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# Essentials of the Natural Model of the Hydrogen Atom

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## Abstract

The usual *quantum model* for the *hydrogen atom* is a Hamiltonian dynamical system defined over a *complex projective space*; it is based on Schrödinger operator  $H$ . Since the 1920s it is known that, when compared with the physical atom, the quantum model has important shortcomings. First and foremost the quantum system has no trajectories joining stationary electron configurations at different energy levels, as reflected by Schrödinger phrase *diese verdammte quantenspringerei*. The *natural model* is an alternative Hamiltonian system also based on  $H$  but having space of states equal to the *cotangent manifold* of a *real projective space*. This is a system with trajectories that join any pair of given stationary electron configurations. The present paper discusses some basic aspects of the natural model.

## Notation

Let  $h =$  *Planck constant*,  $c =$  *speed of light*,  $m_e =$  *mass of the electron*.

## §1 The quantum system

1. A new, natural, *non-linear Hamiltonian* wave dynamical system based on Schrödinger operator  $H$  will be explained.
2. This system has trajectories joining *any pair* of stationary electron configurations.
3. For present purposes the *real valued square integrable* wave functions  $\psi : \mathbb{R}^3 \rightarrow \mathbb{R}$  suffice. No use is made of complex wave functions.
4. The real wave functions conform an *inner product linear space*  $E$  on which Schrödinger operator acts

$$H : E \rightarrow E$$

5. The *ray* (real line) of  $\psi$  is  $[\psi] = \{\mu\psi \mid \mu \in \mathbb{R}\}$ . Rays are physically dimensionless. The *projective space* of  $E$  is the smooth manifold  $PE$  formed by these rays.
6. Rays are then interpreted as *electron configurations*. The term *state* is reserved for the wave momentums that will appear below.
7. The *configuration energy function*  $e_H : PE \rightarrow \mathbb{R}$  is the *normalized mean value* of Schrödinger operator

$$e_H([\psi]) = \frac{\langle H(\psi), \psi \rangle}{\langle \psi, \psi \rangle}$$

8. The vectors  $\xi$  tangent to  $PE$  at  $[\psi]$  (generalized wave velocities), and the dual cotangent vectors  $\xi^*$ , have *physical dimension*  $\text{Time}^{-1}$ , same than *frequency*.
9. Tangent vectors  $\xi$  and cotangent vectors  $\xi^*$  are the elements of respectively the *tangent manifold*  $TPE$  and the *cotangent manifold*  $T^*PE$ .
10. The *cotangent space* at  $[\psi]$  can be identified (up to being disjoint from tangents at other configurations) with the *orthogonal hyperplane*

$$T_{[\psi]}^*PE = [\psi]^\perp$$

11. The *cotangent manifold* is *symplectic*.
12. The *Riemannian metric* allows to identify  $\xi$  with its dual  $\xi^*$ , hence  $TPE$  with  $T^*PE$ , similarly to the *Dirac bra-ket formalism*. Thus  $\xi^* = \xi$  can be assumed.
13. There is a *vector bundle projection* map  $pr_{PE}^* : T^*PE \rightarrow PE$  that sends  $\xi \in T^*PE$  to its base electron configuration  $[\psi] \in PE$ .
14. The *momentum* of the generalized wave velocity  $\xi$  is the *cotangent vector*

$$\phi = \frac{h}{c} \xi$$

15. The above momentum is a *wave function version* of the classical *de Broglie scalar formula*  $p = h\nu/c$ .
16. A *hydrogen state* is synonym of *momentum*. Each momentum uniquely determines, or is riding on, a unique electron configuration  $[\psi]$
17. It will be assumed that, except for rotational phenomena, the *momentum wave function*  $\phi$  is a faithful mathematical representation of the photon while interacting with the bound electron.
18. The following are respectively the *kinetic*, *potential* and *total* energy functions of the *state*  $\phi$  a hydrogen atom having *base electron configuration*  $[\psi]$

$$e_K(\phi) = \frac{1}{2m_e} \frac{\langle \phi, \phi \rangle}{\langle \psi, \psi \rangle}$$

$$e_H(\phi) = \frac{\langle H(\psi), \psi \rangle}{\langle \psi, \psi \rangle}$$

$$e_T(\phi) = e_H(\phi) + e_K(\phi)$$

19. In words, for a hydrogen state:
- 1.- The *kinetic energy* is half the *square* of the *momentum*, divided by the electron mass and *normalized* by the base configuration wave function.
  - 2.- The *potential energy* is the energy of the electron base configuration (7 above).
  - 3.- The *total energy* is the sum of the potential and kinetic energies.
20. Since  $T^*PE$  is *symplectic*, there is for the total energy function a well defined *Hamiltonian vector field*  $X_{e_T}$  (canonical equations).
21. The *evolution equation* for the *natural hydrogen model* is  $X_{e_T}$ .
22. The *evolution equation* generates the *natural Hamiltonian flow*  $U_t : T^*PE \rightarrow T^*PE$ .
23. The natural Hamiltonian equation and flow are *non-linear* because, due to the qualitative geometric properties of  $PE$  and  $T^*PE$ , these manifolds *cannot be converted* into linear spaces.
24. The natural hydrogen atom is *stable* because the *minimum* of the total energy  $e_T$  is attained at the fundamental state. Thus the electron cannot fall into the nucleus.
25. The natural model of the hydrogen atom then consists of the following *mathematical structures*:
- |                                |                                      |
|--------------------------------|--------------------------------------|
| Natural configurations         | $PE$                                 |
| Natural states                 | $T^*PE$                              |
| Total energy function          | $e_T : T^*PE \rightarrow \mathbb{R}$ |
| Hamiltonian evolution equation | $X_{e_T}$                            |
| Hamiltonian flow               | $U_t$                                |
26. In the natural model of hydrogen the *proton*  $p^+$  is represented by Coulomb potential  $V$ ; the *electron* by the ray  $[\psi]$ ; and the *photon* by a wave momentum  $\phi$  having base configuration equal to  $[\psi]$ .
27. The *time evolution* of an initial state  $\phi_0$  is the natural trajectory  $U_t(\phi_0)$ .
28. With an *initial momentum*  $\phi_0$  having *electron initial configuration*  $[\psi_0] = pr_{PE}^*(\phi_0)$  the *electron movement* is causal, continuous and deterministic, as described by the configuration trajectory  $pr_{PE}^*(U_t(\phi_0))$  in the space  $PE$ .
29. A given initial electron configuration  $[\psi_0] \in PE$  is *not sufficient* to *uniquely specify* the *electron movement*. The trajectory of  $[\psi_0]$  depends on the choice of a photon  $\phi_0 \in T_{[\psi_0]}^*PE$ .
30. Let  $[\psi_m], [\psi_n]$  be stationary electron configurations with stationary energies  $-\lambda_m < -\lambda_n$ . The *upper configuration* and *upper energy* are  $[\psi_n]$  and  $-\lambda_n$ ; the *lower configuration* and *lower energy* are  $[\psi_m]$  and  $-\lambda_m$ .

31. There are *hydrogen states*  $\phi_0^-$  with constant energy trajectory  $U_t(\phi_0^-)$  such that the corresponding *configuration trajectory*  $pr_{PE}^*(U_t(\phi_0^-))$  *starts* at the *upper energy configuration*  $[\psi_n]$  and *ends* at the *lower energy configuration*  $[\psi_m]$ . The electron loses energy  $\lambda_m - \lambda_n$ .
32. There are *hydrogen states*  $\phi_0^+$  with constant energy trajectory  $U_t(\phi_0^+)$  such that the corresponding *configuration trajectory*  $pr_{PE}^*(U_t(\phi_0^+))$  *begins* at the *lower energy configuration*  $[\psi_m]$  and *culminates* at the *higher energy configuration*  $[\psi_n]$ . The electron gains energy  $\lambda_m - \lambda_n$ .
33. The natural model is *causal, continuous* and *deterministic*. Avoids probabilities, indeterminacy and uncertainties, and has no wave packet collapse problem.
34. The *natural model* of the hydrogen atom could be the *foundational stone* for a causal, continuous and deterministic theory of atoms as envisioned by Max Plank, Albert Einstein, Louis de Broglie, Erwin Schrödinger, David Bohm and many others that stubbornly resisted and criticized the quantum ideas.

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 Oteyeva, September 17, 2019