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# COMMENTS ON THE WLP IN DEGREE 3 FOR 8 CUBES IN $\mathbb{P}^6$

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ABSTRACT. We were informed by Rosa Miró-Roig and Hoa Tran Quang that Lemma 5.2 and Proposition 5.5 of our paper [2] were imprecise for the first one and false for the second one. We thank them for pointing out these errors. Here we add the necessary details for Lemma 5.2 and explain why the proof of Proposition 5.5 is wrong. Actually we verify that, contradicting what we wrote first in [2, Proposition 5.5], that

**Proposition.** *The ideal  $I = (L_1^3, \dots, L_8^3)$  has the WLP in degree 3 where  $L_1, \dots, L_8$  be general linear forms on  $\mathbb{P}^6$ .*

## 1. PRECISIONS FOR LEMMA 5.2

Let us recall first this lemma.

**Lemma 5.2.** *Let  $I$  be the ideal  $(L_1^d, \dots, L_r^d)$  where the  $L_j$  are linear forms and  $r < r_d$ . Let  $K$  be its syzygy bundle. Then*

$$h^0(K(i)) = 0 \Leftrightarrow rr_i \leq r_{d+i}.$$

Two precisions in this Lemma are missing. First of all, we must precise that *the  $L_j$  are general linear forms*. We must also precise that  $i < d$ . Indeed, if  $i \geq d$  then many relations  $G_1 L_1^d + \dots + G_r L_r^d = 0$  appear; for instance when  $i = d$  there are the trivial relations

$$G_i L_i^d + G_j L_j^d = L_j^d L_i^d + (-L_i^d) L_j^d = 0.$$

## 2. ON PROPOSITION 5.5

Let us explain where was the mistake in the proof of [2, Proposition 5.5]. We wrote that the dimension of the cokernel of  $A_3 \rightarrow A_4$  consisting in quartics cones with a fixed vertex (the point  $\{L^V\}$ ) and 8 double points was at least 91 by expliciting a particular set of such quartics. This is wrong because there exist linear relations between these 91 cones. Actually, the dimension is exactly 78. Indeed, a quartic of  $\mathbb{P}^6$  with a singular point of order 4 is a cone; having a double point for a cone means that the line joining the double point to the vertex is also double. Then this dimension is the dimension of the space of quartics in  $\mathbb{P}^5$  with 8 double points. It is expected to be  $126 - 6 \times 8 = 126 - 48 = 78$ . It is exactly 78 since 8 double points in general position in  $\mathbb{P}^5$  impose independent conditions to the quartics because this is not one of the exceptional cases listed by Alexander and Hirshowitz in [1].

According to [2, Theorem 5.1], since the dimension is exactly 78, this proves that

**Proposition.** *The ideal  $I = (L_1^3, \dots, L_8^3)$  has the WLP in degree 3 where  $L_1, \dots, L_8$  be general linear forms on  $\mathbb{P}^6$ .*

## REFERENCES

- [1] James Alexander and André Hirshowitz, Polynomial interpolation in several variables, *J. Alg. Geom.* (1995), 4(2): 201–222.
- [2] Roberta Di Gennaro, Giovanna Ilardi and Jean Vallès, Singular hypersurfaces characterizing the Lefschetz properties, *J. London Math. Soc.* (2014) 89(1):194–212.