Optimal Participation of Residential Aggregators in Energy and Local Flexibility Markets

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Abstract—This paper presents an optimization model for Home Energy Management Systems from an aggregator's standpoint. The aggregator manages a set of resources such as PV, electrochemical batteries and thermal energy storage by means of electric water heaters. Resources are managed in order to participate in the day-ahead energy and local flexibility markets, also considering grid constraint support at the Point of Common Coupling. The resulting model is a Mixed-Integer Linear Programming problem in which the objective is to minimize day-ahead operation costs for the aggregator while complying with energy commitments in the day-ahead market and local flexibility requests. Three sources of uncertainty are considered: energy prices, PV production and load. Adjustable Robust Optimization is used to find a robust counterpart of the problem for including uncertainty. The results obtained show that using robust optimization allows strategic bidding to capture uncertainties while complying with obligations in the wholesale and local market. Data from a real-life energy community with 25 households is used to validate the proposed robust bidding methodology.

Index Terms—Aggregator, bidding, local flexibility market, prosumers, robust optimization.

NOMENCLATURE

$\hat{}$	Marker to identify central forecasts
h	index for household, $h = 1, 2, ..., N$
i	index for uncertain quantities, $i = \{c, -, +, pv, d, th\}$
s	index for segment of the piece-wise cycling cost function of the battery, $s = 1, 2, ..., S$
t	index for time step, $t = 1, 2, ..., T$
$\bar{H}_h$	TES device maximum power [kW]
$\bar{P}_h^c$	Battery's maximum charging power [kW]
$\bar{P}_h^d$	Battery's maximum discharging power [kW]
$\bar{X}_h$	Battery's maximum SOC [kWh]
$\bar{Y}_h$	TES device maximum SOC [kWh]
η^c	Battery's charging efficiency
η^d	Battery's discharging efficiency
Γ	Robustness parameter
$\hat{D}_{t,h}$	Forecasted Electrical load

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$\mu_t^{-/+}$	Negative/Positive imbalance price [EUR/kWh] with maximum and minimum forecasted values $\bar{\mu}_t$ and $\underline{\mu}_t$
π^{dso}	Remuneration for providing constraint support at the PCC [EUR]
π^{fl}	Remuneration for providing flexibility service [EUR]
π_t	Spot price [EUR/kWh] with maximum and minimum forecasted values $\bar{\pi}_t$ and $\underline{\pi}_t$
$\underline{X}_h$	Battery's minimum SOC [kWh]
$\underline{Y}_h$	TES device minimum SOC [kWh]
$a_{h,s}, b_{h,s}$	Parameters of piecewise cost functions
C_h	Thermal capacitance of TES device
$D_{t,h}^{q\%}$	q-th quantile of electrical load
$l_s^{min/max}$	Minimum/maximum DoD value for each linearized segment s of the cycling cost function
$P_{t,h}^{q\%}$	q-th quantile of PV production
$Q_{t,h,s}$	EWH load [kW]
$Q_{t,h}^{q\%}$	q-th quantile of thermal load
R_h	Thermal resistance of TES device
$H_{t,h}$	EWH input [kW]
$I_t^{-/+}$	Negative/Positive energy imbalance [kWh]
$l_{t,h,s}$	Binary variable to detect active segment of cycling cost function
$P_{t,h}^c$	Battery charging power [kW]
$P_{t,h}^d$	Battery discharging power [kW]
$P_{t,h}^{pv}$	PV panel production [kW]
$P_t^{pcc_{min/max}}$	Minimum/Maximum allowed power at the PCC [kW]
P_t^g	Day-ahead energy commitment in the wholesale market [kWh]
R_t	Net ramping rate at the PCC [kW/h]
$u_{t,h}$	Binary variable. Equals "1" if battery is charging, "0" otherwise
$X_{t,h,s}^{Ds}$	Battery DoD at the beginning of a charging cycle in segment s [p.u.]
$X_{t,h}$	Battery SOC [kWh]
$x_{t,h}$	Binary variable to detect beginning of a charging cycle
$X_{t,h}^D$	Battery DoD at the beginning of a charging cycle [p.u.]
$X_{t,h}^{Df}$	Auxiliary variable that helps capturing Battery DoD at the beginning of a charging cycle
$Y_{t,h}$	SOC of TES device
$z^{(i)}, q^{(i)}, y^{(i)}$	Dual and auxiliary variables of the robust counterpart

I. INTRODUCTION

A. Motivation

The flexibility potential extends beyond HV- and MV-grids and also reaches building and residential levels [1]. Some research has pointed out the need for providing a suitable environment to promote renewable distributed penetration and integration of prosumers into the energy market [2]. In particular, authorities at the European level have highlighted the importance of promoting fair deals for consumers in the context of energy transition and especially the importance of citizens' participation as active consumers [3], [4].

In this context, renewable- and storage-based energy communities and aggregators emerge as potential entities to participate in local markets, help overcome regulatory barriers for end-users and facilitate interactions. However, in this evolving field of research consensus seems to be on the necessity of adapting regulatory frameworks and adjusting grid operation rules and market architectures to this reality [5]. Hence, the importance of exploring different alternatives to integrate end-users and aggregate them in local energy markets, and provide new mathematical and optimization tools for energy community decision-making processes, in which storage systems are key role players.

Given that prosumers are physically connected to the distribution grid, the market environment and products must be geographically defined to allow local trading [6]. For instance, directions to exploit local flexibility include: 1) taking advantage of current markets (Day-Ahead, intraday, balancing); 2) creating new and separate markets; and 3) contracting flexibility as a system reserve. In addition, communication and coordination of Distribution System Operators (DSO) and new local energy agents are necessary to avoid upstream operative problems. Probabilistic flexibility metrics are also currently subject to debate [7], considering that intermittent sources and disturbance are governed by uncertain behavior, and this uncertainty could lead to technical and economic issues.

B. Current Research

Research that articulates local flexibility and prosumers has emerged in recent years. One common practice is to directly include local flexibility scheduling in distribution power flow calculations to solve voltage and congestion issues [8]–[12]. This approach assumes that the DSO and aggregator form a unique entity, and decisions about flexibility exploitation are driven by grid state analysis through power flows. However, these scenarios might not be applicable in all real-life cases, due to the inherent separation of agents' activities, ownership of the resources connected to the grid, and privacy-related constraints. For instance, the authors in [13] discuss these privacy issues in solving the problem of locational marginal pricing of building aggregators to alleviate grid congestion problems.

In contrast to the references above, [14] presents an optimization model that receives external signals from the DSO and schedules resources to provide the required flexibility service. This service is provided in the form of upward or downward regulation with respect to a baseline scenario.

Moreover, it proposes a local market platform in which energy cooperatives and prosumers offer flexibility and the DSO purchases the product.

Authors in [15] present a mathematical optimization model to analyze the participation of an energy community controlled by an aggregator, in wholesale and local markets. This work is centered on the market modelling and clearing process in a transactive environment in which multiple aggregators interact with each other and with the wholesale market. However, in this paper, local market prices are assumed to be a percentage of wholesale energy prices.

The research presented in [16] analyzes a microgrid that schedules devices taking distribution net ramping into account in the model. These constraints act as a service required by the utility. In addition, this study integrates inter-hour and intra-hour interactions, grid connected and islanded operation, however, it disregards uncertainty and considers resources and load from a broad perspective without detailing building- or home- level integration. Other studies have also addressed the importance of managing ramping at the distribution level by smoothing net exchanges [17], [18]. In the case of [18], ramping capabilities are not traded locally but in traditional wholesale and ancillary markets.

Reference [19] proposes a local energy system in which an aggregator acts as an intermediate between multi-energy resources and the wholesale market. Although the aggregation is local, the market interaction takes place with the wholesale market only, and assumes that local flexibility is traded in this centralized environment. Similarly [20], [21] propose bidding schemes in traditional wholesale markets.

To tackle uncertainty, an approach that has gained substantial attention in recent years is Robust Optimization (RO) [22], which is an interval-based optimization method. RO does not require knowledge of the Probability Density Function (PDF) of the uncertain variables, but instead requires only moderate information, i.e. an uncertainty set for each uncertain variable.

Although most efforts to model uncertainties with RO have been directed at large power systems applications, some work related to medium-size DG/microgrids has begun to be published. For instance, [18] presents a microgrid model for strategic bidding in energy and ancillary markets, in which RO is used to include RES uncertainty, and Stochastic Optimization (SO) is used to tackle price uncertainty. For bidding purposes in day-ahead and real-time markets, reference [20] proposes a hybrid stochastic/robust approach, in which RO captures uncertainty in real-time prices, while stochastic optimization is used to include wind and PV scenarios. References [23], [24] also include robust participation in multiple markets, specifically in energy and ancillary markets.

Robust models for home-level aggregation are still very scarce in the literature. Some proposals aim to minimize electricity bills [25], while others propose real-time decision making for batteries [26] or the management of thermal storage systems [27]. In contrast with [18]–[21], [23], [24], [28], and in line with [14]–[16], we propose a bidding strategy in both wholesale and local energy markets for flexibility offers, considering that Local Flexibility Markets (LFM) constitute an independent trading space/platform with specific bidding

rules, following the recommendations of the literature [2], [5], [6]. Unlike [14], which presents a thorough model and conceptualization for DG participation in LFM, we present a bidding strategy that takes into account uncertainty of multiple sources and includes bidding in the traditional wholesale market, in addition to the local market. We propose a model from the standpoint of a residential aggregator, unlike [15], which solves the problem from the market operator perspective in a transactive environment. In addition, our approach differs from [16], in that: 1) we include uncertainty effects and storage systems; 2) we solve the problem from the perspective of residential storage aggregation and not from MV-level DG standpoint; and 3) in the sense that we include a local flexibility trading strategy, in addition to the local constraint support. Finally, in contrast to [8]–[12] we do not consider power flow calculations to be part of the flexibility aggregator's tasks.

C. About the present work

The motivation of using the robust approach can be summarized as follows:

- Unlike the traditionally used SO formulations, RO requires less detailed probabilistic information for the uncertain variables given that the main input is associated with the confidence interval. In real-life applications, it may be difficult to create high quality scenarios for the forecasted uncertain variables (i.e. PV and residential load) required by SO. To generate these scenarios, spatio-temporal correlations among the variables should be considered. This can be a complex task and remains as an open research field, whereas defining an uncertainty interval for robust optimization is a more straightforward task, which does not necessarily require correlation analysis.

- The load and PV forecasts are obtained with state-of-the art techniques that were developed independently from this publication and that are fed with real-world data measurements in a real rural neighborhood in Portugal, as a part of the European project SENSIBLE [29], [30]. This advanced forecasting methodology and the final available outcome (quantile-based forecast intervals) are efficiently used as inputs in the RO formulation.

Our proposal aims to fill an existing gap in the literature, related to bidding strategies of smart-home aggregators for coordinated participation in both wholesale energy markets and emerging LFM, also considering uncertainties in prices, PV production and demand. To the authors' knowledge, these combined aspects have never been addressed from the standpoint of an aggregator that controls residential flexibilities, in this case, provided by PV panels and thermal and electrochemical storage.

The key contributions to the state of the art are the following:

- A new local flexibility management strategy is proposed, which is based on two products: 1) flexibility bids on a local market; and 2) local constraint support for the DSO in the form of maximum allowed net power and net ramping rate.

- An Adjustable Robust Optimization (ARO) model is proposed for coordinated management of energy community resources and bidding in wholesale and local flexibility markets. The robust counterpart includes uncertainty in energy/imbalance prices, PV production, electrical demand and thermal consumption.

The paper is organized as follows: section II shows the framework of the proposed model. Next, section III presents the details of the mathematical formulation. Section IV presents the results of the proposed approach and concluding remarks are outlined in Section V.

II. DESCRIPTION OF THE PROPOSED FRAMEWORK

In this paper we propose a bidding strategy for an aggregator of smart-homes, which are present in an energy community connected to the main distribution grid. Some of the households are equipped with solar panels, li-ion batteries and heat storage devices. The strategy involves interactions between the aggregator and three entities, i.e. the wholesale market, the DSO and the local flexibility market. The interaction with the wholesale market is established in a traditional manner, in which the aggregator commits a certain amount of energy to the day-ahead market; during the operation day, deviations are settled in the form of negative and positive imbalances.

The interaction with the DSO is given in terms of operational constraint support at the Point of Common Coupling (PCC). Concretely, two types of constraint/product that might be activated by the DSO, if needed, are considered in this work: 1) ramping constraints (R_t [MW/h]) and 2) Power-Max, in which the aggregator ensures that its local portfolio will not exceed P_t^{PCCmax} [kW]. Ramping products are motivated by the need to offset variability of the increasing renewable penetration in distribution grids; and Power-Max allows peak modulation to control overloads or promote investment deferral [31], [32]. Given the difficulty of creating a tuple to describe the temporality/quantity of these products so that they can fit into traditional auction architectures, bilateral contracts are considered between the two agents [2], [31] to remunerate the service.

The aggregator and the Local Market Operator (LMO) interact in such a way that when flexibility is needed by the DSO (or other third party. i.e. Balancing Responsible Party (BRP)), then the LMO communicates with the aggregator in order to request a flexibility bid. If the flexibility is awarded to the aggregator, it will receive the bid price (pay-as-bid) for providing the service, which is commonplace in ancillary and service markets, and also avoids price increase of the local flexibility services. The specifics of the market design concept and architecture, and the flexibility clearing algorithm, which is a task performed by an LMO, do not come into the scope and objectives of this paper.

A. Main steps of the proposed framework

The main steps and timeline of the process that involves the participation of the aggregator, are described in the sequence depicted in Fig. 1 and detailed as follows:
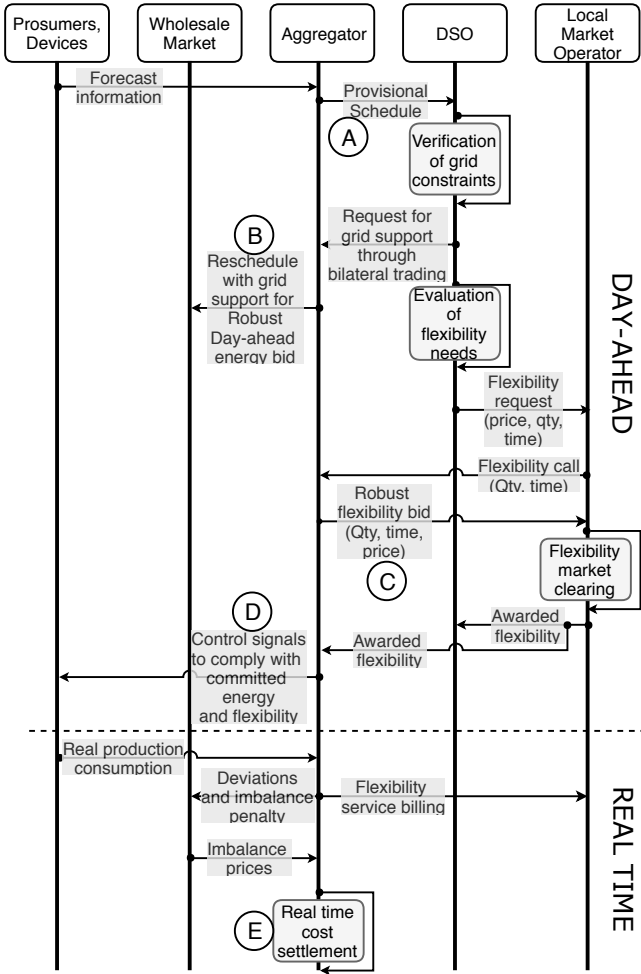
Fig. 1. Sequence of actions for the proposed framework

First, the aggregator gathers the information related to PV forecasts, device availability, consumption forecasts and energy price forecast. With this information, the aggregator determines a baseline (unconstrained) or provisional schedule (in Fig. 1, referenced as action Ⓐ) that minimizes total operation costs presuming that no operational constraints are imposed by the DSO at the PCC. The DSO proceeds to determine the expected operation state of the grid and if needed sends ramping or power-limit constraints to the aggregator (and/or all distributed resources connected to the distribution grid) if needed, as detailed in subsection III-B. If no constraint support is required by the DSO, there are no changes in the provisional schedule. On the other hand, if constraint support is needed, a bilateral transaction takes place between the DSO and the aggregator, whereby the aggregator must be paid the incurred extra-cost of rescheduling its devices to provide the adjustments needed by the DSO. After this, with or without constraint support, the aggregator sends its definitive day-ahead energy commitment to the wholesale market (action Ⓑ).

Next, the local flexibility market opens and receives requests from the DSO or other parties (i.e. BRP) specifying a tuple with location, time-frame (t^{fl}) and quantity (P^{fl}) of the required flexibility. With this information, the LMO calls for

flexibility bids from all potential flexibility providers, including the aggregator. If the aggregator has available flexibility to offer, a bid featuring the quantity and price for the required time frame is sent to the LMO (action Ⓒ). This bid has to be robust enough to withstand: energy- and imbalance-price uncertainties, PV production and demand uncertainties; additionally, it has to be robust towards the acceptance or non-acceptance of the flexibility bid. After this, the LMO clears all flexibility bids and informs the aggregator whether its bid is accepted.

The aggregator's schedule (action Ⓓ) is such that takes into account the mentioned sources of uncertainty, the awarded flexibility and the constraint support, while minimizing total operational costs. This action is the result of extracting the set-points of the devices after solving action Ⓒ with the result of the awarded flexibility.

Next, during the operation day, given the fluctuations in PV production and demand, deviations have to be met by additional purchases/sales of energy. This leads to negative/positive imbalances settled by the wholesale market. The total real-time operation cost (action Ⓔ) is given by a combination of the day-ahead energy commitment, the imbalance penalizations, the bilateral trading with the DSO (if called upon) and the flexibility service provided through the LFM (if called upon). Given the uncertain nature of the imbalance prices, the aggregator's bidding strategy in all markets has to be robust in the sense of predicting potential deviations, and thus, price the services to supply accordingly.

B. Forecast information

Electrical demand of individual households is predicted through quantile smoothing spline fitting. For PV power forecasts, a quantile approach is also employed, using a conditional Kernel estimator based on irradiance level forecasts. After this, extreme quantiles (10%-90%) for both demand and PV production are obtained in order to be used as inputs for the optimization model. For further details on these forecast models, readers are invited to review reference [33].

Electricity prices are taken from the ENTSOE database [34], using data from the three months prior to the day of dispatch. With this input, a Kernel Density Estimation (KDE) is performed to obtain a non-parametric density function of prices. KDE is selected given that it is appropriate for the creation of forecast intervals, that are required to generate the confidence intervals needed for the robust optimization approach [35]. To perform the KDE, Python package scikit-learn [36] was used.

The mathematical model which served as the base for the main actions Ⓐ to Ⓔ in Fig. 1 and interactions of the aggregator, are detailed in the following subsections.

III. MATHEMATICAL FORMULATION AND COORDINATED BIDDING STRATEGY

A. Deterministic model

1) Objective function: The day-ahead operation of the house aggregation aims to minimize the costs of energy purchases on the wholesale market, imbalance penalization,

and the equivalent cycling cost of the batteries. This objective function is shown in (1).

$$\text{minimize } \sum_{t=1}^{T}[\hat{\pi}_t^+ P_t^E + \hat{\mu}_t^- I_t^- - \hat{\mu}_t^+ I_t^+ \\ + \sum_{h=1}^{N}\sum_{s=1}^{S}(a_{h,s}X_{t,h,s}^{Ds} + b_{h,s}l_{t,h,s})] \tag{1}$$

The second term in the objective function corresponds to the piece-wise linearization of the battery degradation equivalent cost, in line with [37] and supported on constraints (9)-(17).

2) Operational constraints: Equation (2) describes the energy balance of the physical system and the net exchange at the PCC with the distribution grid.

$$P_t^E + I_t^- - I_t^+ + \Delta t \sum_{h=1}^{N}(P_{t,h}^{pv} - P_{t,h}^c + P_{t,h}^d - \hat{D}_{t,h} - H_{t,h}) = 0, \forall t \tag{2}$$

3) Device constraints: PV production is limited to the forecasted values as expressed in constraint (3). In this case, the total forecasted power of all panels in each time frame is compacted in $\hat{P}_t^{pv}$.

$$\sum_{h=1}^{N} P_{t,h}^{pv} \le \hat{P}_t^{pv}, \forall t \tag{3}$$

Equations (4)-(8) describe the behaviour of the Battery Energy Storage Systems (BESS) in terms of the intertemporal charging/discharging pattern and the energy/power limits of each device. Equations (6) and (7) and binary variable $u_{t,h}$ ensure that charging and discharging of the battery are mutually exclusive.

$$X_{t,h} = X_{t-1,h} + \eta^c \Delta t P_{t-1,h}^c - \Delta t P_{t-1,h}^d / \eta^d, \forall t, t \ne 1, \forall h \tag{4}$$

$$X_{1,h} = X_{T,h}, \forall h \tag{5}$$

$$0 \le P_{t,h}^c \le \bar{P}_h^c \cdot u_{t,h}, u_{t,h} \in \{0,1\}, \forall t, t \ne 1, \forall h \tag{6}$$

$$0 \le P_{t,h}^d \le \bar{P}_h^d \cdot (1 - u_{t,h}) \tag{7}$$

$$\underline{X}_h \le X_{t,h} \le \bar{X}_h \tag{8}$$

The model used to identify the adequate cost segment of the batteries' degradation linearized cost function is expressed by equations (9)-(17). Special ordered sets and auxiliary constraints/variables are used to identify the beginning of charging cycles, expressed by binary variable $x_{t-1,h}$.

$$x_{t-1,h} - y_{t-1,h} = u_{t,h} - u_{t-1,h}, \forall t, t \ne 1, \forall h \tag{9}$$

$$x_{T,h} - y_{T,h} = u_{0,h} - u_{T,h}, \forall h \tag{10}$$

$$x_{t,h} + y_{t,h} \le 1, x_{t,h}, y_{t,h} \in \{0,1\}, \forall t, \forall h \tag{11}$$

$$X_{t,h}^D + X_{t,h}^{Df} = 1 - X_{t,h}/E_h^{rated}, \forall t, \forall h \tag{12}$$

$$X_{t,h}^D \le x_{t,h}, \forall t, \forall h \tag{13}$$

$$X_{t,h}^{Df} \le 1 - x_{t,h}, \forall t, \forall h \tag{14}$$

$$\sum_{s=1}^{S} X_{t,h,s}^{Ds} = X_{t,h}^D, \forall t, \forall h \tag{15}$$

$$l_s^{min}l_{t,h,s} \le X_{t,h,s}^{Ds} \le l_s^{max}l_{t,h,s}, l_{t,h,s} \in \{0,1\} \, \forall t, \forall h \tag{16}$$

$$\sum_{s=1}^{S} l_{t,h,s} = x_{t,h}, \forall t, \forall h \tag{17}$$

A more detailed description of the methodology for including equivalent degradation battery costs can be found in [37]. The presence of the BESS' binary variables in the model increases complexity and turns it into a Mixed-Integer Linear Programming (MILP) problem.

The Thermal Energy Storage (TES) capabilities provided by the Electric Water Heaters (EWHs) that are present in the energy community are modeled by constraints (18)-(21), in which inter-temporal energy storage behavior, energy loss and variable limits are described.

$$Y_{t,s,h} = Y_{t-1,s,h} + \Delta t H_{t-1,s,h} - Y_{t-1,s,h}/R_h C_h \\ - \Delta t \hat{Q}_{t-1,s,h}, \forall t, t \ne 1, \forall s, \forall h \tag{18}$$

$$Y_{1,s,h} = Y_{T,s,h}, \forall s, \forall h \tag{19}$$

$$\underline{Y}_{t,h,s} \le Y_{t,h,s} \le \bar{Y}_{t,h,s} \tag{20}$$

$$0 \le H_{t,h,s} \le \bar{H}_{t,h,s} \tag{21}$$

B. Grid operational requirements

When the DSO directly requests grid support, additional equations must be included in the day-ahead scheduling of the aggregator. Constraints (22)-(25) model the allowed ramping, which represents the maximum allowed net power change in consecutive time steps. Constraint (22) ensures that the day-ahead commitment respects maximum ramping, and similarly, constraint (24) also includes negative and positive imbalance exchanges. Constraints (23), (25) ensure continuity between the first and last time steps on the operation day. Constraints (26)-(27) model the maximum allowed power exchange. Parameters $P_t^{PCC_{min}}/P_t^{PCC_{max}}$ are the min/max net power at the PCC.

$$|P_t^E - P_{t-1}^E|/\Delta t \le R_t \Delta t, \forall t, t \ne 1 \tag{22}$$

$$|P_1^E - P_T^E|/\Delta t \le R_t \Delta t \tag{23}$$

$$|P_t^E + I_t^- - I_t^+ - (P_{t-1}^E + I_{t-1}^- - I_{t-1}^+)|/\Delta t \le R_t \Delta t, \forall t, t \ne 1 \tag{24}$$

$$|P_1^E + I_1^- - I_1^+ - (P_T^E + I_T^- - I_T^+)|/\Delta t \le R_t \Delta t \tag{25}$$

$$P_t^{PCC_{min}} \le P_t^E \le P_t^{PCC_{max}}, \forall t \tag{26}$$

$$P_t^{PCC_{min}} \le P_t^E + I_t^- - I_t^+ \le P_t^{PCC_{max}}, \forall t \tag{27}$$

As explained above, if these constraints were activated to support grid operation, their inclusion would most likely lead to a different schedule and operation point for the aggregator, when compared to the provisional schedule. Hence, any extra-cost due to these constraints will determine the minimum fixed cost that the DSO has to pay the aggregator for providing this service, and settled through a bilateral contract between these two parties, as explained in subsection III-D.

C. Robust counterpart

The previous deterministic model possesses multiple sources of uncertainty, namely: energy and imbalance prices, PV production and electrical and thermal demand. Strong duality theorem is used to determine an adjustable robust optimization (ARO) counterpart [38] and rewrite all constraints with uncertain parameters and coefficients.

These uncertain parameters introduce a potential deviation in the objective function, hence an optimal solution must be protected against parameter uncertainty without jeopardizing optimality and feasibility. The details of the formulation of the robust counterpart and the relation with the model in this subsection is provided in the Appendix.

For the case of the objective function (1), the cost coefficients of P_t^E, I_t^- and I_t^+ present uncertainty and the equivalent dual robust counterpart is represented by constraints (28)-(35).

$$
\sum_{t=1}^{T} \sum_{h=1}^{N} \sum_{s=1}^{S} (a_{h,s} X_{t,h,s}^{Ds} + b_{h,s} l_{t,h,s})
$$
$$
+ \sum_{t=1}^{T} (\hat{\pi}_t P_t^E + \hat{\mu}_t^- I_t^- - \hat{\mu}_t^+ I_t^+) \tag{28}
$$
$$
+ \sum_{t=1}^{T} (q_t^c + q_t^- + q_t^+) + \Gamma^c z^c + \Gamma^- z^- + \Gamma^+ z^+
$$

$$
z^c + q_t^c \geq \frac{1}{2}(\bar{\pi}_t - \underline{\pi_t}) y_t^c, \forall t \tag{29}
$$

$$
-y_t^c \leq P_t^E \leq y_t^c, \forall t \tag{30}
$$

$$
z^- + q_t^- \geq \frac{1}{2}(\bar{\mu}_t^- - \underline{\mu_t}^-) y_t^-, \forall t \tag{31}
$$

$$
-y_t^- \leq I_t^- \leq y_t^-, \forall t \tag{32}
$$

$$
z^+ + q_t^+ \geq \frac{1}{2}(\bar{\mu}_t^+ - \underline{\mu_t}^+) y_t^+, \forall t \tag{33}
$$

$$
-y_t^+ \leq I_t^+ \leq y_t^+, \forall t \tag{34}
$$

$$
z^c, q_t^c, y_t^c, z^-, q_t^-, y_t^-, z^+, q_t^+, y_t^+ \geq 0, \forall t, \forall h \tag{35}
$$

To control conservatism regarding uncertainty in energy, and negative and positive imbalance prices, three parameters appear: Γ^c, Γ^- and Γ^+, respectively. Each of these parameters can take values from zero (deterministic case) to T, in correspondence with the maximum number of coefficients that can deviate from the central forecast. Dual variables z and q result from strong duality and represent an increase in problem size. For instance, there are 3 additional z's and 24 additional q^+'s, q^-'s and q^c's according to the dual problem in (63)-(65).

Constraints (2) and (3) include additional uncertain parameters, i.e. electrical demand and PV production, respectively. The robust counterpart includes the robust parameters Γ_t^D and Γ_t^{pv} for controlling conservatism of demand and PV uncertainty. In line with the analysis presented in the Appendix for the modeling of uncertainty in right-hand side parameters (equations (80)-(84)), constraints (36)-(40) represent the robust counterpart that results from strong duality theorem.

$$
P_t^E + I_{t,s}^- - I_{t,s}^+ + \Delta t \sum_{h=1}^{N} P_{t,h}^{pv} + P_{t,h}^d - P_{t,h}^c - H_{t,h}
$$
$$
= \hat{D}_t + q_t^D + (\Gamma_t^D)^2 z_t^D, \forall t, \forall s \tag{36}
$$

$$
z_t^D + q_t^D \geq \frac{1}{2}(D_t^{90\%} - D_t^{10\%}) y_t^D, \forall t \tag{37}
$$

$$
\sum_{h=1}^{N} P_{t,h}^{pv} = \hat{P}_t^{pv} - q_t^{pv} - \Gamma_t^{pv} z_t^{pv}, \forall t \tag{38}
$$

$$
z_t^{pv} + q_t^{pv} \geq \frac{1}{2}(P_t^{90\%} - P_t^{10\%}) y_t^{pv}, \forall t \tag{39}
$$

$$
z_t^D, q_t^D, z_t^{pv}, q_t^{pv} \geq 0, y_t^D, y_t^{pv} \geq 1, \forall t \tag{40}
$$

In this case, there is one additional variable z and q, per equation with uncertainty. Modelling of demand uncertainty protects the optimal solution against a demand increase within the confidence interval, whereas PV modeling protects against a decrease in available renewable resources within the confidence interval. The presence of $(\Gamma_t^D)^2$ instead of Γ_t^D corresponds to an alternative to counter conservatism in load aggregation schemes due to the portfolio effect, as presented in [39].

Constraints (41)-(43) are the resulting robust counterpart of considering uncertainty in thermal load, with Γ_t^{th} controlling thermal load conservatism and with cardinality [0,1].

$$
Y_{t,h} = Y_{t-1,h} + \Delta t H_{t-1,h} - Y_{t-1,h}/R_h C_h -
$$
$$
\Delta t(\hat{Q}_{t-1,h} + q_{t-1,h}^{th} + (\Gamma_{t-1,h}^{th})^2 z_{t-1,h}^{th}), \forall t, \forall h \tag{41}
$$

$$
z_{t,h}^{th} + q_{t,h}^{th} \geq \frac{1}{2}(Q_{t,h}^{90\%} - Q_{t,h}^{10\%}) y_{t,h}^{th}, \forall t, \forall h \tag{42}
$$

$$
z_{t,h}^{th}, q_{t,h}^{th} \geq 0, y_{t,h}^{th} \geq 1, \forall t, \forall h \tag{43}
$$

The above equations constitute the building block to optimally manage the aggregator's resources in multiple markets and taking into account uncertainty. The relation of this mathematical model with the actions Ⓐ-Ⓔ depicted in Fig. 1, is explained in the following subsections.

D. Provisional and robust reschedule with grid support

The provisional schedule (action Ⓐ) corresponds to the robust optimization problem when no grid constraint support is considered. Action Ⓐ is achieved by solving the following adjustable robust counterpart MILP (ROMILP):

$$
\text{minimize } C^A = (28) \tag{44}
$$
s.t.
$$
\text{Constraints} : (4) - (17), \tag{45}
$$
$$
(19) - (21), (29) - (43) \tag{46}
$$

where:

- (28): is the robust counterpart of the objective function.
- (4)-(17): are the BESS constraints.
- (19)-(21): are the EWH constraints.
- (29)-(43): are the remaining robust counterpart constraints related to PV production, electrical demand and heat demand.

When this unconstrained baseline robust schedule is determined, the net power exchange is communicated to the DSO to check if constraint support is needed: action Ⓑ. If this is

the case, R_t and $P_t^{PCC max/min}$ values are received and the following ROMILP is solved:

$$\text{minimize } C^B = (28) \tag{47}$$

$$\text{s.t.}$$

$$\text{Constraints} : (22) - (27), \tag{48}$$

$$(45), (46) \tag{49}$$

where (22)-(27) are the local flexibility constraints and (45)-(46) are the constraints of action (A), which correspond to the complete robust scheduling of the aggregation. Note that the only difference between (A) and (B) is the presence of the local flexibility constraints in the case the DSO calls for this service. In the case where these constraints are included, the set-points of (A) will be overwritten according to the result of problem (B) in order to comply with the DSO's request.

The minimum remuneration that the aggregator involved in the bilateral transaction receives for providing the service of ramping and max. power, is given by the difference: $\pi^{dso} = C^B - C^A$.

E. Algorithm for bidding local flexibility

When the LMO asks the aggregator for a flexibility bid, the process followed by the aggregator, i.e. action (C), is the following:

Step 1 The aggregator determines the cost of the baseline (constrained) case, C^B, by solving (47)-(49).

Step 2 The flexibility request sent by the LMO is received by the aggregator as the tuple: time-step of required flexibility, t^{fl}; required power flexibility, P^{fl}.

Step 3 The aggregator determines the robust cost of providing the required flexibility (C^{fl}) by solving (47)-(49) and including constraint:

$$\Delta t \sum_{h=1}^{N} P_{t^{fl},h}^{pv} + P_{t^{fl},h}^{d} - P_{t^{fl},h}^{c} - H_{t^{fl},h} \\ - \hat{D}_{t^{fl}} - q_{t^{fl}}^{D} - \Gamma_{t^{fl}}^{D} z_{t^{fl}}^{D} = P^{fl} \tag{50}$$

Step 4 The aggregator computes the flexibility bid: $\pi^{fl} = C^{fl} - C^B$, and sends the bid to the LMO.

The minimum price that should be paid to the aggregator for providing the required flexibility is given by π^{fl}. Constraint (50) ensures that the flexibility will be met by adjusting the device settings of the smart-homes. This set of control signals, settings and expected penalties is called action (D) in the timeline of Fig 1. Additionally, the robust characteristic of the formulation makes it possible to include potential imbalances and the corresponding penalties paid to the wholesale market, which are assumed by the aggregator. In this approach, these costs are accounted for in the bidding process to protect the aggregator against uncertainties. Note that when no local constraint support is needed by the DSO, calculated C^B in step 1, becomes the same C^A.

This bidding methodology is general enough to bid in local market schemes that accept not only single point bids, but also bidding curves. The characteristic of the robust bid will depend upon the capabilities of the clearing algorithm used by the LMO. To construct a flexibility bidding curve for a specific time-frame t^{fl}, steps 1-4 need to be repeated for a range of values of $P^{fl} = [P^{fl min}, P_{max}^{fl}]$, given by the LMO or determined by the technical flexibility capabilities of the aggregator for the specific time period.

F. Real time performance evaluation

This part of the proposal aims to evaluate the ability of the aggregator to comply with: 1) the committed day-ahead energy; 2) the constraint support; and 3) the local awarded flexibility, while minimizing the total operation cost in the face of multiple sources of uncertainty. Monte Carlo simulation is used to test the robustness of the proposed approach for several patterns of random generated prices, consumption and PV production during the operation day. The total cost calculation when these random values are generated and used as input is given by the day-ahead energy payments, the equivalent cycling cost of the batteries, the revenues for providing constraint support to the DSO, the revenues for providing local flexibility, and the penalization due to imbalances produced by real-time production/consumption in each household. Monte Carlo simulation returns a measure of the performance in terms of average cost and standard deviation (SD), as a measure of the risk related to a particular robust bidding strategy.
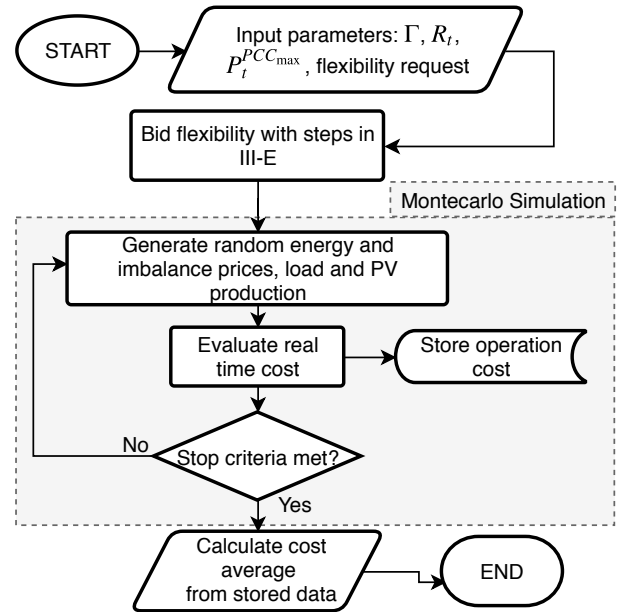

Fig. 2. Real time performance evaluation: action E

The outline of the performance methodology is presented in Fig. 2. The number of simulations (stop criteria) is limited to the maximum between a) 1000 trials, and b) the number of trials in achieving a margin of error of maximum 1% with a confidence interval of 95%.

The cost of each trial is given the following expression:

$$C^{trial} = C^{ws} - \pi^{dso} - \pi^{fl} + C^{cyc} \tag{51}$$

where C^{ws} is the result of solving the following optimization problem:

$$\text{minimize } C^{ws} = (1) \tag{52}$$

s.t.

$$\text{Constraints} : (2) - (27), (50) \tag{53}$$

where constraints (2)-(27) are related to the deterministic problem and constraint (50) ensures the provision of required flexibility, neglecting dual robust variables. Each trial corresponds to a deterministic optimization problem. Cycling constraints (9)-(17) are included in the model, and the equivalent degradation cost (C^{cyc}) is calculated after the optimization process with the last term of equation (1).

The previous mathematical models and optimization problems are related to each of the actions described in section II and Fig. 1. The summary and relation of these actions and the respective optimization models to be solved in the sequence are presented in Table I.

TABLE I
SUMMARY OF MAIN ACTIONS AND THE CORRESPONDING
MATHEMATICAL MODELS

Action	Related optimization model	Details
(A)	Robust problem (44)-(46)	Provisional scheduling without grid support
(B)	Robust problem (47)-(49)	Reschedule with grid support (if necessary)
(C)	Solve steps 1-4 in subsection III-E	Robust bidding in LFM
(D)	Device settings resulting from robust problem (47)-(50) including awarded flexibility	Control signals from setpoints of batteries and EWHs
(E)	Multiple trials in deterministic problem (52)-(53)	Average cost calculation with (51) with Monte Carlo simulation in Fig. 2

The actions described in the table I and the related robust optimization problems, are fed with the Γ parameters to control robustness through the complete process of scheduling and bidding, hence different levels of conservatism can be analyzed as shown in the next section. This is in line with the principle of the proposed adjustable robust optimization methodology, by allowing to evaluate different levels of robustness, and also calculating performance (action E) to demonstrate advantages of the presented bidding strategy.

IV. RESULTS

The proposed algorithm is coded in Python and the robust optimization problems are solved with Gurobi. The 25 houses in the energy community include 25 PV panels, 16 BESS and 15 EWHs. A normalized thermal load pattern is taken from [40] and adjusted to the consumption level of the Evora testcase.

The charging and discharging efficiency of the storage system (including inverter) is assumed to be 90%. 15 batteries are rated 3kW / 3.3 kWh, and the remaining battery is a 10kW / 20 kWh device. All PV panels are rated 1.5 kWp.

The rated power/energy for all EWH is 1.5 kW / 3 kWh and thermal resistance/capacitance are 568 (°C/kW)/0.3483 (kWh/°C) in line with [41]. For all simulations and for simplicity, we assume $\Gamma^{th} = \Gamma^D$ and $\Gamma^{DA} = \Gamma^c = \Gamma^- = \Gamma^+$.

A. Robust participation in the wholesale market and local constraint support

If the provisional schedule (action (A)) does not jeopardize grid operation, this schedule will remain as the robust day-ahead bid (action (B)). Fig. 3 shows the demand bidding curves for t=14, when energy prices vary in the range 10%-90% percentile of the priced predicted by the KDE. These curves show how local net power interacts with the price variations in the wholesale market. They can also be interpreted as the bids in the case where flexibility can be traded on the centralized wholesale market, as proposed in [19]–[21].

The results show that increasing robustness (higher values of Γ) results in bids that can withstand higher purchase energy prices for similar values of bidding power, and for most of the bidding range. Note that the green curve (full robustness) tends to be the right-most.
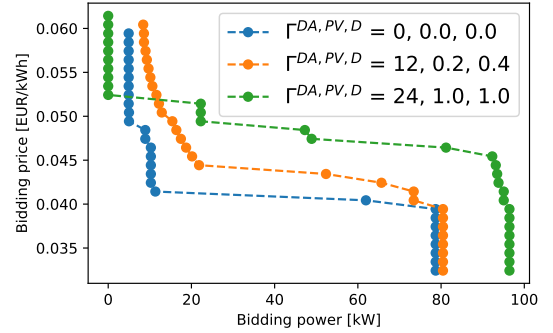

Fig. 3. Demand bidding curves in the wholesale market for t = 14h and different levels of robustness without local constraint support (Action A = Action B).

When local constraint support is needed by the DSO, the fixed remuneration (π^{dso}) component is shown in Table III for different values of $P_t^{PCC_{max}}$ and R_t, and for the case in which full robustness is considered ($\Gamma^{DA} = \Gamma^{PV} = \Gamma^D = 1$).

TABLE II
REMUNERATION π^{dso} FOR THE AGGREGATOR FOR DIFFERENT VALUES
OF ALLOWED RAMPING AND MAXIMUM POWER

R_t	$P_t^{PCC_{max}}$		
	0.04 MW	0.05 MW	0.07 MW
0.01 MW/h	1.13	1.10	1.10
0.03 MW/h	1.04	1.00	0.99
0.06 MW/h	1.04	1.00	0.97

B. Robust bidding in the local flexibility market

When the local flexibility bidding algorithm described in subsection III-E (action (C)) is run for a range of requested flexibility values, the bidding curves in Fig. 4 are obtained. From these figures it can be seen that when flexibility is

requested at the PCC, the remuneration increases as the requested deviation from he original day-ahead schedule increases. For instance, the original day-ahead schedule for the aggregator when Γ^{DA}=12, Γ^{PV}=0.2, Γ^D=0.4 (blue dotted curve) in hour 21h is 29.2 kW, which is the point with the lowest flexibility bidding price, meaning that maintaining this power will be less costly for the aggregator. When other values of flexibility are requested, the remuneration starts increasing due to the activation of home flexibility and the potential imbalance costs that will need to be settled in the wholesale market and foreseen by the aggregator.
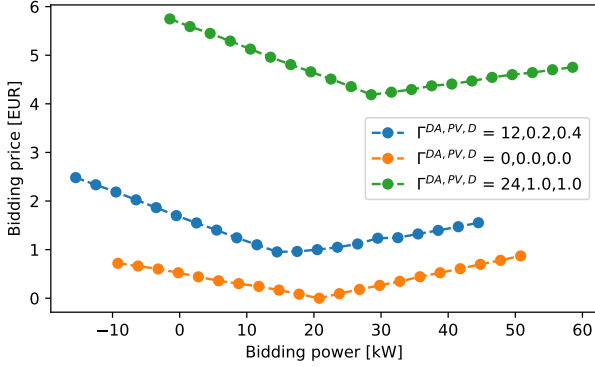

Fig. 4. Local flexibility bidding curves for t = 21h

Given that the aggregator has to manage residential resources to supply the flexibility offered and also foresee PV and demand deviations, imbalances might occur and have to be accounted for in the flexibility bids. Due to the fact that the aggregator bids also in the energy market, the flexibility offered in Fig. 4 (blue dotted curve), includes the robust imbalances expected to be capable of both adjusting the device settings and supplying local electrical and thermal demand. The aggregator is responsible for negotiating these quantities in the wholesale market, which acts as a form of penalization or deviation from the original day-ahead committed energy.

The fact of bidding different quantities in the local market in the presence of uncertainty, results in imbalances and changes in the devices' settings. The average behavior of storage devices (all HEMS batteries aggregated) resulting from the points of the bidding curve is shown in Fig. 5. The energy stored in the batteries changes slightly during most of the time frames, but changes more actively during the 21h and 22h time frames to provide the necessary flexibility. Similarly, the energy stored in the EWHs presents higher variations during time frames prior to t=21h, to adjust the operation for provision of flexibility and supply local thermal residential load.

C. Assessing performance when facing uncertainties

To measure performance, a first test is developed by comparing deterministic and robust approaches when the aggregator participates in both wholesale energy and local flexibility markets. This test considers an arbitrary flexibility request of -10 kW by the LMO for demonstration purposes. After
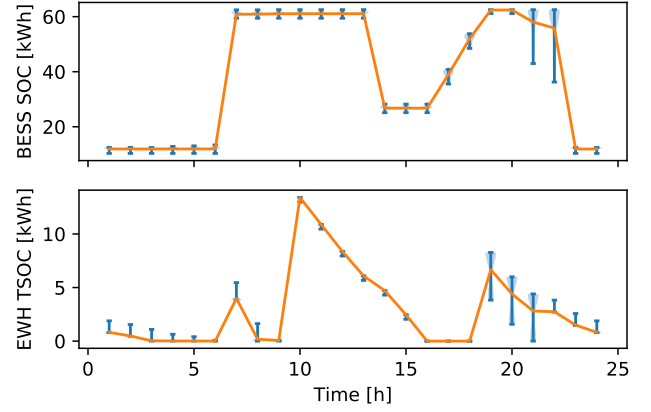

Fig. 5. Average (orange) state of charge and boxplots (blue) of batteries and EWHs for the bidding quantities in t=21h

running Monte Carlo simulation by setting the stop criteria at a margin of error of maximum 1% with a confidence interval of 95%, the expected real-time operation costs are 26.15€ for the deterministic solution and 22.05€, 22.42€, 22.48€ for ARO1 ($\Gamma^{DA,PV,D} = 12, 0.2, 0.4$), ARO2 ($\Gamma^{DA,PV,D} = 18, 0.2, 0.2$) and ARO3 ($\Gamma^{DA,PV,D} = 12, 0.4, 0.6$) respectively. Improvement of the proposed robust approach in expected operation cost ranges from 14.0% to 15.7% when compared to the deterministic case, showing the ability of the robust formulation to optimize participation in the wholesale market and comply with the local flexibility despite the uncertainty in prices, PV and load.

The Cumulative Density Function (CDF) of the performance is depicted in Fig. 6. This CDF shows that the leftmost curves are related to the proposed ARO approach, which in turn shows that the probability of obtaining lower associated costs is higher for ARO cases.

The obtained Standard Deviation (SD) values are 3.47€, 2.15€, 2.91€, and 2.49€ for the deterministic, ARO1, ARO2 and ARO3 cases respectively. In all cases, robust solutions improve the deterministic SD in a range of 16.1% to 38.0%.
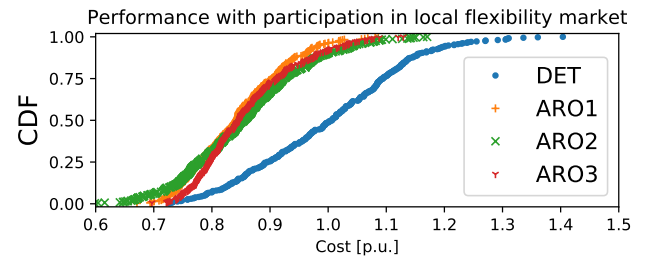

Fig. 6. Performance CDF of different bidding strategies : deterministic, ARO1: $\Gamma^{DA,PV,D} = 12, 0.2, 0.4$, ARO2: $\Gamma^{DA,PV,D} = 18, 0.2, 0.2$ ARO3: $\Gamma^{DA,PV,D} = 12, 0.4, 0.6$, for november 15th and flexibility activated of -10 kW (up regulation).

In addition to the analysis of participation in energy and local markets (case 1 in Table III) we evaluated performance for local constraint activation (case 2) and for the case in which flexibility bid is not accepted by the local market (case 3). For

case 3, performance is not jeopardized by the rejection of the flexibility offer, given that ARO bidding strategy outperforms the deterministic solution in the range of 9.1%-9.7%.

However, if the local flexibility offer is accepted, dispatched and remunerated, there are improvements in the operation. For the ARO-1 case, for example, acceptance of the local flexibility offer (case 1) implies a cost reduction when compared to the case of offer rejection (case 3). This reduction represents a 6.4% improvement in real-time expected cost.

ARO approaches also behave better when local constraint support (case 2) is included, and improvement is achieved in comparison to participation in the energy-only market.

Cost reduction exists in all cases when ARO approaches are compared to the deterministic solution. Additionally, when each robust case is analyzed separately, improvements persist for the cases in which local flexibility market participation is included. This shows the ability of the proposed approach to optimally bid in multiple market platforms even with different settlement schemes.

TABLE III
PERFORMANCE FOR PARTICIPATION IN DIFFERENT MARKETS WITH DIFFERENT ROBUST BIDDING STRATEGIES. $*R_t = 30kW/h$, $P_t^{PCC_{max}} = 50kW$

Case	Market	Det.	ARO1	ARO2	ARO3
1	Energy + Local Market	26.15	22.05	22.42	22.48
2	Energy + Local Market + Local Constraint Support*	26.21	22.25	22.43	22.81
3	Only Energy	26.13	23.57	22.69	23.74

D. Analysis of the probability of bid acceptance

The expected operation cost is also dependent on whether or not the LMO accepts the bid. Acceptance of the bid depends on many factors, such as the bidding price, which also depends on the desired level of robustness, time-frame, quantity, etc. The greater the robustness, the higher the bidding prices (as seen in Fig. 4). The trade-off is that the lower the price, the greater the probability that the flexibility will be awarded and dispatched. However, the remuneration is lower.
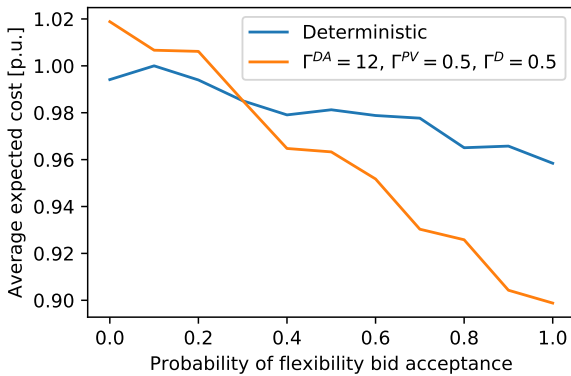
Fig. 7. Average expected costs for different levels of flexibility bid acceptance. Deterministic versus ARO case. Flexibility of -10 kW activated.

For the sake of example, this fact can be seen in Fig. 7. Two cases are shown: average cost of the deterministic case, and an arbitrary ARO case with $\Gamma^{DA,PV,D} = 12, 0.5, 0.5$. The simulation consists in generating bids for hour $t = 21$ and -10 kw of flexibility required by the LMO. Analyzing the performance of each alternative requires using Monte Carlo simulation for price, PV and demand values, as well as generating uniform random values to compare with different bid acceptance probabilities. If the random value is lower than the predefined probability, then the bid is accepted and dispatched. The results obtained show that the robust bidding strategy is only more appealing than the deterministic one when the probability that the bid will be accepted is higher than 27%.

Another case is shown in Fig. 8, in which the deterministic and full robust cases are compared. An analysis of this comparison indicates that the probability of acceptance has to be higher in order for the robust approach to be more attractive than the deterministic one. In addition, the cost of the robust case follows a steeper slope. This is explained by the fact that greater robustness means that the cost of providing the flexibility will be higher hence resulting in higher remuneration when flexibility is awarded.
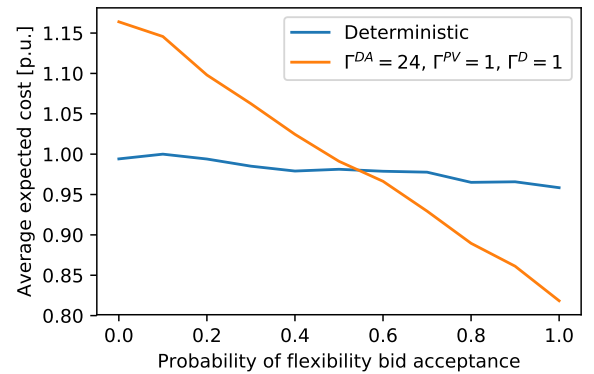
Fig. 8. Comparison of average expected costs for different levels of flexibility bid acceptance. Simulation for deterministic and full ARO cases.

The combination of the three preceding cases is depicted in Fig. 9. This figure shows that the complete robust approach becomes the most interesting alternative only for high acceptance probabilities ($\geq 70\%$). However, as stated above, the robust approach bids are more expensive and hence are less likely to be accepted. Moreover, for these three cases, the deterministic approach results are more attractive for lower bid acceptance probabilities. The figure also evidences the difference in the slopes of both proposed robust bidding strategies, showing that case $\Gamma^{DA,PV,D} = 12, 0.5, 0.5$ has a descending behavior due to the remuneration of flexibility, but that it is not as steep as the complete robust scheme.

When the robust strategy ARO1: $\Gamma^{DA,PV,D} = 12, 0.2, 0.4$ already analyzed in Table III is subjected to the same analysis taking into account the probability of bid acceptance, the evolution of average costs obtained is shown in Fig. 10. In this case, strategic bidding has lower average costs than the deterministic case, despite the probability of bid acceptance.
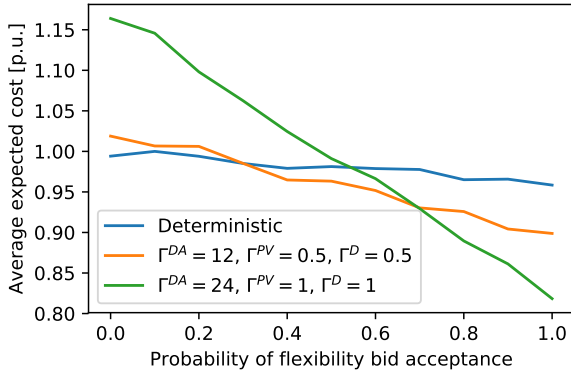
Fig. 9. Comparison of average expected costs for different levels of flexibility bid acceptance. Deterministic, full robust and one ARO case.

This is a very important result given that full certainty of bid acceptance cannot be foreseen when sending the bid to the LMO. This not only means that the proposed strategy is robust enough to withstand uncertainties coming from demand, prices and PV, but also that it performs towards bid acceptance uncertainty.
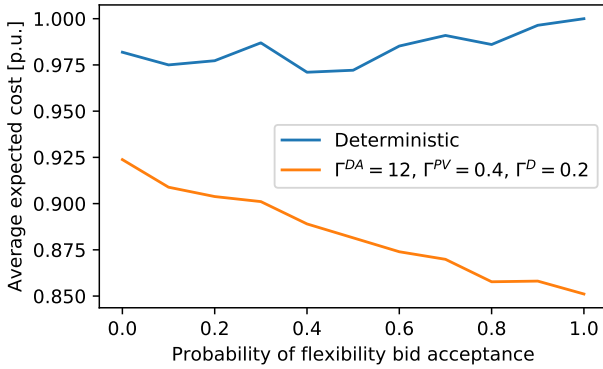

Fig. 10. Average expected costs for different levels of flexibility bid acceptance and ARO versus deterministic case.

E. Remarks on computational time

MILP problems, such as those presented in this work, can be difficult to solve depending on the problem structure, the number of constraints and the number of binary variables. For this specific case, the binary variables are related to the modeling of the degradation costs of the battery (equations (4)-(17)) and impose additional computational complexity. The test case results in 3,456 binary variables in order to include the piece-wise cost linearization. For the simulations run, actions Ⓐ and Ⓑ have similar simulation times, presenting mean values of 32 s. The bidding (action Ⓒ) on the local market presents higher computational time due to a more constrained search to comply with the flexibility request in equation (50). The average computational times measured for the bids presented in each point in Fig. 4 are 624 s. In spite of the higher complexity for action Ⓒ, times remain feasible in order to make day-ahead bidding decisions.

Regarding performance evaluation (action Ⓔ) the mean computational time to obtain cost performance values is 1,856 s (as presented in Table III), with an associated average of 563 LPs solved before achieving stop criteria. The LPs are presented in equations (52)-(53).

V. CONCLUSIONS

The proposed approach presents a mathematical model to include local flexibility in the form of 1) maximum allowed net power exchange and ramping at the PCC and 2) participation in local flexibility markets. The simulations analyzed different cases in which the aggregator participates in wholesale and flexibility markets and determines the changes in the schedule to achieve minimum operation costs while complying with DSO's flexibility constraints, flexibility requests, and energy committed in the day-ahead market.

For the robust cases analyzed, the expected cost outperforms the deterministic case up to 15.7% when participation in multiple markets is allowed. In addition, participation in LFM decreases operational costs due to remuneration for providing flexibility services. The robust approach places bids in all markets and schedule devices in such a way that cost is minimized while facing uncertainties produced by energy prices, PV production and load.

The remuneration of the flexibility is related to the level of robustness. If robust parameters are set to high values, the cost for providing flexibility also increases provided that the aggregator includes the expected cost for potential deviations of prices, demand and PV production. In addition, there is a trade-off between the level of robustness and the possibility of being awarded the flexibility service, given that when the cost of providing the service is higher, it is less likely to be dispatched by the LMO. However, the case analyzed features a combination of robust parameters that yield better costs than the deterministic scheme, despite the probability of bid acceptance.

This framework allows an aggregator to participate in the electricity market while cooperating with the local DSO to enhance network operations and promote decentralization of the electrical system.

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APPENDIX

To introduce uncertainty in the decision making process through RO, the following canonical optimization problem is defined:

$$\text{minimize } \mathbf{c'x} \tag{54}$$

$$\text{s.t.}$$

$$\mathbf{Ax} \leq \mathbf{b} \tag{55}$$

$$\mathbf{x} \geq \mathbf{0} \tag{56}$$

If uncertainty in cost coefficients ($\mathbf{c}$) is present in the model in such a way that maximum deviation for a given coefficient

j is given by $(c_j + c_j^{max})$, an optimal solution $(\mathbf{x}^*)$ must satisfy the worst case scenario (robust solution):

$$\text{minimize } \mathbf{c'x} + \max \left\{ \sum_{j \in J_0} c_j^{max} |x_j| \right\} \quad (57)$$

s.t.

$$\mathbf{Ax} \leq \mathbf{b} \quad (58)$$
$$\mathbf{x} \geq \mathbf{0} \quad (59)$$

A quantity $\Gamma \in [0, |J_0|]$ is introduced such that $|J_0|$ is the maximum number of uncertain coefficients. For a vector $(\mathbf{x}^*)$, the following problem is defined:

$$\text{maximize } \sum_{j \in J_0} c_j^{max} |x_j^*| w_i \quad (60)$$

s.t.

$$\sum_{j \in J_0} w_j \leq \Gamma \quad (61)$$
$$0 \leq w_j \leq 1, \ j \in J_0 \quad (62)$$

In the previous problem, Γ aims to measure the level of protection of an optimal solution $(\mathbf{x}^*)$ in the original problem, against Γ deviations of (c). Auxiliary variable w_j takes values between 0 and 1 in order to impact cost coefficients and maximize deviation for a given Γ.

Next, by strong duality, if problem (60)-(62) is feasible and bounded, the dual problem is also feasible and bounded, and their objective function values are the same. The equivalent dual problem is then:

$$\text{minimize } \sum_{j \in J_0} q_j + \Gamma z \quad (63)$$

s.t.

$$q_j + \Gamma z \geq c_j^{max} |x_j^*|, \ j \in J_0 \quad (64)$$
$$z, q_j \geq 0, \ j \in J_0 \quad (65)$$

If the previous dual problem is substituted in (57)-(59), and auxiliary variable y_i is introduced, the following equivalent problem is obtained:

$$\text{minimize } \mathbf{c'x} + \sum_{j \in J_0} q_j + \Gamma z \quad (66)$$

s.t.

$$\mathbf{Ax} \leq \mathbf{b} \quad (67)$$
$$q_j + \Gamma z \geq c_j^{max} y_i, \ j \in J_0 \quad (68)$$
$$-y_i \leq x_j \leq y_i, \ j \in J_0 \quad (69)$$
$$z, q_j, y_i \geq 0, \ j \in J_0 \quad (70)$$
$$\mathbf{x} \geq 0 \quad (71)$$

where z, q_j are dual variables. The problem in (66)-(71) is the robust counterpart when cost coefficients present uncertainty. The robust control parameter Γ controls conservatism of the solution. If $\Gamma = 0$, the resulting problem is the deterministic one in (54)-(56). When this procedure is applied to the objective function (1) to reflect uncertainty in coefficients π_t, μ_t^- and μ_t^+, the resulting equations are (28)-(35), reflecting the corresponding additional constraints, auxiliary and dual variables.

When the uncertainty is related to the right-hand side (RHS) ($\mathbf{b}$) of the optimization problem (i.e. uncertainty in PV and load), the equivalent optimization problem that has to be solved to satisfy the worst case scenario is the following:

$$\text{minimize } \mathbf{c'x} \quad (72)$$

s.t.

$$\sum_j a_{ij} x_j - \max \{ b_i^{max} \} \leq b_i, \ \forall i \quad (73)$$
$$x_j \geq 0, \ \forall j \quad (74)$$

After defining the maximization problem for the deviation of the RHS, the equivalent dual problem associated with each constraint i, is the following:

$$\text{minimize } q_i + \Gamma z_i \quad (75)$$

s.t.

$$q_i + \Gamma z_i \geq b_i^{max} y_i \quad (76)$$
$$z_i, q_i \geq 0 \quad (77)$$
$$y_i \geq 1 \quad (78)$$

After substitution of the previous robust counterpart in each constraint i of (72)-(74), the equivalent takes the following form:

$$\text{minimize } \mathbf{c'x} \quad (79)$$

s.t.

$$\sum_j a_{ij} x_j \leq b_i + q_i + \Gamma z_i, \ \forall i \quad (80)$$
$$q_i + \Gamma z_i \geq b_i^{max} y_i, \ \forall i \quad (81)$$
$$z_i, q_i \geq 0, \ \forall i \quad (82)$$
$$y_i \geq 1, \ \forall i \quad (83)$$
$$x_j \geq 0, \ \forall j \quad (84)$$

In the formulation of this paper, the following RHS quantities present uncertain behavior: electrical demand $\hat{D}_{t,h}$, PV production $\hat{P}_t^{pv}$ and thermal demand $\hat{Q}_{t-1,s,h}$, present in equations (2), (3) and (18), respectively. When the robust counterpart in equations (80)-(84) is introduced in the formulation, then the equations (36)-(43). Appropriate signs are selected for uncertainty in PV production (equation (38)), given that protection against uncertainty for this quantity is related to the scarcity of the resource. The opposite happens in the case of electrical and thermal demand.

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