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# On DEF expansion modelling in concrete structures under variable hydric conditions

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## Abstract

Delayed Ettringite Formation (DEF) in concrete is likely to develop in massive civil engineering structures such as bridges, nuclear plants, and dams with major security issues. In many cases, DEF pathology can lead to swelling and cracking which may significantly impact mass transfer and mechanical properties. It is then of major importance to build predictive tools for engineering conceptions and expertises. In this contribution, the chemical swelling evolution is integrated within the overall constitutive law of concrete that, besides, can experience other phenomena such like damage, plasticity, and long term creep, not all considered here. On another hand, as DEF is activated by environmental humidity above a certain threshold, we introduce the notion of effective time that takes into account the cumulative exposition above this threshold. Hence, a special care is taken with regards to the chem-

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ical irreversibility, together with the humidity-drying cycles. Computations are used to calibrate various sets of model parameters with the help of results from the literature, on the one hand, and from an experimental campaign where a calcareous aggregates-based concrete is studied, on the other hand. We show the efficiency of the developed numerical tool through a series of numerical examples.

*Keywords:* DEF pathology modelling, Variable humidity, Effective time concept

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## 1. Introduction

Delayed ettringite formation (DEF) in concrete can have significant impact on the long term behaviour of massive concrete structures. The swelling induced by the development of this pathology generates cracking and so **can in general affect the mechanical properties, the transfer properties, for instance see [1, 2], and the durability of the concrete structures, for instance [3]**. It is then of major importance to build numerical tools that can simulate this expanding kinematics and its consequences on the structural serviceability. Ettringite formation is delayed under a particular set of conditions, that have been unequally thoroughly studied: cement compositions ([4],[5],[6],[7] among others), aggregate mineralogical nature ([8],[9],[10],[11], among others), curing ([12],[13],[7]) or environmental humidity. Few authors have focused on the study the influence of the ambient humidity of the material. Water has a significant role in the delayed ettringite formation. It has been noticed that the concrete structures with DEF pathology were in an environment with a high ambient relative humidity (submerged parts, inflows of

17 water, or exposition to high humidity). Water seems to be both a reaction  
18 agent and environment, as ettringite crystallisation requires 32 molecules of  
19 water ([14]), but it also allows the transport of other agents that influence the  
20 formation of DEF ([15]). Indeed, it boosts the leaching of alkalines out of the  
21 interstitial water in the material, leading to a decrease of pH which favours  
22 the formation of ettringite. It is nowadays unanimously accepted that the  
23 DEF rate and amplitude is increased by the increase of saturated water, thus  
24 environmental humidity, e.g. see for example [16], [15], [17], [18], [19]. For  
25 instance in [17] and [19], the authors have even demonstrated experimentally  
26 this impact by putting reactive mortar and concrete in different controlled  
27 environments from 75%RH to 100%RH. Based on macroscopic observations,  
28 they identified a RH threshold of about 92% to initiate DEF.

29 From the mathematical point of view, extensive research has been con-  
30 ducted to model expansions caused by DEF, see for example [18, 13, 20, 21,  
31 among others] where expansions range around 0.25% to 1.3%, **even, up to**  
32 **3% [22]**, and further developments are nowadays still ongoing. The models  
33 are mostly based on phenomenological approaches formulated in terms of in-  
34 ternal variables. The expansion is in general characterized by two important  
35 ingredients:

- 36 (i) An amplitude that depends on the temperature history at early age  
37 due to hydration and/or on heating conditions if a curing process is  
38 employed such as for pre-casting;
- 39 (ii) A kinetics that depends on the time, more precisely on the cumulated  
40 time, of exposure in contact with water.

41 Of interest in this work is the introduction of a new notion; the concept  
42 of *effective* time, that we denote throughout by  $\tilde{t}$ , and that is controlled by  
43 the amount of water (humidity) during time and, consequently, that influ-  
44 ences the kinetics of expansion, i.e. the above ingredient (ii). This extends  
45 the applicability of the existing expansion models not only for cases of full  
46 saturation, but for varying environmental conditions as well. Among others,  
47 the effective time depends on a threshold below which the swelling process  
48 stops, and above which swelling strongly depends on humidity as well as on  
49 temperature. Within the continuum, the effective time is an internal field  
50 variable since each material point  $\boldsymbol{x}$  has its own humidity history during the  
51 real time  $t$ ;  $\tilde{t} \equiv \tilde{t}(\boldsymbol{x}, t)$ .

52 An outline of the remainder of this paper is as follows: we first recall  
53 the basic constitutive equations together with the kinematical assumption  
54 we adopt in this work. The notion of effective time is then motivated and  
55 detailed with a focus on the most relevant points involved by the present for-  
56 mulation. Finally, we present a set of numerical simulations to illustrate the  
57 effectiveness of the proposed framework that compare satisfactorily against  
58 experimental data from an experimental campaign and from results in the  
59 literature.

## 60 **2. Swelling kinematics**

61 The kinematical choice is as usual based on an additive split of the *total*  
62 strain tensor  $\boldsymbol{\varepsilon}$  into an elastic part  $\boldsymbol{\varepsilon}^e$  and complementary parts, each one  
63 corresponding to a phenomenon. To be as clear as possible, let us consider

64 the simplest choice,

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}_{\text{th}} + \boldsymbol{\varepsilon}_{\text{hyd}} + \boldsymbol{\varepsilon}_{\chi} , \quad (1)$$

65 where  $\boldsymbol{\varepsilon}_{\text{th}}$  and  $\boldsymbol{\varepsilon}_{\text{hyd}}$  are respectively the thermal and hydric dilatations, and  
 66  $\boldsymbol{\varepsilon}_{\chi}$  the chemical expansion tensor, in our case due to DEF. We can use the  
 67 classical relations for the formers as:

$$\boldsymbol{\varepsilon}_{\text{th}} = \alpha (T - T_0) \mathbf{1} , \quad \boldsymbol{\varepsilon}_{\text{hyd}} = \varpi (S_r - S_{r0}) \mathbf{1} , \quad (2)$$

68 where  $T$  is the temperature,  $S_r$  is the saturation,  $\alpha$  and  $\varpi$  are respectively the  
 69 thermal and hydric dilatation coefficients, assumed constant for simplicity,  
 70 and  $\mathbf{1}$  is the second-order identity tensor. Here  $T_0$  and  $S_{r0}$  are the initial  
 71 temperature and the initial saturation, respectively. Notice that a form based  
 72 on the relative humidity can equivalently be used instead of the saturation-  
 73 based form (2)<sub>2</sub>. **In future contributions, inelastic deformations through a**  
 74 **plastic strain  $\boldsymbol{\varepsilon}^P$  together with creep through a viscoelastic strain  $\boldsymbol{\varepsilon}^V$  will be**  
 75 **appended to the decomposition (1) in a classical fashion, i.e.  $\boldsymbol{\varepsilon} \equiv \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}_{\text{th}} +$**   
 76  **$\boldsymbol{\varepsilon}_{\text{hyd}} + \boldsymbol{\varepsilon}_{\chi} + \boldsymbol{\varepsilon}^P + \boldsymbol{\varepsilon}^V$  , see for example [23, 24] for similar couplings.**

77 Now if we consider a purely volumetric chemical expansion, we write:

$$\boldsymbol{\varepsilon}_{\chi} = \varepsilon_{\chi} \mathbf{1} , \quad (3)$$

78 where the scalar functional  $\varepsilon_{\chi} \equiv \varepsilon_{\chi}(S_r, T, t \dots)$  is the so-called free chemical  
 79 expansion, the expression of which depends on the humidity, the temperature,  
 80 and of course on the time as well.

81 In fully saturated conditions ( $S_r = 1$  all the time), one has the nowadays  
 82 well known expression used for both of DEF and Alkali-Aggregate Reaction

83 (AAR) expansions phenomena, see for example [10, 25, 18, among others] :

$$\varepsilon_\chi = \varepsilon_\infty^0 \frac{1 - e\left(-\frac{t}{\tau_c}\right)}{1 + e\left(-\frac{t-\tau_l}{\tau_c}\right)} \left(1 - \frac{\phi}{t + \delta}\right), \quad (4)$$

84 where,

- 85 •  $\varepsilon_\infty^0$  is the potential chemical strain that constitutes the amplitude of
- 86 expansion, i.e. ingredient (i) in the Introduction Section,
- 87 •  $\tau_c$  and  $\tau_l$  are respectively the characteristic and latent times that char-
- 88 acterize the kinetics of expansion,
- 89 • and  $\phi$  and  $\delta$  are parameters that control the long term kinetics. They
- 90 are such that  $\delta > \phi$ . Equation (4) reduces to Larive's law [26] when  $\phi$
- 91 is set to zero.

92 For the case of a DEF analysis,  $\varepsilon_\infty^0$  depends on the thermal history at

93 *early-age*. To fix the ideas the following definition can be used for its mod-

94 elling, see for instance [25, 27]:

$$\varepsilon_\infty^0 = \bar{\alpha} \int_0^{t_m} \begin{cases} 0 \, dt, & \text{if } T \leq T_{\text{def}} \\ e\left[-\frac{E_a^{\text{def}}}{R} \frac{1}{T - T_{\text{def}}}\right] dt, & \text{if } T > T_{\text{def}} \end{cases} \quad (5)$$

95 where the constant  $\bar{\alpha}$  is a material parameter,  $T_{\text{def}}$  is the threshold of temper-

96 ature above which DEF can be generated (about  $65^\circ\text{C}$ ),  $E_a^{\text{def}}$  is the activation

97 energy relative to DEF expansion, and  $t_m$  is the maturation time (few days

98 to few weeks depending on the size of the considered structure).

99 Now to take into account the influences of both of the thermal and hydric

100 conditions, relation (4) must be adapted. To be exhaustive, let us assume

101 the following choices that not all will be considered in this step of the devel-

102 opments due to the lack of experimental results.

103 *2.1. Variable thermal conditions*

104 It has been shown experimentally that the ambient temperature has an  
 105 influence on the kinetics of expansion. Among other choices, we can consider  
 106 that the two characteristic times be thermo-activated with the forms

$$\tau_l = \bar{\tau}_l e^{\left[\frac{U_l}{R} \left(\frac{1}{T} - \frac{1}{\bar{T}}\right)\right]}, \quad \tau_c = \bar{\tau}_c e^{\left[\frac{U_c}{R} \left(\frac{1}{T} - \frac{1}{\bar{T}}\right)\right]}, \quad (6)$$

107 as suggested in [26, 28] for AAR. Here  $\bar{\tau}_l$  and  $\bar{\tau}_c$  are reference characteristic  
 108 times for a reference temperature  $\bar{T}$ , and  $U_l$  and  $U_c$  are activation energies.  
 109 Notice that, even possible, the amplitude  $\varepsilon_\infty^0$  is not affected by temperature  
 110 in our DEF case.

111 *2.2. Variable hydric conditions*

112 Experimental evidences have also shown that the expansion stops below  
 113 a certain threshold of humidity, see for example [19]. The relation (4) must  
 114 then be adapted to take into account this strong dependency. In terms of  
 115 saturation, denoting the threshold below which the expansion reaction stops  
 116 by  $\bar{S}_r$ , we can formally write:

$$\begin{cases} \dot{\varepsilon}_\chi = 0, & \text{if } S_r \leq \bar{S}_r, \\ \dot{\varepsilon}_\chi > 0, & \text{if } S_r > \bar{S}_r, \\ \text{and } \varepsilon_\chi \text{ is given by (4),} & \text{if } S_r = 1. \end{cases} \quad (7)$$

117 Now for  $\bar{S}_r < S_r < 1$ , we need to establish a *continuous* link between the  
 118 extreme situations (7)<sub>1</sub> and (7)<sub>3</sub>. We introduce for this an *effective* time that  
 119 we denote by  $\tilde{t}$  and such that,

$$\tilde{t} \equiv \tilde{t}(S_r, t, \dots) \in [0, 1]. \quad (8)$$

120 A possible choice would be:

$$\dot{\tilde{t}} = \left( \frac{\langle S_r - \bar{S}_r \rangle_+}{1 - \bar{S}_r} \right)^{\bar{m}} \implies \tilde{t} = \int_0^t \left( \frac{\langle S_r - \bar{S}_r \rangle_+}{1 - \bar{S}_r} \right)^{\bar{m}} dt, \quad (9)$$

121 where the Macauley bracket  $\langle \cdot \rangle_+$  denotes the positive part function, and the  
 122 exponent parameter most probably depends on the saturation, i.e.

$$\bar{m} \equiv \bar{m}(S_r). \quad (10)$$

123 Hence, by replacing the real time  $t$  by the effective time  $\tilde{t}$  into the expres-  
 124 sion (4), we obtain a free expansion law that covers the requirements (7) for  
 125 variable humidity.

### 126 3. Constitutive equations and mechanical balance

127 The kinematic decomposition, Eq. (1), must now be embedded into a  
 128 constitutive relation. At this point, the simplest choice is to consider an  
 129 elastic relation for the reversible behaviour:

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon}^e, \quad (11)$$

130 where  $\boldsymbol{\sigma}$  is the stress tensor and the elastic part of the strain tensor  $\boldsymbol{\varepsilon}^e$  is  
 131 the one that has been used in the decomposition (1).  $\mathbf{C}$  is the fourth-order  
 132 elasticity tensor that can in turn be affected by chemical damage as

$$\mathbf{C} = (1 - d_\chi) \mathbf{C}_0, \quad (12)$$

133 where  $\mathbf{C}_0$  is the elastic modulus for the undamaged concrete (Hooke's law)  
 134 and  $d_\chi$  is a damage variable in the sense of continuum damage mechanics, see  
 135 for example [24, 29]. Intuitively, it can be driven by the chemical expansion

136 itself, then explicitly given as a function of the quantity  $\varepsilon_\chi$ , for instance, the  
 137 following form as adopted in [25]:

$$d_\chi = 1 - e^{\left[-\omega \langle \varepsilon_\chi - \varepsilon_{\text{trs}} \rangle_+\right]}, \quad (13)$$

138 where  $\varepsilon_{\text{trs}}$  is the strain-like chemical damage threshold below which no dam-  
 139 age occurs, and  $\omega \geq 0$  a convenient parameter, i.e. no chemical damage takes  
 140 place if we take  $\omega = 0$ .

141 With (11) we have a minimalist modelling framework with the simplest  
 142 possible resolution procedure. The mechanical balance is linear since the  
 143 thermal and hydric fields are a priori known at every time step, i.e. assuming  
 144 a weak coupling between the thermo-hydric diffusion and the mechanical  
 145 equilibrium. *Let us notice however that when inelastic strains (plasticity)*  
 146 *will be taken into account together with creep and mechanical damage, the*  
 147 *problem clearly becomes nonlinear. Furthermore, the thermal and hydric*  
 148 *parts of the modelling are left out of the scope of this paper. Nevertheless,*  
 149 *let us stress that for the hydric part at least, care must be taken when solving*  
 150 *for the saturation field. Indeed, on the one hand, the hydric diffusion is not*  
 151 *constant and, on the other hand, concrete materials are characterized by*  
 152 *typical hysteretic sorption-desorption responses that strongly influence the*  
 153 *soaking-drying kinetics, to mention a few. Developments on this topic are*  
 154 *still in progress.*

155 At the actual time  $t_{n+1}$  for instance, the weak form of the mechanical

156 balance at hand is given by:

$$\begin{aligned} \int_{\mathcal{B}} \nabla^s \delta \mathbf{u} : \mathbf{C}_{n+1} : \nabla^s \mathbf{u}_{n+1} \, dV &= G_{n+1}^{\text{ext}}(\delta \mathbf{u}) \\ + \int_{\mathcal{B}} \nabla^s \delta \mathbf{u} : \mathbf{C}_{n+1} : (\boldsymbol{\varepsilon}_{\text{th}_{n+1}} + \boldsymbol{\varepsilon}_{\text{hyd}_{n+1}} + \boldsymbol{\varepsilon}_{\chi_{n+1}}) \, dV &, \end{aligned} \quad (14)$$

157 which must hold for any displacement variation  $\delta \mathbf{u}$ , and where  $\nabla^s(\cdot)$  is the  
 158 symmetric gradient operator. Here  $G_{n+1}^{\text{ext}}$  is a short hand notation for the  
 159 virtual work of the external loads embedding both of the volumetric forces  
 160 in the body  $\mathcal{B}$  and traction forces on part of its boundary  $\partial_t \mathcal{B} \subset \partial \mathcal{B}$  applied  
 161 at time  $t_{n+1}$ .

162 Equation (14) is to be solved for the actual displacement field  $\mathbf{u}_{n+1}$ . How-  
 163 ever, care must be taken in evaluating the chemical strain  $\boldsymbol{\varepsilon}_{\chi_{n+1}}$  on the right  
 164 hand-side of (14). Indeed, this latter is computed with the effective time  $\tilde{t}_{n+1}$   
 165 that must in turn be updated locally at each time step as:

$$\tilde{t}_{n+1} = \tilde{t}_n + \left( \frac{\langle S_{r_{n+1}} - \bar{S}_r \rangle_+}{1 - \bar{S}_r} \right)^{\bar{m}_{n+1}} \Delta t, \quad (15)$$

166 where  $\Delta t = t_{n+1} - t_n$  is the *real* time increment, and the exponent parameter  
 167  $\bar{m}_{n+1}$  is evaluated with  $S_{r_{n+1}}$ , i.e.  $\bar{m}_{n+1} \equiv \bar{m}(S_{r_{n+1}})$  in the form (10). Conse-  
 168 quently, the effective time is treated as an internal field variable ;  $\tilde{t} \equiv \tilde{t}(\mathbf{x}, t)$ .

#### 169 4. Simulations of the influence of the relative humidity on DEF 170 expansions

171 In this section we present simulations with regards to experimental tests  
 172 given in [19] on the study of the influence of relative humidity on expansion  
 173 associated with DEF in concrete. A set of  $11 \times 22$  cm cylindrical samples

174 made with a siliceous-based aggregate concrete were heat-treated at early age  
175 so as to potentially trigger DEF expansion, and then separated in different  
176 groups, each group submitted to a controlled constant relative humidity RH,  
177 see [19] for details:

- 178 - Samples that have been stored at 94%, 96%, 98% and 100% RH, re-  
179 spectively denoted by RH94, RH96, RH98 and RH100. ;
- 180 - Samples that have been immersed continuously, denoted by IM ;
- 181 - Samples that have been stored at 91% RH and then immersed in water  
182 at the age of 334 days, denoted by RH91-IM.

183 The experimental results are shown in figure 1 where interesting observa-  
184 tions are to be pointed out:

- 185 - Expansions under continuous immersion and RH100 (not shown here)  
186 are almost identical;
- 187 - With a relative humidity of 91%, the saturation is under the threshold  
188  $\bar{S}_r$ ;
- 189 - Above the saturation threshold, the expansion is very sensitive to the  
190 ambient humidity, i.e. from 94 to 100% RH.

191 From the point of view of the modelling, the curve obtained at saturation  
192  $S_r = 1$  that corresponds to the case of RH100 or IM, is used to identify the  
193 parameters of the free-expansion law (4). They are summarized in Table 1  
194 where the asymptotic parameters  $\phi$  and  $\delta$  are used for a better description  
195 of the long term expansion kinetics.

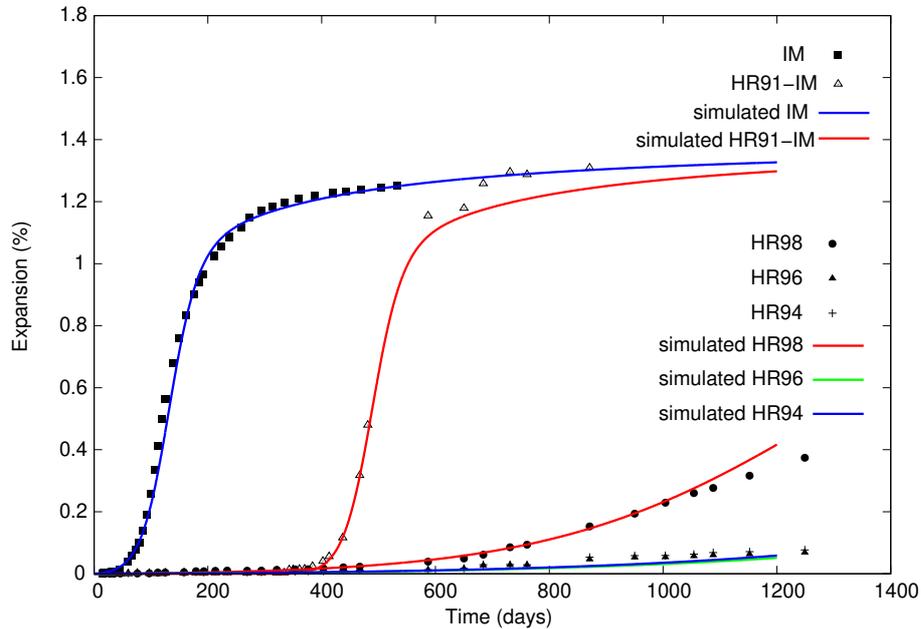


Figure 1: Expansions of all concrete specimens. Experimental results from (Al Shamaa & al., 2015). Superposition with the results of the numerical simulations.

196 Now for the effective time, the property  $(7)_2$  is active in this example,  
 197 so the expression (9) must be used. We take  $\bar{S}_r = 0.93$  for the saturation  
 198 threshold, and the exponent parameter  $\bar{m}$  has the form (10) chosen here as  
 199 a quadratic function given by,

$$\bar{m}(S_r) = 1 + 42.5(S_r - \bar{S}_r) + 1750(S_r - \bar{S}_r)^2. \quad (16)$$

200 With this set of parameters at hand, we show in figure 1 the numerical  
 201 results predicted by the model for the different exposures to humidity. One  
 202 can notice that the model is able to represent the experimental behaviour.

Table 1: Material parameters with the expansion law (4) for the siliceous concrete of figure 1.

Definition	identified values
expansion amplitude $\varepsilon_{\infty}^0$	1.4%
latent time $\tau_l$	128 days
characteristic time $\tau_c$	25 days
parameter $\phi$	68 days
parameter $\delta$	100 days

## 203 5. DEF expansions under cyclic soaking-drying cycles

204 An experimental campaign has been performed in this work as well. Con-  
 205 crete samples were designed for their known expansive behaviour when heat-  
 206 treated and immersed in water after casting. The concrete formulation used  
 207 is detailed in Table 2; A Portland cement CEMII/A L with 6% of calcare-  
 208 ous additions (42.5 MPa) is used with a 0.57 water-cement ratio. Calcareous  
 209 aggregates were used (Boulonnais aggregates).

210 Now to simulate the heating induced by cement hydration in massive  
 211 structures, the samples undergo a thermic treatment as shown in figure 2:  
 212 the temperature was maintained at  $20^{\circ}C$  for 2h, then increased from  $20^{\circ}C$   
 213 to  $80^{\circ}C$  in 24 h, then maintained at  $80^{\circ}C$  for three days, and then decreased  
 214 from  $80^{\circ}C$  to  $20^{\circ}C$  in three days. After this treatment, half of the samples  
 215 were immersed in water at  $20^{\circ}C$ ; the other half undergo soaking and drying  
 216 cycles. In all cases,  $11 \times 22$  cm cylindrical samples have been used, and are  
 217 monitored following the LCPC method n°66.

Table 2: Concrete composition.

Materials	Composition
Cement	350 kg
Water	210 kg
Sand (0-5 mm)	858 kg
Aggregates (5-12.5 mm)	945 kg
Viscosity modifying admixture	6.6 kg
W/C	0.57
A/S	1.1
Density	2354 kg/m <sup>3</sup>

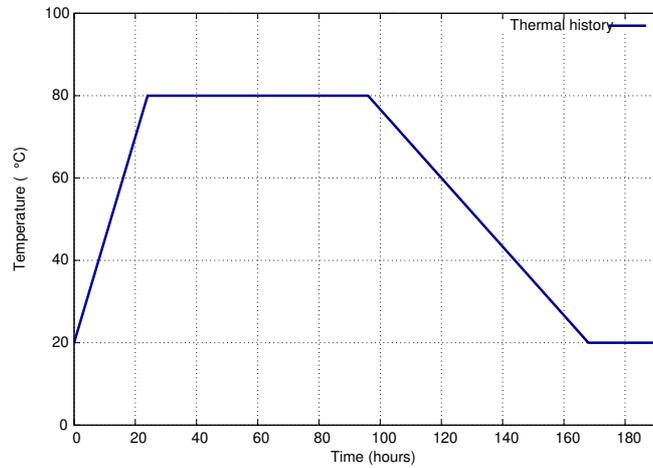


Figure 2: Heating treatment of the 11×22 cm cylindrical samples during first hours after curing.

218 *5.1. Simulations for the immersed samples*

219 The experimental evolution of the mean expansions for the samples is  
 220 presented in figure 3, in square-dots-points. The phase of accelerated degra-  
 221 dation is finishing and the stabilisation phase is taking place (about 0.23%  
 222 maximum expansion).

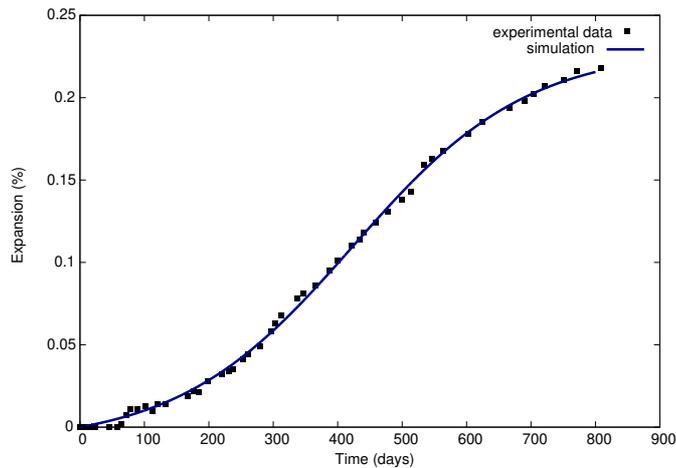


Figure 3: Swelling of concrete samples with calcareous aggregates. Experimental data and numerical simulation.

223 Curve fitting with the free-expansion law (4) gives the parameters sum-  
 224 marized in Table 3 where Larive’s law is sufficient, i.e. with  $\phi = 0$ .

225 With these values, the simulated expansion is superposed in figure 3 where  
 226 a good agreement is to be noticed. As the samples are immersed during  
 227 the whole time, uniform saturation  $S_r = 1$  is taken and, consequently, the  
 228 effective time is identical to the real time.

Table 3: Material parameters with the expansion law (4) for the calcareous-based concrete.

Definition	identified values
expansion amplitude $\varepsilon_\infty^0$	0.23%
latent time $\tau_l$	423.5 days
characteristic time $\tau_c$	135.48 days
parameter $\phi$	0
parameter $\delta$	0

229 *5.2. Simulations of the effect of soaking-drying cycles on DEF*

230 A second set of samples with the above calcareous aggregates was submit-  
 231 ted to cycles of drying (at 50 % of relative humidity and 23°C) alternated  
 232 with soaking (at 23°C) to represent a more realistic environment of some  
 233 structures which are not permanently in contact with water. The cycles are  
 234 recorded in Table 4.

235 The swelling monitoring of this new study is plotted in figure 4 (the  
 236 dashed line with circle dots), and compared to the previous experimental  
 237 curve of figure 3 with the samples kept continuously immersed in water  
 238 (square-dots-points curve). One can notice that the drying cycles restrict  
 239 the delayed ettringite formation in the material and have an important im-  
 240 pact on the expansion kinetics.

241 From the modelling point of view, this falls within the scope of the prop-  
 242 erties (7)<sub>1</sub> for drying, and (7)<sub>3</sub> for soaking. For the material parameters of  
 243 the expansion law (4), we use the ones of Table 3. And for the effective time  
 244 (9), we fix the saturation threshold to  $\bar{S}_r = 0.95$ . Here the computation is

Table 4: Soaking-drying cycles for the calcareous-based concrete samples.

Cycle	Soaking time	Drying time
$n^\circ$	(days)	(days)
1	9	14
2	22	20
3	7	7
4	10	12
5	20	12
6	35	9
7	22	22
8	10	7
9	23	19
10	16	7
11	34	10
12	19	21
13	13	22
14	12	7
15	37	22
16	13	21
17	13	17
18	38	23
19	41	24
20	14	17
21	30	20
22	38	38
23	42	24

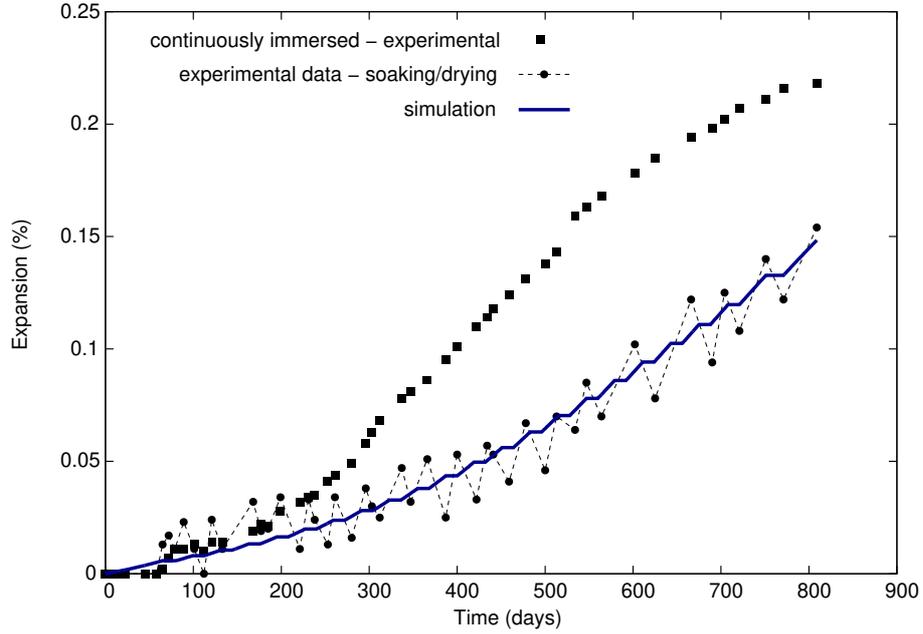


Figure 4: Impact of drying cycles on the swelling of concrete samples with calcareous aggregates. Experimental data and numerical simulation.

245 independent of the exponent parameter  $\bar{m}$  since:

$$\tilde{t} = \begin{cases} 1 & \text{for soaking, with } S_r = 1, \\ 0 & \text{for drying, since } S_r \ll \bar{S}_r. \end{cases} \quad (17)$$

246 In the numerical computation, uniform saturation  $S_r = 1$  is taken during  
 247 the first 72 days, then cycles of drying for 13 days at  $S_r = 0.48$  followed by  
 248 soaking for 19 days at  $S_r = 1$  are prescribed. In total, the sample is immersed  
 249 514 days and dried 295 days.

250 The simulated expansion is superposed in figure 4 where good agreement  
 251 is again observed. Notice further that the expansion after 809 days of alter-  
 252 nate soaking/drying cycles is almost the same as the 514 days' expansion in  
 253 continuous immersion conditions.

## 254 **6. Conclusions**

255 In this study, we have presented and numerically tested an efficient tool  
256 for the modelling of the influence of relative humidity on the development  
257 of DEF expansions. The simple notion of effective time implicitly takes into  
258 account the amount of water during time in variable environmental conditions  
259 that strongly influence the kinetics of DEF expansions.

260 From the numerical point of view, we have briefly presented a numerical  
261 design for the integration of the chemical strain and the update of the effective  
262 time. In particular, this latter must be performed at the integration points  
263 level when use is made of the finite element method.

264 Numerical simulations have shown good agreements with the experimen-  
265 tal results issued from an experimental compaign, and others from the lit-  
266 erature. However, further studies are ongoing **to include this notion within**  
267 **more detailed and complex formulations, i.e. including creep, plasticity and**  
268 **mechanical damage among others.**

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