

How Potential BLFs Can Help to Decide under Incomplete Knowledge

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Introduction

- Qualitative decision problem
- Under incomplete knowledge
- Bipolar evaluation
- BLF = good visualization of
 - Connection between decision **alternatives** and **goals**,
 - **Importance** and **Polarities** of goals
- Aim: take into account **completeness** of knowledge.

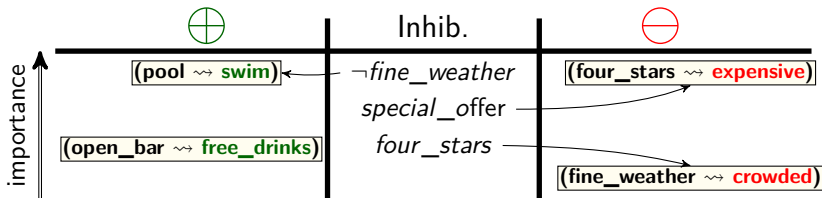
Bipolar Layered Frameworks (BLFs)

Agent's information:

- set \mathcal{P} of **decision principles** (DP) in $\mathcal{L}_F \times LIT_G$
(φ, g) = defeasible rule
"when φ (*features*) holds then g (*goal*) is generally achieved"
- **polarity of goals**: \oplus (desirable) and \ominus (undesirable)
- **relative importance of goals**: \preceq
- exceptions that **inhibit** defeasible rules:
 $\mathcal{R} = \{(\psi, p) \mid \psi \in \mathcal{L}_F, p \in \mathcal{P}\}$

Example of BLF: Find a hotel

- goals: $\text{swim} \simeq \text{expensive} \succ \text{free_drinks} \succ \text{crowded}$
- features: $\{\text{pool}, \text{open_bar}, \text{four_stars}, \text{fine_weather}, \text{special_offer}\}$



K -Valid-BLF

- Decision maker: knowledge K about alternative a
- $K \subseteq \mathcal{L}_F$ (features) **supposed consistent**
- Three kinds of knowledge about a :
 - φ holds for candidate a (i.e., $K \models \varphi$)
 - φ does not hold ($K \models \neg\varphi$)
 - φ is unknown for a ($K \not\models \varphi$ and $K \not\models \neg\varphi$)

Definition (K -Valid-BLF)

Given K and $BLF=(\mathcal{P}, \mathcal{R}, pol, \preceq)$

A K -Valid-BLF is a quadruplet $(\mathcal{P}_K, \mathcal{R}_K, pol, \preceq)$

- $\mathcal{P}_K = \{(\varphi, g) \in \mathcal{P}, \text{ s.t. } K \models \varphi\}$
DPs whose reason holds in K : K -valid-DPs.
- $\mathcal{R}_K = \{(\psi, p) \in \mathcal{R}, \text{ s.t. } K \models \psi\}$: *valid inhibitions wrt to K .*

Comparing alternatives

Definition (realized goal)

A goal g is *realized* w.r.t. a K -Valid-BLF $(\mathcal{P}_K, \mathcal{R}_K, \text{pol}, \preceq)$ iff

$$\exists(\varphi, g) \in \mathcal{P}_K \text{ and } (\varphi, g) \text{ not inhibited in } \mathcal{R}_K$$

Definition (BiLexi decision rule (Bonneton, Dubois, Fargier'08))

Alternatives	a	a'
Knowledge	K	K'
Realized goals	R	R'

$$R^\oplus = R \cap \overline{\oplus}, R^\ominus = R \cap \overline{\ominus}.$$

$$a \succeq_{\text{BiLexi}} a' \quad \text{iff } |R_\delta^\oplus| \geq |R'_\delta{}^\oplus| \text{ and } |R_\delta^\ominus| \leq |R'_\delta{}^\ominus|$$

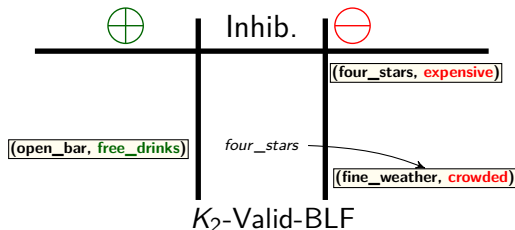
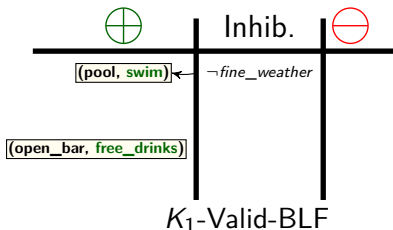
where $\delta = \text{argmax}_\lambda (\{|R_\lambda^\oplus| \neq |R'_\lambda{}^\oplus| \text{ or } |R_\lambda^\ominus| \neq |R'_\lambda{}^\ominus|\})$

Example of K -Valid-BLF

- Decision Maker: compare two hotels

$K_1 = \neg \text{fine_weather}, \text{pool}, \text{open_bar}$

$K_2 = \text{fine_weather}, \text{four_stars}, \text{open_bar}$



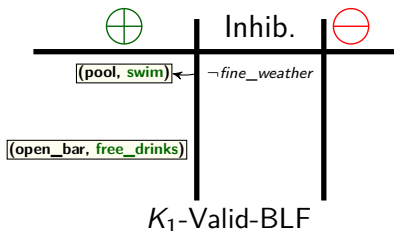
- hotel 1 \succeq_{BiLexi} hotel 2 :

$$\begin{aligned} R_{K_1}^{\oplus} &= \{ \text{free_drinks} \} & R_{K_1}^{\ominus} &= \emptyset \\ R_{K_2}^{\oplus} &= \{ \text{free_drinks} \} & R_{K_2}^{\ominus} &= \{ \text{expensive} \} \end{aligned}$$

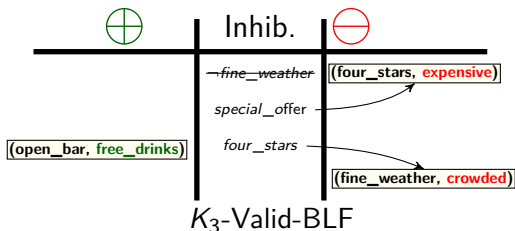
Weakness of K -Valid-BLF

The **completeness** of the agent knowledge is not taken into account

$K_1 = \neg \text{fine_weather}, \text{pool},$
 open_bar



$K_3 = \text{fine_weather}, \text{four_stars},$
 $\text{open_bar}, \text{special offer}$



K -Potential-BLF

Definition (K -Potential-BLF)

Given K and $BLF=(\mathcal{P}, \mathcal{R}, pol, \preceq)$, the K -Potential-BLF is the quadruplet $(\hat{\mathcal{P}}_K, \hat{\mathcal{R}}_K, pol, \preceq)$

- $\hat{\mathcal{P}}_K$: potential DPs

$$\hat{\mathcal{P}}_K = \{p \in \mathcal{P} \mid K \cup \{reas(p)\} \text{ is consistent} \}$$

- $\hat{\mathcal{R}}_K$: potential inhibitions relation

$$\hat{\mathcal{R}}_K = \{(\psi, p) \in \mathcal{R} \mid K \cup \{reas(p) \wedge \psi\} \text{ is consistent} \}$$

K_1 -Potential-BLF: $\hat{\mathcal{P}}_{K_1} = \{p_1, p_2, p_3\}$

$\hat{\mathcal{R}}_{K_1} = \{(\neg fine_weather, p_1), (special_offer, p_3), (four_stars, p_4)\}$

Potential realization

status	Realized	Not realized
notation	$g \in \mathbb{R}_K$	$g \in \bar{\mathbb{R}}_K$
definition	$\exists p \in \mathcal{P}_K \text{ } concl(p) = g \text{ and } \nexists(\psi, p) \in \mathcal{R}_K$	$\forall p \in \mathcal{P}_K \text{ either } concl(p) \neq g \text{ or } \exists(\psi, p) \in \mathcal{R}_K$
status	Necessarily realized	Possibly not realized
notation	$g \in \mathbb{NR}_K$	$g \in \Pi \bar{\mathbb{R}}_K$
definition	$\exists p \in \mathcal{P}_K \text{ } concl(p) = g \text{ and } \nexists(\psi, p) \in \hat{\mathcal{R}}_K$	$\forall p \in \mathcal{P}_K \text{ either } concl(p) \neq g \text{ or } \exists(\psi, p) \in \hat{\mathcal{R}}_K$
status	Potentially realized	Potentially not realized
notation	$g \in P\mathbb{R}_K$	$g \in P\bar{\mathbb{R}}_K$
definition	$\exists p \in \hat{\mathcal{P}}_K \text{ } concl(p) = g \text{ and } \nexists(\psi, p) \in \hat{\mathcal{R}}_K$	$\forall p \in \hat{\mathcal{P}}_K \text{ either } concl(p) \neq g \text{ or } \exists(\psi, p) \in \hat{\mathcal{R}}_K$
status	Possibly realized	Necessarily not realized
notation	$g \in \Pi \mathbb{R}_K$	$g \in \mathbb{NR}_K$
definition	$\exists p \in \hat{\mathcal{P}}_K \text{ } concl(p) = g \text{ and } \nexists(\psi, p) \in \mathcal{R}_K$	$\forall p \in \hat{\mathcal{P}}_K \text{ either } concl(p) \neq g \text{ or } \exists(\psi, p) \in \mathcal{R}_K$

Link between K -Potential-BLF and K -Valid-BLF

Proposition

For any BLF $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ and any knowledge base K

$$\begin{array}{ll} NR_K \subseteq R_K \subseteq \Pi R_K & \Pi \bar{R}_K \subseteq \bar{R}_K \subseteq N\bar{R}_K \\ NR_K \subseteq PR_K \subseteq \Pi R_K & \Pi \bar{R}_K \subseteq P\bar{R}_K \subseteq N\bar{R}_K \end{array}$$

Definition (Sensitivity)

Sensitivity of a BLF $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ wrt K :

$$s(BLF, K) = \left| \frac{\Pi R_K}{NR_K} \right|$$

Awareness

Definition

K is *perfect knowledge* wrt a BLF $(\mathcal{P}, \mathcal{R}, \text{pol}, \preceq)$ iff

$\forall \varphi \in \bigcup_{p \in \mathcal{P}} \{\text{reas}(p)\} \cup \bigcup_{(\psi, p) \in \mathcal{R}} \{\psi\}$, either $K \models \varphi$ or $K \models \neg \varphi$.

Proposition

For all BLF $(\mathcal{P}, \mathcal{R}, \text{pol}, \preceq)$,

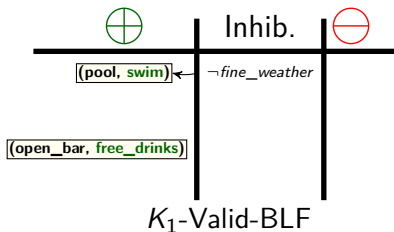
$(K \text{ is a perfect knowledge wrt BLF}) \Rightarrow (\hat{\mathcal{P}}_K = \mathcal{P}_K \text{ and } \hat{\mathcal{R}}_K = \mathcal{R}_K)$

$\Rightarrow s(\text{BLF}, K) = 0$.

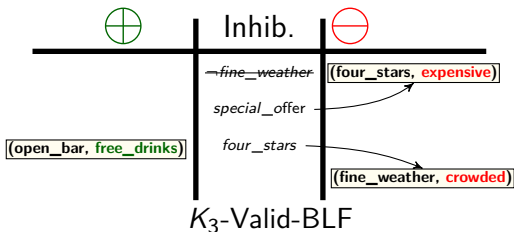
But the converse does not necessarily hold.

Refining the ordering of Candidates

$K_1 = \neg \text{fine_weather}, \text{pool}, \text{open_bar}$



$K_3 = \text{fine_weather}, \text{four_stars}, \text{open_bar}, \text{special_offer}$



1 \sim_{BiLexi} 3 can be refined by using either NR_K or ΠR_K .

- NR_K is taken if the DM is skeptical
- ΠR_K if the DM is a believer

With NR_K : 1 \sim_{BiLexi} 3 while With ΠR_K , 3 \succ_{BiLexi} 1.

Acquisition of Knowledge to Discriminate Alternatives

We consider that decision maker able to acquire knowledge

Definition

Given a BLF, $a \in \mathcal{A}$ and K and K' describing alternatives a and a' ,

- a is ε -sensitivity-comparable to a' iff

$$|s(BLF, K) - s(BLF, K')| \leq \varepsilon$$

- a is *BiLexi-preferred to a' with ε -sensitivity awareness* iff they are ε -sensitivity-comparable and $a \succ_{\text{BiLexi}} a'$

Acquisition of Knowledge to Discriminate Candidates

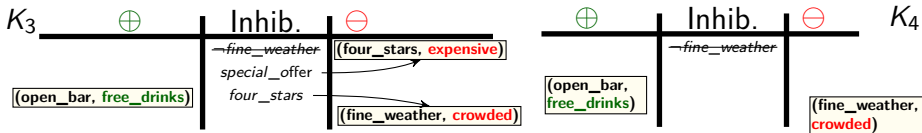
- “What **most important goal** could be necessarily realized
 - by adding only one formula φ to K
 - s.t. not possibly realized by adding $\neg\varphi$?”
- “What is the simplest formula φ that could do that?”

Definition

Given a BLF $(\mathcal{P}, \mathcal{R}, \text{pol}, \preceq)$ and K ,
a **best-discriminating formula** is:

$$\varphi^* = \arg \max_{\varphi \in \mathcal{L}_F} \left\{ k : \left\{ \begin{array}{l} \text{level}(g) = k \text{ and} \\ g \in \text{NR}_{KU\{\varphi\}} \text{ and} \\ g \notin \text{PR}_{KU\{\neg\varphi\}} \end{array} \right\} \right\}$$

Example with 4 hotels



$$1 \sim_{BiLexi} 3 \succ_{BiLexi} 4 \succ_{BiLexi} 2 \quad s(BLF, K_1) = 1 \quad s(BLF, K_2) = 2$$

$$s(B, K_3) = 1 \quad s(B, K_4) = 3$$

The DM can investigate on *pool*:

- If Yes, then $swim \in NR_{K_4 \cup \{pool\}}$, 4 becomes most preferred hotel
- otherwise $swim \notin \Pi R_{K_4 \cup \{\neg pool\}}$

Conclusions and Perspectives

Conclusions

- BLF = visual tool for decision making
- Use **potential**-BLFs to refine comparison between candidates
- **potential**-BLFs integrates **what could additionally be learned** about the candidates.

Perspectives

- Enhance expressiveness by adding supports and weights.
- Take into account uncertainty on the features of candidates.