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Christophe André, Benjamin Boissier, Luc Fillaudeau. Modelling and measurement of residence time distribution in tubular Joule effect heaters with and without geometric modifications. TRACER 4, Oct 2006, Autrans/Grenoble, France. hal-02310114

HAL Id: hal-02310114
https://hal.science/hal-02310114v1

Submitted on 4 Jun 2020

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MODELLING AND MEASUREMENT OF RESIDENCE TIME DISTRIBUTION IN TUBULAR JOULE EFFECT HEATERS WITH AND WITHOUT GEOMETRIC MODIFICATIONS

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Abstract. *In food industry, heat treatment of highly viscous fluids in continuous process is more and more common and the process should perform homogenous thermal treatment in order to ensure quality and safety of the final product. To improve treatment homogeneity, geometric modifications could be used even in laminar regime inducing flow perturbation and mixing. In this work, our objectives were (i) to investigate residence time distribution (RTD) for industrial indirect Joule effect heaters (JEH) with smooth (ST) and modified (MT) tubes, (ii) to demonstrate and quantify the efficiency of geometrical modifications and (iii) to propose a general reactor model including flow regime (10<Re<2000) and tube diameter (18 and 23mm). The analysis demonstrates that the geometrical modifications improve treatment homogeneity by increasing the plug flow contribution and reducing the value of reduced variance. These benefit effects increase when the Reynolds number is increased, the nominal diameter is reduced and the modified tubes are used. Proposed model enabled to predict RTD in JEH with an accurate degree of confidence.*

1. INTRODUCTION

In food industry, heat treatment in continuous process should perform homogenous thermal treatment in order to ensure quality and safety of the final product whereas flow regime is often laminar and mixed convection occurs. To improve treatment homogeneity in tubular Joule Effect Heater (JEH), geometric modifications could be used even in laminar regime inducing flow perturbation and mixing. As a response variable, RTD is an important parameter and it has been commonly used in determining the performances of industrial heat exchangers ***[2, 3, 4]***. RTD analyse provides information about the degree of mixing, cooking and shearing which play an important role in the final product quality. RTD are used for scale-up and improving equipment design.

In a first step, the friction curves and the Residence Time Distribution (RTD) study of industrial exchangers made by tubular JEH (6 tubes and junctions) with and without geometric modifications for different sizes of exchangers ($\varnothing_{int/ext}$ = 18/20 and 23/25) were measured at room temperature. From experimental data, the normalised and reduced experimental x(θ) and y(θ) signals with $\theta = t/\bar{t}_s$ were deduced ***[5]***.

In a second step, the reactor behaviour was described with a RTD analyse based on DTS Pro 4.2 software ***[1]***. The analytical solution of *E(t)* was obtained from the data analyse and the experimental criteria (mean residence time, standard deviation) have been quantified versus tube diameters, geometric modifications and flow regimes ***[6]***.

Finally, a systemic analyse ***[5]*** compared the dispersed plug flow model (DPF) and a reactor model (Plug flow + 2 perfect mixers in series). The second model enabled to quantify the plug-flow contribution and the reduced variance versus flow regime and geometrical criteria.

2. THEORY

2.1. Friction curves

Darcy and Reynolds numbers are defined as follows:

$$Da = \frac{2.d_h}{\rho.U^2}.\frac{\Delta P}{L} \quad \textbf{Eq. 1} \qquad Re = \frac{\rho.U.d_h}{\mu} \quad \textbf{Eq. 2}$$

The friction curve is the representation of *Da* against *Re*. The whole curve ((laminar and the turbulent contributions, Eq.3 and 4) can be described using a simplified Churchill's model (Eq.5). This equation is modelled by a 3 parameters (8ξ, a and b) which are experimentally identified.

$$Da_{lam} = \frac{8.\xi}{Re} \quad \textbf{Eq. 3} \qquad Da_{turb} = a \cdot Re^b \quad \textbf{Eq. 4} \qquad Da_{global} = \left(Da_{lam}^2 + Da_{turb}^2\right)^{1/2} \quad \textbf{Eq. 5}$$

Critical Reynolds numbers, Rec_1 *and* Rec_2 were identified when difference between experimental and modelled Darcy numbers exceeds 10%.

2.2. Residence time distribution

Flow patterns in continuous systems are usually too complex to be experimentally measured or theoretically predicted from solutions of the Navier-Stokes equation or statistical mechanical considerations. The residence time of an element of fluid is defined as the time elapsed from its entry into the system until it reaches the exit. The distribution of these times is called the RTD function of the fluid E, or E-curve, and represents the fraction of fluid leaving the system at each time ***[6]***. We define *x(t)* and *y(t)*: experimental inlet and outlet normalised signals. The mathematical relation between *x(t)* and *y(t)* is described by Eq.6. This product of convolution can be replaced in Laplace domain by a simple product and *X(s), Y(s)* and *G(s)* are the Laplace transforms of *x(t), y(t)* and *E(t)*.

$$y(t) = \int_0^t E(u).x(t-u).du \quad \textbf{Eq. 6} \qquad Y(s) = G(s).X(s) \quad \textbf{Eq. 7}$$

The function *E(t)* is characterized by the mean residence time, t_s (Eq.8). Mean holding time, τ is calculated as the ratio between the volume of the corresponding test section and the flow rate (Eq.9).

$$t_s = \int_0^\infty t.E(t)\,dt \quad \textbf{Eq. 8} \qquad \tau = \frac{V}{Q} \quad \textbf{Eq. 9} \qquad \sigma^2 = \int_0^\infty (t - \bar{t}_s)^2.E(t).dt \quad \textbf{Eq. 10}$$

For such experimental set-up, t_s and τ are equal. If not, experimental results must be rejected. It is an indication that a channelling in the fluid circuit has occurred. Variance (σ^2) and reduced variance (β^2) are defined by Eq.10 and 11:

$$\beta^2 = \frac{\sigma^2}{t_s^2} \quad \textbf{Eq. 11} \qquad E(t) = \frac{1}{2}.\left(\frac{Pe}{\pi.\tau.t}\right)^{1/2}.\exp\left(-\frac{Pe.(\tau - t)^2}{4.\tau.t}\right) \quad \textbf{Eq. 12}$$

Simple models such as cascade of N continuous stirred tank reactors (CSTR) or a dispersed plug flow (DPF) are used for modelling RTD experiments ***[2, 3, 6]***. The DPF model was often found to yield the best agreement between numerical and experimental results. Furthermore, it has the advantage of requiring the estimation of only one parameter (Eq.12), the Peclet number (*Pe*).

3. MATERIAL AND METHODS

3.1. Experimental set-up and injection device

The experimental set-up (Figure 1) consisted of an agitated vessel (600l), a volumetric feed pump (Albin, MR 25I2-207548) and a tubular Joule effect heater. The JEH is a horizontal tubular heat exchanger delivered by the French company Actini (P=20kW, U=18V, I=600A). From a hydraulic point of view, the JEH consists of 6 tubes of 1.40 or 1.50m length with 1.20m heating length (Table 1). Smooth tubes were regular circular straight tubes with an internal diameter of 18 and 23mm and 1mm wall thickness. Modified tubes are made from smooth tubes ; the geometrical modifications consist in three pinching on a section with 120° angle between them. This motive is repeated along the heating length with a regular space (100 or 140mm) and an alternated angle (60°). For each pinching, the tube wall was pushed inside the tube with a depth ranging from 4 to 5mm on a length of 25mm. An injection and detection device was developed to realise homogenous injections of tracer even in a laminar flow. Tracer was injected by applying a backpressure on a ceramic microfiltration membrane (19 channels, permeable length L=2cm, Figure 1). The objective is to realise an ideal pulse ***[6]***, and the tracer was quantified by electrical conductivity measurements of the JEH.

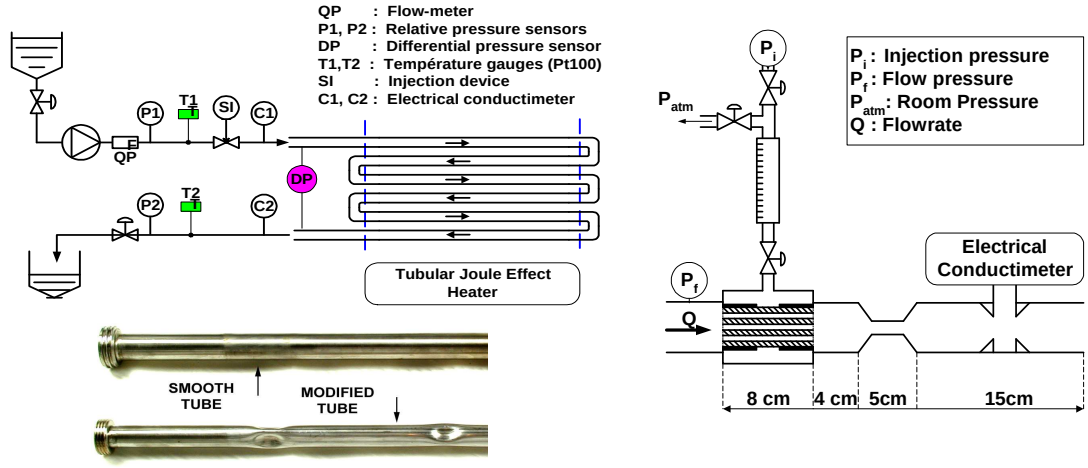


Figure 1 : Experimental set-up and injection device (SI) - image of ST and MT.

Tube	L, [mm]	d_h, [mm]	nb	E, [mm]	L_m /l_m/e_m, [mm]	V, [l]	ΔV, [%]
ST 18/20	1400	17,91	0	-	-/-/-	2,37	-
MT 18/20	1400	17,91	9	140	25/13/4	2,31	2,5%
ST 23/25	1500	22,93	0	-	-/-/-	4,18	-
MT 23/25	1500	22,93	12	100	25/14/5	3,94	5,7%

Table 1 : Geometrical characteristics of ST and MT.

Experimental measurements were: flow-rate, temperature, electric conductivity, differential and relative pressure and electrical conductivity. The flow rate was measured using electromagnetic flow-meter (KHRONE, type X1000/6, precision ±1%), the temperatures by use of platinum resistance probes (Pt 100Ω) placed at the entrance and exit of each exchange zone (precision ±0.5°C) and the pressure with relative pressure sensors (JPB, type TB233, precision 0,2%). The differential pressure (Bayley Fischer Porter: 0-75mBar and 0-900mBar and Schlumberger: 0-750mBar and 0-2Bar) were measured to establish the friction curve. Electrical conductivity was monitored with two sensors at the inlet and outlet of the exchanger (Stratos, type 9117/93,

n°31308, K=0,3790 and type 9111/93, n°31403, K=0,2340, range 0,2µS to 1000mS, precision ±1%). All signals were electrically conditioned (module SCX-1) and collected using a data acquisition card (AT-MOI-16E-10). A software driver Ni-DAQ made the configuration and control of data acquisition system possible. Measurements were saved on a PC (PC Pentium 200 MHz) with a specific software (Labview).

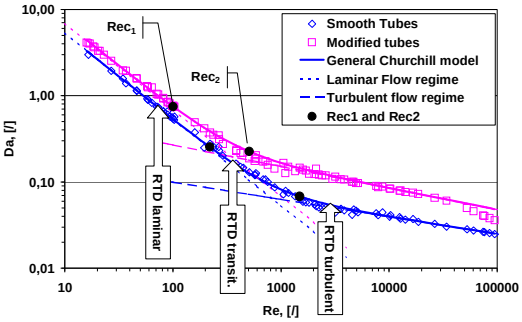
3.2. Fluids and Experimental conditions

Friction curves and RTD experiments were conducted at room temperature (20°C±5) with water and sucrose solutions. NaCl was used as tracer for RTD. For friction curves determination, he flow rate ranged between 100 to 7000l.h^{-1}, whereas for RTD experiments, the flow-rates remained constant and close to 100l.h^{-1}. For RTD, three Reynolds numbers were investigated corresponding to turbulent, transition and laminar regime (Table 2). Each operating condition is repeated at least 3 times.

4. RESULTS AND DISCUSSIONS

4.1. Friction curves

Critical Reynolds numbers, Re_{c1} *and* Re_{c2} are determined from the friction curves and Churchill's model defining the laminar, transitory and turbulent regimes (Table 2).

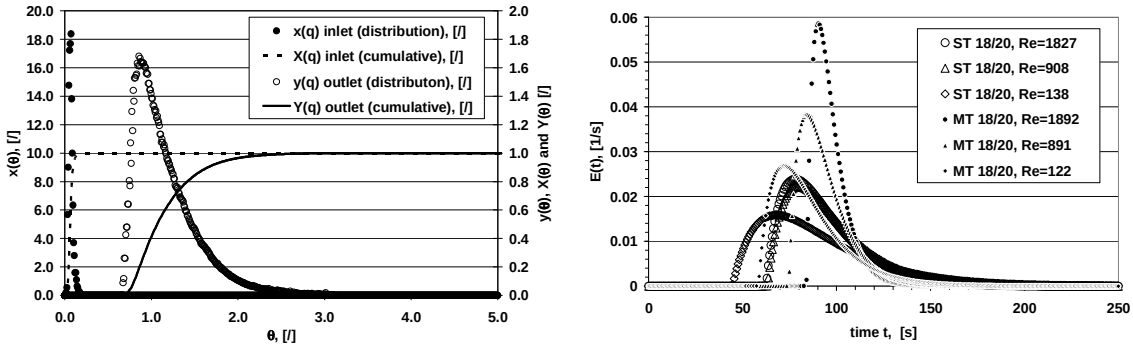


	Laminar		Turbulent		
	$8*\zeta$	Re_{c1}	a	b	Re_{c2}
Tube	64	<2000	0.316	-0.25	>4000
ST18/20	53.8	200	0.25	-0.20	1500
MT18/20	75.1	100	0.85	-0.25	500
ST23/25	60.1	300	0.25	-0.17	1300
MT23/25	78.7	150	0.85	-0.21	450

Figure 2 : Friction curves of ST and MT 18/20.

Table 2 : Friction curves parameters and critical Reynolds numbers

4.2. Data analysis and RTD formulation: analytical solution.



A B

Figure 3 : Inlet and outlet normalised signals, x(t) and y(t) versus θ for ST 18/20, Re=138 (A) and RTD curves versus Reynolds numbers for ST and MT 18/20 (B).

Three flow regimes (*Re ≈ 110, 925 and 1700*) were scrutinised in agreement with established friction curves and corresponding to the industrial practise. Average experimental data (mean value of at least 3 runs) are reported in table 3. Experimental results where the mean holding time, τ differed significantly from the mean residence time, t_s obtained from RTD curve (above 10%) were rejected. The normalised and reduced experimental, *x(θ) and y(θ)* signals with $\theta = t/\bar{t}_s$ enabled to determined the extremum point (x_{max} *and* $\theta(x_{max})$) and the values of θ for *F=0.05, 0.50 and 0.95* of RTD signal. Inlet and outlet normalised signals (Figure 3) are used to determined *E(t)* through convolution product in Laplace domain as follows :

$$\left.\begin{array}{l} X(s) = L(x(t)) \\ Y(s) = L(y(t)) \end{array}\right\} G(s) = \frac{Y(s)}{X(s)} \Rightarrow E(t) = L^{-1}(G(s))$$

	Q [l.h^{-1}]	T [°C]	V [l]	τ [	Re [/]	$X_{max}(\theta)$	$\theta(X_{max})$	$\theta_{0,05}$	$\theta_{0,50}$	$\theta_{0,95}$
ST 18/20	96.59	18.56	2.37	88.45	1827	2.68	0.946	0.845	1.04	1.95
	92.19	24.67	2.37	92.57	908	2.17	0.925	0.797	1.03	2.41
	97.19	25.81	2.37	87.79	138	1.76	0.833	0.754	1.04	1.82
MT 18/20	93.09	21.58	2.31	89.33	1893	4.62	1.102	1.016	1.13	1.58
	95.31	22.84	2.31	87.29	892	3.28	1.040	0.925	1.08	1.47
	98.08	22.74	2.31	84.79	123	2.41	0.919	0.804	1.01	1.61
ST 23/25	108.79	16.19	4.18	138.35	1518	2.64	0.909	0.823	1.01	2.10
	98.19	23.09	4.18	153.27	868	2.49	0.909	0.798	1.00	1.92
	108.39	22.68	4.18	138.84	89	1.87	0.8228	0.6883	0.966	1.5910
MT 23/25	107.74	16.05	3.94	131.66	1494	4.66	1.037	0.948	1.065	1.360
	110.04	24.26	3.94	128.93	1032	2.82	1.051	0.921	1.107	1.730
	107.11	22.96	3.94	132.44	90	2.21	0.821	0.715	0.948	1.572

Table 3 : Average operating conditions and experimental data.

4.3. Systemic analyse

DPF model was compared to a cascade of a plug reactors (τ_p) in series with 2 CSTR (τ_m). The DPF model (Eq.13) has been widely applied to describe the flow in a tube, and is the most frequently selected to simulate flow in holding tubes in aseptic processes ***[2, 3]***. Peclet number, *Pe* were determined by curve fitting and minimizing the sum of squares of residuals (SSR) as reported in Table 4. Analysis of specific RTD curves were found to yield inconsistent and inaccurate results in our study.

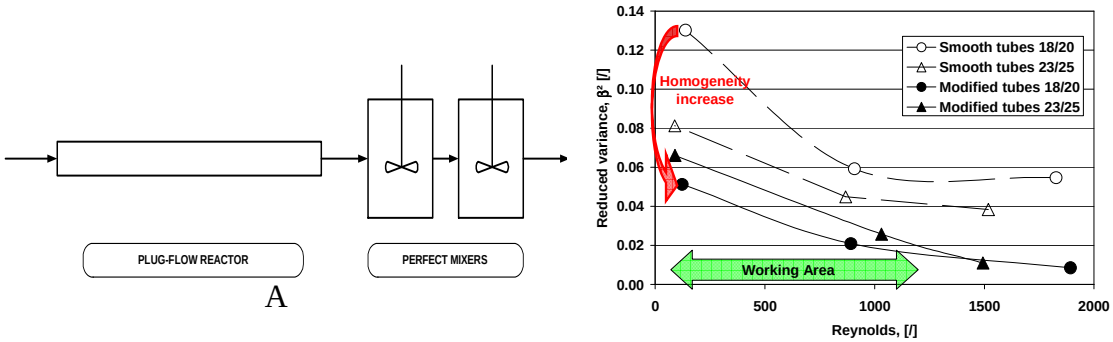


Figure 4 : Cascade of a plug reactor (τ_p) in series with 2 CSTR (A) and modelled reduced variance versus Reynolds numbers for ST and MT (B).

A cascade of a plug reactor (τ_p) in series with 2 CSTR (τ_m) was chosen, the transfer function *G(s)* of the cascade is formulated by Eq.13. DTS Progepi software was used in

order to determine the plug reactor contribution, α in term of residence time. Van Laar's relation leads to a simple relation between α and β^2 (Eq.14 and 15). Model is in agreement with experiments and β^2 shows the difference between ST and MT (Figure4).

Tube	*Re*	*Pe-ST*	*Pe-MT*
	1860	39	264
18/20	900	35	105
	130	15	41
	1506	56	203
23/25	950	47	84
	90	25	31

$$G(s) = \frac{\exp(-s.\tau_p)}{\left(1+\left(\frac{s.\tau_m}{2}\right)\right)^2} \qquad \textbf{Eq.13}$$

$$\alpha = \frac{\tau_p}{\tau_p + \tau_m} \textbf{ Eq.14} \text{ and } \beta^2 = \frac{(1-\alpha)^2}{2} \textbf{ Eq.15}$$

Table 4 : Peclet numbers issued from DPF model with ST and MT.

5. CONCLUSION

In conclusion, this work demonstrates that (i) the geometrical modifications improve treatment homogeneity by increasing the plug flow contribution and reducing the value of reduced variance, (ii) these benefit effects increase when the Reynolds number is increased, the nominal diameter is reduced and the modified tubes are used, (iii) the proposed model enabled to predict RTD in JEH with an accurate degree of confidence, (iv) these benefits should be compared to pressure drop increased due to a higher Darcy number. In the future, the impact of heat transfer should be taken into account and investigated with Newtonian and non-Newtonian fluids.

Nomenclature :

a, b	coefficient	*V*	Volume, [m^3]
Da	Darcy number, [/]	*x, y*	Inlet and oulet normalised signal
d_h	Hydraulic diameter, [m]	*X, Y*	Laplace transform of x and y.
e	Space, [m]	α	Plug flow contribution, [/]
E	RTD function, [s^{-1}] or [/]	ξ	Geometrical factor, [/]
F	Cumulative RTD function, [/]	β^2	Reduced variance, [/]
G	Laplace transform of E	θ	Reduced time, [/]
L	Length, [m]	ΔP	Pressure drop, [bar]
Pe	Peclet number, [/]	μ	Viscosity, [Pa s]
Q	Volume flow rate, [$m^3.s^{-1}$]	ρ	Volume mass, [$kg.m^{-3}$]
Re	Reynolds number, [/]	σ^2	Variance, [s^2]
t	Time, [s]	τ	Mean holding time, [s]
t_s	Mean residence time, [s]	*lam*	Laminar
T	Temperature, [°C]	*m*	modification
U	Velocity, [$m.s^{-1}$]	*turb*	turbulent

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