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# Spatially regularized multi-exponential transverse relaxation times estimation from magnitude MRI images under Rician noise.

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#### Synopsis

This work aims at improving the estimation of multi-exponential transverse relaxation times from noisy magnitude MRI images. A spatially regularized Maximum-Likelihood estimator accounting for the Rician distribution of the noise was introduced. This approach is compared to a Rician corrected least-square criterion with the introduction of spatial regularization. To deal with the large-scale optimization problem, a majoration-minimization approach was used, allowing the implementation of both the maximum-likelihood estimator and the spatial regularization. The importance of the regularization alongside the rician noise incorporation is shown both visually and numerically on magnitude MRI images acquired on fruit samples.

#### Purpose

Multi-exponential relaxation times  $T_2$  and their associated amplitudes  $I_0$  in an MRI image provide very useful information for assessing the constituents of the imaged sample. Typical examples are the detection of water compartments of plant tissues<sup>1</sup> and the quantification of myelin water fraction for multiple sclerosis disease diagnosis<sup>2</sup>. The estimation of the multi-exponential signal model from magnitude MRI images faces the problem of a relatively low signal to noise ratio (SNR), with a Rician distributed noise and a large-scale optimization problem when dealing with the entire image. Actually,  $T_2$  maps are composed of coherent regions with smooth variations between neighboring voxels. This study proposes an efficient reconstruction method of  $T_2$  values and amplitudes from magnitude images by incorporating this information in order to reduce the noise effect. The main feature of the method is to use a regularized maximum likelihood estimator derived from a Rician likelihood and a Majorization-Minimization approach coupled with the Levenberg-Marquardt algorithm to solve the large-scale optimization problem. Tests were conducted on apples and the numerical results are given to illustrate the relevance of this method and to discuss its performances.

#### Methods

For each voxel j of the MRI image, the measured signal  $y_i(\tau)$  at echo time  $\tau$  is represented by a multi-exponential model:

$$M_j( au, heta_j) = \sum_{c=1}^{N_c} I_{0_{(c,j)}} e^{-rac{ au}{T_{2_{(c,j)}}}}$$

with  $heta_j = [I_{0_{(1,j)}}, T_{2_{(1,j)}} \dots I_{0_{(N_{c,j})}}, T_{2_{(N_{c,j})}}]$ 

The data are subject to an additive Gaussian noise in the complex domain and therefore magnitude MRI data follows a Rician distribution<sup>3</sup>:

$$P_{Rician}\left(y_{j}\left( au
ight),M_{j}\left( au, heta_{j}
ight),\sigma
ight)=rac{y_{j}\left( au
ight)}{\sigma^{2}}e^{-\left\lfloorrac{y_{j}\left( au^{2}+M_{j}\left( au, heta_{j}
ight)^{2}
ight
ceil}{2\sigma^{2}}
ight
ceil}J_{0}\left(rac{y_{j}\left( au
ight)M_{j}\left( au, heta_{j}
ight)}{\sigma^{2}}
ight
ceil$$

 $J_0$  is the first kind modified Bessel function of order 0 and  $\sigma$  is the standard deviation of the noise which is usually estimated from the image background.

For an MRI image with  $N_v$  voxels, the model parameters are usually estimated by minimizing the least-squares (LS) criterion  $\sum_{j=1}^{N_v} ||y_j(\tau) - M_j(\tau, \theta_j)||_2^2$  under the assumption of a Gaussian noise using nonlinear LS solvers such as Levenberg-Marquardt (LM). However, this approach does not yield satisfying results when applied to magnitude data<sup>5</sup>. Several solutions to overcome this issue are proposed by adding a correction term to the LS criterion<sup>3,4</sup>. In this study, the retained correction uses the expectation value of data model under the hypothesis of Rician distribution<sup>5</sup> since it outperforms the other correction strategies:

$$J_{RCLS} = \sum_{j=1}^{N_v} \left\| y_j\left( au
ight) - E_{rice}\left[M_j\left( au, heta_j
ight),\sigma
ight] 
ight\|_2^2$$

 $\|\cdot\|_2^2$  stands for the sum of squares. We refer to this method as Rician corrected LS (RCLS).

A more direct way for solving this estimation problem is to use a maximum likelihood (ML) estimator<sup>5</sup> which comes down to minimize:

$$J_{ML} = \sum_{j=1}^{N_v} \sum_{ au= au_0}^{ au_n} \left[ rac{M_j( au, heta_j)^2}{2\sigma^2} - \logigg(J_0\left(rac{y_j\left( au
ight)M_j\left( au, heta_j
ight)}{\sigma^2}
ight)igg)
ight].$$

To solve this optimization problem when dealing with the entire image, a majorization-minimization (MM) technique was adopted<sup>6</sup>. The resulting MM-ML algorithm is summarized in figure 1, the LM algorithm used in this method minimizes a set of LS criteria derived from the quadratic majorization strategy.

A spatial regularization term based on a cost function  $\psi(\theta_j(i) - \theta_k(i))$  was also added to both criteria ( $J_{ML}$  and  $J_{RCLS}$ ) to ensure spatial smoothness of the estimated maps. In order to reduce the numerical complexity by maintaining variable separability between each voxel j and it's neighboring voxels k, the function  $\psi$  is majorized by<sup>7</sup>:

$$\psi( heta_j(i) - heta_k(i)) \leq rac{1}{2}\psi(2 heta_j(i) - ( heta_j^n(i) + heta_k^n(i))) + rac{1}{2}\psi(2 heta_k(i) - ( heta_j(i)^n + heta_k(i)^n)) \ for \ i = 1, 2...2*N_c$$

where n stands for the iteration number of the iterative optimization algorithm.

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The criterion to be minimized at each iteration  $\boldsymbol{n}$  of the MM algorithm is thus given by :

$$(J_{RCLS} \ or \ J_{ML}) + \sum_{i=1}^{2*Nc} \sum_{j=1}^{Nv} \sum_{k \in V_j} eta( heta(i)) \psi(2 heta_j(i) - ( heta_j(i)^n + heta_k(i)^n))$$

 $V_j$  contains the indices of voxel j's neighboors

The resulting methods are referred to by RCPLS and MM-PML. The regularization function  $\psi$  and the weight vector  $\beta$  should be adapted to each problem.

## Results

We first acquired very high SNR images (128\*128 voxels) on an apple with a 1.5T MRI (Avanto, Siemens) using a multiple-spin echo sequence<sup>8</sup> allowing the acquisition of up to 512 echo times per relaxation decay. Then we estimated the reference tri-exponentiels  $T_2$  and  $I_0$  maps which were used afterwards for 100 Monte-Carlo signal simulations with a SNR of 108. Four methods were compared : RCLS, MM-ML, RCPLS and MM-PML. A quadratic penalty function  $\psi$  was used with  $\beta = 0.16 * 10^{-2}$  for all the variables. Figure 2 gives the relative bias, the relative standard deviation and the relative MSE per parameter. Figure 3 shows the reference  $T_2$  maps and the estimated ones.

Results in figure 2 show that even though the bias is higher in the regularized version, the standard deviation was reduced significantly yielding to lower MSE. Figure 3 confirms the results by showing that the regularized version yields to visually better results. The fruit structures can thus be identified, and the  $T_2$  maps are no longer drowned in the noise.

Concerning the numerical complexity, the MM-PML algorithm ran faster (41.5 s) than the RCPLS (107.96 s).

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# **Figures**



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	Bias %		Standard Deviation %		MSE	
	RCLS	MM-LM	RCLS	MM-LM	RCLS	MM-LM
I <sub>01</sub>	9,61	4,65	41,96	35,64	1852,99	1291,83
T21	-0,55	-3,15	34,82	30,07	1212,73	914,13
I <sub>02</sub>	14,75	4,28	29,26	19,55	1073,71	400,52
T22	6,96	0,09	24,92	19,38	669,45	375,59
I <sub>03</sub>	-8,48	-1,16	34,18	22,86	1240,18	523,93
T23	8,71	6,01	18,90	11,79	433,07	175,12
Mean	5.17	1.79	30.67	23.22	967.38	542.37

	Bias %		Standard Deviation %		MSE	
	RCPLS	MM-PLM	RCPLS	MM-PLM	RCPLS	MM-PLM
I <sub>01</sub>	12,78	12,71	2,14	2,11	167,91	166,00
T21	9,23	9,30	4,21	4,10	102,92	103,30
I02	2,33	2,22	0,49	0,48	5,67	5,16
T22	5,28	5,22	0,92	0,92	28,72	28,09
I <sub>03</sub>	1,64	1,75	0,45	0,44	2,89	3,26
T <sub>23</sub>	0,99	0,97	0,48	0,48	1,21	1,17
Mean	5,38	5,36	1,45	1,42	31,05	30,75

The relative bias is given in percentage and is computed across the image and the 100 Monte-Carlo realizations. The standard deviation is also given in percentage and is computed across the 100 Monte-Carlo realizations and the average standard deviation of the different voxels is shown. Table 1 shows the results of both algorithms without regularization; Table 2 shows the results of the regularized version.



First column contains the real  $T_2$  maps used for this study, and then respectively :  $T_2$  maps reconstructed using :RCLS, RCPLS and MM-PML algorithm. The MM-ML is not shown to avoid redundancy.