathcal{O}' \in \text{Max}(\text{Opt}(T))$. Assume that $\text{Arg}(\mathcal{O}) = \text{Arg}(\mathcal{O}')$.
 1510 It follows that $\text{Option}(\text{Arg}(\mathcal{O})) = \text{Option}(\text{Arg}(\mathcal{O}'))$. From item 2, it follows that
 1511 $\mathcal{O} = \mathcal{O}'$. \square

1512 *Proof of Theorem 11* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1513 $T = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency, strict
 1514 precedence and closure under both strict rules and sub-arguments.

1515 Let us show that $\text{Max}(\text{Opt}(T)) \subseteq \text{Max}(\text{POpt}(T))$. Let $\mathcal{O} \in \text{Max}(\text{Opt}(T))$. From
 1516 (item 1) of Theorem 10, $\text{Arg}(\mathcal{O}) \in \text{Ext}_n(\mathcal{H})$. From Theorem 8, $\text{Concs}(\text{Arg}(\mathcal{O})) =$
 1517 $\text{CN}(\text{Option}(\text{Arg}(\mathcal{O})))$. From Corollary 1, $\text{Option}(\text{Arg}(\mathcal{O})) = \mathcal{O}$. Hence, Concs
 1518 $(\text{Arg}(\mathcal{O})) = \text{CN}(\mathcal{O})$. From Theorem 4, there exists $\mathcal{O}' = (\mathcal{F}, \mathcal{S}, \mathcal{Z})$ such that

$$1521 \quad \mathcal{Z} = \left(\bigcup_{(d,x) \in \text{Arg}(\mathcal{O})} \text{Def}(d) \right) \cup \{r \mid r \in \mathcal{D} \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\text{Th}(\text{Arg}(\mathcal{O})))\}$$

1522 and $\text{Concs}(\text{Arg}(\mathcal{O})) = \text{CN}(\mathcal{O}')$. Since $\text{Concs}(\text{Arg}(\mathcal{O})) = \text{CN}(\mathcal{O})$ and using Proposition
 1523 12, $\text{Concs}(\text{Arg}(\mathcal{O})) = \text{CN}(\text{Th}(\text{Arg}(\mathcal{O})))$, then $\text{CN}(\text{Th}(\text{Arg}(\mathcal{O}))) = \text{CN}(\mathcal{O})$ and we get

$$1524 \quad Z = \left(\bigcup_{(d,x) \in \text{Arg}(\mathcal{O})} \text{Def}(d) \right) \cup \{r \mid r \in \mathcal{D} \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\mathcal{O})\}$$

1525 and $\text{CN}(\mathcal{O}) = \text{CN}(\mathcal{O}')$. From Proposition 1, it follows that $\mathcal{O} = \mathcal{O}'$. Furthermore, $\mathcal{O} \in$
 1526 $\text{Max}(\text{Opt}(\mathcal{T}))$ and is maximal (for set inclusion) up to consistency and contains the strict
 1527 part of \mathcal{T} , then $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$.

1528 Let us now show that $\text{Max}(\text{POpt}(\mathcal{T})) \subseteq \text{Max}(\text{Opt}(\mathcal{T}))$. Let $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$. By
 1529 definition of preferred option, $\text{CN}(\mathcal{O})$ is consistent. Since \mathcal{R} is conflict-dependent, $\text{Arg}(\mathcal{O})$
 1530 is conflict-free by Proposition 5. Assume now that $\text{Arg}(\mathcal{O}) \notin \text{Ext}_n(\mathcal{H})$. Thus, $\exists a \in$
 1531 $\text{Arg}(\mathcal{T}) \setminus \text{Arg}(\mathcal{O})$ such that $\text{Arg}(\mathcal{O}) \cup \{a\}$ is conflict-free. Consequently, $\exists \mathcal{E} \in \text{Ext}_n(\mathcal{H})$
 1532 such that $\text{Arg}(\mathcal{O}) \cup \{a\} \subseteq \mathcal{E}$. Thus, $\text{Concs}(\text{Arg}(\mathcal{O}) \cup \{a\}) \subseteq \text{Concs}(\mathcal{E})$. Since $\text{CN}(\mathcal{O})$ is
 1533 consistent, $\text{Concs}(\text{Arg}(\mathcal{O})) = \text{CN}(\mathcal{O})$. Thus, $\text{CN}(\mathcal{O}) \cup \{\text{Conc}(a)\} \subseteq \text{Concs}(\mathcal{E})$. From The-
 1534 orem 8, $\exists \mathcal{O}' \in \text{Max}(\text{Opt}(\mathcal{T}))$ such that $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O}')$. Then, $\text{CN}(\mathcal{O}) \cup \{\text{Conc}(a)\} \subseteq$
 1535 $\text{CN}(\mathcal{O}')$. This means that $\mathcal{O}' \in \text{POpt}(\mathcal{T})$ (since it contains all consequences of the strict
 1536 part of \mathcal{T}). This contradicts the fact that \mathcal{O} is a maximal preferred option. Consequently,
 1537 $\text{Arg}(\mathcal{O}) \in \text{Ext}_n(\mathcal{H})$. From Theorem 8, $\mathcal{O} \in \text{Max}(\text{Opt}(\mathcal{T}))$. □

1538 *Proof of Theorem 12* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1539 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency and closure under sub-
 1540 arguments. From Property 11, $\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{E}_i \in \text{Ext}_n(\mathcal{H})} \text{Concs}(\mathcal{E}_i)$. From Theorem 8, for
 1541 all $\mathcal{E}_i \in \text{Ext}_n(\mathcal{H})$, there exists a unique $\mathcal{O}_i \in \text{Max}(\text{Opt}(\mathcal{T}))$ such that $\text{Concs}(\mathcal{E}_i) = \text{CN}(\mathcal{O}_i)$.
 1542 Also, Corollary 1 guarantees that $\text{Max}(\text{Opt}(\mathcal{T}))$ does not have any additional elements that
 1543 do not have a mapping in $\text{Ext}_n(\mathcal{H})$. Thus,

$$1544 \quad \text{Output}(\mathcal{H}) = \bigcap_{\mathcal{O}_i \in \text{Max}(\text{Opt}(\mathcal{T}))} \text{CN}(\mathcal{O}_i).$$

1545 □

1546 **Lemma 2** Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory $\mathcal{T} =$
 1547 $(\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates. For any
 1548 $\mathcal{E} \in \text{Ext}_s(\mathcal{H})$, it holds that $(\mathcal{F}, \mathcal{S}, Z) \in \text{Max}(\text{POpt}(\mathcal{T}))$ whenever

$$1549 \quad Z = \left(\bigcup_{(d,x) \in \mathcal{E}} \text{Def}(d) \right) \cup \{r \mid r \in \mathcal{D} \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\text{Th}(\mathcal{E}))\}.$$

1550 *Proof* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory \mathcal{T} such that
 1551 \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates. Let $\mathcal{E} \in \text{Ext}_s(\mathcal{H})$ and $\text{Th}(\mathcal{E}) =$
 1552 (X, Y, Z) . Let $\mathcal{O} = (\mathcal{F}, \mathcal{S}, Z \cup Z')$ where $Z' = \{r \mid r \in \mathcal{D} \setminus Z \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\text{Th}(\mathcal{E}))\}$.
 1553 Clearly, $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$. From Theorem 4, $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$. Since \mathcal{H} satisfies consistency,
 1554 then $\text{Concs}(\mathcal{E})$ is consistent, and thus, $\text{CN}(\mathcal{O})$ is consistent as well. Since $(\mathcal{F}, \mathcal{S}, \emptyset) \sqsubseteq \mathcal{O}$
 1555 and $\text{CN}(\mathcal{O})$ is consistent, from Property 7, $\exists \mathcal{O}' \in \text{POpt}(\mathcal{T})$ such that $\mathcal{O} \sqsubseteq \mathcal{O}'$. From Propo-
 1556 sition 3, $\text{Arg}(\text{Th}(\mathcal{E})) \subseteq \text{Arg}(\mathcal{O}) \subseteq \text{Arg}(\mathcal{O}')$. From Proposition 9, $\mathcal{E} = \text{Arg}(\text{Th}(\mathcal{E}))$.
 1557 Hence, $\mathcal{E} \subseteq \text{Arg}(\mathcal{O}) \subseteq \text{Arg}(\mathcal{O}')$. Since \mathcal{R} is conflict-dependent, $\text{CN}(\mathcal{O})$ and $\text{CN}(\mathcal{O}')$ are
 1558 consistent, then from Proposition 5 $\text{Arg}(\mathcal{O})$ and $\text{Arg}(\mathcal{O}')$ are both conflict-free. From Prop-
 1559 erty 1, $\mathcal{E} \in \text{Ext}_n(\mathcal{H})$. Then, \mathcal{E} is maximal (for set inclusion) among conflict-free sets.

1560 Thus, $\mathcal{E} = \text{Arg}(\mathcal{O}) = \text{Arg}(\mathcal{O}')$. From consistency of $\text{CN}(\mathcal{O})$ and $\text{CN}(\mathcal{O}')$, it follows that
 1561 $\text{Concs}(\text{Arg}(\mathcal{O})) = \text{CN}(\mathcal{O})$ and $\text{Concs}(\text{Arg}(\mathcal{O}')) = \text{CN}(\mathcal{O}')$. Then, $\text{CN}(\mathcal{O}) = \text{CN}(\mathcal{O}')$ and
 1562 $\mathcal{O} \in \text{POpt}(\mathcal{T})$.

1563 Let us now show that $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$. Assume that $\exists \mathcal{O}' \in \text{POpt}(\mathcal{T})$ such that
 1564 $\text{CN}(\mathcal{O}) \subseteq \text{CN}(\mathcal{O}')$. Since $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$, $\text{Concs}(\mathcal{E}) \subseteq \text{CN}(\mathcal{O}')$. Let $x \in \text{CN}(\mathcal{O}')$ and
 1565 $x \notin \text{Concs}(\mathcal{E})$. Since $\text{CN}(\mathcal{O}')$ is consistent, there exists an argument $(d, x) \in \text{Arg}(\mathcal{O}')$, i.e., d
 1566 is a derivation of x from \mathcal{O}' . Clearly, $(d, x) \notin \mathcal{E}$. Thus, $\exists (d', x') \in \mathcal{E}$ such that $(d', x')\mathcal{R}(d, x)$.
 1567 Since \mathcal{R} is conflict-dependent, $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent. But, $\text{Seq}(d') \subseteq \text{CN}(\text{Th}(\mathcal{E}))$
 1568 and $\text{Seq}(d) \subseteq \text{CN}(\mathcal{O}')$. Proposition 13 gives $\text{CN}(\text{Th}(\mathcal{E})) = \text{Concs}(\mathcal{E})$. Then, $\text{CN}(\text{Th}(\mathcal{E})) \subseteq$
 1569 $\text{CN}(\mathcal{O}')$. Finally, $\text{Seq}(d) \cup \text{Seq}(d') \subseteq \text{CN}(\mathcal{O}')$. This contradicts the fact that $\text{CN}(\mathcal{O}')$ is
 1570 consistent. \square

1571 *Proof of Theorem 13* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1572 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency, exhaustiveness, strict prece-
 1573 dence and closure under both strict rules and sub-arguments. Assume that $\text{Ext}_s(\mathcal{H}) \neq \emptyset$.
 1574 Let $\mathcal{E} \in \text{Ext}_s(\mathcal{H})$ and $\text{Th}(\mathcal{E}) = (X, Y, Z)$. From Lemma 2, the option $\mathcal{O} = (\mathcal{F}, \mathcal{S}, Z') \in$
 1575 $\text{Max}(\text{POpt}(\mathcal{T}))$ with $Z' = Z \cup \{r \mid r \in \mathcal{D} \setminus Z \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\text{Th}(\mathcal{E}))\}$. Since $X \subseteq \mathcal{F}$,
 1576 $Y \subseteq \mathcal{S}$ and $Z \subseteq Z'$, $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$. From Theorem 4, $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$.

1577 Let us show that \mathcal{E} has a unique corresponding preferred maximal option. Assume that
 1578 $\exists \mathcal{O}_1, \mathcal{O}_2 \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}_1$, $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O}_1)$, $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}_2$
 1579 and $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O}_2)$. Obviously, $\text{CN}(\mathcal{O}_1) = \text{CN}(\mathcal{O}_2)$. However, $\mathcal{O}_1, \mathcal{O}_2 \in \text{POpt}(\mathcal{T})$
 1580 according to Property 6 hence Proposition 1 gives $\mathcal{O}_1 = \mathcal{O}_2$.

1581 Let us now show that $\mathcal{E} = \text{Arg}(\mathcal{O})$. Since $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$, from Proposition 3, $\text{Arg}(\text{Th}(\mathcal{E})) \subseteq$
 1582 $\text{Arg}(\mathcal{O})$. From Proposition 15, $\mathcal{E} \subseteq \text{Arg}(\mathcal{O})$. Assume now that $\exists a = (d, x) \in \text{Arg}(\mathcal{O})$ such
 1583 that $a \notin \mathcal{E}$. Thus, $\exists b = (d', x') \in \mathcal{E}$ and $b\mathcal{R}a$. Since \mathcal{R} is conflict-dependent, $\text{Seq}(d) \cup$
 1584 $\text{Seq}(d')$ is inconsistent. Besides, $\text{Seq}(d) \subseteq \text{CN}(\mathcal{O})$ and $\text{Seq}(d') \subseteq \text{CN}(\text{Th}(\mathcal{E}))$. Since \mathcal{H}
 1585 is closed under sub-arguments, $\text{CN}(\text{Th}(\mathcal{E})) = \text{CN}(\mathcal{O})$ by Proposition 12. Thus, $\text{Seq}(d) \cup$
 1586 $\text{Seq}(d') \subseteq \text{CN}(\mathcal{O})$. This contradicts the fact that \mathcal{O} is an option. \square

1587 *Proof of Theorem 14* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1588 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies consistency, strict precedence
 1589 and closure under both strict rules and sub-arguments. Assume that $\text{Ext}_s(\mathcal{H}) \neq \emptyset$. Let
 1590 $\mathcal{E}, \mathcal{E}' \in \text{Ext}_s(\mathcal{H})$. From Theorem 13, $\exists \mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$
 1591 and $\exists \mathcal{O}' \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\text{Concs}(\mathcal{E}') = \text{CN}(\mathcal{O}')$. If $\mathcal{O} = \mathcal{O}'$, then $\text{Concs}(\mathcal{E}) =$
 1592 $\text{Concs}(\mathcal{E}')$. Assume that $\exists a = (d, x) \in \mathcal{E} \setminus \mathcal{E}'$. Thus, $\exists b = (d', x') \in \mathcal{E}'$ such that $b\mathcal{R}a$.
 1593 Since \mathcal{R} is conflict-dependent, $\text{Seq}(d) \cup \text{Seq}(d')$ is inconsistent. But \mathcal{H} is closed under
 1594 sub-arguments. Thus, $\text{Concs}(\mathcal{E}) = \text{CN}(\text{Th}(\mathcal{E}))$ and $\text{Concs}(\mathcal{E}') = \text{CN}(\text{Th}(\mathcal{E}'))$. Besides,
 1595 $\text{Seq}(d) \subseteq \text{CN}(\text{Th}(\mathcal{E}))$ and $\text{Seq}(d') \subseteq \text{CN}(\text{Th}(\mathcal{E}'))$. Since $\text{CN}(\text{Th}(\mathcal{E}')) = \text{CN}(\text{Th}(\mathcal{E}))$,
 1596 $\text{Seq}(d) \cup \text{Seq}(d') \subseteq \text{CN}(\text{Th}(\mathcal{E}))$. Thus, $\text{CN}(\text{Th}(\mathcal{E}))$ is inconsistent. This contradicts the
 1597 fact that \mathcal{H} satisfies consistency. The same reasoning holds for $a = (d', x') \in \mathcal{E}' \setminus \mathcal{E}$. \square

1598 *Proof of Theorem 15* Any argumentation system $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ that satisfies strict
 1599 precedence should have \mathcal{F} as plausible conclusions, i.e., $\mathcal{F} \subseteq \text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Output}(\mathcal{H})$.
 1600 Since $\top \in \mathcal{F}$, then $\text{Output}(\mathcal{H}) \neq \emptyset$. However, since $\mathcal{R} \in \mathfrak{R}_{s_1}$, $\text{Ext}_s(\mathcal{H}) = \emptyset$. Thus,
 1601 $\text{Output}(\mathcal{H}) = \emptyset$. \square

1602 *Proof of Theorem 16* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system over a theory
 1603 \mathcal{T} such that $\mathcal{R} \in \mathfrak{R}_{s_2}$. Let $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$. Since $|\text{Ext}_s(\mathcal{H})| = |\text{Max}(\text{POpt}(\mathcal{T}))|$,
 1604 from Theorems 13 and 14, $\exists \mathcal{E} \in \text{Ext}_s(\mathcal{H})$ such that $\mathcal{E} = \text{Arg}(\mathcal{O})$, hence $\text{Arg}(\mathcal{O}) \in$
 1605 $\text{Ext}_s(\mathcal{H})$. \square

1606 *Proof of Theorem 17* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a
 1607 theory \mathcal{T} such that $\mathcal{R} \in \mathfrak{R}_{s_2}$. From Corollary 5, $\text{Ext}_s(\mathcal{H}) = \text{Ext}_{s,s}(\mathcal{H})$. Assume that
 1608 $\exists \mathcal{E} \in \text{Ext}_p(\mathcal{H}) \setminus \text{Ext}_s(\mathcal{H})$. From Theorem 22, there exists $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that
 1609 $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$. Since $|\text{Ext}_s(\mathcal{H})| = |\text{Max}(\text{POpt}(\mathcal{T}))|$, from Theorem 16, $\text{Arg}(\mathcal{O}) \in \text{Ext}_s(\mathcal{H})$.
 1610 From Theorems 13 and 14, $\mathcal{O} = \text{Option}(\text{Arg}(\mathcal{O}))$. From Theorem 24, $\mathcal{E} = \text{Arg}(\mathcal{O})$. \square

1611 *Proof of Theorem 18* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system such that $\mathcal{R} \in \mathfrak{R}_{s_2}$.
 1612 If \mathcal{H} satisfies all the postulates under naive semantics, then from Corollary 1 and Theorem 11,
 1613 there is a bijection between $\text{Ext}_n(\mathcal{H})$ and $\text{Max}(\text{POpt}(\mathcal{T}))$. From Theorems 13 and 16,
 1614 $|\text{Ext}_s(\mathcal{H})| = |\text{Max}(\text{POpt}(\mathcal{T}))|$. Since every stable extension is a naive one, $\text{Ext}_n(\mathcal{H}) =$
 1615 $\text{Ext}_s(\mathcal{H})$. \square

1616 *Proof of Theorem 19* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1617 \mathcal{T} such that $\mathcal{R} \in \mathfrak{R}_{s_2}$. From Property 11,

$$\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{E}_i \in \text{Ext}_s(\mathcal{H})} \text{Concs}(\mathcal{E}_i).$$

1619 From Theorems 13 and 14, for all $\mathcal{E}_i \in \text{Ext}_s(\mathcal{H})$, there exists a unique $\mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T}))$
 1620 such that $\text{Concs}(\mathcal{E}_i) = \text{CN}(\mathcal{O}_i)$. Thus,

$$\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{O}_i \in \text{Max}(\text{Opt}(\mathcal{T}))} \text{CN}(\mathcal{O}_i).$$

1622 \square

1623 *Proof of Theorem 20* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1624 \mathcal{T} such that $\mathcal{R} \in \mathfrak{R}_{s_3}$. From Property 11,

$$\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{E}_i \in \text{Ext}_s(\mathcal{H})} \text{Concs}(\mathcal{E}_i).$$

1626 From Theorem 13, for all $\mathcal{E}_i \in \text{Ext}_s(\mathcal{H})$, there exists a unique $\mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T}))$ such
 1627 that $\text{Concs}(\mathcal{E}_i) = \text{CN}(\mathcal{O}_i)$. Since $\mathcal{R} \in \mathfrak{R}_{s_3}$, $|\text{Ext}_s(\mathcal{H})| < |\text{Max}(\text{POpt}(\mathcal{T}))|$. Thus,

$$\text{Output}(\mathcal{H}) = \bigcap_{\mathcal{O}_i \in \mathcal{X}} \text{CN}(\mathcal{O}_i)$$

1629 with $\mathcal{X} = \{\mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T})) \mid \mathcal{E}_i = \text{Arg}(\mathcal{O}_i) \in \text{Ext}_s(\mathcal{H})\}$. \square

1630 *Proof of Theorem 21* Let \mathcal{H} be an argumentation system built over a theory $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$
 1631 such that \mathcal{H} satisfies the strict precedence postulate, i.e., $\mathcal{F} \subseteq \text{Output}(\mathcal{H})$. Since $\top \in \mathcal{F}$,
 1632 $\text{Output}(\mathcal{H}) \neq \emptyset$. Hence, $\text{Ext}_p(\mathcal{H}) \neq \{\emptyset\}$. \square

1633 *Proof of Theorem 22* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1634 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates. Let $\mathcal{E} \in \text{Ext}_p(\mathcal{H})$ and
 1635 $\text{Th}(\mathcal{E}) = (X, Y, Z)$. From Theorem 4, $\text{Concs}(\mathcal{E}) = \text{CN}(\mathcal{O})$ where $\mathcal{O} = (\mathcal{F}, \mathcal{S}, Z \cup Z')$ and
 1636 $Z' \subseteq \mathcal{D} \setminus Z$. Clearly $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$. From consistency, $\text{Concs}(\mathcal{E})$ is consistent. Then, $\text{CN}(\mathcal{O})$
 1637 is consistent as well. Then, there exists $\mathcal{O}' \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\mathcal{O} \sqsubseteq \mathcal{O}'$. Therefore,
 1638 $\text{CN}(\mathcal{O}) \subseteq \text{CN}(\mathcal{O}')$. Thus, $\text{Concs}(\mathcal{E}) \subseteq \text{CN}(\mathcal{O}')$. \square

1639 *Proof of Theorem 23* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1640 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates
 1641 under preferred semantics. Since $\text{Ext}_s(\mathcal{H}) \subseteq \text{Ext}_p(\mathcal{H})$, \mathcal{H} satisfies the postulates under
 1642 stable semantics. Consequently, from Theorem 15, $\text{Ext}_s(\mathcal{H}) \neq \emptyset$. Let $\mathcal{E} \in \text{Ext}_s(\mathcal{H})$. From
 1643 Theorem 13, $\exists \mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\mathcal{E} = \text{Arg}(\mathcal{O})$. \square

1644 *Proof of Theorem 24* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1645 \mathcal{T} such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies exhaustiveness and closure under sub-
 1646 arguments. Let $\mathcal{E}, \mathcal{E}' \in \text{Ext}_p(\mathcal{H})$ and $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$ and
 1647 $\text{Th}(\mathcal{E}') \sqsubseteq \mathcal{O}$. We show that $\mathcal{E} \cup \mathcal{E}'$ is a preferred extension (which contradicts the fact that \mathcal{E}
 1648 and \mathcal{E}' are preferred extensions).

1649 From Proposition 3, $\text{Arg}(\text{Th}(\mathcal{E})) \subseteq \text{Arg}(\mathcal{O})$ and $\text{Arg}(\text{Th}(\mathcal{E}')) \subseteq \text{Arg}(\mathcal{O})$. Since \mathcal{H} satis-
 1650 es exhaustiveness and closure under sub-arguments, from Proposition 9, $\text{Arg}(\text{Th}(\mathcal{E})) = \mathcal{E}$
 1651 and $\text{Arg}(\text{Th}(\mathcal{E}')) = \mathcal{E}'$. Thus, $\mathcal{E} \cup \mathcal{E}' \subseteq \text{Arg}(\mathcal{O})$. Since $\text{CN}(\mathcal{O})$ is consistent and \mathcal{R} is
 1652 conflict-dependent, from Proposition 5 $\text{Arg}(\mathcal{O})$ is conflict-free. Consequently, $\mathcal{E} \cup \mathcal{E}'$ is also
 1653 conflict-free. Moreover, $\mathcal{E} \cup \mathcal{E}'$ defends its elements since \mathcal{E} and \mathcal{E}' are preferred extensions.
 1654 Thus, $\mathcal{E} \cup \mathcal{E}'$ is an admissible set. Due to \mathcal{E} and \mathcal{E}' being preferred extensions, it follows that
 1655 $\mathcal{E} \cup \mathcal{E}' = \mathcal{E} = \mathcal{E}'$. □

1656 *Proof of Theorem 25* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1657 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$ such that \mathcal{R} is conflict-dependent and privileges strict arguments, and \mathcal{H}
 1658 satisfies consistency, exhaustiveness, strict precedence and closure under sub-arguments.
 1659 From consistency and strict precedence, it follows by Proposition 8 that $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset))$ is
 1660 consistent.

1661 The conclusion of the theorem, i.e., $\text{Arg}(\text{Free}(\mathcal{T})) \subseteq \bigcap_{\mathcal{E}_i \in \text{Ext}_p(\mathcal{H})} \mathcal{E}_i$, is trivial in the
 1662 case that $\text{Arg}(\text{Free}(\mathcal{T}))$ is empty. Consider $a \in \text{Arg}(\text{Free}(\mathcal{T}))$. Let $\mathcal{E} \in \text{Ext}_p(\mathcal{H})$.

1663 Let us show that $\mathcal{E} \cup \{a\}$ is conflict-free. Assume that $\exists b = (d_2, x_2) \in \mathcal{E}$ such that $a\mathcal{R}b$
 1664 or $b\mathcal{R}a$. From Lemma 1, there exists $a' \in \text{Sub}(a)$ such that $a' = (d'_1, x'_1) \in \text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$
 1665 and $a'\mathcal{R}b$. Then, $\text{Seq}(d'_1) \cup \text{Seq}(d_2)$ is inconsistent. Since \mathcal{H} satisfies strict precedence and
 1666 exhaustiveness, $\text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \mathcal{E}$ by Proposition 10, so $a' \in \mathcal{E}$. Consequently, $\text{Seq}(d'_1) \cup$
 1667 $\text{Seq}(d_2) \subseteq \text{CN}(\text{Th}(\mathcal{E}))$ by Proposition 12. Since \mathcal{H} satisfies consistency and closure under
 1668 sub-arguments, by Proposition 6 $\text{Concs}(\mathcal{E}) = \text{CN}(\text{Th}(\mathcal{E}))$ is consistent. Contradiction.

1669 Let us show that \mathcal{E} defends a . Consider $b \in \text{Arg}(\mathcal{T})$ such that $b\mathcal{R}a$. From Lemma 1,
 1670 there exists $a' \in \text{Sub}(a)$ such that $a' \in \text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset))$ and $a'\mathcal{R}b$. Since \mathcal{H} satisfies strict
 1671 precedence and exhaustiveness, $\text{Arg}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \mathcal{E}$, thus $a' \in \mathcal{E}$.

1672 Summing up, $\mathcal{E} \cup \{a\}$ is an admissible set. However, $\mathcal{E} \in \text{Ext}_p(\mathcal{H})$ means that \mathcal{E} is a
 1673 maximal admissible set; hence, $\mathcal{E} \cup \{a\} \subseteq \mathcal{E}$. Therefore, $a \in \mathcal{E}$. □

1674 *Proof of Theorem 26* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be a system built over a theory \mathcal{T} such that
 1675 \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates. From Theorem 22, for all $\mathcal{E} \in$
 1676 $\text{Ext}_p(\mathcal{H})$, $\exists \mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$ such that $\text{Th}(\mathcal{E}) \sqsubseteq \mathcal{O}$ and $\text{Concs}(\mathcal{E}) \subseteq \text{CN}(\mathcal{O})$. From
 1677 Theorem 24, there cannot exist two maximal preferred extensions \mathcal{E} and \mathcal{E}' such that $\text{Th}(\mathcal{E}) \sqsubseteq$
 1678 \mathcal{O} and $\text{Th}(\mathcal{E}') \sqsubseteq \mathcal{O}$ for some $\mathcal{O} \in \text{Max}(\text{POpt}(\mathcal{T}))$. Thus, every maximal preferred option is
 1679 captured by at most one preferred extension. Then, $|\text{Ext}_p(\mathcal{H})| \leq |\text{Max}(\text{POpt}(\mathcal{T}))|$. □

1680 *Proof of Theorem 27* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system built over a theory
 1681 $\mathcal{T} = (\mathcal{F}, \mathcal{S}, \mathcal{D})$. Assume that $\mathcal{R} \in \mathfrak{R}_{p_2}$. The following equalities hold by Theorems 16
 1682 and 17: $\text{Ext}_p(\mathcal{H}) = \text{Ext}_s(\mathcal{H}) = \text{Ext}_{ss}(\mathcal{H}) = \{\text{Arg}(\mathcal{O}_i) \mid \mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T}))\}$. Let us
 1683 now show the equality

$$1684 \bigcap_{\mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T}))} \text{Arg}(\mathcal{O}_i) = \text{Arg}\left(\bigcap_{\mathcal{O}_i \in \text{Max}(\text{POpt}(\mathcal{T}))} \mathcal{O}_i\right).$$

1685 Let $\text{Max}(\text{POpt}(\mathcal{T})) = \{\mathcal{O}_1 = (\mathcal{F}, \mathcal{S}, \mathcal{D}_1), \dots, \mathcal{O}_n = (\mathcal{F}, \mathcal{S}, \mathcal{D}_n)\}$. Assume that $(d, x) \in$
 1686 $\bigcap_{i=1}^n \text{Arg}(\mathcal{O}_i)$. For any $i = 1, \dots, n$, $(d, x) \in \text{Arg}(\mathcal{O}_i)$ and thus

$$1687 (\text{Facts}(d), \text{Strict}(d), \text{Def}(d)) \sqsubseteq \mathcal{O}_i.$$

1688 This means that $\text{Def}(d) \subseteq \bigcap_{i=1}^n \mathcal{D}_i$. Consequently, d is also a derivation schema from
 1689 $(\mathcal{F}, \mathcal{S}, \bigcap_{i=1}^n \mathcal{D}_i) = \bigcap_{i=1}^n \mathcal{O}_i$. Finally, $(d, x) \in \text{Arg}(\bigcap_{i=1}^n \mathcal{O}_i)$.

1690 Assume now that $(d, x) \in \text{Arg}(\bigcap_{i=1}^n \mathcal{O}_i)$. Then, d is a derivation schema from
 1691 $(\mathcal{F}, \mathcal{S}, \bigcap_{i=1}^n \mathcal{D}_i)$. Hence, $\text{Def}(d) \subseteq \bigcap_{i=1}^n \mathcal{D}_i$. Hence, for any $i = 1, \dots, n$, $\text{Def}(d) \subseteq \mathcal{D}_i$.
 1692 Thus, d is a derivation schema from each theory \mathcal{O}_i and (d, x) is an argument in each
 1693 $\text{Arg}(\mathcal{O}_i)$.

1694 From above, it follows that

$$1695 \bigcap_{\mathcal{O}_i \in \text{Max}(\text{POpt}(T))} \text{Arg}(\mathcal{O}_i) = \text{Arg}(\text{Free}(T)).$$

1696 □

1697 *Proof of Theorem 28* Let \mathcal{H} be an argumentation system which satisfies the five postulates.
 1698 From strict precedence and the fact that $\text{Output}(\mathcal{H}) = \text{Concs}(\text{IE}(\mathcal{H}))$, it holds that
 1699 $\text{CN}((\mathcal{F}, \mathcal{S}, \emptyset)) \subseteq \text{Concs}(\text{IE}(\mathcal{H}))$. From Theorem 4, $\text{Concs}(\text{IE}(\mathcal{H})) = \text{CN}(\mathcal{O})$ such that
 1700 $\mathcal{O} = (\mathcal{F}, \mathcal{S}, Z)$ where

$$1701 Z = \left(\bigcup_{(d,x) \in \text{IE}(\mathcal{H})} \text{Def}(d) \right) \cup \{r \mid r \in \mathcal{D} \text{ and } \text{Body}(r) \not\subseteq \text{CN}(\text{Th}(\text{IE}(\mathcal{H})))\}.$$

1702 It holds that $\text{Th}(\text{IE}(\mathcal{H})) \sqsubseteq \mathcal{O}$. From consistency postulate, it follows that $\text{CN}(\mathcal{O})$ is consistent
 1703 (since $\text{Concs}(\text{IE}(\mathcal{H}))$ is consistent). Thus, there exists $\mathcal{O}' \in \text{POpt}(T)$ such that $\mathcal{O} \sqsubseteq \mathcal{O}'$.
 1704 From Property 4, $\text{CN}(\mathcal{O}) \subseteq \text{CN}(\mathcal{O}')$. Consequently, $\text{Concs}(\text{IE}(\mathcal{H})) \subseteq \text{CN}(\mathcal{O}')$. □

1705 *Proof of Theorem 29* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a
 1706 theory T . Assume that $\mathcal{R} \in \mathfrak{R}_{p_2}$ and privileges strict arguments. From Theorem 27,
 1707 $\bigcap_{\mathcal{E}_i \in \text{Ext}_p(\mathcal{H})} \mathcal{E}_i = \text{Arg}(\text{Free}(T))$. From Theorem 2, $\text{Arg}(\text{Free}(T))$ is an admissible
 1708 extension of \mathcal{H} . Thus, $\text{IE}(\mathcal{H}) = \text{Arg}(\text{Free}(T))$. □

1709 *Proof of Theorem 30* The proof is similar to that of Theorem 28. □

1710 *Proof of Corollary 1* It follows directly from Theorems 8 and 10. □

1711 *Proof of Corollary 2* It follows directly from Theorem 11 and Corollary 1. □

1712 *Proof of Corollary 3* It follows from Corollary 1. □

1713 *Proof of Corollary 4* It follows from Corollary 3, i.e., the equality $|\text{Ext}_n(\mathcal{H})| = |\text{Max}(\text{Opt}$
 1714 $(T))|$ and the fact that if a theory T is finite, then it has a finite number of options, thus of
 1715 maximal options. □

1716 *Proof of Corollary 5* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be such that $\mathcal{R} \in \mathfrak{R}_{s_2} \cup \mathfrak{R}_{s_3}$. From Theorem 15,
 1717 $\text{Ext}_s(\mathcal{H}) \neq \emptyset$. From Property 1, $\text{Ext}_s(\mathcal{H}) = \text{Ext}_{ss}(\mathcal{H})$. □

1718 *Proof of Corollary 6* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1719 T such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates. From Theorem 15,
 1720 $\text{Ext}_s(\mathcal{H}) \neq \emptyset$. From Theorem 13, $|\text{Ext}_s(\mathcal{H})| \leq |\text{Max}(\text{POpt}(T))|$. □

1721 *Proof of Corollary 7* Let $\mathcal{H} = (\text{Arg}(T), \mathcal{R})$ be an argumentation system built over a theory
 1722 T such that \mathcal{R} is conflict-dependent and \mathcal{H} satisfies the five postulates. If T is finite, then T
 1723 has a finite number of maximal preferred options. From Corollary 6, \mathcal{H} has a finite number
 1724 of stable extensions. □

- 1725 *Proof of Corollary 8* It follows immediately from Theorem 26. □
- 1726 *Proof of Corollary 9* It follows immediately from Theorems 17, 13 and 16. □
- 1727 *Proof of Corollary 10* Let $\mathcal{H} = (\text{Arg}(\mathcal{T}), \mathcal{R})$ be an argumentation system over a theory \mathcal{T} such that $\mathcal{R} \in \mathfrak{R}_{p_2}$ and privileges strict arguments. From Theorem 29, $\text{IE}(\mathcal{H}) = \text{Arg}(\text{Free}(\mathcal{T}))$. Then, $\text{Output}(\mathcal{H}) = \text{Concs}(\text{IE}(\mathcal{H})) = \text{Concs}(\text{Arg}(\text{Free}(\mathcal{T})))$. Since $\text{CN}(\text{Free}(\mathcal{T}))$ is consistent, $\text{CN}(\text{Free}(\mathcal{T})) = \text{Concs}(\text{Arg}(\text{Free}(\mathcal{T})))$. □

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