

# Eliciting preferences on multiattribute societies

Patrick Meyer & Grégory Ponthière

TELECOM Bretagne, France & University of Liège, Belgium

16 june 2008  
3rd Decision Deck Workshop  
Coimbra, Portugal

## Some questions beforehand

Are all **human societies** equally good?

Are **standards of living** better in some societies than in others?

Since the early stages of Economics

How to **measure** standards of living?

## Some questions beforehand

**Subjectivity:** individuals value standards of living in different ways!

**Multidimensionality:**

- It is widely accepted that living conditions are made of various **components**.

*Classical* components of standards of living:

- consumption
- environmental quality
- health
- leisure time
- longevity

- Existence of interactions between those components.

## UN's Human Development Index (HDI)

countries	life expect. index	education index	gross dom. prod. index	HDI
Iceland	0.941	0.978	0.985	0.968
Norway	0.913	0.991	1.000	0.968
Australia	0.931	0.993	0.962	0.962
Canada	0.921	0.991	0.970	0.961
Ireland	0.890	0.993	0.994	0.959
Sweden	0.925	0.978	0.965	0.956
Switzerland	0.938	0.946	0.981	0.955
Japan	0.954	0.946	0.959	0.953
Netherlands	0.904	0.988	0.966	0.953
France	0.919	0.982	0.954	0.952
weight	1/3	1/3	1/3	

Table: International ranking, Human Development Index, 2007

Difficulties

**Arbitrariness** of the weights

No consideration of **interactions**

No **subjectivity**

# Use of Choquet integral-based MAVT

## MAVT:

- Aggregation of multiple indicators of standards of living
- Extraction of **individuals'** preferences in a non-arbitrary way

## Theory

## Choquet integral:

- *Simple* and *natural* extension of the weighted mean
- No necessity to suppose mutual preferential independence
- Modelling of **interaction** phenomena

## MAVT

$X := X_1 \times \dots \times X_n$  is a set of objects of interest described by a set  
 $N := \{1, \dots, n\}$  of attributes

MAVT aims at modelling the preferences on  $X$  of a DM, represented by  $\succeq$ ,  
by means of an **overall value function**  $U : X \rightarrow \mathbb{R}$  s.t.

$$x \succeq y \iff U(x) \geq U(y)$$

Here

$$U(x) := F(u_1(x_1), \dots, u_n(x_n)),$$

where  $F$  is the *aggregation function* and  $u_i : X_i \rightarrow \mathbb{R}$  are the *marginal value functions*.

## Choquet integral-based MAVT

$$x \succeq y \iff C_\mu((u(x)) \geq C_\mu(u(y))),$$

where  $u(x) := (u_1(x_1), \dots, u_n(x_n))$  and

$$C_\mu(u(x)) := \sum_{i=1}^n u_{\sigma(i)}(x_{\sigma(i)})[\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i+1)})],$$

where  $\sigma$  is a permutation on  $N$  such that  $u_{\sigma(1)}(x_{\sigma(1)}) \leq \dots \leq u_{\sigma(n)}(x_{\sigma(n)})$ ,  
 $\mu$  is a capacity on  $N$ ,  $A_{\sigma(i)} := \{\sigma(1), \dots, \sigma(n)\}$ , for all  $i \in \{1, \dots, n\}$ , and  
 $A_{\sigma(n+1)} := \emptyset$ .

# Choquet integral-based MAVT

Main **advantages**:

- Great flexibility via the capacity  $\mu$ ;
- Consideration of interaction phenomena.

Main **difficulties**:

- Large number of parameters for  $\mu$  (up to  $2^n$ );
- Construction of the value functions by taking into account interaction phenomena.

Necessity to **identify** the parameters of the capacity  $\mu$  via preferential information given by the DM.

## The capacity identification problem

We assume that the marginal value functions have been determined **beforehand**.

### Goal

Search for a **capacity** such that the Choquet integral w.r.t. this capacity numerically represents the **preferences** of the DM.

### Receipe

Ask the DM to express some **preferential information** on a subset of  $X$ , on importances, on interactions ... and **exploit** it.

## The capacity identification problem

**Resolution of an optimisation problem:**

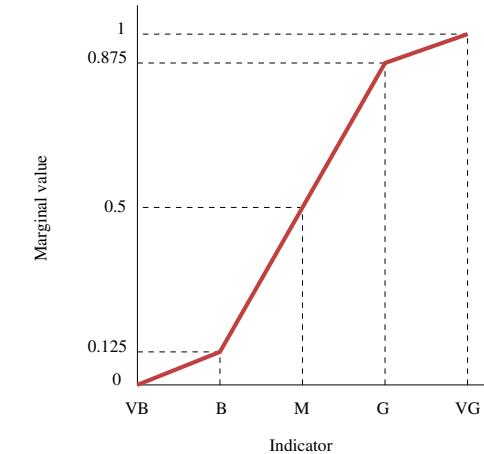
min or max  $f$

$$\text{subject to } \left\{ \begin{array}{l} \mu(S \cup i) - \mu(S) \geq 0, \forall i \in N, \forall S \subseteq N \setminus i, \\ \mu(N) = 1, \\ C_\mu(u(x)) - C_\mu(u(x')) \geq \delta_C, \\ \vdots \\ \phi_\mu(i) - \phi_\mu(j) \geq \delta_{Sh}, \\ \vdots \\ I_\mu(ij) - I_\mu(kl) \geq \delta_I, \\ I_\mu(kl) \geq 0 \\ \vdots \end{array} \right.$$

where  $f$  is an objective function that distinguishes the different identification methods.

## Illustration: preferences on two-dimensional societies

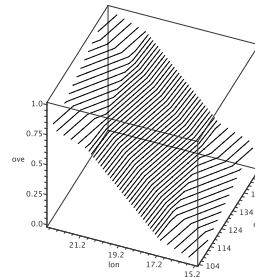
**S-shaped** partial value function on “consumption” and “longevity”:



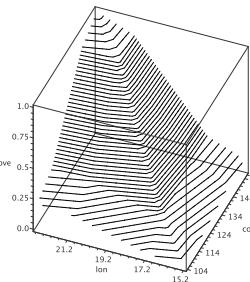
## Illustration: preferences on two-dimensional societies

**Indifference maps** for 3 situations:

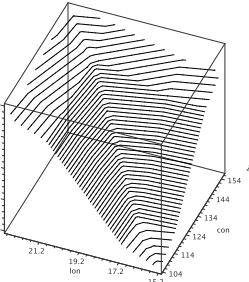
No interactions:



Complementarity:



Substitutivity:



## Experiment

## Making the world better: a simple classroom experiment

Small sized **standardised questionnaire** submitted to a set of 14 students<sup>1</sup>

Asked to **rank** hypothetical multidimensional societies

**Goal**  
Elicitate **individual preferences** via Choquet integral-based MAVT

## Dimensions of standards of living

**Psychology:** human beings can handle at most  $7 \pm 2$  items at the same time

To simplify the task: **5**-dimensional societies

- consumption
- environmental quality
- health
- leisure time
- longevity

**Simplification**, but a significant achievement on each of those dimensions is required to realise anyone's life plan!

<sup>1</sup>Master in Environmental Sciences and Management, University of Liège, Belgium, 2007-2008.

# Attributes

Selection of an **indicator** for each dimension of standards of living

Two rules:

- **Generality:** to capture well the dimension under study;
- **Understandability:** to allow a DM to easily express preferences on the object under study.

dimension	attribute	description
consumption	€ per month	
environmental quality	CO <sub>2</sub> tonnes per capita	
health	healthy life expectancy at age 65	years
leisure time	labour time	working hours per week
longevity	life expectancy at age 65	years

# Societies (alternatives)

**Hypothetical** multidimensional societies

No “real” societies to avoid “stereotypical” answers

Starting from a **society of reference**:

Attributes	'very bad' society (-20 %)	'bad' society (-10 %)	society of reference	'good' society (+ 10 %)	'very good' society (+ 20 %)
<b>Consumption</b> (€ per month)	1040 €	1170 €	1300 €	1430 €	1560 €
<b>Environment</b> (CO <sub>2</sub> tonnes per capita)	12 tonnes	11 tonnes	10 tonnes	9 tonnes	8 tonnes
<b>Health</b> (healthy life expectancy at age 65)	12 years	13.5 years	15 years	16.5 years	18 years
<b>Labour time</b> (working hours per week)	45.6 hours	41.8 hours	38 hours	34.2 hours	30.4 hours
<b>Longevity</b> (life expectancy at age 65)	15.2 years	17.1 years	19 years	20.9 years	22.8 years

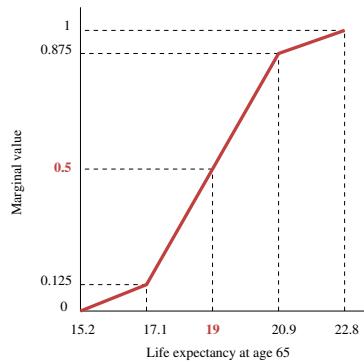
## Marginal value functions

Proper construction is a non-trivial task in Choquet integral-based MAVT

Not compatible with a classroom experiment (limited time)

Strong hypothesis

We suppose that the marginal value function are **s-shaped**



## Model

**Identification method:** minimise the variance of the parameters of the capacity

Motivations:

- The Choquet integral with respect to this capacity exploits the most, on average, its arguments;
- Closest to the simple arithmetic mean;
- Unique solution.

Lowest possible level of **k-additivity**

# Questionnaire

## Sections 1 & 2:

Goal of the study and what is expected from the respondents

## Section 3:

Description of the multiattribute societies

## Section 4.1:

10 groups of 6 societies to be ranked

ID	Consumption	Environment	Health	Labour	Longevity	Rank
S019	1430 €	8 t	15 y	30.4 h	19 y	
S020	1560 €	9 t	15 y	30.4 h	19 y	
S021	1560 €	8 t	15 y	34.2 h	19 y	
S022	1170 €	12 t	15 y	41.8 h	19 y	
S023	1170 €	11 t	15 y	45.6 h	19 y	
S024	1040 €	11 t	15 y	41.8 h	19 y	

## Results

## Section 4.2:

Rank the ten first-ranked societies and the ten last-ranked societies of Section 4.1

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Introduction

Theory

Multiattribute societies

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21 / 30

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22 / 30

Conclusion

## General observations

1 respondent is not compatible with a Choquet integral-based MAVT representation

Identification based on Section 4.2

01	02	03	04	06	07	08	09	10	11	12	13	14
2	3	2	3	2	2	4	3	2	2	2	4	2

Table: Levels of  $k$ -additivity for the 13 respondents

### Observation 1

Additive model has to be rejected!

## Shapley importance indexes of the different dimensions

resp.	con.	env.	hea.	lab.	lon.	resp.	con.	env.	hea.	lab.	lon.
01	0.26	0.24	0.15	0.20	0.15	09	0.19	0.24	0.17	0.22	0.18
02	0.21	0.12	0.16	0.31	0.20	10	0.20	0.22	0.21	0.23	0.15
03	0.19	0.19	0.20	0.23	0.19	11	0.23	0.19	0.21	0.20	0.16
04	0.29	0.16	0.26	0.15	0.15	12	0.20	0.16	0.19	0.18	0.27
06	0.20	0.21	0.18	0.24	0.18	13	0.22	0.17	0.15	0.24	0.21
07	0.20	0.22	0.18	0.20	0.20	14	0.22	0.21	0.13	0.20	0.24
08	0.23	0.23	0.18	0.18	0.18						

Table: The values of the Shapley indexes for the 13 respondents

No equal weights!

### Observation 2

Indicators based on equal weights (e.g. HDI) seem to misrepresent the complexity of human preferences!

## Shapley importance indexes of the different dimensions

resp.	con.	env.	hea.	lab.	lon.	resp.	con.	env.	hea.	lab.	lon.
01	5.0	4.0	1.5	3.0	1.5	09	3.0	5.0	1.0	4.0	2.0
02	4.0	1.0	2.0	5.0	3.0	10	2.0	4.0	3.0	5.0	1.0
03	2.0	2.0	4.0	5.0	2.0	11	5.0	2.0	4.0	3.0	1.0
04	5.0	3.0	4.0	1.5	1.5	12	4.0	1.0	3.0	2.0	5.0
06	3.0	4.0	1.5	5.0	1.5	13	4.0	2.0	1.0	5.0	3.0
07	3.0	5.0	1.0	3.0	3.0	14	4.0	3.0	1.0	2.0	5.0
08	4.5	4.5	2.0	2.0	2.0						

Table: Fractional ranking of the Shapley indexes for the 13 respondents

**Little homogeneity** across individuals!

## Interactions of the different dimensions

complementarity (> 0)				redundancy (< 0)				
env.	hea.	lab.	lon.	env.	hea.	lab.	lon.	
con.	6	4	6	7	4	8	6	5
env.		6	7	2		6	3	7
hea.			6	0			5	10
lab.				6				7

Table: Number of strictly positive and strictly negative interactions

**No homogeneity** across individuals!

## Compatibility with the first part of the questionnaire

grp.	01	02	03	04	06	07	08
01	0.73	0.87	1.00	0.73	0.33	-0.20	0.60
02	0.73	0.60	0.87	1.00	0.73	-0.07	0.73
03	0.60	0.73	0.87	0.87	0.73	0.73	0.60
04	0.60	0.60	0.73	0.87	0.47	0.47	0.47
05	0.47	0.47	0.33	0.47	0.47	0.87	0.47
06	1.00	0.47	0.33	0.73	0.60	0.87	0.73
07	0.60	0.73	0.73	0.87	0.87	0.47	0.73
08	0.47	0.60	0.87	0.47	0.87	0.60	0.47
09	0.87	0.60	0.87	0.73	0.87	0.60	0.73
10	0.60	0.73	0.87	0.87	0.87	0.87	0.60
grp.	09	10	11	12	13	14	
01	0.73	0.47	0.33	0.73	0.60	0.33	
02	1.00	0.87	0.73	0.33	0.73	0.87	
03	0.47	0.60	0.60	0.87	0.47	0.73	
04	0.60	0.87	0.87	-0.33	0.47	0.60	
05	0.60	0.60	0.33	-0.60	0.60	0.73	
06	0.60	0.73	0.60	-0.33	0.87	0.60	
07	0.60	0.87	0.73	-0.47	0.47	0.33	
08	0.33	0.33	0.47	-0.60	0.47	0.60	
09	0.33	0.60	0.73	0.47	0.47	0.47	
10	0.47	0.87	0.87	-0.73	0.33	0.47	

Table: Kendall's taus between the obtained rankings and the ones given in Section 4.1 of the questionnaire

## Further analyses

**Sensitivity** of the results to the postulated marginal value functions

Ranking of **actual** societies:

Country	01	02	03	04	06	07	08	09	10	11	12	13	14
Austria	6	6	6	6	6	6	6	6	6	6	6	6	6
Denmark	3	4	4	4	3	3	3	3	5	3	3	3	3
Germany	5	5	5	5	5	4	5	4	4	5	4	5	4
Greece	1	2	1	2	1	1	1	1	1	1	1	1	1
Italy	4	3	3	3	4	5	4	5	3	4	5	4	5
Netherlands	7	7	7	7	7	7	7	7	7	7	7	7	7
Spain	2	1	2	1	2	2	2	2	2	2	2	2	2

Table: Ranks of the 7 actual societies for each respondent

## Conclusion and perspectives

Different dimensions of standards of living matter to **unequal** extents

Significant **complementarities** and **redundancies** arise between the dimensions

**Weighted sum** is inadequate

**Thank you for your attention**

Mere **illustration**

Problematic **postulate** on the marginal value functions

More detailed study of **homogeneities** among a given sample of respondents

Construction of an **overall social indicator** of standards of living