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# Error Estimate and Fairness in Resource Allocation with Inaccurate Information Sharing

Francesca Fossati, Deep Medhi, *Fellow, IEEE*, Stefano Moretti, Stefano Secci, *Senior, IEEE*

**Abstract**—In resource allocation systems, inaccurate information sharing situations are such that users can be aware, up to a small error, about the other users’ demands and the available global resource (which can be insufficient to meet the overall demand). Consequently, given an allocation rule, users can predict an allocation that will not necessarily coincide with the actual one. In this work, we provide an estimation of the error for a number of allocation rules and compare their robustness in inaccurate information sharing settings.

## I. BACKGROUND

Resource allocation protocols in communication networks and computing systems are commonly used to ensure fairness and efficiency when dividing resources among different agents or users. A challenging situation is the choice of the allocation rule when the amount of available resource is limited and not sufficiently high to fully satisfy the entire demand.

Classically, the network setting is such that users have little information about the available resources and demands of other users. Nonetheless, with the emergence of new networking features such as 5G infrastructure sharing and programmability in SDN, and for auditability requirements (i.e., to ensure tenants fair sharing), network setting is evolving toward a complete information sharing situation so that all users can be aware of the demands of the other users and of the available resources for resource allocation systems [1], [2].

As an intermediate context between the classical no information sharing and the complete information sharing, we consider the scenario in which the information is known to have inaccuracy. In this work, we study the behavior of resource allocation rules under this scenario that we call *inaccurate information sharing context*, where the amount of available resource is known up to a constant (Fig. 1); we also highlight the impact of inaccurate information sharing on the demand of the other users. Indeed, in certain practical situations, such as in radio resource availability or in systems over/under provisioned by the infrastructure provider, it is likely to suffer from inaccurate information on the available resources to be shared. Furthermore, complete sharing may not be possible since this may require a lot of exchanges of updates causing a large overhead. While the problem of

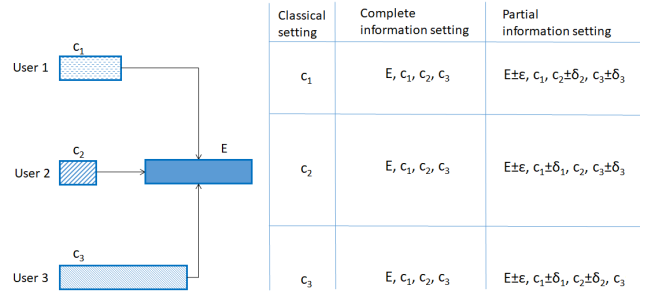


Fig. 1: Information sharing contexts in resource allocation.

inaccurate information in networks has been studied before [3], a formal treatment on the error estimate and fairness has not been studied for different allocations schemes.

Formally, a *resource allocation problem* is characterized by a pair  $(c, E)$ , where  $c$  is the demand vector for users in  $N = \{1, \dots, n\}$ , with  $n > 1$ , and  $E$  is the resource to split among them. Resource allocation is challenging when  $E$  is less than the global demand of users ( $\sum_{i=1}^n c_i \geq E$ ).

An *allocation*  $x \in \mathbb{R}^N$  is a  $n$ -dimensional vector where  $x_i$  represents the quantity of resource  $E$  assigned to each user  $i \in N$ , and an *allocation rule* is a function that associates a unique allocation  $x$  to each pair  $(c, E)$ . In this work, we consider three allocation rules discussed below.

**Weighted proportional allocation rule:** this is the result of the maximization of  $\sum_{i=1}^n w_i \log x_i$  subject to demand boundedness ( $x_i \leq c_i, \forall i \in N$ ) and efficiency constraints ( $\sum_{i=1}^n x_i = E$ ) [4]. The weights  $w_i$  can be chosen equal to 1 to get the proportional allocation rule or equal to  $c_i$  to obtain the allocation that assigns the same proportion of demand to all users. We only consider the second case with  $w_i = c_i$ , that is the allocation that maximizes the *Jain's index of fairness* [5].

**Max-Min Fair (MMF) allocation rule:** This is an egalitarian solution that privileges users with small demands [6]. After ordering the users according to their increasing demand, i.e.,  $c_1 \leq c_2 \leq \dots \leq c_n$ , the MMF allocation for user  $i$  is given by:  $MMF_i(c, E) = \min \left( c_i, \frac{E - \sum_{j=1}^{i-1} MMF_j(c, E)}{n - i + 1} \right)$ .

**Mood value:** It revisits user satisfaction taking into account the awareness of other users’ demands and the amount of available resources [1], [2]. Each user  $i \in N$  is assigned  $min_i + m(max_i - min_i)$  as its resource share, where  $min_i$  is what remains if all the other users in  $N \setminus \{i\}$  are fully satisfied ( $min_i = \max\{0, E - \sum_{j \in N \setminus \{i\}} c_j\}$ ),  $max_i$  is the maximum user  $i$  can get, i.e., its own demand or the available resource if the demand overcomes it, and  $m = (E - \sum_{i=1}^n min_i) / (\sum_{i=1}^n max_i - \sum_{i=1}^n min_i)$  takes value in  $[0, 1]$ .

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## II. ERROR ASSESSMENT

We are interested in evaluating the error on the allocation when users are in a inaccurate information context (Fig. 1). We treat first the case where there is error on the available resource. For each user  $i \in N$  the error  $ERR_i$  is defined as  $|\hat{x}_i - x_i|$ , where  $x_i$  is the share obtained by  $i$  when the resource is  $E$  and  $\hat{x}_i$  is the allocation for user  $i$  when resource is  $E \pm \varepsilon$ .

In the following, we calculate the value of the error, considering the allocations described in the previous section.

*Weighted proportional allocation:* This allocation coincides with the allocation that assigns the resource proportionally to the demand, i.e.,  $x_i^p = c_i E / \sum_{i=1}^n c_i$  when the resource is  $E$ . If users believe that the available resource is  $E \pm \varepsilon$  then the allocation is  $\hat{x}_i^p = c_i(E \pm \varepsilon) / \sum_{i=1}^n c_i$ , which implies that the error on the allocation for each user is:

$$ERR_i = \pm \frac{c_i}{\sum_{i=1}^n c_i} \varepsilon \quad (1)$$

The error  $\varepsilon$  is divided between the users proportionally to their demands.

*MMF allocation:* We consider the hypothesis that  $\varepsilon$  is small enough not to change the nature of the user. This means that if  $c_i < \frac{E}{n}$ , it holds also that  $c_i < \frac{E-\varepsilon}{n}$ . It follows that the users with small demands receive the same amount of resource (i.e., their demand), while the excess  $\varepsilon$  is equally divided between the users that receive less than their demand. When the first  $k$  users receive their demands, the error is:

$$ERR_i = \begin{cases} 0, & \text{if } i = 1, \dots, k \\ \frac{\pm \varepsilon}{n-k}, & \text{if } i = k+1, \dots, n. \end{cases} \quad (2)$$

*Mood value:* For the mood value allocation, we need to consider that we have four types of users when we take into account the minimum and the maximum value they can get. Table I shows that they are categorized checking (i) what remains if the other users received what they demand and (ii) the maximum quantity of resource they can get, i.e.,  $c_i$  if they ask less than the available resource,  $E$  otherwise. In [1], [2] it was shown that there exists just six types of scenarios combining the different type of users. The six combinations of users are:

- GM: All the players are in scenario GM.
- GG: All the players are in scenario GG.
- MM: All the players are in scenario MM.
- GM-GG: Some players of type GM, some of type GG.
- GM-MM: Some players of type GM, some of type MM.
- GM-MG: One player of type MG, the others of type GM.

We again consider the hypothesis that  $\varepsilon$  is small enough not to change the nature of the user. This means that, e.g., if  $c_i \geq E$

	$\max_i = c_i$ ( $c_i < E$ )	$\max_i = E$ ( $c_i \geq E$ )
$\min_i = 0$ ( $\sum_{j \neq i} c_j > E$ )	GM	GG
$\min_i \neq 0$ ( $\sum_{j \neq i} c_j < E$ )	MM	MG

TABLE I: User types in complete/inaccurate information sharing.

for a user  $i \in N$  it also holds  $c_i \geq E + \varepsilon$ , if  $\sum_{j \neq i} c_j < E$  it holds also  $\sum_{j \neq i} c_j < E - \varepsilon$ , and so on.

1) *Case GM:* This case coincides with the weighted proportional allocation. For each user  $i$  the error is given by (1).

2) *Case GG:* In this case, if the resource is  $E$  it holds that  $\min_i = 0$ ,  $\max_i = E$  for each user  $i$ . The value of the mood is  $m = \frac{E-0}{nE-0} = \frac{1}{n}$ , and the mood value is  $x_i^m = \frac{E}{n}$ . If the value of the available resource is  $E \pm \varepsilon$  the value of the mood  $\hat{m}$  is again equal to  $\hat{m} = \frac{1}{n}$  and the mood value is  $\hat{x}_i^m = \frac{E \pm \varepsilon}{n}$ . It follows that for each user  $i$  the error is equal to:  $ERR_i = \pm \varepsilon / n$  (3).

In this case, the error is divided equally between the users without considering the value of their demands.

3) *Case MM:* In this case, if the resource is  $E$  it holds that  $\min_i \neq 0$ ,  $\max_i = c_i$  for each user  $i$ . The value of the

mood is  $m = \frac{E-n(E)+(n-1) \sum_{i=1}^n c_i}{\sum_{i=1}^n c_i - n(E)+(n-1) \sum_{i=1}^n c_i} = \frac{n-1}{n}$ , and the mood

value is  $x_i^m = E - \sum_{j \neq i} c_j + \frac{n-1}{n} (\sum_{i=1}^n c_i - E)$ . If the value

of the available resource is  $E \pm \varepsilon$ , the value of the mood  $\hat{m}$  is given by:  $\hat{m} = \frac{E \pm \varepsilon - n(E \pm \varepsilon) + (n-1) \sum_{i=1}^n c_i}{\sum_{i=1}^n c_i - n(E \pm \varepsilon) + (n-1) \sum_{i=1}^n c_i} = \frac{n-1}{n}$  and the

mood value is  $\hat{x}_i^m = E \pm \varepsilon - \sum_{j \neq i} c_j + \frac{n-1}{n} (\sum_{i=1}^n c_i - E \mp \varepsilon)$ . It follows that for each user  $i$  the error is:  $ERR_i = \pm \varepsilon / n$  (4)

The error is equally divided between the users also here.

4) *Case GM-GG:* Let  $N = N_1 \cup N_2$  be partitioned into two disjoint sets  $N_1$  and  $N_2$  representing the set of user of type GM and GG, respectively. When the resource is  $E$ , the value of the mood is  $m = \frac{E}{\sum_{i \in N_1} c_i + n_2 E}$ , and the mood value is

$x_i^m = \frac{E}{\sum_{i \in N_1} c_i + n_2 E} c_i$  if  $i \in N_1$  and  $x_i^m = \frac{E^2}{\sum_{i \in N_1} c_i + n_2 E}$  if  $i \in N_2$ .

If the value of the available resource is  $E \pm \varepsilon$ , only the maximum value for the user GG is changing. The value of the

mood is  $\hat{m} = \frac{E \pm \varepsilon}{\sum_{i \in N_1} c_i + n_2 E \pm n_2 \varepsilon}$ , and the mood value is  $\hat{x}_i^m =$

$\frac{E \pm \varepsilon}{\sum_{i \in N_1} c_i + n_2 E \pm n_2 \varepsilon} c_i$  if  $i \in N_1$  and  $\hat{x}_i^m = \frac{(E \pm \varepsilon)^2}{\sum_{i \in N_1} c_i + n_2 E \pm n_2 \varepsilon}$  if

$i \in N_2$ . Called  $D$  the denominator of  $m$ , the error is :

$$ERR_i = \begin{cases} \pm \frac{\sum_{i \in N_1} c_i \varepsilon c_i}{(\sum_{i \in N_1} c_i + n_2 E)(\sum_{i \in N_1} c_i + n_2 E \pm n_2 \varepsilon)}, & i \in N_1 \\ \left( \frac{\varepsilon \pm 2E}{D \pm n_2 \varepsilon} \mp \frac{E^2 n_2}{D(D \pm n_2 \varepsilon)} \right) \varepsilon, & i \in N_2 \end{cases} \quad (5)$$

5) *Case GM-MM:* Let  $N = N_1 \cup N_2$  be partitioned into two disjoint sets  $N_1$  and  $N_2$  representing the set of user of type GM and MM, respectively. When the resource is  $E$ , the

value of the mood is  $m = \frac{(n_2-1)(\sum_{i \in N_2} c_i - E) + n_2(\sum_{i \in N_1} c_i)}{n_2(\sum_{i \in N_2} c_i - E) + (n_2+1)(\sum_{i \in N_1} c_i)}$ , and

the mood value is  $x_i^m = m c_i$  if  $i \in N_1$  and  $x_i^m = E - \sum_{j \neq i} c_j + m(\sum_{i \in N} c_i - E)$  if  $i \in N_2$ . When the available resource

is  $E \pm \varepsilon$  the mood and the mood value are, respectively:  $\hat{m} = \frac{(n_2-1)(\sum_{i \in N_2} c_i - E \mp \varepsilon) + n_2(\sum_{i \in N_1} c_i)}{n_2(\sum_{i \in N_2} c_i - E \mp \varepsilon) + (n_2+1)(\sum_{i \in N_1} c_i)}$ ,  $\hat{x}_i^m = \hat{m} c_i$  if  $i \in N_1$ ,  $\hat{x}_i^m =$

$E \pm \varepsilon - \sum_{j \neq i} c_j + \hat{m}(\sum_{i \in N} c_i - E \mp \varepsilon)$  if  $i \in N_2$ . Called  $A$  the denominator of  $m$  and  $B$  the one of  $\hat{m}$ , the error is:

$$ERR_i = \begin{cases} \pm \sum_{i \in N_1} c_i \varepsilon c_i / (AB), & i \in N_1 \\ \pm \left( \frac{\sum_{i \in N_1} c_i - E \mp \varepsilon}{B} + \frac{(\sum_{i=1}^n c_i - E) \sum_{i \in N_1} c_i}{AB} \right) \varepsilon, & i \in N_2 \end{cases} \quad (6)$$

6) *Case GM-MG*: Let  $N = N_1 \cup N_2$  be partitioned into two disjoint sets  $N_1$  and  $N_2$  representing the set of user of type GM and the only one MG user, respectively. When the resource is

$E$ , the value of the mood is  $m = \frac{E - E + \sum_{i \in N_1} c_i}{\sum_{i \in N_1} c_i + E - E + \sum_{i \in N_1} c_i} = \frac{1}{2}$ ,

and the mood value is  $x_i^m = \frac{1}{2}c_i$  if  $i \in N_1$  and  $x_i^m = E - \sum_{i \in N_1} c_i + \frac{1}{2}(E - E + \sum_{i \in N_1} c_i) = E - \frac{1}{2}(\sum_{i \in N_1} c_i)$  if  $i \in N_2$ .

If the value of the available resource is  $E \pm \varepsilon$ , due to the hypothesis that we consider, only the minimum value for the user MG is changing. The value of the mood  $\hat{m}$  is again equal to  $\frac{1}{2}$  and the mood value is  $\hat{x}_i = \frac{1}{2}c_i$  if  $i \in N_1$  and  $\hat{x}_i = E \pm \varepsilon - \frac{1}{2}(\sum_{i \in N_1} c_i)$  if  $i \in N_2$ . It follows:

$$ERR_i = \begin{cases} 0, & i \in N_1 \\ \pm \varepsilon, & i \in N_2. \end{cases} \quad (7)$$

Concerning the boundness of the error in case of the three allocation policy we can state the following proposition.

**Proposition 1.** If the allocation rule is proportional, MMF or mood value, the error to the users is less than or equal to  $\varepsilon$ .

*Proof.* The error boundness in case of proportional and MMF allocation is easily proof from the error formulas (1), (2). The mood value corresponds to the  $\tau$ -value solution of bankruptcy games as proved in [2] and satisfies the monotonicity property as proved in [7]. We show that  $|\hat{x}_i - x_i| \leq \varepsilon$ . When the resource is  $E + \varepsilon$  due to the monotonicity it holds:

$$x_i^m(E, c) \leq x_i^m(E + \varepsilon, c), \forall i \in N \quad (8)$$

and due to the efficiency it holds:

$$\sum_{i=1}^n x_i^m(E, c) = E, \sum_{i=1}^n x_i^m(E + \varepsilon, c) = E + \varepsilon \quad (9)$$

From (8) and (9) follows that  $x_i^m(E + \varepsilon, c) - x_i^m(E, c) \leq \varepsilon, \forall i \in N$ . In similar way when the resource is  $E - \varepsilon$  due to the monotonicity it holds:

$$x_i^m(E - \varepsilon, c) \leq x_i^m(E, c), \forall i \in N \quad (10)$$

and due to the efficiency it holds:

$$\sum_{i=1}^n x_i^m(E, c) = E, \sum_{i=1}^n x_i^m(E - \varepsilon, c) = E - \varepsilon \quad (11)$$

Given (10), (11) then  $x_i^m(E - \varepsilon, c) - x_i^m(E, c) \leq -\varepsilon, \forall i \in N$ .  $\square$

### III. ALLOCATED ERROR AND FAIRNESS CONSIDERATIONS

When each user has the same misknowledge of the available resource (i.e., the same  $\varepsilon$ ) equations (1)-(7) explain how the error  $\varepsilon$  is distributed among them. In this section, we are interested in the analysis of the fairness in the error sharing for the allocation rules presented in section II.

As already noticed, the error, for each allocation, is bounded by  $\varepsilon$ , i.e., the error is split between the users without anyone

being severely disadvantaged. Furthermore, considering the fairness policy behind each error allocation, we can notice that it is close to the one of the resource allocation. In fact:

- the *weighted proportional allocation rule* splits the error proportionally to the users demands;
- the *MMF allocation* protects weak users, i.e., users with a smaller demand compared to the other users, not allocating them the error. No differences exist between the other users, receiving the same proportion of the error;
- the *mood value* takes into account the nature of each user and of the others.

In particular the mood value in the GM case allocates the error as the proportional rule does; if the users are all of type GG, or all of type MM, it does not make difference between the user and that is a good property due to the fact that they have close demands; in the case of mixed users, it assigns the error considering the group to which a user belongs.

We now look at the variation of the user satisfaction between the two scenarios with and without misknowledge on the available resource value using the three different allocation rules<sup>1</sup>. The user satisfaction, when users can collect information about other users' demands and the available resource, has to be measured as  $S_i = \frac{x_i - \min_i}{\max_i - \min_i}$  where  $\min_i$  and  $\max_i$  are equal to are the smallest and the biggest possible allocation for the user  $i$  [1], [2]. We can state the following:

**Theorem 1.** If each user has a full knowledge of the other user demands and the same misknowledge on the available resource ( $E \pm \varepsilon$  instead of  $E$ ), the mood value is the only scheme that:

- 1) equalizes the satisfaction of the users,
- 2) equalizes the error on the user satisfaction.

*Proof.* The proof of the first part is in [1], [2]. Due to the fact that the value of the satisfaction for all the user is the same for both the case in which the resource is  $E$  and  $E \pm \varepsilon$ , the error on the satisfaction, i.e., the difference between the satisfaction in case without and with the misknowledge on the available resource, is the same.  $\square$

Tables II and III show two counterexamples where proportional allocation and MMF allocation do not allocate the same satisfaction and the same error on the user satisfaction.

Figure 2 shows the variation of the satisfaction for the 6 possible cases. We can notice, as the theorem states, that the value of the satisfaction is the same for each user and for each value of  $E$  because it coincides with the mood  $m$  and

<sup>1</sup>In this analysis we consider the fairness concept linked to the users satisfaction but other fairness properties can be analyzed, such as the envy-freeness or other generalized measure of fairness, not strictly linked to the concept of satisfaction [8] can be used.

	$x_i$	$S$	$\hat{x}_i$	$\hat{S}$	$\Delta S (\hat{S} - S)$
User 1	1.3333	0.66665	1.3335	0.66675	$10^{-4}$
User 2	4	0.6	4.0004	0.6	0
User 3	4.6667	0.53334	4.6671	0.53333	$-10^{-5}$

TABLE II: Error on user satisfaction with the proportional allocation -  $x_i$  and  $\hat{x}_i$  are the allocation when  $E = 10$  and  $E = 10 + \varepsilon$ ,  $S$  and  $\hat{S}$  are the satisfaction when the allocations are  $x_i$  and  $\hat{x}_i$ ,  $\varepsilon = 10^{-3}$ ,  $c = (2, 6, 7)$ .

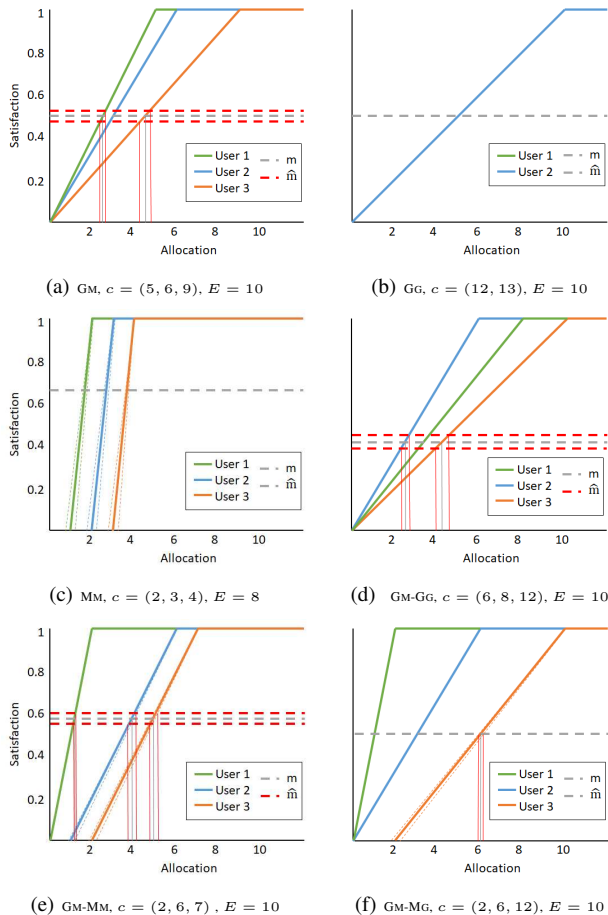


Fig. 2: Users satisfaction with and without misknowledge on the available resource - mood value case.

$\hat{m}$ . Furthermore, in Figures 2a, 2e, 2d, we clearly see that the gap between the satisfaction value when the resource is  $E$  (i.e.,  $m$ ) and when the resource is  $\hat{E}$  (i.e.,  $\hat{m}$ ) is the same for each user. In addition to the two properties stated in the theorem, we can see that in Figures 2b, 2c, 2f the value of the satisfaction does not increase or decrease when we consider the error on the available resource. In this case the satisfaction of the users, called also mood, depends only by the number of users and not by the value of the demands. That are in fact situations (i) in which each single user has the same nature of the coalition of the other ones, or (ii) in which there is only one greedy user. As already explained, in the first case the error is split uniformly between the users and in second one the greedy users keeps all the error. Another interesting fact is that the slope of the satisfaction line for users with smaller demands is not smaller than the one of users with bigger demands. This imply that for these users the allocated error cannot be bigger than the other ones.

	$x_i^{MMF}$	$S$	$\hat{x}_i^{MMF}$	$\hat{S}$	$\Delta S (\hat{S} - S)$
User 1	2	1	2	1	0
User 2	4	0.6	4.0005	0.60002	$2 \cdot 10^{-5}$
User 3	4	0.4	4.0005	0.39998	$-2 \cdot 10^{-5}$

TABLE III: Error on user satisfaction with the MMF allocation -  $x_i$  and  $\hat{x}_i$  are the allocation when  $E = 10$  and  $E = 10 + \varepsilon$ ,  $S$  and  $\hat{S}$  are the satisfaction when the allocations are  $x_i$  and  $\hat{x}_i$ ,  $\varepsilon = 10^{-3}$ ,  $c = (2, 6, 7)$ .

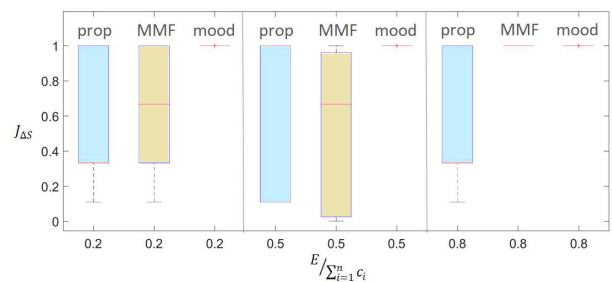


Fig. 3:  $J_{\Delta S}$  for three congestion levels as a function of  $E$ .

We now look at a global measure of fairness called Jain index [5]. The fairness is maximized using the mood value allocation, when we define the index as follows [1], [2]:

$$J_S = \left[ \frac{\sum_{i=1}^n (S_i)}{n} \right]^2 / \sum_{i=1}^n (S_i)^2 \quad (12)$$

where  $S_i = \frac{x_i - \min_i}{\max_i - \min_i}$ . We are then interested into evaluate the global fairness on the error. In particular we can re-define Jain index as follows:

$$J_{\Delta S} = \left[ \frac{\sum_{i=1}^n (\Delta S_i)}{n} \right]^2 / \sum_{i=1}^n (\Delta S_i)^2 \quad (13)$$

where  $\Delta S_i$  is the difference of the satisfaction calculated when the resource is  $E$  and when the resource is  $E \pm \varepsilon$ . The redefinition of the index is necessary to evaluate the fairness on the satisfaction error. We can derive the following theorem.

**Theorem 2.**  $J_S$  and  $J_{\Delta S}$  are maximized when the resource allocation is based on the mood value.

*Proof.* The maximization of  $J_S$  happens when each user receives the same satisfaction  $S_i$ , and the maximization of  $J_{\Delta S}$  happens when each user receives the same  $\Delta S_i$  [1], [2]. From Theorem 1, it follows that the mood value is the allocation that maximizes the two indices of fairness.  $\square$

We now test the behavior of the three considered allocation schemes in term of  $J_{\Delta S}$  by simulating 100 resource allocation problems with random demands belonging to  $[1,10]$  while varying the value of  $E$ . We set the error equal to  $10^{-2}$ . Fig. 3 shows the boxplot of  $J_{\Delta S}$  when  $E$  is the 20%, 50% and 80% of the sum of users demands. We note that the mood value maximizes the fairness on the satisfaction error. In fact, the index takes value in  $[0, 1]$ , and the higher is its value, the higher is the fairness. The proportional and the MMF allocation can produce inequality between users, presenting median values different from one and high variability.

In summary, when users are in the analyzed inaccurate information scenario, they prefer the mood value allocation because:

- it equalizes the user satisfaction and the satisfaction error and it maximizes the Jain index of fairness on the allocation and on the error;
- it takes into account the user nature in error splitting;
- in some cases, it allocates a portion of resource that provides exactly the expected satisfaction.

#### IV. OTHER CASES OF INACCURATE INFORMATION SHARING

Additionally, we consider (i) the case in which each user has misknowledge on the available resource but the error is

weighted proportional	$\mp \frac{(n-1)c_i \delta E}{(\sum_{i=1}^n c_i \pm (n-1)\delta)(\sum_{i=1}^n c_i)}$
MMF	$\begin{cases} 0, & i = 1, \dots, k \\ \mp k\delta, & i = k+1, \dots, n \end{cases}$
Mood value	$\mp \frac{(n-1)c_i \delta E}{(\sum_{i=1}^n c_i \pm (n-1)\delta)(\sum_{i=1}^n c_i)}$
GM	0
GG	0
MM	$\mp \frac{n-1}{n} \delta$

TABLE IV: Evaluation errors for errors on users demands.

not equal among the users and (ii) the case in which users has misknowledge of the other users' demands.

When we have misknowledge on the available resources, interestingly (1)-(7) still provide the evaluation of the error for an user  $i$ , but clearly it depends on  $\varepsilon_i$ . We can notice that, for each allocation and for each group of user, again the error depends linearly on the value of the error, but compared to the case analyzed in Section II, in which the error  $\varepsilon$  is shared between the users so that the sum of the users error is equal to  $\varepsilon$ , here it does not happen. The coefficient of dependency varies between the users, taking into account the nature of the user, i.e., the absolute value of the demand and the demand compared to the other users. Because of the error of each user depends on different variables, we can not compare in general the allocations errors but from (1)-(7) we can see that the error is always limited by  $\varepsilon_i$ , so that each of the allocation considered does not strongly advantages/disadvantages an user.

In contrast, for scenario (ii) the error is not always limited by  $\varepsilon$  and most of the time increases with the number of users. Table IV shows the value of the error in case (ii) when the error for each user is the same and equal to  $\delta$  (for the mood value we do not report mixed scenarios GM-GG, GM-MM, GM-MG). For example, in the MMF case, the error can be greater than  $\delta$  but bounded by  $k\delta$ . Furthermore, we can notice one more time that the mood value assigns an error that depends on the nature of the problem: it assigns the same error to users belonging to cases GG and MM, while it differentiates users belonging the group GM.

Looking at the satisfaction  $S$ , we can state the following theorem.

**Theorem 3.** If each user has a full knowledge of the available resource and the same misknowledge on the other users demand, the mood value is the only scheme that equalizes the error on the satisfaction for the same type of user.

*Proof.* We calculate the value of the  $\Delta S$  in each case for each type of user. We report the evaluation in Table V, where we can see that for users of same type the error on the satisfaction, i.e.,  $\Delta S (\hat{S} - S)$  is the same.  $\square$

From Table V we can also see that in some resource allocation problem types (e.g. GG, MM,GM-MG), as it was happening in case of misknowledge on the available resource, the users receive an allocation that satisfies them at the same level of the case of complete information.

## V. CONCLUSION AND OPEN QUESTIONS

In this work, we analyze resource allocation with inaccurate information sharing. We present three theorems that show that

User type	$S$	$\hat{S}$
GM	$\frac{E}{\sum_{i=1}^n c_i}$	$\frac{E}{\sum_{i=1}^n c_i \pm (n-1)\delta}$
GG	$1/n$	$1/n$
MM	$(n-1)/n$	$(n-1)/n$
GM	$\frac{E}{\sum_{i \in N_1} c_i + n_2 E}$	$\frac{E}{\sum_{i \in N_1} c_i + n_2 E \pm (n_1-1)\delta}$
GG	$\frac{E}{\sum_{i \in N_1} c_i + n_2 E}$	$\frac{E}{\sum_{i \in N_1} c_i + n_2 E \pm n_1 \delta}$
GM	$\frac{(n_2-1)a+n_2b}{n_2a+(n_2+1)b}$	$\frac{(n_2-1)(a \pm n_2\delta) + n_2(b \pm (n_1-1)\delta)}{n_2(a \pm n_2\delta) + (n_2+1)(b \pm (n_1-1)\delta)}$
MM	$\frac{(n_2-1)a+n_2b}{n_2a+(n_2+1)b}$	$\frac{(n_2-1)(a \pm (n_2-1)\delta) + n_2(b \pm n_1\delta)}{n_2(a \pm (n_2-1)\delta) + (n_2+1)(b \pm n_1\delta)}$
GM	$1/2$	$1/2$
MG	$1/2$	$1/2$

TABLE V: Evaluation of  $\hat{S}$  and  $S$  in case of full knowledge of the available resource and the same misknowledge on the other users demand.  $N_1$ =set of users GM,  $N_2$ =set of users of the other type.  $\sum_{i \in N_2} c_i - E = a$ ,  $\sum_{i \in N_1} c_i = b$ .

the mood value allocation is superior to the proportional and MMF allocation in terms of fairness.

A number of questions and research paths remain open for further work. Consider the analysis of the scenario in which the error is on both the resource and users' demand and the multi-resource scenario, i.e., when the resources to share are more than one and there exists a dependency between resources. In this case allocation rules, as the DRF [9], are obtained as results of an optimization problem [10]. Differently from the case single-resource in which there are rules with a direct formula to calculate the allocation, the estimation of the error becomes complex and will be studied separately.

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