

Online compressed sensing MR image reconstruction for high resolution T_2^* imaging

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ISMRM 2019, Montreal, Canada



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High resolution MRI:

- improves the spatial definition → helps early diagnosis
- requires longer acquisition time

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Compressed Sensing MRI

Provides theoretical guarantees of exact reconstruction under three main pillars:

- Sparse decomposition in a dictionary (Wavelet, Total Variation, Frames, ...)

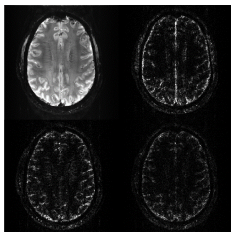


Figure: Sparse decomposition using wavelet basis

Compressed Sensing MRI

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- Asymptotically incoherent acquisition with respect to this sparse decomposition⁷

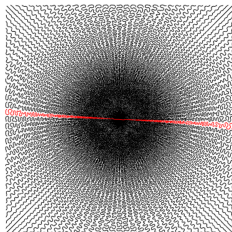


Figure: Under sampled K-space used to accelerate the acquisition

Compressed Sensing MRI

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- Sparse decomposition in a dictionary (Wavelet, Total Variation, Frames, ...)
- Asymptotically incoherent acquisition with respect to this sparse decomposition⁷
- Reconstruction that promotes the sparsity.

$$\hat{x} = \underset{x \in \mathbb{C}^N}{\operatorname{argmin}} \frac{1}{2} \|y - F_{\Omega}x\|_2^2 + \lambda \|\Psi x\|_1$$

with:

- Ψ : sparse decomposition
- x : MR image to be recovered
- y : under-sampled k-space data
- F_{Ω} : under-sampled Fourier operator on the support Ω
- $\lambda > 0$: hyper-parameter

⁷Adcock et al. 2017.

Compressed sensing limitations

Reconstruction time is long especially for:

- Highly accelerated acquisition with non-Cartesian sampling schemes
- when nonuniform Fourier transform^{8,9} is needed

Our proposition:

- Start the reconstruction from incomplete data
- Interleave acquisition and reconstruction

This will allow us to give a continuous feedback to the radiologist along the scan.

⁸Fessler and Sutton 2003, *IEEE Transactions on Signal Processing*.

⁹Keiner, Kunis, and Potts 2009, *ACM Transactions on Mathematical Software (TOMS)*.

Our approach for online MR image reconstruction

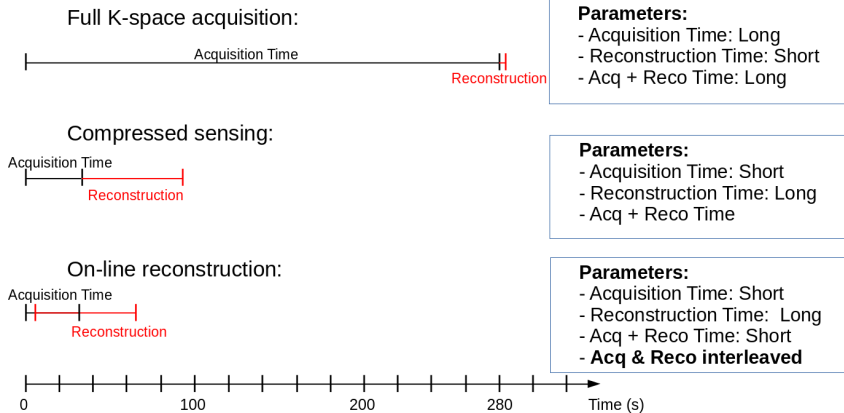


Figure: *Online* MR image reconstruction framework

Online reconstruction: Problem statement

Online MR image reconstruction is formulated as follows:

$$\forall j \in \mathbb{N}, 0 < j \leq n_b; \quad \hat{\mathbf{x}}^j = \operatorname{argmin}_{\mathbf{x} \in \mathbb{C}^N} \frac{1}{2\#\Omega_j} \left\{ \|\mathbf{F}_{\Omega_j} \mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\boldsymbol{\Psi} \mathbf{x}\|_1 \right\}$$

With:

- n_b : the number of batches
- s_b : the number of spokes in a batch
- n_j : the number of iterations in each batch
- Γ_i : the support of the i^{th} shot
- $\Omega_j = \bigcup_{0 \leq i \leq j s_b} \Gamma_i$ is the *cumulative* set of the $j s_b$ collected spokes

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At the end of the acquisition the online and offline problems are equivalent.

Optimization algorithm

Primal dual optimization

We aim to find:

$$\hat{\mathbf{x}} \in \arg \min_{\mathbf{x} \in \mathbb{C}^N} [f(\mathbf{x}) + g(\Psi \mathbf{x})] \quad (1)$$

where:

- f is convex, differentiable on \mathbb{C}^N and its gradient is β -Lipschitz
- $g \in \Gamma_0(\mathbb{C}^{N_\Psi})$ with a closed form proximity operator, given by:

$$\text{prox}_g(\mathbf{z}) = \arg \min_{\mathbf{v} \in \mathbb{C}^{N_\Psi}} \frac{1}{2} \|\mathbf{z} - \mathbf{v}\|^2 + g(\mathbf{v}) \quad (2)$$

Note: Those are standard assumptions in optimization-based image reconstruction methods.

The problem is convex (i.e. does not depend on the initialization)

Condat-Vũ Algorithm

We adapted and implemented Condat¹⁰-Vũ¹¹ algorithm as follows:

Algorithm 1: Condat-Vũ algorithm

```
1 initialize  $i = 1, j = 1, \mathbf{x}_1^1, \mathbf{z}_1^1$ ;  
2 while  $j \leq n_b$  do  
3    $\kappa_j := \frac{\beta_j}{2\|\mathbf{T}\|^2}$ ;  
4    $\tau_j := \frac{1}{\beta_j}$ ;  
5   while  $i \leq n_j$  do  
6      $\mathbf{x}_{i+1}^j := \mathbf{x}_i^j - \tau_j (\nabla f_{\Omega_j}(\mathbf{x}_i^j) + \mathbf{T}^* \mathbf{z}_i^j)$ ;  
7      $\mathbf{w}_{i+1}^j := \mathbf{z}_i^j + \kappa_j \mathbf{T} (2\mathbf{x}_{i+1}^j - \mathbf{x}_i^j)$ ;  
8      $\mathbf{z}_{i+1}^j := \mathbf{w}_{i+1}^j - \kappa_j \text{prox}_{g/\kappa_j} \left( \frac{\mathbf{w}_{i+1}^j}{\kappa_j} \right)$ ;  
9      $i := i + 1$ ;  
10  end  
11   $\mathbf{x}_1^{j+1} := \mathbf{x}_{n_k}^j$ ;  
12   $\mathbf{z}_1^{j+1} := \mathbf{z}_{n_k}^j$ ;  
13   $j := j + 1$ ;  
14 end
```

with:

- $\mathbf{z} = \Psi \mathbf{x}$
- β_j the Lipschitz constant of the spectral norm of f_{Ω_j}

Figure: Optimization algorithm

¹⁰Condat 2013, *Journal of Optimization Theory and Applications*.

¹¹Vũ 2013, *Advances in Computational Mathematics*.

Experiments parameters

Acquisition parameters:

- T2*-weighted ex-vivo baboon brain
- scanned at 7T
- Resolution: $0.4 \times 0.4 \times 3\text{mm}^3$
- FOV: 20.4cm
- Base resolution: 512×512
- TR: 550 ms (11 slices)
- TE: 30 ms
- FA: 25°

Reconstruction parameters:

- decimated Bi-Orthogonal 7/9 Wavelet transform
- Hyper-parameter λ was set retrospectively
- Final number of iterations was set to 200
- Open source code available on PySAP
- 128 GB of RAM and an 8-core (2.40 GHz) Intel Xeon E5-2630 v3 Processor

Retrospective Cartesian under-sampling

Parameter setting

- Sampling mask: 187 lines of 512 samples each
- Under-sampling factor: 2.7
- 12 central lines were acquired first and the others in pseudo random order next
- FFT was used
- Time per iteration $T_{it} = 0.12s$

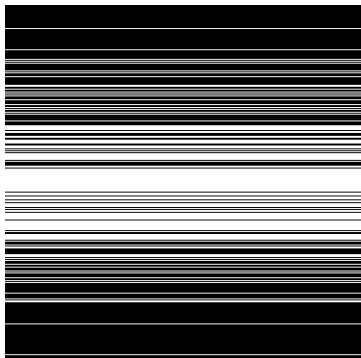


Figure: Retrospective under-sampling Cartesian mask.

Retrospective Cartesian acquisition

Batch setup

Table: Parameter setting for Cartesian acquisitions.

	Batch size s_b	Iterations n_j
$s_b = 2$	[2, 4, 6, ..., 182, 184, 187]	[9, 9, 9, ..., 9, 200]
$s_b = 23$	[23, 46, 69, 92, 115, 138, 161, 187]	[100, 100, ..., 100, 200]
$s_b = 46$	[46, 92, 138, 187]	[200, 200, 200, 200]
$s_b = 92$	[92, 187]	[400, 200]
Offline	[187]	[200]

Results: SSIM¹² scores

Cartesian under-sampling

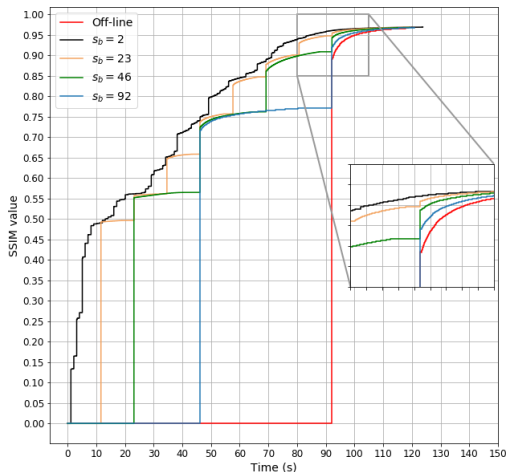


Figure: Comparison of SSIM scores for different batch sizes.

¹²Wang et al. 2004, *IEEE Transactions on Image Processing*.

Results: Images by the end of the acquisition

Cartesian under-sampling

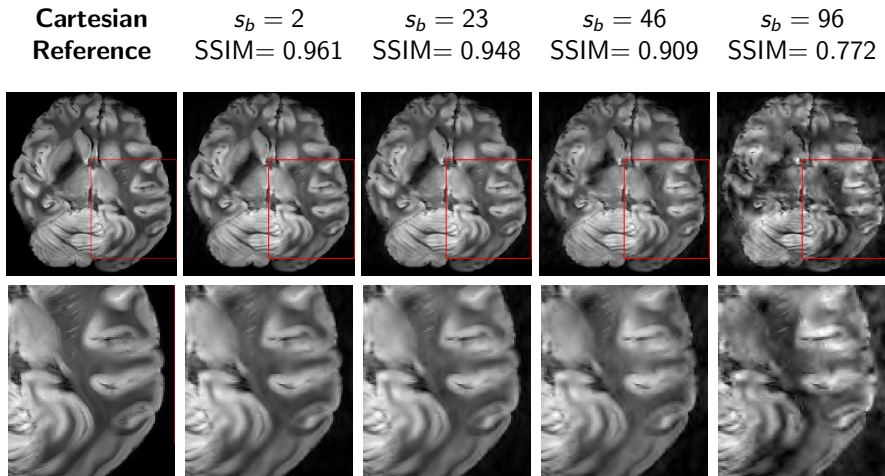


Figure: MR images delivered by the end of acquisition.

Prospective non-Cartesian under-sampling

Parameter setting

A modified T2* weighted GRE sequence was implemented based on the multi-shot Sparkling^a trajectories:

- Number of shots: 43
- Number of samples per shots: 3072
- Acceleration factor: 12 in time
- Under-sampling factor: 2
- Sequence was implemented using a golden angle approach ($\approx 112^\circ$ between consecutive shots)
- NFFT^b was used
- Time per iteration: 0.25s

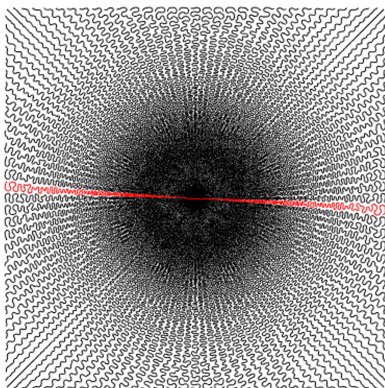


Figure: Prospective Sparkling under-sampling scheme.

^aLazarus et al. 2019, *Magnetic Resonance in Medicine*.

^bKeiner, Kunis, and Potts 2009, *ACM Transactions on Mathematical Software (TOMS)*.

Prospective non-Cartesian acquisition

Batch setup

Table: Parameter setting for non-Cartesian acquisitions.

	Batch size s_b	Iterations n_j
Offline	[43]	[200]
H_1	[5, 15, 29, 43]	[22, 30, 30, 200]
H_2	[7, 14, 21, 28, 35, 43]	[15, 15, 15, 15, 17, 200]
H_3	[4, 8, 12, 16, ..., 40, 43]	[8, 8, 8, ..., 8, 6, 200]

Results: SSIM scores

Non-Cartesian under-sampling

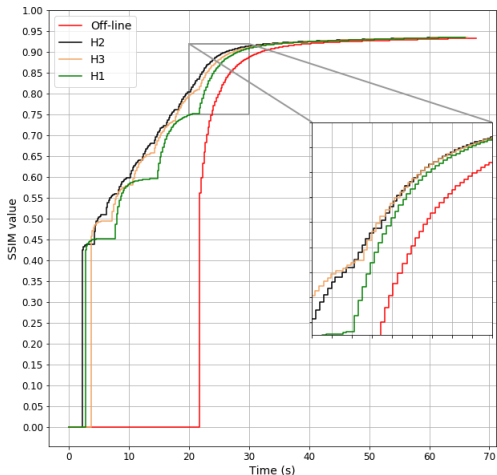


Figure: Comparison of SSIM scores for different batch setups.

Results: Images by the end of the acquisition

Non-Cartesian under-sampling

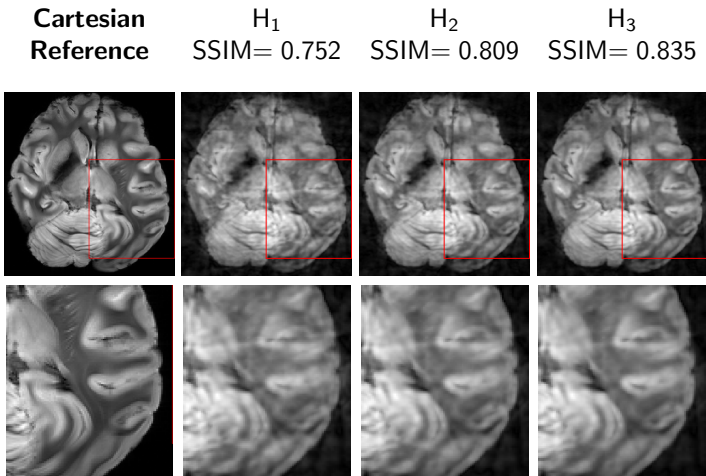


Figure: MR images delivered by the end of acquisition.

Conclusions & Outlook

Conclusions:

- We proposed a new image reconstruction framework that takes the sequential structure of multi-shot MR acquisition into account.
- This methods provides an online feedback during MR acquisition.
- Compared to offline CS reconstruction, our approach is able to provide online feedback by the end of MR acquisition, both for Cartesian and non-Cartesian sampling.
- We compared multiple batch sizes to get the best reconstruction by the end of the acquisition → small batch sizes give improved results.
- In the given allocated acquisition time, our approach achieves better image quality for Cartesian under-sampling as the time per iteration is cheaper.

Perspectives:

- Extension to the multi-channel acquisition (calibration-less, beyond ℓ_1 -norm regularization)
- Integration in the Gadgetron framework to enable this feedback directly on the scanner.



Acknowledgements

This project has been granted by the mobility grant of the SFRMBM and the FLI society



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