



HAL
open science

Comments on the WLP in degree 3 for 8 cubes in P^6

Roberta Di Gennaro, Giovanna Ilardi, Jean Vallès

► **To cite this version:**

Roberta Di Gennaro, Giovanna Ilardi, Jean Vallès. Comments on the WLP in degree 3 for 8 cubes in P^6 . 2019. hal-02321118

HAL Id: hal-02321118

<https://hal.science/hal-02321118v1>

Preprint submitted on 20 Oct 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

COMMENTS ON THE WLP IN DEGREE 3 FOR 8 CUBES IN \mathbb{P}^6

ROBERTA DI GENNARO, GIOVANNA ILARDI AND JEAN VALLÈS

ABSTRACT. We were informed by Rosa Miró-Roig and Hoa Tran Quang that Lemma 5.2 and Proposition 5.5 of our paper [2] were imprecise for the first one and false for the second one. We thank them for pointing out these errors. Here we add the necessary details for Lemma 5.2 and explain why the proof of Proposition 5.5 is wrong. Actually we verify that, contradicting what we wrote first in [2, Proposition 5.5], that

Proposition. *The ideal $I = (L_1^3, \dots, L_8^3)$ has the WLP in degree 3 where L_1, \dots, L_8 be general linear forms on \mathbb{P}^6 .*

1. PRECISIONS FOR LEMMA 5.2

Let us recall first this lemma.

Lemma 5.2. *Let I be the ideal (L_1^d, \dots, L_r^d) where the L_j are linear forms and $r < r_d$. Let K be its syzygy bundle. Then*

$$h^0(K(i)) = 0 \Leftrightarrow rr_i \leq r_{d+i}.$$

Two precisions in this Lemma are missing. First of all, we must precise that *the L_j are general linear forms*. We must also precise that $i < d$. Indeed, if $i \geq d$ then many relations $G_1 L_1^d + \dots + G_r L_r^d = 0$ appear; for instance when $i = d$ there are the trivial relations

$$G_i L_i^d + G_j L_j^d = L_j^d L_i^d + (-L_i^d) L_j^d = 0.$$

2. ON PROPOSITION 5.5

Let us explain where was the mistake in the proof of [2, Proposition 5.5]. We wrote that the dimension of the cokernel of $A_3 \rightarrow A_4$ consisting in quartics cones with a fixed vertex (the point $\{L^V\}$) and 8 double points was at least 91 by expliciting a particular set of such quartics. This is wrong because there exist linear relations between these 91 cones. Actually, the dimension is exactly 78. Indeed, a quartic of \mathbb{P}^6 with a singular point of order 4 is a cone; having a double point for a cone means that the line joining the double point to the vertex is also double. Then this dimension is the dimension of the space of quartics in \mathbb{P}^5 with 8 double points. It is expected to be $126 - 6 \times 8 = 126 - 48 = 78$. It is exactly 78 since 8 double points in general position in \mathbb{P}^5 impose independent conditions to the quartics because this is not one of the exceptional cases listed by Alexander and Hirshowitz in [1].

According to [2, Theorem 5.1], since the dimension is exactly 78, this proves that

Proposition. *The ideal $I = (L_1^3, \dots, L_8^3)$ has the WLP in degree 3 where L_1, \dots, L_8 be general linear forms on \mathbb{P}^6 .*

REFERENCES

- [1] James Alexander and André Hirshowitz, Polynomial interpolation in several variables, *J. Alg. Geom.* (1995), 4(2): 201–222.
- [2] Roberta Di Gennaro, Giovanna Ilardi and Jean Vallès, Singular hypersurfaces characterizing the Lefschetz properties, *J. London Math. Soc.* (2014) 89(1):194–212.