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1 **Influence of membrane resistance on swelling and removal of colloidal filter cake after**
2 **filtration pressure release**

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8 **Highlights**

9 Impact of filtration membrane resistance on colloidal filter cake swelling is modeled
10 Possibility of filtrate uptake across the membrane during the swelling is considered
11 The lower is the membrane resistance the faster is the filter cake swelling
12 Concentration profile in the cake becomes bell-like when membrane resistance is low
13 Appearance of bell-like solid concentration profile can facilitate the cake removal
14

15 **Abstract**

16 The influence of filtration membrane resistance on colloidal filter cake swelling after the
17 filtration pressure release is discussed in frame of filtration-consolidation theory. It is argued that
18 in the case of reversibly compressible filter cake, the solid pressure relaxation can result in filtrate
19 inflow across the membrane with the following absorption by the filter cake part adjacent to the
20 membrane surface. According to the model results, the filtrate inflow and the filter cake swelling
21 rates increase with decreasing of the membrane hydraulic resistance. At low membrane
22 resistance, the swelling can result in the non-monotonous bell-like solid concentration
23 distribution in the filter cake (solid concentration on the cake-membrane surface is lower than the
24 maximal solid concentration). In contrast, the filter cake swelling in the absence of filter inflow
25 across the membrane (or the deposit swelling on the impermeable surface) results in the
26 monotonous decrease of the solid concentration. The non-monotonous solid concentration

27 distribution can facilitate the membrane rinsing and cleaning and lead to the filter cake removal
28 before its complete swelling.

29
30 **Keywords:** filtration-consolidation theory, gel swelling, membrane rinsing, membrane fouling
31 reversibility, membrane hydraulic resistance.

33 **Nomenclature**

34 C_e modified average consolidation coefficient ($\text{m}^2 \cdot \text{s}^{-1}$)
35 c local solid concentration ($\text{kg} \cdot \text{m}^{-3}$)
36 c_0 solid concentration in bulk suspension ($\text{kg} \cdot \text{m}^{-3}$)
37 c_{mi} initial solid concentration on cake-membrane interface (at the end of filtration) ($\text{kg} \cdot \text{m}^{-3}$)
38 e void ratio (dimensionless)
39 h gap thickness, thickness of a layer on the membrane surface which can flow relatively the
40 membrane and the rigid gel under the applied external shear stress (m)
41 K parameter of Herschel-Bulkley equation ($\text{Pa} \cdot \text{s}^N$)
42 l length (m)
43 N parameter of Herschel-Bulkley equation (dimensionless)
44 ΔP applied pressure (Pa)
45 p_l liquid pressure (Pa)
46 p_m solid pressure at the membrane surface (Pa)
47 p_{mi} initial solid pressure at the membrane surface (at the end of filtration) (Pa)
48 p_s compressive solid pressure (Pa)
49 p_{si} initial solid pressure (at the end of filtration) (Pa)
50 r_{ci} initial filter cake resistance (cake resistance at the end of filtration) (m^{-1})
51 r_{in} outside-in membrane hydraulic resistance (m^{-1})
52 r_m inside-out membrane hydraulic resistance (m^{-1})
53 r_r hydraulic resistance of residual gel (m^{-1})
54 S cross-sectional area of filtration membrane (m^2)
55 t time (s)
56 U completeness of filter cake swelling (dimensionless)
57 u apparent liquid velocity relative to solids ($\text{m} \cdot \text{s}^{-1}$)

58	V_f	filtrate volume (m^3)
59	$V_{f\infty}$	final filtrate volume (m^3)
60	v	tangential velocity of rigid gel relative to membrane ($\text{m}\cdot\text{s}^{-1}$)
61	x	distance to membrane surface in Cartesian coordinates (m)
62	Greek letters	
63	α	specific filtration resistance ($\text{m}\cdot\text{kg}^{-1}$)
64	α_e	average specific filtration resistance at the membrane surface ($\text{m}\cdot\text{kg}^{-1}$)
65	α_m	specific filtration resistance of a layer on a cake-membrane interface ($\text{m}\cdot\text{kg}^{-1}$)
66	$\dot{\gamma}$	shear rate (s^{-1})
67	θ	required swelling duration, duration of swelling required in order to attain $\varphi_m = \varphi_{sg}$ (s)
68	κ	local permeability (m^2)
69	λ	variable, root of Eq. (A5) (dimensionless)
70	μ	filtrate viscosity ($\text{Pa}\cdot\text{s}$)
71	ξ	variable (dimensionless)
72	ρ_s	specific weight of solids ($\text{kg}\cdot\text{m}^{-3}$)
73	τ_0	local yield stress (Pa)
74	τ_{ext}	external applied shear stress (wall shear stress) (Pa)
75	φ	solid volume fraction (dimensionless)
76	φ_m	solid volume fraction on cake-membrane interface (dimensionless)
77	φ_{max}	maximal instant solid volume fraction in the system (dimensionless)
78	φ_{sg}	solid volume fraction of sol-gel transition (dimensionless)
79	ω	distance to membrane surface in material coordinates (m)
80	ω_0	position of cake-bulk interface in material coordinates (m)
81	ω_r	thickness of swelled gel in material coordinates (m)
82		

83 **1. Introduction**

84 In the literature, it is commonly accepted to distinguish two components of the external
85 membrane fouling (fouling resistance): reversible fouling (which is removed from the surface by
86 shear after the filtration pressure release/decrease or after the increasing of the shear rate) and
87 irreversible fouling (that corresponds to the difference between the total fouling resistance and
88 the resistance of reversible fouling). The scientific background for this distinction is DLVO
89 theory applied to the case of a filter cake [1-4]. Increasing of the local compressive pressure
90 applied to cake particles (solid pressure) in the direction of membrane surface results in the
91 decreasing of local inter-particle distance (increasing of local particle concentration). At the
92 distance to membrane surface, where the local compressive force overcomes the interparticle
93 repulsion force, cake particles undergo an aggregation (sol-gel transition). In contrast to outer
94 part of the cake (sol, concentration polarization layer), the inner part of the cake (rigid gel,
95 deposit) is not removed from the membrane surface by crossflow shear until the shear force
96 overcomes the particle attraction force (Coulomb failure criterion applied to filter cake removal
97 [5]). Therefore, appearance of irreversible external membrane fouling is associated with the gel
98 (or deposit, dense structure) formation on membrane surface.

99 The extent of irreversible membrane fouling is commonly characterized with the help of
100 cyclic variation of filtration conditions: applied wall shear stress and transmembrane pressure (or
101 imposed filtrate flux) [5-11]. The hysteresis of fouling, which is observed in coordinates fouling-
102 resistance vs. pressure (or vs. shear stress), is used in order to determine critical fouling
103 conditions (filtration conditions that result in gel formation). It should be noticed that duration of
104 filtration cycles (fouling formation followed by fouling removal) is typically short, and its
105 influence on results of the experiment (extent of irreversible fouling, irreversibility of fouling in
106 general) is out of the scope of the majority of studies.

107 However, a number of articles demonstrated that both the rate and the extent of fouling
108 removal after a cross-flow and dead-end filtration increases with the increasing of duration of the
109 pressure relaxation step [11-18]. In other words, sufficient pressure relaxation duration can result
110 in the following removal of “irreversible” fouling (at least partial). For example, it was
111 demonstrated that the casein micelles gel, which is considered to cause irreversible fouling during
112 the dead-end filtration-stirring experiment [19], can swell and redisperse in course of time after
113 the osmotic compression experiments [20] or in a different filtration experiment [21].

114 Articles concerning the modelling of the influence of pressure relaxation duration on the
115 fouling extent and removal rate are rather rare in filtration literature [18]. However, significant
116 progress is attained in the modelling of gel dissolution and the hydrodynamic cleaning (i.e.,
117 rinsing) of impermeable surface from foulants (deposits), where models consider the time-
118 dependent gel swelling (solvent diffusion into a deposit) that precedes the particles/molecules
119 redispersion by applied flow (e.g., [22-36]). In addition, behavior of filter cakes (and
120 consolidated filter cakes) after the filtration pressure decrease (or release) was modeled in the
121 frame of conventional filtration-consolidation theory and experimentally characterized in a series
122 of papers by Iritani, Iwata, Katagiri, Murase, and Shirato [37-45] and Banks [46]. Particularly, it
123 was experimentally demonstrated that colloidal filter cakes (and hydrogels) can swell and restore
124 lower values of local particle concentration and permeability (at least partially) after the filtration
125 pressure release [40, 42-45] and a new definition of filter cake reversibility, based on filtration
126 resistance recovery, was introduced [44]. It was demonstrated that the response of filter cake
127 properties (accessed from the experimentally measured filter cake resistance) on the variation of
128 applied pressure is not immediate, and kinetics of cake relaxation is related with local cake
129 properties (compressibility and permeability), initial cake thickness and values of compressive
130 pressure applied during the filter cake formation and cake relaxation [37-45]. Unfortunately,
131 previous articles did not take into consideration any possible influence of filtration membrane
132 resistance on the filter cake relaxation and the following membrane rinsing.

133 The present article is devoted to the modeling of colloidal filter cake swelling after the
134 filtration pressure release in the frame of filtration-consolidation theory. It is assumed that the
135 filter cake compression is reversible, in particular, the local filter cake concentration can decrease
136 below the concentration of sol-gel transition during the swelling. In this case, the sol-gel
137 transition is defined as the transition from liquid-like to solid-like rheological behavior. The
138 present article considers the influence of the membrane resistance to filtrate inflow into the cake
139 on the cake swelling kinetics, and analyzes the influence of parameters (the filter cake thickness,
140 compressibility and permeability, the membrane hydraulic resistance, the swelling duration, flow
141 behavior of swelled material) on the following membrane rinsing by tangential shear. Several
142 possible methods for the experimental verification of model results are proposed and the
143 importance of swelling duration and membrane resistance for the fouling removal is discussed.

144 **2. Description of a filter cake swelling on membrane surface**

145 When a liquid flows across a porous body under a pressure difference ΔP (e.g., across a
 146 filter cake and a membrane during a constant pressure dead-end filtration of particulate
 147 suspension, Fig. 1a), the liquid pressure p_l in capillaries between particles and in membrane pores
 148 decreases in the flow direction.

149

150 Fig. 1a,b.

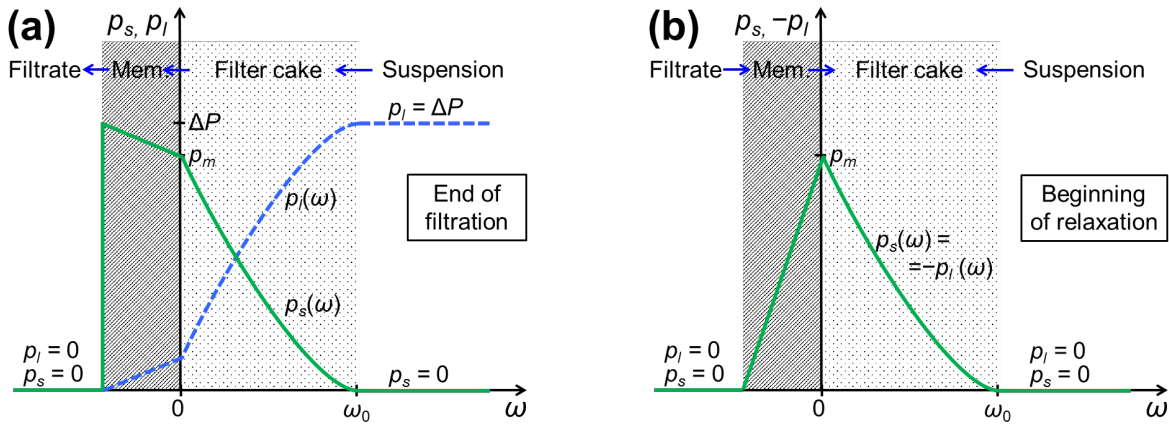


Fig. 1. Liquid p_l and solid p_s pressures (dashed and solid curves, respectively) in a system of a particulate filter cake on a membrane surface at the moments (a) before and (b) after the release of filtration pressure ΔP ; arrows show the liquid flow direction. The initial solid pressure in the cake p_{si} varies from the membrane-cake ($\omega = 0$) to the cake-suspension interface ($\omega = \omega_0$), where is ω the material coordinate. For colloidal objects, distribution of the local osmotic pressure π in cake and suspension is equivalent to distribution of p_s in particulate filter cake, while the liquid pressure p_l is not determined.

152

153 The liquid pressure decreases because of the friction (viscous drag), which exists between
 154 the moving liquid and stationary particles (pores surface). The same force with an opposite sign
 155 is exerted on solids by the moving liquid. In the conventional filtration-consolidation theory, the
 156 net force applied to particles or porous body is divided by the cross-sectional area of the whole
 157 system and represented as an effective (fictive) solid compressive pressure p_s (e.g., [47]). In the
 158 case of the pressure filtration, the local force balance (i.e., force balance in a thin cross-sectional

159 layer of the filter cake or filtration membrane) is presented by Eq. (1):

$$dp_s = - dp_l \quad (1)$$

160 Eq. (1) is used in the majority of pressure filtration models [47-49]. Given that in suspension $p_l =$
161 ΔP and $p_s = 0$ (there is no relative solid-liquid movement and friction), integration of Eq. (1)
162 across the system of a filter cake and filtration membrane yields

$$\Delta P = p_l + p_s \quad (2)$$

163 Eqs. (1) implies that p_s increases with decreasing of p_l (in the flow direction, Fig. 1a). The
164 solid compressive pressure increases because the compressive force is transmitted and
165 accumulated in the flow direction across the porous body (from particle to particle, from filter
166 cake to membrane, and along the membrane pores). At the given moment of filtration, the solid
167 pressure within the cake increases from zero at the cake-suspension interface to a maximum at
168 the cake-membrane interface (which is denoted herein as p_m). It further increases across the
169 membrane (linearly with the depth, if the membrane is homogeneous and incompressible) and
170 reaches $p_s = \Delta P$ at the external membrane surface. In filtrate, both $p_l = 0$ and $p_s = 0$ because the
171 applied pressure is transmitted to a membrane support (Fig. 1a).

172 Eq. (1) allows substituting the liquid pressure gradient dp_l/dx in the Darcy equation

$$u \propto dp_l/dx \quad (3)$$

173 with the solid pressure gradient dp_s/dx and applying the Darcy equation for description of liquid
174 flow in compressible filter cakes (i.e., filter cakes with p_s -dependent permeability and particle
175 concentration):

$$u = - \frac{1}{\mu \alpha(p_s) \rho_s} \frac{dp_s}{d\omega} \quad (4)$$

176 where u is the apparent liquid velocity relative to solids, μ is liquid viscosity, α is local specific
177 resistance, ρ_s is density of the solids, ω is material coordinate. Therefore, Eq. (4) is widely used
178 for description of formation and consolidation of particulate filter cakes [47-49].

179 Usually, membrane filtration of colloidal objects that have a considerable osmotic pressure
180 π is not discussed in terms of local values of p_l and p_s , which characterize porous bodies and
181 particulate filter cakes; in this case Eq. (4) can not be applied immediately. However, Elimelech
182 and Bhattacharjee [50] demonstrated that the drag force, which is applied by moving liquid to a
183 layer of colloidal particles or solute molecules retained by the membrane during filtration is equal
184 to a thermodynamic force, which is exerted by solvent molecules on these particles (solute

185 molecules). The thermodynamic force is related with the solvent chemical potential (and not with
186 the solvent pressure alone as the drag force applied to the liquid in conventional filtration-
187 consolidation theory for particulate and porous systems), and it is directly proportional to a local
188 osmotic pressure gradient $d\pi/dx$ [50]. Since the drag force is directly proportional to the apparent
189 liquid velocity u (according to Stokes equation), it follows that

$$u \propto -d\pi/dx \quad (5)$$

190 during the liquid flow across the layer of colloidal particles (solute molecules) [50]. Eq. (5) can
191 be transformed into Eq. (4), where p_s stands for π and local properties of colloidal particle layer
192 are characterized in terms of conventional filtration-consolidation theory (specific cake resistance
193 and solid volume fraction) instead of collective diffusion coefficient and number particle
194 concentration, which are usually employed for description of colloids with considerable osmotic
195 pressure. That is why Eq. (4) was applied for description of membrane filtration and
196 consolidation of colloids with considerable osmotic pressure and without a defined capillary
197 structure: protein solutions [51] and hydrogel particles (polymer networks) [52, 53]. Distribution
198 of π during filtration of colloidal objects is equivalent to distribution of p_s (which is presented in
199 Fig. 1a), while p_l is usually not determined for colloidal filter cakes.

200 For majority of materials the increasing of solid pressure causes the filter cake
201 compression, i.e., plastic and elastic structure deformation and increasing of solids concentration
202 (because of the decreasing of inter-particle distance together with deformation and squeezing of
203 aggregates and porous particles). For many materials (especially, for colloids with considerable
204 osmotic pressure, e.g., repulsing nanoparticles, polymer molecules, soft particles having elastic
205 polymer networks), the structure deformation under compression can be reversible, at least
206 partially. After release of the applied pressure, which maintained a solid pressure in the reversibly
207 compressible filter cake, the remaining solid pressure tends to dissipate via the structure
208 expansion, as a compressed spring after the load release. Consequently, the liquid flows inside
209 the filter cake, the filter cake swells (expands) (Fig. 1b). In the case of reversibly compressible
210 particulate filter cake, this liquid inflow can be seen as a result of the momentary liquid pressure
211 reduction in the filter cake: after the release of ΔP , p_l in the filter cake becomes negative (relative
212 to the atmospheric pressure) in accordance to Eq. (2). This decreasing of p_l can be seen as result
213 of a tension created in liquid-filled pores and capillaries by expanding elastic solid structure of
214 compressed particulate filter cake. In the case of reversibly compressible colloidal filter cakes

215 with considerable osmotic pressure, the liquid inflow after the pressure release can be seen as
 216 osmotic effect. The liquid is absorbed by colloidal filter cake because its excessive osmotic
 217 pressure (relative to the osmotic pressure of filtrate and bulk suspensions, which are assumed to
 218 be negligible herein) is not compensated by the applied pressure after the pressure release. In
 219 both cases, the liquid flows inside the cake, in the direction of higher solid compressive (or
 220 osmotic) pressure areas after the release of applied pressure (Fig. 1b).

221 Basic equations of filtration-consolidation theory (which are normally used to describe cake
 222 formation and compression [47, 54, 55]) hold to describe the filter cake swelling as well:

223 a) the continuity equation for the liquid flow in an infinitesimal layer of a filter cake

$$\left(\frac{\partial e}{\partial t}\right)_{\omega} = -\left(\frac{\partial u}{\partial \omega}\right)_t \quad (6)$$

224 (where t is time, ω is material coordinate, e is local void ratio, which is equal to porosity divided
 225 by solids volume fraction (see Eq. (B3) in Appendix B), u is the apparent liquid velocity relative
 226 to solids);

227 b) the Darcy equation (4);

228 c) and the force balance Eq. (1).

229 Their combination yields the basic consolidation equation [56]

$$\frac{\partial p_s}{\partial t} = -\frac{\partial p_s}{\partial e} \cdot \frac{\partial}{\partial \omega} \left(\frac{1}{\mu \rho_s \alpha} \frac{\partial p_s}{\partial \omega} \right) \quad (7)$$

230 which can describe the filter cake swelling, if appropriate initial and boundary conditions are
 231 used. In the case of colloidal filter cakes with considerable osmotic pressure, p_s in Eq. (7) and in
 232 the following discussion denotes π .

233 It should be underlined that derivation of Eq. (7) from Eq. (6) is based on the assumption
 234 that the local void fraction explicitly depends on the solid pressure only (and does not depend on
 235 time). Therefore, it is assumed that the filter cake behaves a Terzaghi element [52]; creep is not
 236 considered during the swelling in this model.

237 Fig. 1b presents the filter cake on the membrane surface at the moment of filtration pressure
 238 release. At end of filtration there is a certain solid pressure distribution within the filter cake – the
 239 initial solid pressure distribution in the swelling cake $p_{si}(\omega)$

$$p_s = p_{si}(\omega) \quad \text{at } t = 0 \quad (8)$$

240 At the cake-suspension surface the solid pressure is naught (if suspension is diluted and the
 241 sample weight is negligible)

$$p_s = 0 \quad \text{at } \omega = \omega_0 \quad (9)$$

242 At the moment after filtration pressure release the solid pressure at the cake-membrane surface p_m
 243 is maximal (as it was during the filter cake formation), while it is absent at the filtrate side of the
 244 membrane (Fig. 1b). It is reasonable to assume that during the swelling, the liquid (filtrate) can
 245 flow across the membrane into the filter cake under the solid pressure gradient, which is equal to
 246 p_m (this is also a usual presentation of osmotic flow across a semipermeable membrane). If it is
 247 assumed that membrane is a thin semipermeable incompressible continuum, this flow can be
 248 described using the Darcy equation as

$$u = \frac{p_m}{\mu r_{in}} \quad \text{at } \omega = 0 \quad (10)$$

249 as soon as $p_s = 0$ in the filtrate. In Eq. (10), r_{in} is the outside-in membrane hydraulic resistance
 250 (resistance to the liquid flow across the membrane from the filtrate side into the cell). The value
 251 of r_{in} can differ from the value of the usually discussed inside-out membrane resistance r_m (e.g.,
 252 [57]). The liquid flow also penetrates an infinitesimal filter cake layer adjacent to the membrane
 253 surface that can be described as

$$u = \frac{1}{\mu \alpha_m \rho_s} \frac{dp_s}{d\omega} \quad \text{at } \omega = 0 \quad (11)$$

254 where α_m is specific resistance of this layer. Combination of Eqs. (10) and (11) yields the second
 255 boundary condition required for solution of Eq. (7):

$$\frac{dp_s}{d\omega} = \frac{\alpha_m \rho_s}{r_{in}} p_m \quad \text{at } \omega = 0 \quad (12)$$

256 In Eq. (12) p_m and α_m change with time (decrease) during the swelling.

257 Eq. (12) differs the present paper from important works on the filter cake behavior after the
 258 filtration pressure release [37-39, 41, 42, 45, 46, 52], because previous models did not consider
 259 the membrane as a hindrance for filtrate inflow into the cake during the swelling (though the
 260 value of membrane resistance is usually included in the modeling of cake formation and cake
 261 expression [44, 51, 58-61]).

262 **3. On the possibility of filtrate inflow and absorption across the membrane**

263 In the previous section it was assumed that the liquid can flow inside the filter cake after
 264 the filtration pressure release due to the existence of solid pressure (or osmotic pressure) gradient
 265 across the semipermeable membrane. As it follows from Eqs. (1) and (2), this implies that the

266 liquid pressure in the membrane pores is negative $p_l < 0$ (relative to the atmospheric pressure in
267 filtrate). In the case of particulate filter cakes, the liquid pressure in pores of relaxing cake is
268 negative too (while the liquid pressure in osmotic systems is not determined). Surprisingly, the
269 negative values of liquid pressure in relaxing filter cake or filtration membrane were not pointed
270 out in previous studies on filter cakes expansion during the relaxation [37]. However, this
271 phenomenon is recognized in discussions on osmotic flow across the semipermeable membrane
272 [62, 63, 64, 65] and clay swelling in soil mechanics [66].

273 It is necessary to discuss the conditions, which enable the water flow across the membrane
274 (and inside the particular filter cake) under the negative liquid pressure. Though purified and
275 degassed water can exist for a certain time even at pressure below 0 Pa (i.e., under the tension
276 above $1.01 \cdot 10^5$ Pa), it is expected that in a real case application of high tension to the liquid inside
277 porous system will result in formation of cavities or dissolved gas bubbles [62, 65]. This would
278 reduce the tension and interrupt or slow down the liquid transport across the membrane [62, 65,
279 67-69]. The probability of bubbles and cavities formation must be highly dependent on the actual
280 membrane pore structure. We can speculate that it will less likely happen in very small pores
281 (when the pore size is smaller than the critical size of bubble formation) and in membranes with
282 large distant pores (when the actual liquid pressure gradient across the pore is lower than the
283 liquid pressure gradient calculated from the nominal membrane resistance). In any case, the risk
284 of bubbles formation and flow interruption increases with decreasing of the liquid tension (i.e., it
285 increases with the increasing of the pressure applied during the filter cake formation ΔP). For this
286 reason, all calculations in this article were performed for $\Delta P = 10^5$ Pa, which resulted in even
287 lower maximal values of p_m and lower absolute values of p_l inside the membrane during the filter
288 cake swelling (both depend on assumed filter cake thickness and membrane resistance).

289 **4. Solution of basic consolidation equation for compressible filter cakes**

290 Analytical solution of Eq. (7) for actual compressible filter cakes with pressure-dependent
291 specific filtration resistance and void ratio is difficult to obtain. Therefore, these dependencies are
292 usually substituted with a constant effective parameter (an average modified consolidation
293 coefficient). Following this tradition, analytical solution of Eq. (7) with two constant effective
294 parameters was obtained; the solution is presented and the reason of its restricted applicability is
295 discussed in Appendix A.

296 Eq. (7) was also solved numerically for the actual case of compressible filter cakes, as it is
 297 discussed in Appendix B. The values of parameters (pressure dependencies of specific filtration
 298 resistance and solids volume fraction, local rheological properties, initial filter cake thickness, as
 299 well as applied pressure and membrane resistance) were typical for ultrafiltration of colloids that
 300 form highly compressible filter cakes with low permeability. Experimentally obtained
 301 constitutive equations for permeability, compressibility and shear stress of aqueous dispersions of
 302 casein micelles were used for this purpose; they are discussed in Appendix C.

303 Both solutions were obtained for a constant value of ω_0 : it was assumed that the weight of
 304 solids in a filter cake holds during the swelling. This is a weak assumption for a filter cake that
 305 swells on horizontal membrane in the absence of agitation; the assumption means that the swelled
 306 part of the cake remains at the cake-suspension interface.

307 5. Results and discussion

308 5.1. Particle concentration distribution

309 Fig. 2 presents two examples of solution of Eq. (7): filter cake swelling kinetics without (a)
 310 and with (b) filtrate absorption across the membrane.

311

312 *Fig. 2a, b.*

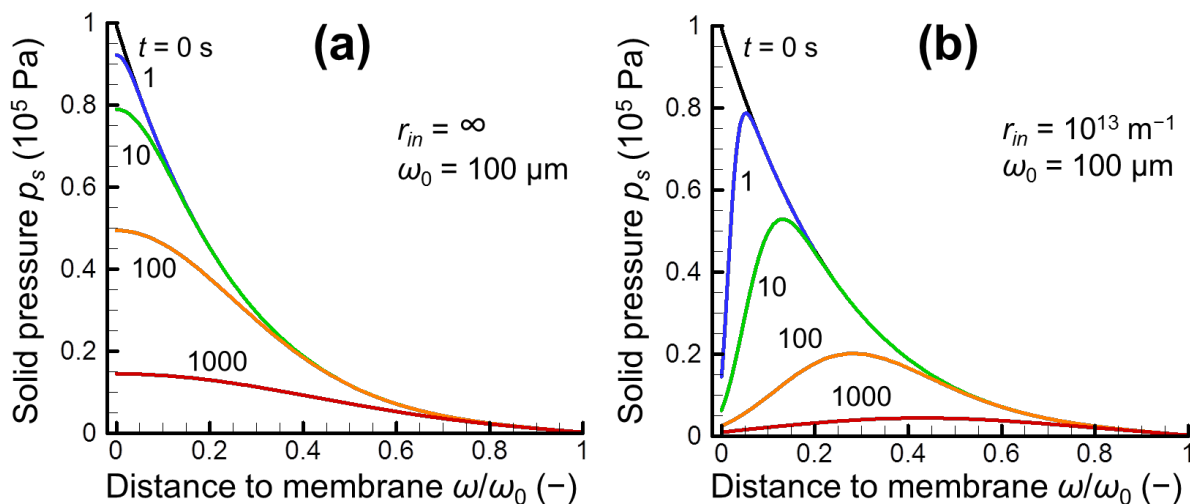


Fig. 2. Solid pressure distribution during the swelling of a thick filter cake ($\omega_0 = 100 \mu\text{m}$) formed at the membrane surface with $r_m = 10^{13} \text{ m}^{-1}$: (a) without filtrate inflow across the membrane ($r_{in} =$

∞); (b) with filtrate inflow across the membrane with $r_{in} = 10^{13} \text{ m}^{-1}$. The time after the pressure release (in seconds) is shown near the curves.

314 In the case when filtrate inflow into the cake does not happen for any reason (as it would be, for
 315 example, for impermeable surface, $r_{in} = \infty$), the solid pressure decreases continuously from the
 316 cake-membrane surface to the cake-bulk interface (Fig. 2a). However, when the membrane
 317 surface is permeable for filtrate, the solid pressure distribution in the swelling cake is non-
 318 monotonic (Fig. 2b), with the lower p_s values near the cake-membrane interface, which is
 319 explained by faster solid pressure relaxation in this area because of filtrate inflow across the
 320 membrane. Other observed trends (gradual decreasing of the solid pressure with time, slowdown
 321 of the pressure relaxation rate) are common for two cases and typical for relaxation processes. It
 322 must be noticed that in best of the authors' knowledge similar bell-like $p_s(\omega, t)$ behavior was for
 323 the first time reported by Murase [37] in the paper devoted to the modeling of expansion of
 324 consolidated material after the load release and later discussed in [39]. However, as soon as the
 325 membrane resistance was not considered (which is equal to the case $r_{in} = 0$), the solid pressure at
 326 the drainage (internal cake-membrane) surface attained $p_s = 0$ and the bell-like $p_s(\omega)$ and $\varphi(\omega)$
 327 profiles appeared immediately on the beginning of cake expansion.

328 Fig. 3 presents the evolution of local solid concentration in the filter cake caused by the
 329 solid pressure decrease during the swelling with consideration of membrane resistance.

330

331

Fig. 3a – c.

332

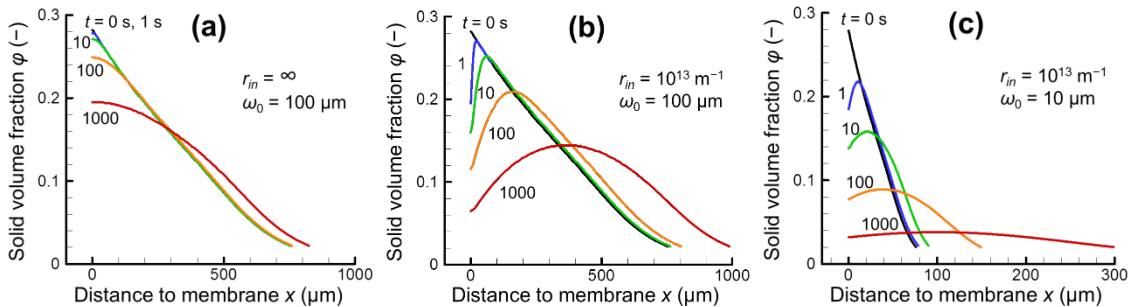


Fig. 3. Solid volume fraction distribution during the filter cake swelling: (a) thick filter cake without filtrate inflow across the membrane ($r_{in} = \infty$); (b) thick filter cake with filtrate inflow across the membrane with $r_{in} = 10^{13} \text{ m}^{-1}$; (c) thin filter cake with filtrate inflow across the

membrane with $r_{in} = 10^{13} \text{ m}^{-1}$. The time after the pressure release (in seconds) is shown near the curves; in each case $r_m = 10^{13} \text{ m}^{-1}$.

333 In the absence of filtrate inflow across the membrane ($r_{in} = \infty$, Fig. 3a), the profiles $\varphi(x)$ have no
334 peculiarities: monotonic decrease of solid pressure with x results in the monotonic decrease of
335 particle concentration from the membrane surface to the cake-bulk interface. While in the case of
336 filtrate inflow across the membrane (Figs. 3b,c), appearance of the low p_s area near the membrane
337 results in the bell-like distribution of solid concentration, i.e., the maximal concentration is
338 observed at a certain distance from the membrane; this distance increases with swelling duration.
339 The maximum of the $\varphi(x)$ curve is more pronounced in the case of initially thick filter cake with
340 $\omega_0 = 100 \text{ }\mu\text{m}$ as compared to thin cake with $\omega_0 = 10 \text{ }\mu\text{m}$, which is explained by expectedly faster
341 relaxation of thin cakes due to the faster liquid absorption across the cake-bulk interface. It can be
342 noticed that in both cases ($r_{in} = \infty$ and $r_{in} \neq \infty$) the swelling of filter cakes results in rather small
343 increase on the filter cake thickness at the beginning of the swelling ($t < 1000 \text{ s}$), which can be
344 difficult to detect by currently available methods for the filter cake surface visualization (e.g.,
345 optical coherence tomography [70], fluid dynamic gauging [71], electrochemical sensors [72],
346 particle image velocimetry [73, 74], x-ray radiography [75], magnetic resonance imaging [76-
347 78], ultrasonic [79], simple optical [80], etc. [81, 82]). However, the remarkable difference in
348 local solid concentration distribution near the membrane surface in the case of thick filter cakes
349 with and without filtrate inflow across the membrane could be observed, for example, by the
350 method of *in situ* SAXS analysis of local solid concentration in filter cakes, described in [74, 83-
351 85]. In the case of thin filter cakes, the available spatial resolution of SAXS (about $25 \text{ }\mu\text{m}$ [86-
352 88]) can be insufficient for observation of the bell-like shape of the $\varphi(x)$ [74]. Therefore, the
353 general model verification requires an application of indirect methods of the filter cake analysis.

354 **5.2. Particle concentration on membrane-cake interface**

355 Fig. 4 presents the evolution of particle concentration on the internal membrane surface φ_m
356 during the swelling of thick filter cakes at different values of membrane resistance r_{in} .

357

358 *Fig. 4.*

359

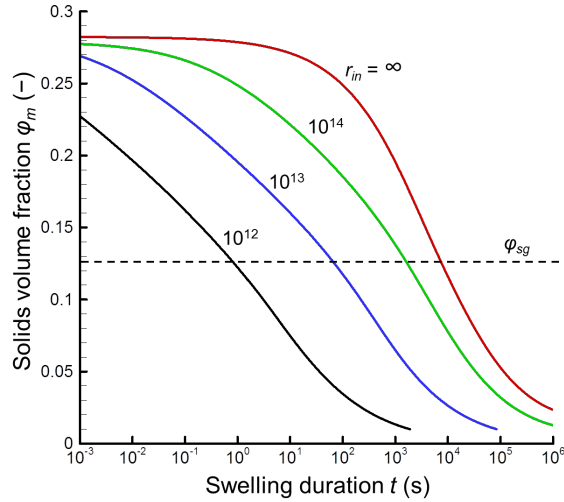


Fig. 4. Solid volume fraction at the cake-membrane interface versus the swelling duration for the thick filter cake (example for $\omega_0 = 100 \mu\text{m}$) for different values of outside-in membrane resistance r_{in} (shown near the curves). The inside-out membrane resistance is $r_m = r_{in}$ (except of $r_m = 10^{13} \text{ m}^{-1}$ for $r_{in} = \infty$). Dashed horizontal corresponds to the concentration of sol-gel transition φ_{sg} for the studied system.

360 For any nonzero value of membrane resistance (including $r_{in} = \infty$) the value of φ_m gradually
 361 decreases with time: the lower is r_{in} , the faster is the concentration decrease that is explained by
 362 faster filtrate inflow and its absorption by the filter cake part adjacent to the membrane surface. In
 363 the limiting case of $r_{in} = 0$, the solid volume fraction at the membrane surface decreases to the
 364 value $\varphi(p_s = 0)$ at the first instance of the swelling [37, 39]. However, in the current model it is
 365 assumed that the filter cake compression is reversible ($\varphi \rightarrow 0$ when $p_s \rightarrow 0$), and the value of φ_m
 366 can decrease below φ_{sg} , the concentration of sol-gel transition (i.e., transition from liquid-like to
 367 solid-like rheological behavior).

368 Fig. 5 presents the time θ , which is required in order to attain $\varphi_m = \varphi_{sg}$ at different
 369 conditions of swelling.

370

371 Fig. 5a – b.

372

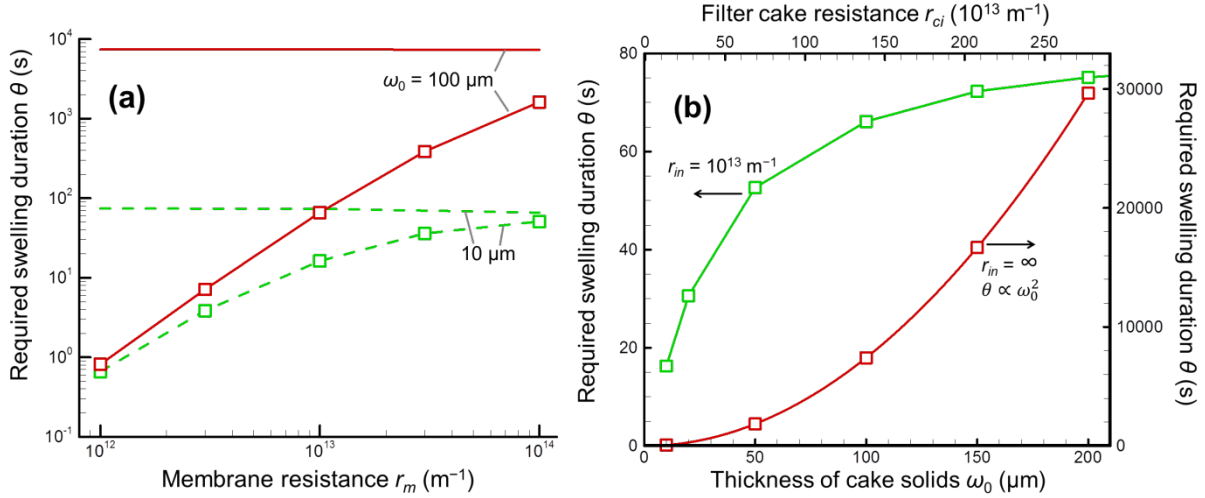


Fig. 5. Time required for filter cake swelling θ at different conditions: (a) θ versus the membrane hydraulic resistance r_m for thin ($\omega_0 = 10 \mu m$, dashed curves) and thick ($\omega_0 = 100 \mu m$, solid curves) filter cakes for the cases with ($r_{in} = r_m$, curves) and without ($r_{in} = \infty$, horizontals) filtrate inflow across the membrane; (b) θ versus the filter cake thickness ω_0 and initial filter cake resistance r_{ci} for the cases with (example for $r_{in} = 10^{13} m^{-1}$) and without ($r_{in} = \infty$) filtrate inflow across the membrane with $r_m = 10^{13} m^{-1}$.

373 For any filter cake thickness the value of θ increases with r_{in} and asymptotically approaches the
 374 value expected for the case of filter cake swelling in the absence of filtrate inflow across the
 375 membrane (cake swelling on impermeable surface with $r_{in} = \infty$, horizontals in Fig. 5a). The
 376 difference in swelling kinetics in the absence and presence of filtrate inflow is more remarkable
 377 for the thick filter cakes as compared to thin filter cakes. It is interesting to note that in the case
 378 when filtrate inflow is possible ($r_{in} \neq \infty$), the required swelling duration θ is far less sensitive to
 379 the filter cake thickness increase as compared to the case, when filtrate is not absorbed across the
 380 membrane ($r_{in} = \infty$). For example, when $r_{in} = 10^{13} m^{-1}$ the increasing of θ quickly slows down as
 381 ω_0 increases, while θ continuously increases as ω_0^2 in the case of impermeable membrane
 382 (Fig. 5b). This can be explained as follows: in first case the filtrate can be absorbed via two
 383 surfaces of the filter cake (Fig. 1), and its flow towards the high p_s area is impeded by membrane
 384 resistance r_{in} (which is assumed to be constant) and resistance of the filter cake r_{ci} (which is
 385 proportional to ω_0 , Fig. 5b). Therefore, the increasing of r_{ci} in the case of a permeable membrane
 386 has only limited impact on the swelling kinetics at the membrane-cake interface. While in the

387 case of impermeable surface with $r_{in} = \infty$, the swelling follows the usual law for swelling and
 388 diffusion into a semi-infinite medium $t \propto l^2$ [53, 89].

389 5.3. Membrane rinsing after the filter cake swelling

390 It must be noticed that for any value of θ , when $\varphi_m = \varphi_{sg}$, the $\varphi(x)$ distribution in the
 391 swelling cake is distinct in case of the absence and presence of filtrate inflow across the
 392 membrane: in the case of $r_{in} = \infty$ the highest particle concentration exists at the membrane surface
 393 ($\varphi_{max} = \varphi_m = \varphi_{sg}$), while in the case with filtrate inflow φ_{max} is situated at the certain distance from
 394 the membrane surface and a gelled part of the filter cake with $\varphi_{max} > \varphi_{sg}$ remains in the system
 395 event when $t > \theta$. In principle, this difference can be used for the model verification via the
 396 following experiment. When a filter cake with $\varphi_m > \varphi_{sg}$ is present on the membrane, certain
 397 minimal value of tangential force is required in order to sweep it from the surface (e.g., [5]). It
 398 can be expected that both attraction between cake particles and adhesion of the cake to the
 399 membrane decrease with decreasing of local particle concentration [90-92] (for example, due to
 400 the swelling [12]). Below, it is assumed that the force, required for the sweeping of the filter cake
 401 layer with a local particle concentration φ , significantly decreases, when $\varphi \leq \varphi_{sg}$ (i.e., the gel is
 402 rigid and strongly attached to membrane surface, while the sol is liquid and can be easily
 403 removed). These are reasonable assumptions for a reversibly compressible filter cake, which is in
 404 the focus of the current paper. Fig. 6 schematically presents two filter cakes (after the swelling
 405 but before the rinsing): cake 1 with $\varphi_m > \varphi_{sg}$ and cake 2 with $\varphi_m < \varphi_{sg}$.

406

407 *Fig. 6.*

408

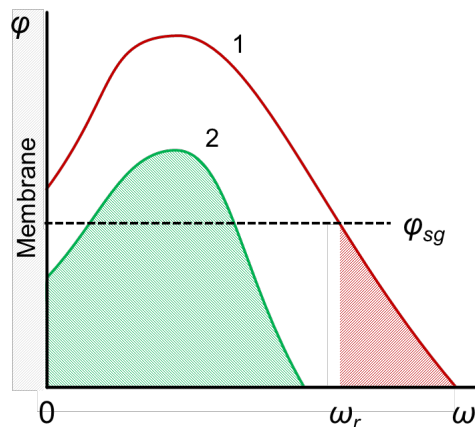


Fig. 6. To the experiment on membrane rinsing after the cake swelling: filter cakes with different particle concentration distribution (cake 1 with $\varphi_m > \varphi_{sg}$ and cake 2 with $\varphi_m < \varphi_{sg}$); dashed horizontal corresponds to the concentration of sol-gel transition, φ_{sg} , shaded parts can be removed from the surface (at left) by small tangential force, because $\varphi \leq \varphi_{sg}$ at their left borders.

409 According to the previous reasoning, if small tangential force is applied, only shaded part of the
 410 cake 1 with $\varphi \leq \varphi_{sg}$ is removed, the quantity of solids remaining on the membrane surface after
 411 the swelling-sweeping operation is defined by the solid pressure distribution profile attained
 412 during the sweeping and it is denoted as ω_r , $\omega_r > 0$ (Fig. 6). However, when the value $\varphi_m \leq \varphi_{sg}$ is
 413 attained during the swelling (cake 2, Fig. 6), the cake is removed entirely because the adhesion of
 414 its part adjacent to membrane surface is small. If the hydraulic resistance of the residual part of
 415 the filter cake r_r is measured by liquid percolation at the same applied pressure ΔP as it was used
 416 for the cake formation, the value of r_r can be calculated as that of r_{ci} (using Eqs. (B8) – (B9) with
 417 the value of ω_0 replaced by ω_r).

418

419 Fig. 7a, b.

420

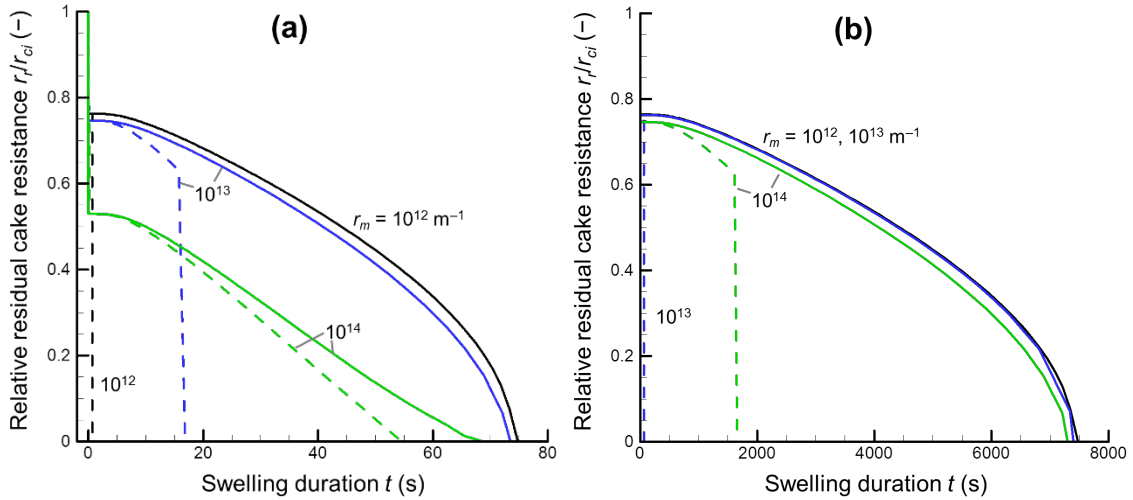


Fig. 7. Relative residual cake resistance r_r/r_{ci} when the sweeping force is applied after the swelling duration t in the absence (solid curves $r_{in} = \infty$) and presence (dashed curves, $r_{in} = r_m$) of filtrate inflow across the membrane: (a) thin filter cakes, $\omega_0 = 10 \mu\text{m}$; (b) thick filter cakes, $\omega_0 = 100 \mu\text{m}$. The values of r_m are shown near the curves.

421 Fig. 7a compares the values of r_r/r_{ci} attained after the swelling of initially thin ($\omega_0 = 10 \mu\text{m}$) filter
422 cakes different in the absence (solid curves) and presence (dashed curves) of filtrate inflow and
423 absorption across the membrane. The observed immediate decrease of r_r/r_{ci} at $t = 0$ for all filter
424 cakes is explained by the fact that the outer part of the cake has $\varphi < \varphi_{sg}$ and can be easily removed
425 by sweeping without swelling. In the absence of filtrate inflow and absorption across the
426 membrane ($r_{in} = \infty$), the difference in $r_r(t)$ behavior is explained by initially different filter cake
427 structure, when the cakes are formed at different membrane resistances: the lower is r_m , the lower
428 is the initial $\varphi(x)$ distribution (under constant ΔP and ω_0) and the faster is the cake swelling and
429 removal (that corresponds to the small negative slope of horizontals in Fig. 5a). An important
430 common feature for the cakes swelling at $r_{in} = \infty$ is that the decrease of $r_r(t)$ to 0 is gradual. In
431 contrast, when the filtrate inflow and absorption across the membrane are possible, the $r_r(t)$
432 behavior is different: gradual decreasing is followed by an abrupt drop to 0, because at $t = \theta$ the
433 gelled part of the swelling cake is not in a direct contact with the membrane. This difference is
434 less obvious for thin filter cakes at high values of r_{in} (at low ratio of r_{ci}/r_{in} , e.g. for $r_m = 10^{14} \text{ m}^{-1}$ in
435 Fig. 7a) because in this case the swelling behavior at large t approaches to that in the absence of
436 filtrate inflow (Fig. 3c). However, this difference is distinct in the case of low r_{in} ($r_{in} = 10^{12}$ and
437 10^{13} m^{-1}). For thick cakes ($\omega_0 = 100 \mu\text{m}$), which swell with filtrate inflow across the membrane,
438 an abrupt drop of r_r is observed even in the case of high r_{in} (Fig. 7b).

439 Though an implementation of the experiment on the swelling followed by the sweeping
440 described above can put in evidence the influence of the membrane resistance on the filter cake
441 swelling kinetics, it is based on the assumption that the swelled gel (with $\varphi_m < \varphi_{sg}$) is
442 instantaneously removed from the surface under the application of a small (infinitesimal)
443 tangential shear force. However, in practice, removal of the swelled gel from the surface in a
444 reasonably short time will require an application of certain minimal force. This can also provoke
445 a tangential movement (“sliding”) of the non-swelled gel that must be considered in predicting of
446 the $r_r(t)$ behavior. It can be assumed that when an external tangential shear force τ_{ext} is applied to
447 the gel after a certain swelling duration, the entire gel can move along the membrane surface in
448 the direction of τ_{ext} , if a part of the swelled gel in vicinity of the membrane attained a
449 concentration, which corresponds to the local yield stress $\tau_0 < \tau_{ext}$ (at $x = 0..h$, Fig. 8).

450

451 Fig. 8.
 452

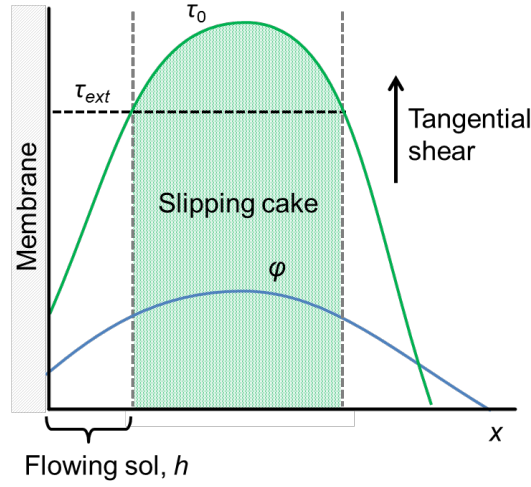


Fig. 8. Solid concentration distribution $\varphi(x)$ and corresponding yield stress distribution $\tau_0(x)$ in a swelled gel. In the vicinity of membrane the part of the gel with $\tau_0(x) < \tau_{ext}$ can flow under the external shear force, that cause the movement of the denser part of the gel with $\tau_0(x) > \tau_{ext}$.

453 If it is assumed that the part of the gel having local yield stress $\tau_0(x) > \tau_{ext}$ behaves as a rigid body
 454 (shaded area in Fig. 8), its velocity can be estimated from the equation for the plate, which moves
 455 relatively a parallel fixed surface (membrane) with the gap filled with non-Newtonian liquid
 456 having rheological properties depending on a local solid concentration. If the local rheological
 457 properties in flowing sol (Fig. 8) are described by Herschel-Bulkley model (Eq. (C3)), the
 458 relative velocity v under the external shear stress τ_{ext} can be calculated as

$$v = \int_0^h \left(\frac{\tau_{ext} - \tau_0(x)}{K(x)} \right)^{\frac{1}{N(x)}} dx \quad (13)$$

459 where h is the thickness of the gap between the membrane and the gel ($\tau_0 = \tau_{ext}$ at $x = h$, Fig. 8),
 460 and the local values of parameters of Herschel-Bulkley equation $K(x)$, $N(x)$ and $\tau_0(x)$ are
 461 determined by the local solid concentration $\varphi(x)$.

462 Fig. 9 presents values of the gap thickness h and the velocity v calculated for different
 463 swelling durations t . The concentration dependencies of Herschel-Bulkley equation parameters,
 464 determined by Jin et al. [83] for casein micelles dispersion and presented in the Appendix C,
 465 were used for calculations. The data presented in Fig. 9 are calculated for $\tau_{ext} = 20$ Pa, which is a

466 reasonable estimate for the upper level of τ_{ext} available for conventional lab scale tangential and
 467 stirred dead-end filtration equipment. At the same time $\tau_{ext} = 20$ Pa is significantly lower than the
 468 yield stress of the gelled part of the filter cake on the membrane surface before the swelling (Fig.
 469 C2)

470

471 Fig. 9a, b.

472

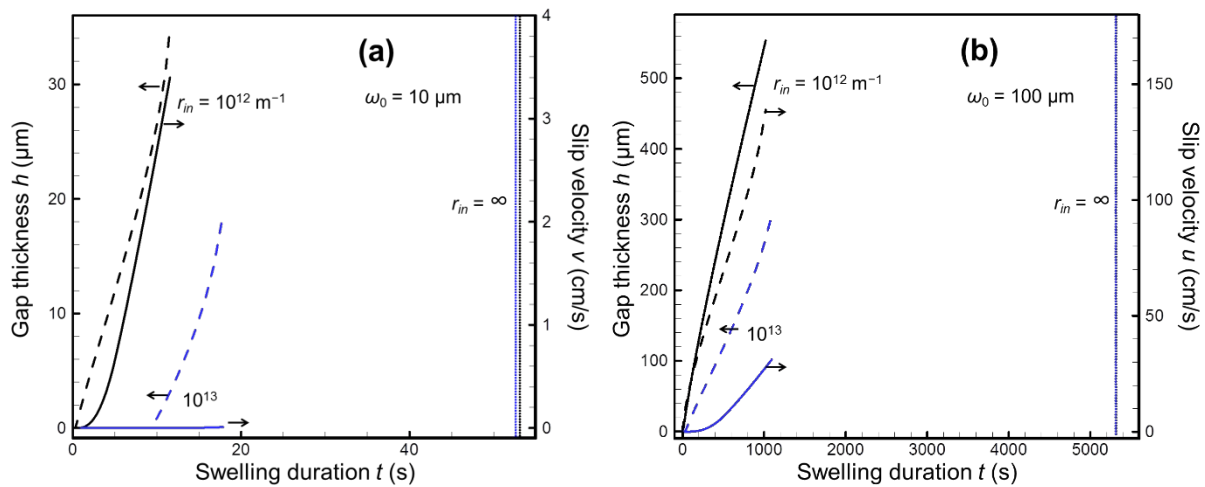


Fig. 9. Estimated values of the gap thickness h (dashed curves) and the tangential velocity of the gel v (solid curves) versus the swelling duration for (a) thin ($\omega_0 = 10 \mu\text{m}$) and (b) thick ($\omega_0 = 100 \mu\text{m}$) filter cakes under the external tangential shear $\tau_{ext} = 20$ Pa. The values of the membrane resistance to filtrate inflow r_{in} are indicated near the curves. The curves end, when the solid gel disappears because of the swelling: $\varphi_{max} = \varphi(\tau_0 = \tau_{ext})$. Horizontals correspond to the moment of the solid gel disappearance in the absence of filtrate inflow across the membrane: $r_{in} = \infty$, $\varphi_m = \varphi(\tau_0 = \tau_{ext})$.

473 The lower are r_{in} and ω_0 , the faster is the gel swelling (Fig. 5a), and the faster is the
 474 increasing of h and v with t . In the case of initially thin filter cake ($\omega_0 = 10 \mu\text{m}$, Fig. 9a) the
 475 values of h attained before the moment of the solid gel disappearance (final points of the curves
 476 in Fig. 9) are of the order of tenth of microns. These low values of h are close to the surface
 477 roughness of actual membranes that conflicts with the assumption about the parallel membrane-
 478 gel alignment (Fig. 8). It can be speculated that in the case of thin filter cakes, the part of the gel
 479 with $\varphi(x) > \varphi(\tau_0 = \tau_{ext})$ can not move along the membrane surface because of the membrane

480 roughness. Therefore, in this case $\omega_0 = 10 \text{ } \mu\text{m}$, the swelled cake is swept from the membrane
 481 surface only after sufficient swelling duration, when $\varphi(x) < \varphi(\tau_0 = \tau_{ext})$: $t \approx 10 \text{ s}$ at $r_{in} = 10^{12} \text{ m}^{-1}$
 482 and $t \approx 20 \text{ s}$ at $r_{in} = 10^{13} \text{ m}^{-1}$. This noticeably increases the expected time required for complete
 483 gel removal from the membrane in the presence of filtrate inflow, from $\approx 1 \text{ s}$ to $\approx 10 \text{ s}$ at
 484 $r_{in} = 10^{12} \text{ m}^{-1}$ and from $\approx 15 \text{ s}$ to $\approx 18 \text{ s}$ at $r_{in} = 10^{13} \text{ m}^{-1}$. Nevertheless, these values are lower as
 485 compared to the time required for the gel swelling in the absence of filtrate inflow across the
 486 membrane ($\approx 50 \text{ s}$ at $r_{in} = \infty$, Fig. 9a), which suggests the positive influence of filtrate inflow on
 487 the gel removal. At high filter cake thickness $\omega_0 = 100 \text{ } \mu\text{m}$, the gel swelling is slow, however, the
 488 gap thickness h can reach the values $h > 100 \text{ } \mu\text{m}$ and expected gel tangential velocity can exceed
 489 $10 \text{ cm}\cdot\text{s}^{-1}$ before the gel disappearance (Fig. 9b). Therefore, in the presence of filtrate inflow and
 490 absorption by filter cake, an abrupt decrease of residual resistance r_r (Fig. 7a, b) can be also
 491 expected in the case of application of non-infinitesimal τ_{ext} for the gel sweeping both for thick and
 492 thin filter cakes. This also implies that in the presence of filtrate inflow and absorption across the
 493 membrane, the experiment on the swelling-sweeping of thick filter cakes can result in the
 494 appearance of gel lumps in the flushing liquid [93, 94] (in contrast to the case with $r_{in} = \infty$, where
 495 the membrane is efficiently cleaned, when the gel swelling is completed). Though the swelling
 496 duration required for the thick cake sweeping from the surface is too high to have practical
 497 application (Fig. 9b), the proposed experiment can be used for the elucidation of the influence of
 498 membrane resistance on the filter cake swelling.

499 Tangential movement (“sliding”) of the filter cake, which is discussed above, is different
 500 from the apparent wall slip (or wall slip) [95], which is not included in the current simplified
 501 model of the filter cake removal. The discussed “sliding” onsets, when the stress applied to the
 502 swelling gel exceeds a minimal local value of the true yield stress in the internal swelling part of
 503 the gel ($\tau_{ext} > \min(\tau_0)$). The wall slip appears, when τ_{ext} exceeds an apparent local yield stress τ_{0a}
 504 ($\tau_{ext} > \tau_{0a}$). For homogeneous yield stress materials with a given particle volume fraction φ , the
 505 apparent yield stress is lower than the true one, $\tau_{0a}(\varphi) < \tau_0(\varphi)$ [96]. The value of τ_{0a} is not a
 506 material property, it decreases with the increasing of the smoothness on the fixed wall [96]. The
 507 information on τ_{0a} is scarce, however, it was reported that for some materials τ_{0a} (as well as τ_0)
 508 strongly decreases with decreasing of φ [96]. Hence, the value of τ_{0a} should also decrease during
 509 the filter cake swelling on the membrane surface. If the membrane smoothness is not dependent
 510 on membrane resistance, the decreasing of τ_{0a} should be faster for the membrane with lower

511 resistance to filtrate inflow r_{in} because of the faster decreasing of φ in the vicinity of the
512 membrane surface (and φ_m , in particular). Hence, qualitatively, whatever the reason of tangential
513 movement of the swelled filter cake under the external shear stress (wall slip or “sliding”), the
514 decreasing of r_{in} facilitates both (increases the tangential velocity, or decreases τ_{ext} and swelling
515 time required for the onset of tangential movement). The conclusion about easier removal of filter
516 cake in the case of higher membrane permeability and filtrate inflow holds in case of wall slip.

517 **6. Conclusions and final remarks**

518 The model of one-dimensional swelling of colloidal filter cake after the filtration pressure
519 release is developed in the frame of conventional filtration-consolidation theory. The model
520 considers the filter cake as a Terzaghi element, and it takes into account possible filtrate inflow
521 across the membrane under the solid pressure gradient and its uptake by the cake.

522 The model predicts that the filter cake swelling mechanism and swelling kinetics are
523 determined by the filter cake properties (compressibility, permeability), the initial filter cake
524 resistance r_{ci} and the membrane resistance to filtrate inflow r_{in} . The rate of the cake swelling on
525 the impermeable surface ($r_{in} = \infty$) significantly decreases with increasing of r_{ci} (or initial cake
526 thickness). While in the presence of filtrate inflow across the membrane ($r_{in} \neq \infty$), the swelling
527 rate is less sensitive to r_{ci} , and it increases with decreasing of the membrane resistance to filtrate
528 inflow r_{in} . This is explained by relatively fast filtrate inflow and absorption across the membrane
529 as compared to slower filtrate flow from the bulk suspension throughout the inner part of the
530 filter cake.

531 At high values of r_{ci}/r_{in} ratio (relatively thick filter cakes on permeable membrane surface),
532 the swelling results in the non-uniform (bell-like) solid concentration distribution in the swelling
533 cake (in contrast to monotonic decreasing of local solid concentration in the cake swelling on an
534 impermeable surface). At a reasonably high initial cake thickness ($\omega_0 = 100 \mu\text{m}$) this prediction
535 can be verified with the help of in situ SAXS-filtration method [74, 83-88].

536 It is interesting to notice that the positive influence of non-uniform deposit swelling on its
537 removal from non-porous surface is widely discussed, but outside of filtration literature [97-102].
538 It was demonstrated that formation of more voluminous internal layer under the rigid outer layer
539 of the deposit can result in formation of surface instabilities, which facilitate the deposit
540 detachment. Moreover, an interesting article recently demonstrated that the wrinkles formation

541 during the alginate gel swelling after the reverse osmosis and forward osmosis filtration
542 facilitates the fouling gel detachment and removal from the membrane [103].

543 Further calculations, based on actual rheological properties of colloidal dispersions of
544 casein micelles, show that filtrate inflow across the membrane under the solid pressure gradient
545 can have a noticeable influence on the rate of membrane surface rinsing by external shear force,
546 which is applied after the cake swelling: the decreasing of r_{in} results in the decreasing of the
547 swelling duration required for the filter cake elimination. Because of the bell-like solid
548 concentration distribution in the swelling cake with initially high thickness ($\omega_0 = 100 \mu\text{m}$), it can
549 be swept from the surface under the tangential shear stress, which is lower than the yield stress of
550 the gel. This peculiar dependence of the strength of the filter cake adhesion to the membrane on
551 the swelling duration, the filter cake thickness and the membrane resistance to filtrate inflow can
552 be verified with the help of available methods for analysis of deposit properties [71, 90-92, 104-
553 109].

554 It should be noted that the positive effect of filtrate inflow into the cell (filtrate back
555 permeation) on the membrane surface cleaning is widely demonstrated and modeled in the
556 literature on osmotic backwash or reverse osmosis and forward osmosis membranes (e.g., [110-
557 114]) and hydraulic back pulsing of ultrafiltration membranes (e.g., [115-117]). It is discussed
558 that liquid permeation across the membrane (either under the hydraulic pressure application or
559 the osmotic pressure difference between the filtrate and saline bulk solution) reduces the deposit
560 attachment to the membrane and shifts it away from the membrane surface. This facilitates the
561 deposit removal by tangential flow. The present model results suggest that the same effect
562 (filtrate backflow, gel softening, and its lifting from the membrane surface) can be expected
563 without the hydraulic back-pressure application or increasing the bulk osmotic pressure.

564 A number of assumptions was used in the current model of filter cake swelling: completely
565 reversible cake compression, absence of creep, one-dimensional cake expansion during the
566 swelling, and simplified consideration of cake adhesion to membrane surface and simplified
567 presentation of filtration membrane. Hence, further development of this model can be focused on
568 partial irreversibility of filter cake compression (e.g., as in [44, 45]) and its limited redispersion
569 (as in [20]). It also can include more realistic membrane model (porous surface instead of
570 semipermeable continuum) and account for 2D and 3D swelling (in order to be able to describe
571 possible wrinkles formation [103]).

572 Despite of used assumptions and conclusion that the influence of the membrane resistance
573 to filtrate inflow on the filter cake swelling and removal is less important for thin filter cakes
574 (which are met in membrane filtration applications) as compared to thick filter cakes (that are
575 mainly used for laboratory analysis of filter cakes), we hope that conclusions of current work can
576 be useful in different cases: for example, during the interpretation of experimental data on
577 reversibility of cake formation and compression ([18, 44, 45, 118, 119]), filter cake adhesion to
578 membrane [120], and critical conditions of cake removal from the membrane surface ([21, 94]).
579

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586 **Appendix A. Analytical solution of basic consolidation equation for cake swelling**

587 Solution of Eq. (7) with conditions (8), (9) and (12) requires the knowledge of pressure
 588 dependencies of void ratio $e(p_s)$ and specific filtration resistance $\alpha(p_s)$. In the filtration-
 589 consolidation literature it became conventional to replace these dependencies with an effective
 590 constant, the modified average consolidation coefficient C_e

$$C_e = \frac{1}{\mu\alpha\rho_s(-de/dp_s)} \quad (\text{A1})$$

591 which value can quantitatively characterize the kinetics of entire consolidation (or swelling)
 592 process for moderately compressible filter cakes. Following the traditional approach and
 593 introducing another effective constant, the average specific filtration resistance on the membrane
 594 surface α_e , instead of $\alpha_m(p_s)$ into Eq. (12), Eq. (7) can be solved analytically [37, 41, 43, 45, 52],
 595 and solid pressure distribution during the swelling can be presented as

$$p_s(\omega, t) = \int_0^1 p_{si}(\xi) \sum_{k=1}^{\infty} \frac{2\lambda_k \sin^2 \lambda_k}{\lambda_k - \sin \lambda_k \cos \lambda_k} f_n(\omega) f_n(\xi) \exp\left(-\lambda_k^2 \frac{C_e}{\omega_0^2} t\right) d\xi \quad (\text{A2})$$

596 where functions $f_n(\omega)$ and $f_n(\xi)$ are

$$f_n(\omega) = \cos\left(\lambda_k \frac{\omega}{\omega_0}\right) - \cot \lambda_k \sin\left(\lambda_k \frac{\omega}{\omega_0}\right) \quad (\text{A3})$$

$$f_n(\xi) = \cos(\lambda_k \xi) - \cot \lambda_k \sin(\lambda_k \xi) \quad (\text{A4})$$

597 and λ_k are roots of equation

$$-\frac{\tan \lambda}{\lambda} = \frac{r_{in}}{\rho_s \alpha_e \omega_0} \quad (\text{A5})$$

598 in the order of increase of their absolute values.

599 Eq. (A2) is obtained for an arbitrary initial solid pressure distribution in the filter cake
 600 $p_{si}(\omega)$. As soon as the current approach is justified only for moderately compressible filter cakes,
 601 one can focus only on the linear initial solid pressure distribution:

$$p_{si}(\omega) = p_{mi}(1 - \omega/\omega_0) \quad (\text{A6})$$

602 where p_{mi} is the initial value of solid pressure at the membrane surface. Introducing Eq. (A6) into
 603 (A2) yields

$$p_s(\omega, t) = p_{mi} \sum_{k=1}^{\infty} \frac{2(\sin^2 \lambda_k - \lambda_k \sin \lambda_k \cos \lambda_k)}{\lambda_k^2 - \lambda_k \sin \lambda_k \cos \lambda_k} f_n(\omega) \exp\left(-\lambda_k^2 \frac{C_e}{\omega_0^2} t\right) \quad (\text{A7})$$

604 Using Eq. (A2) the solid pressure at cake-membrane surface ($\omega = 0$) can be determined as

$$p_m(t) = p_{mi} \sum_{k=1}^{\infty} \frac{2(\sin^2 \lambda_k - \lambda_k \sin \lambda_k \cos \lambda_k)}{\lambda_k^2 - \lambda_k \sin \lambda_k \cos \lambda_k} \exp\left(-\lambda_k^2 \frac{C_e}{\omega_0^2} t\right) \quad (\text{A8})$$

605 The value of p_m asymptotically approaches 0 during the filter cake swelling. The volume of
 606 filtrate absorbed by the swelling filter cake can be obtained with the help of integration of Darcy
 607 equation (10) over the swelling time:

$$\frac{V_f(t)}{S} = \frac{p_{mi}}{\mu r_{in} C_e} \sum_{k=1}^{\infty} \frac{2}{\lambda_k^2} \cdot \frac{\sin^2 \lambda_k - \lambda_k \sin \lambda_k \cos \lambda_k}{\lambda_k^2 - \lambda_k \sin \lambda_k \cos \lambda_k} \cdot \left[1 - \exp\left(-\lambda_k^2 \frac{C_e}{\omega_0^2} t\right)\right] \quad (\text{A9})$$

608 where S is the membrane surface.

609 The volume of filtrate absorbed by the cake across the membrane increases with time and
 610 asymptotically approaches to a certain value $V_{f\infty}$ that is defined as

$$\frac{V_{f\infty}}{S} = \frac{p_{mi}}{\mu r_{in} C_e} \sum_{k=1}^{\infty} \frac{2}{\lambda_k^2} \cdot \frac{\sin^2 \lambda_k - \lambda_k \sin \lambda_k \cos \lambda_k}{\lambda_k^2 - \lambda_k \sin \lambda_k \cos \lambda_k} \quad (\text{A10})$$

611 The “completeness” of filtrate absorption by the filter cake can be characterized with the help of
 612 a ratio U

$$U = \frac{V_f(t)}{V_{f\infty}} \quad (\text{A11})$$

613 which changes from 0 to 1 during the filter cake swelling.

614 Equations analogous to Eqs. (A9) – (A11) are frequently applied for description of different
 615 types of consolidation of moderately compressible materials. Therefore, pressure dependencies of
 616 $\alpha(p_s)$ and $e(p_s)$ are generally ignored during the modelling: it is assumed that C_e is constant
 617 despite the fact that the solid pressure changes with time and across the sample.

618 The assumption about the constancy of C_e can be acceptable for the cases where the
 619 average solid pressure, that governs the process, does not change significantly with time: for
 620 example, during mechanical compression with small pressure increments.

621 However, in contrast to different consolidation processes, description of a filter cake
 622 swelling on a membrane surface requires an additional parameter, the average specific filtration
 623 resistance α_e of the filter cake layer adjacent to the membrane surface (parameter α_e appears in
 624 Eq. (A2) and following equations via Eq. (A5)). It substitutes an actual pressure dependency
 625 $\alpha(p_m)$, where p_m is the solid pressure on the cake-membrane surface. The latter decreases from p_{mi}
 626 to 0 (Eq. (A8)), which implies significant decrease of α during the swelling. In the present paper
 627 devoted to the role of membrane resistance in the filter cake swelling, this decrease can not be

628 ignored, because, in accordance to Eq. (A5), the value of specific filtration resistance α on the
 629 membrane surface is of the same importance for description of cake swelling as the value of the
 630 membrane resistance r_{in} .

631 Therefore, numerical solution of Eq. (7) was preferred in the current paper in order to avoid
 632 the ambiguities concerning the role of membrane resistance in cake swelling.

633 Nevertheless, it was observed that same trends are predicted for the solid pressure evolution
 634 with the help of numerical solution of Eq. (7) and with application of Eqs. (A2) – (A11).

635 **Appendix B. Basic consolidation equation for cake swelling, numerical solution**

636 The dependencies of osmotic pressure (solid pressure) $p_s(c)$ and hydraulic permeability $\kappa(c)$
 637 and on the mass concentration of solid c reported by Bouchoux et al. [121] for aqueous
 638 dispersions of casein micelles were used in numerical solution of Eq. (7):

$$639 \quad p_s(c) = 8c + b_p c^2 + c_p c^2 + f_p c^6 \quad (B1)$$

$$640 \quad \kappa(c) = (a_\kappa c + b_\kappa c^2 + f_\kappa c^6)^{-1} \quad (B2)$$

639 Here c is expressed in kg/m^3 , and numerical values of polynomial coefficients are $a_p = 8$, $b_p =$
 640 $3 \cdot 10^{-2}$, $c_p = 10^{-5}$, $f_p = 3 \cdot 10^{-11}$, $a_\kappa = 9.2 \cdot 10^{14}$, $b_\kappa = 1.1 \cdot 10^{12}$, $f_\kappa = 4.6 \cdot 10^3$. These dependencies are
 641 discussed in Appendix C.

642 Introducing the dependencies $\kappa(c)$ and $p_s(c)$ into Eq. (7) with the help of relations (C1) –
 643 (C2) and the definition of the void ratio

$$644 \quad e = (1 - \varphi) / \varphi \quad (B3)$$

yield handy forms of basic consolidation equation

$$645 \quad \frac{\partial c}{\partial t} = \frac{c^2}{\mu \rho_s^2} \cdot \frac{\partial}{\partial \omega} \left(c \kappa(c) \frac{\partial p_s(c)}{\partial c} \cdot \frac{\partial c}{\partial \omega} \right) \quad (B4)$$

645 as well as initial and boundary conditions. When the swelling is preceded by the constant
 646 pressure dead-end filtration that results in formation of a filter cake with the solid concentration
 647 distributed from c_{mi} (on the membrane surface where $\omega = 0$) to c_0 (on the cake-suspension
 648 interface where $\omega = \omega_0$), the initial condition (8) can be rewritten as

$$649 \quad \int_c^{c_{mi}} c \kappa(c) \frac{\partial p_s(c)}{\partial c} dc = \frac{\omega}{\omega_0} \int_{c_0}^{c_{mi}} c \kappa(c) \frac{\partial p_s(c)}{\partial c} dc \quad \text{at } t = 0 \quad (B5)$$

The boundary conditions (9) and (12) can be represented as

$$c = c_0 \quad \text{at } \omega = \omega_0 \quad (\text{B6})$$

$$\frac{dc}{d\omega} = \frac{p_s(c)}{r_{in}} \cdot \frac{\rho_s}{c \kappa(c) \frac{\partial p_s(c)}{\partial c}} \quad \text{at } \omega = 0 \quad (\text{B7})$$

650 Eq. (B4) can be solved with initial and boundary conditions (B5) – (B7) for known
 651 dependencies $\kappa(c)$ and $p_s(c)$ and given values of parameters r_{in} , c_{mi} and ω_0 . The initial solid
 652 concentration on the cake-membrane interface of c_{mi} (in Eq. (B5)) is defined by the pressure
 653 applied during the cake formation (i.e., dead-end filtration) ΔP and the ratio of inside-out
 654 membrane resistance r_m to the filter cake resistance attained at the end of filtration r_{ci} as

$$p_s(c_{mi}) = \Delta P / (1 + r_m / r_{ci}) \quad (\text{B8})$$

655 where r_{ci} is defined from

$$r_{ci} = \rho_s \omega_0 \cdot \frac{p_s(c_{mi})}{\int_{c_0}^{c_{mi}} c \kappa(c) \frac{\partial p_s(c)}{\partial c} dc} \quad (\text{B9})$$

656 Eq. (7) was solved (dependencies $c(\omega/\omega_0, t)$ were obtained) by the Crank-Nicholson
 657 method with the help of Maple 18 (Maplesoft). Temporal and spatial step sizes were chosen such
 658 that their reduction would have had no observable effect on the result of calculation of local
 659 properties (compressibility, permeability, and further tangential velocity) of the swelling cake.

660 In order to study the influence of the possibility and intensity of filtrate inflow across the
 661 membrane on the filter cake swelling, Eq. (7) was solved for different values of r_{in} , including the
 662 case $r_{in} = \infty$, which corresponds to the absence of filtrate absorption by the cake. The value of ω_0 ,
 663 used for solution, was varied from 10 to 200 μm . Particular attention was focused on the cases
 664 $\omega_0 = 10 \mu\text{m}$ and $\omega_0 = 100 \mu\text{m}$. The former corresponds to “thin” filter cakes, which can be met in
 665 membrane filtration applications, while the latter simulates rather “thick” filter cakes (or
 666 deposits) that are usually obtained in laboratory studies on filtration and/or surface cleaning [74,
 667 83-88, 90-92]. Other parameters were fixed at reasonable values: the pressure applied during the
 668 cake formation $\Delta P = 10^5 \text{ Pa}$, the filtrate viscosity $\mu = 10^{-3} \text{ Pa}\cdot\text{s}$, the bulk concentration of casein
 669 micelles dispersion $\varphi_0 = 0.02$.

670 **Appendix C. Casein micelles dispersions, material properties**

671 Casein micelles, which properties are used in order to describe colloids typically met in
 672 filtration, are well characterized natural spherical porous colloidal particles, highly compressible
 673 and deformable, stable against the aggregation at neutral pH [122]. Using the methods of osmotic

674 stress and dead-end filtration, Bouchoux et al. obtained dependencies of hydraulic permeability κ
 675 and osmotic pressure of casein micelles dispersions on the solids concentration c for a wide range
 676 of solid pressure [121]. These dependencies are presented by Eqs. (B1) – (B2). The osmotic
 677 pressure of casein micelles dispersions attained in osmotic stress experiments is equivalent to
 678 their compressive pressure (solid pressure). The specific filtration resistance and the solids
 679 volume fraction can be obtained from the data of Bouchoux et al. [121] as

$$\alpha = 1/\kappa c \tag{C1}$$

$$\varphi = c/\rho_s \tag{C2}$$

680 The density of solids of casein micelles can be calculated from the values of micelles
 681 voluminosity and the micelles density reported by de Kruif and Huppertz [123]: $\rho_s = 1.35$ g/ml
 682 (which is close to the value provided by Morris et al. [124]). The obtained dependencies $\alpha(p_s)$ and
 683 $\varphi(p_s)$ presented in Fig. C1.

684

685

Fig. C1.

686

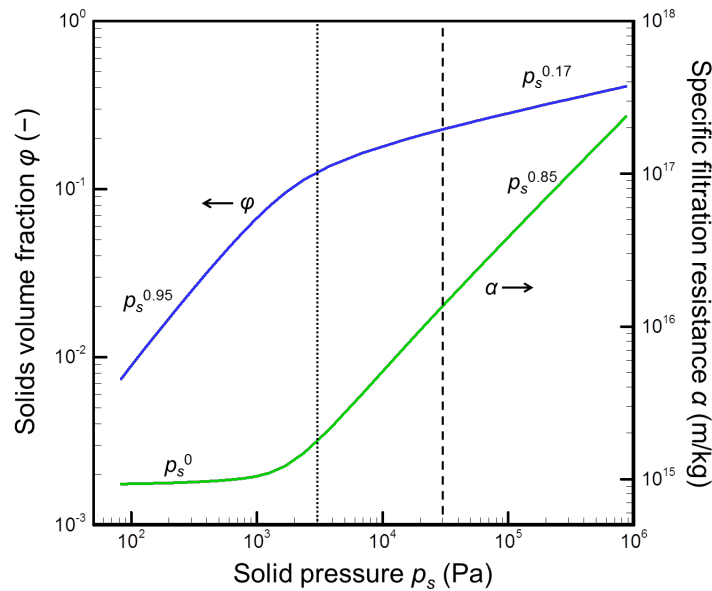


Fig. C1. Pressure dependency of solids volume fraction and specific filtration resistance for aqueous dispersions of casein micelles (calculated from the data reposted by Bouchoux et al. [121]). Dotted vertical corresponds to the solid pressure in the point of sol-gel transition; dashed vertical corresponds to the solid pressure required for “irreversible” gel formation.

687 At low particle concentration, casein micelles dispersions are sols with practically linear
 688 dependence between solids volume fraction and solid pressure and practically constant and low
 689 specific filtration resistance. According to the data of Bouchoux et al. [125, 126], the point of
 690 fluid sol to solid gel transition of casein micelles dispersions corresponds to the solid pressure
 691 about $p_s \approx 3$ kPa. Further, the increase of solid pressure above 30 kPa results into the
 692 “irreversible” gel formation; that is determined from the dead-end ultrafiltration experiments
 693 [19]). As well as many colloidal filter cakes, at high solid pressure, casein micelles gels have
 694 moderate compressibility ($\varphi \propto p_s^{0.17}$) and high specific filtration resistance that strongly depends
 695 on p_s ($\alpha = 10^{16} \dots 10^{17}$ m/kg, $\alpha \propto p_s^{0.95}$).

696 According to Jin et al. [83], for a wide range of solid volume fraction $\varphi = 0.02 - 0.147$
 697 rheological properties of aqueous dispersions of casein micelles can be described by Herschel-
 698 Bulkley model

$$\tau = \tau_0 + K\dot{\gamma}^N \quad (C3)$$

699 where τ is the shear stress, $\dot{\gamma}$ is the shear rate, and K , N and τ_0 (yield stress) are parameters that
 700 depend on the solid volume fraction. Figure C2 presents these dependencies obtained via the
 701 least-square fitting of experimental $\tau(\dot{\gamma})$ data reported by Jin et al. [83] for different casein
 702 micelles concentrations.

703

704 *Fig. C2.*

705

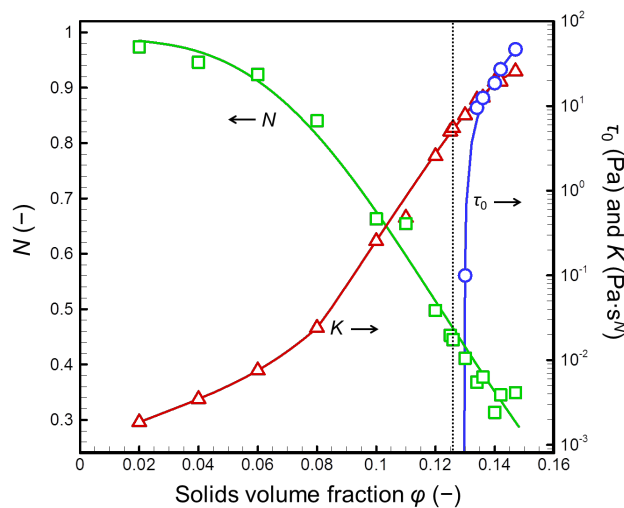


Fig. C2. Values of parameters of Herschel-Bulkley model for casein micelles dispersions at

different solid concentrations: symbols – data obtained by least-square fitting of experimental dependencies $\tau(\dot{\gamma}, \varphi)$ (presented by Jin et al. [83]) with the help of Eq. (C3), curves – result of best curve fitting for these data. Dotted vertical corresponds to the concentration of sol-gel transition.

706 Though dependencies presented in Figs. C1 – C2 were obtained either for equilibrium
707 dispersions (prepared via powder rehydration or osmotic concentration), or dynamically
708 compressed filter cakes (during the dead-end filtration), it was assumed that they are valid for
709 description of local properties of filter cakes during the swelling.
710

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1036 **Figure captions**

1037 Fig. 1. Liquid p_l and solid p_s pressures (dashed and solid curves, respectively) in a system
1038 of a particulate filter cake on a membrane surface at the moments (a) before and (b) after the
1039 release of filtration pressure ΔP ; arrows show the liquid flow direction. The initial solid pressure
1040 in the cake p_s varies from the membrane-cake ($\omega = 0$) to the cake-suspension interface ($\omega = \omega_0$),
1041 where is ω the material coordinate. For colloidal objects, distribution of the local osmotic
1042 pressure π in cake and suspension is equivalent to distribution of p_s in particulate filter cake,
1043 while the liquid pressure p_l is not determined.

1044 Fig. 2. Solid pressure distribution during the swelling of a thick filter cake ($\omega_0 = 100 \mu\text{m}$)
1045 formed at the membrane surface with $r_m = 10^{13} \text{ m}^{-1}$: (a) without filtrate inflow across the
1046 membrane ($r_{in} = \infty$); (b) with filtrate inflow across the membrane with $r_{in} = 10^{13} \text{ m}^{-1}$. The time
1047 after the pressure release (in seconds) is shown near the curves.

1048 Fig. 3. Solid volume fraction distribution during the filter cake swelling: (a) thick filter cake
1049 without filtrate inflow across the membrane ($r_{in} = \infty$); (b) thick filter cake with filtrate inflow
1050 across the membrane with $r_{in} = 10^{13} \text{ m}^{-1}$; (c) thin filter cake with filtrate inflow across the
1051 membrane with $r_{in} = 10^{13} \text{ m}^{-1}$. The time after the pressure release (in seconds) is shown near the
1052 curves; in each case $r_m = 10^{13} \text{ m}^{-1}$.

1053 Fig. 4. Solid volume fraction at the cake-membrane interface versus the swelling duration
1054 for the thick filter cake (example for $\omega_0 = 100 \mu\text{m}$) for different values of outside-in membrane
1055 resistance r_{in} (shown near the curves). The inside-out membrane resistance is $r_m = r_{in}$ (except of
1056 $r_m = 10^{13} \text{ m}^{-1}$ for $r_{in} = \infty$). Dashed horizontal corresponds to the concentration of sol-gel transition
1057 φ_{sg} for the studied system.

1058 Fig. 5. Time required for filter cake swelling θ at different conditions: (a) θ versus the
1059 membrane hydraulic resistance r_m for thin ($\omega_0 = 10 \mu\text{m}$, dashed curves) and thick ($\omega_0 = 100 \mu\text{m}$,
1060 solid curves) filter cakes for the cases with ($r_{in} = r_m$, curves) and without ($r_{in} = \infty$, horizontals)
1061 filtrate inflow across the membrane; (b) θ versus the filter cake thickness ω_0 and initial filter cake
1062 resistance r_{ci} for the cases with (example for $r_{in} = 10^{13} \text{ m}^{-1}$) and without ($r_{in} = \infty$) filtrate inflow
1063 across the membrane with $r_m = 10^{13} \text{ m}^{-1}$.

1064 Fig. 6. To the experiment on membrane rinsing after the cake swelling: filter cakes with
1065 different particle concentration distribution (cake 1 with $\varphi_m > \varphi_{sg}$ and cake 2 with $\varphi_m < \varphi_{sg}$);

1066 dashed horizontal corresponds to the concentration of sol-gel transition, φ_{sg} , shaded parts can be
1067 removed from the surface (at left) by small tangential force, because $\varphi \leq \varphi_{sg}$ at their left borders.

1068 Fig. 7. Relative residual cake resistance r_r/r_{ci} when the sweeping force is applied after the
1069 swelling duration t in the absence (solid curves $r_{in} = \infty$) and presence (dashed curves, $r_{in} = r_m$) of
1070 filtrate inflow across the membrane: (a) thin filter cakes, $\omega_0 = 10 \mu\text{m}$; (b) thick filter cakes, $\omega_0 =$
1071 $100 \mu\text{m}$. The values of r_m are shown near the curves.

1072 Fig. 8. Solid concentration distribution $\varphi(x)$ and corresponding yield stress distribution $\tau_0(x)$
1073 in a swelled gel. In the vicinity of membrane the part of the gel with $\tau_0(x) < \tau_{ext}$ can flow under
1074 the external shear force, that cause the movement of the denser part of the gel with $\tau_0(x) > \tau_{ext}$.

1075 Fig. 9. Estimated values of the gap thickness h (dashed curves) and the tangential velocity
1076 of the gel v (solid curves) versus the swelling duration for (a) thin ($\omega_0 = 10 \mu\text{m}$) and (b) thick (ω_0
1077 $= 100 \mu\text{m}$) filter cakes under the external tangential shear $\tau_{ext} = 20 \text{ Pa}$. The values of the
1078 membrane resistance to filtrate inflow r_{in} are indicated near the curves. The curves end, when the
1079 solid gel disappears because of the swelling: $\varphi_{max} = \varphi(\tau_0 = \tau_{ext})$. Horizontals correspond to the
1080 moment of the solid gel disappearance in the absence of filtrate inflow across the membrane: $r_{in} =$
1081 ∞ , $\varphi_m = \varphi(\tau_0 = \tau_{ext})$.

1082 Fig. C1. Pressure dependency of solids volume fraction and specific filtration resistance for
1083 aqueous dispersions of casein micelles (calculated from the data reposted by Bouchoux et al.
1084 [121]). Dotted vertical corresponds to the solid pressure in the point of sol-gel transition; dashed
1085 vertical corresponds to the solid pressure required for “irreversible” gel formation.

1086 Fig. C2. Values of parameters of Herschel-Bulkley model for casein micelles dispersions at
1087 different solid concentrations: symbols – data obtained by least-square fitting of experimental
1088 dependencies $\tau(\dot{\gamma}, \varphi)$ (presented by Jin et al. [83]) with the help of Eq. (C3), curves – result of
1089 best curve fitting for these data. Dotted vertical corresponds to the concentration of sol-gel
1090 transition.