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Effective channel and ungauged braided river discharge estimation by assimilation of multi-satellite water heights of different spatial sparsity

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Abstract

Multi-satellite sensing of continental water surfaces (WS) represents an unprecedented and increasing potential for studying ungauged hydrological and hydraulic processes from their signatures, especially on complex flow zones such as multichannel rivers. However the estimation of discharge from WS observations only is a very challenging inverse problem due to unknown bathymetry and friction in ungauged rivers, measurements nature, quality and spatio-temporal resolutions regarding the flow (model) scales. This paper proposes an effective 1D hydraulic modeling approach of sufficient complexity to describe braided river flows from sparse multisatellite observations using the HiVDI inverse method presented in Larnier et al. [42] with an augmented control vector including a spatially distributed friction law depending on flow depth. It is shown on 71km of the Xingu River (braided, Amazon basin) with altimetric water height timeseries that a fairly accurate upstream discharge hydrograph and effective patterns of channel bathymetry and friction can be inferred simultaneously. The coherence between the sparse observation grid and the fine hydraulic model grid is ensured in the optimization process by imposing a piecewise linear bathymetry profile $b(x)$, which is consistent with the *hydraulic visibility* of WS signatures (Garambois et al. [27], Montazem et al. [46]). The discharge hydrograph and effective bathymetry-friction patterns are retrieved from 8 years of satellite altimetry (ENVISAT) at 6 virtual stations (VS) along flow. Next, the potential of the forthcoming SWOT data, dense in space, is highlighted by inferring a discharge hydrograph and dense patterns of effective river bathymetry and friction; a physically consistent definition of friction by reaches enabling to consider more dense bathymetry controls. Finally a numerical analysis of the friction term shows clear signatures of river bottom slope break in low flows and width variations in high flows which is consistent with the findings of Montazem et al. [46] from WS curvature analysis

35 *Keywords:* Multichannel River, Ungauged River, 1D Hydraulic Model, Data Assimilation, Satellite Altimetry,
36 SWOT, Hydraulic Visibility

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38 1. Introduction

39 Fresh water is a crucial earth's resource and its journey from the clouds to the oceans passes through the
40 hydrographic network. In order to characterize hydrological fluxes, an essential physical variable is river discharge
41 (cf. Global Climate Observing system et al. [22]) representing an integration of upstream hydrological processes.
42 In complement of in situ sensors networks which are declining in some regions (e.g. Fekete and Vorosmarty [23]),
43 increasingly accurate measurements of hydrological and hydraulic variables, and especially river surface variabilities
44 are now enabled by myriads of satellites for earth observations and new generation of sensors (e.g. Vorosmarty
45 et al. [56], Alsdorf and Lettenmaier [2], Calmant et al. [13], Schumann and Domeneghetti [54]).

46 The forthcoming Surface Water and Ocean Topography (SWOT) wide swath altimetric mission (CNES-NASA,
47 planned to be launched in 2021) will provide a quasi global river surfaces mapping with an unprecedented spatial and
48 temporal resolution on Water Surface (WS) height, width and slope - decimetric accuracy on WS height averaged
49 over 1 km², 1 to 4 revisits every 21 days cycle 50, 5. In addition to decades of nadir altimetry (e.g. Frappart
50 et al. [25], Birkett [6], Da Silva et al. [17], Calmant et al. [12]) and imagery (e.g. Allen and Pavelsky [1]) on inland
51 waters, SWOT will enable an unprecedented *hydraulic visibility*, as defined from hydraulic analysis in Garambois
52 et al. [27], Montazem et al. [47], Montazem et al. [46], of hydrological responses and hydraulic variabilities within
53 river networks. Multi-satellite observations of water surfaces from the local to the hydrographic network scale
54 indeed represent an unprecedented observability of hydrological responses through hydraulic processes signatures,
55 especially on complex flow zones such as floodplains or braided rivers. This increased *hydraulic visibility* represents
56 a great potential to learn hydrodynamic behaviors and infer hydrological fluxes.

57 The estimation of river discharge from water surface observations (elevations, top width) remains an open and
58 difficult question, especially in case of unknown or poorly known river bathymetry, friction or lateral fluxes. Several
59 open-channel inverse problems are studied in a relatively recent litterature in a satellite data context with more
60 or less complex flow models and inverse methods (cf. Biancamaria et al. [5] for a review). Few studies started
61 to highlight the benefit of assimilating synthetic SWOT WS observations in simplified hydraulic models with
62 sequential methods, for inferring inflow discharge assuming known river friction and bathymetry (Andreadis et al.
63 [3], Biancamaria et al. [4]) or inferring bathymetry assuming known friction (Durand et al. [19], Yoon et al. [58]).
64 Next, low-complexity methods have been proposed for estimating river discharge in case of unknown bathymetry
65 and friction based on the Manning-Strickler's law (Durand et al. [21], Garambois and Monnier [28]) or hydraulic
66 geometries (Gleason and Smith [32]) or empirical flow models (Durand et al. [20], see also Bjerklie et al. [7]). They

67 are tested on 19 rivers with synthetic “SWOT-like” daily observations in 20 and their robustness and accuracy is
68 found to fluctuate, the importance of good priors is highlighted; none of the tested river is braided.

69 The combined use of dynamic flow models and optimization methods enables to benefit from WS observations
70 for solving hydraulic inverse problems as shown for flood hydrograph inference in Roux and Dartus [51] from WS
71 width time series used to optimize a 1D hydraulic model or in Honnorat et al. [38], Hostache et al. [39], Lai and
72 Monnier [41] by variational assimilation of flow depth time series in a 2D hydraulic model. The variational data
73 assimilation (VDA) approach (see e.g. Cacuci et al. [11] and references therein) is well suited to solve the present
74 inverse problem (see Brisset et al. [10], Oubanas et al. [48], Larnier et al. [42] and references therein).

75 It consists in fitting the hydraulic model response to the observed WS elevations by optimizing the “input
76 parameters” in a variational framework. However, altimetry measurements of WS are relatively sparse in time
77 compared to local flow dynamics. This important aspect of the inverse problem is investigated in Brisset et al. [10]
78 with the introduction of *identifiability maps*. The latter consist to represent in space-time the available information:
79 WS observables, hydraulic waves and an estimation of the misfit with local equilibrium. These “maps” enable to
80 estimate if the sought upstream discharge information has been observed or not within the downstream river surface
81 deformations; also they help to estimate inferable hydrograph frequencies Brisset et al. [10] or inferable hydrograph
82 time windows Larnier et al. [42].

83 The inference of the hydraulic triplet (inflow discharge $Q(t)$, effective bathymetry $b(x)$ and friction coefficient K)
84 from SWOT like WS observations is investigated in recent studies using 1D hydraulic and variational assimilation
85 methods (e.g. Brisset et al. [10], Gejadze and Malaterre [29], Oubanas et al. [48], Larnier et al. [42]). However the
86 inference of the triplet from WS observations remains a very challenging inverse problem because of the correlated
87 influence of temporal (discharge) and spatial (bathymetry-friction) controls on the simulated flow lines. This
88 is especially true because of the bathymetry-friction “equifinality issue”, see the discussions in Garambois and
89 Monnier [28], Larnier et al. [42]. Those recently developed VDA methods enable to infer accurately the inflow
90 discharge from water surface observables, considering unknown/uncertain channel bathymetry-friction, but from
91 accurate prior information and synthetic WS observations. Note that a strong prior such as a known stage-discharge
92 relationship (rating curve) downstream of a river domain as it is done in [48] highly controls the simulated flow
93 lines (fluvial regime); as a consequence the VDA process converge to the discharge hydrograph corresponding to
94 the imposed (almost exact) rating curve. In the present study the downstream boundary condition is an unknown
95 of the inverse problem.

96 A crucial point is the sensitivity of the triplet inference to the prior value from which the inference is started
97 and it is only studied in a SWOT data context in Garambois and Monnier [28], Yoon et al. [59], Larnier et al.
98 [42], Tuozzolo et al. [55]. The sensitivity of the estimated discharge (in the triplet) to the prior is highlighted
99 by recent estimates performed from AirSWOT airborne measurements on the Willamette River (Tuozzolo et al.
100 [55]). The temporal signal is well retrieved at observation times but using a biased prior hydrograph results in

101 a biased hydrograph inference - see detailed investigations in Larnier et al. [42]. In view to infer worldwide river
102 discharge from the future SWOT observations, especially for ungauged cases, a hierarchical modeling strategy
103 HiVDI (Hierarchical Variational Discharge Inversion) is proposed in Larnier et al. [42]. HiVDI approach includes
104 low complexity flow relations (under the assumption of Low Froude and locally steady-state) which improve the
105 robustness of the inferences in particular if an average value of Q is provided. (It may be provided by a database or a
106 large scale hydrological model). Note that if introducing an a-priori information such as a single depth measurement,
107 it enables to reconstruct an effective low-flow bathymetry see 30, 28, 42.

108 All the studies mentioned above address single thread natural rivers ($\sim 100\text{km}$ in length) without lateral inflows
109 and using synthetic datasets (except in Tuozzolo et al. [55] with AirSWOT data). Moreover very few studies address
110 the modeling of effective 1D channels from real satellite data (e.g. Garambois et al. [27], Schneider et al. [52]).

111 The present paper investigates the effective hydraulic modeling of braided river flows from real multi-sensor
112 satellite observations of WS, the challenging inference of the hydraulic triplet $(Q(t), b(x), K(x, h))$ and its sensitivity
113 to observation density in space. Multichannel rivers are characterized by complex hydraulic geometries relationships
114 across flow regimes as shown in Schubert et al. [53] through an analysis of a metric resolution 2D shallow water
115 model of a braided portion of the Platte River, US. The key point is to build up a sufficiently complex model to
116 describe multichannel river flows and in coherence with satellite altimetry measurements spatio-temporal scales.
117 Based on the inverse method presented in Larnier et al. [42], Brisset et al. [10], an effective hydraulic modeling
118 strategy is adapted for tackling multichannel river flows using: (i) effective 1D cross sections based on real multi-
119 satellite data from low to high flows (ii) a spatially distributed friction law depending on modeled water depth
120 h . The inference of distributed hydraulic parameters patterns is investigated on a 71km long reach of the Xingu
121 River (Amazone basin) from real altimetric observations along a single ENVISAT track or from synthetic SWOT
122 observations, low *identifiability index* (as introduced in 10 and detailed in section 4). The influence of the spatial
123 density of WS observations on the identifiability of spatial controls patterns (in the triplet) is studied. A piecewise
124 linear bathymetry representation is introduced along with a friction power law with piecewise constant parameters
125 to put in coherence the observations and the flow model grids. Their constraining effect on the inversions is studied
126 with spatially sparse observations. Furthermore, numerical investigations are performed to test the sensitivity of
127 hydraulic inferences to prior hydraulic values and also assess the correlated influence of bathymetry and friction on
128 the modeled flow lines (equifinality) across flow regimes.

129 This study is organized as follows. Section 2 presents the 1D Saint-Venant flow model and the effective modeling
130 approach for multichannel rivers including: (i) a spatially distributed friction law depending on modeled flow depth,
131 (ii) the construction of an effective channel geometry from multi-satellite observations, (iii) an inverse method based
132 on variational data assimilation. Section 3 focuses on the calibration of the effective model on 8 years of WS
133 observations gained from ENVISAT altimeter on a single track along this braided river. Using this model as a
134 reference, section 4 proposes detailed investigations of hydraulic inferences from real ENVISAT or synthetic SWOT

135 observations considering this braided river as ungauged. Section 5 presents numerical sensitivity analysis to the
 136 hydraulic prior and investigations on the bathymetry friction equifinality.

137 2. Effective hydraulic modeling approach:

138 This section proposes an original 1D modeling approach of adequate complexity for modeling multichannel river
 139 flows across regimes and in coherence with satellite observations. The approach is built on an effective channel
 140 cross section derived from multi-satellite measurements and a spatially distributed friction law depending on the
 141 flow depth.

142 2.1. The flow model

143 River flow is classically modeled using the 1D Saint-Venant shallow water equations involving an integration
 144 of the flow variables over the cross section (see e.g. Chow [15], Guinot [33] for detailed assumptions). In their
 145 non-conservative form in (A, Q) variables, A the wetted-cross section $[\text{m}^2]$, Q the discharge $[\text{m}^3.\text{s}^{-1}]$, the equations
 146 read as follows [15]:

$$147 \begin{cases} \partial_t(A) + \partial_x(Q) & = 0 \\ \partial_t Q + \partial_x \left(\frac{Q^2}{A} \right) & = -gA \partial_x Z - gAS_f \end{cases} \quad (1)$$

148 where g is the gravity magnitude $[\text{m}.\text{s}^{-2}]$, Z is the WS elevation $[\text{m}]$, $Z = (b+h)$ with b is the river bottom elevation
 149 $[\text{m}]$ and h is the water depth $[\text{m}]$. The friction term S_f is parameterized with the classical Manning-Strickler law
 150 such that $S_f = |Q|Q/K^2 A^2 R_h^{4/3}$ with K the Strickler friction coefficient $[\text{m}^{1/3}.\text{s}^{-1}]$, $R_h = A/P_h$ the hydraulic radius
 151 $[\text{m}]$, P_h the wetted perimeter. The discharge Q is related to the average cross-sectional velocity u $[\text{m}.\text{s}^{-1}]$ such as
 152 $Q = uA$. A spatially distributed Strickler friction coefficient is defined as a power law in the water depth h :

$$153 K(x, h(x, t)) = \alpha(x)h(x, t)^{\beta(x)} \quad (2)$$

154 where α and β are two constants. Similar approaches based on hydraulic geometry or power law resistance equations
 155 are developed in the literature for predicting mean flow velocity for example on a wide range of in situ river flow
 156 measurements in Bjerklie et al. [8] or else for gravel bed streams in Ferguson [24]. The friction depends on the flow
 157 depth through the proposed power law relation (2) enabling a variation of friction effect in function of flow regime
 158 for complex flow zones for instance; this spatially distributed friction law is richer than a constant uniform value as
 159 it is often set in the literature from a-priori table of frictions in function of river types for instance (e.g. [14]).

160 The discharge $Q_{in}(t)$ is classically imposed upstream of the river channel. At downstream the Manning-Strickler
 161 equation depending on the unknowns $(A, Q; K)_{out}$ is imposed (it is classically integrated in the Preissmann scheme
 162 equations). The initial condition is set as the steady state backwater curve profile $Z_0(x) = Z(Q_{in}(t_0))$. This 1D

163 Saint-Venant model (1) is discretized using the classical implicit Preissmann scheme (see e.g. 16) on a regular grid
164 of spacing Δx . It is implemented into the computational software DassFlow DassFlow.

165

166 2.2. Effective braided river model from long altimetric time series, satellite images and a hydrological model

167 A $L = 71\text{km}$ long portion of the Rio Xingu containing braided reaches is considered (figure 1, cf. Garambois
168 et al. [27]). WS observations are available at 6 virtual stations along a single ENVISAT track (#263) representing
169 77 samples of WS profiles between mid 2002 and mid 2010 (cf. Da Silva et al. [17]); that is $\{Z_{s,p}^{obs}\}_{S,P}^{env}$ with $S = 6$
170 corresponding to the locations of the virtual stations simultaneously observed at $P = 77$ times (see table 1).

171 An effective hydraulic modeling strategy of this braided river is proposed based on:

- 172 • Cross-sectional water surface widths $\{W\}_{S,2}^{jers}$ obtained from JERS mosaics (Courtesy of GRFM, NASDA/MITI)
173 in low and high flows. The effective water surface width is the sum of the width of all individual river channels
174 for braided reaches.
- 175 • An a priori river bottom $\{b\}_{r_{VS}}$ obtained from altimetric rating curves from Paris et al. [49]. They are
176 determined by adjusting the parameters of a classical stage discharge relationship on WS elevations gained
177 by satellite altimetry and discharge simulated with the large scale hydrological model MGB (de Paiva et al.
178 [18]) on the temporal window of interest - called true discharge in what follows.

179 Effective cross-sections geometries are defined at the 6 virtual stations with the bathymetry b given by altimetric
180 rating curves and from effective widths such that low flow width (resp. high flow) is reached for the first (res.
181 ninth) decile of observed WS elevations for each cross section. The final model geometry is obtained by linear
182 interpolation between these 6 effective cross sections on the model grid with $\Delta x = 50\text{m}$. It is shown in Fig. 1
183 along with ENVISAT and SWOT spatial samplings. The friction law 2 introduced above and depending on the flow
184 depth h is distributed using patches with constant values for each reach between two successive virtual stations.

185 2.3. The computational inverse method

186 This paper investigates the estimation of the hydraulic triplet $(Q(t), b(x), K(x, h))$ from observations of WS
187 variabilities only on a braided river. The employed inverse method is those presented in Larnier et al. [42] (see also
188 Brisset et al. [10]) with an augmented composite control vector c ; it is detailed in Appendix 7. c contains a spatially
189 distributed friction coefficient enabling to model complex flow zones (while it is an uniform friction law $K(h)$ in
190 Larnier et al. [42]). This definition of $K(x, h)$ enables to consider more heterogeneous bathymetry controls.

191 The principle is to estimate (discrete) flow controls minimizing the discrepancy between Z_{obs} the observed flow
192 line and Z the modeled one; the latter depending on the unknown parameters vector c through the hydrodynamic
193 model (1). This discrepancy is quantified through the cost function term $j_{obs}(c) = \frac{1}{2} \|Z_{obs} - Z(c)\|_2^2$, see Appendix

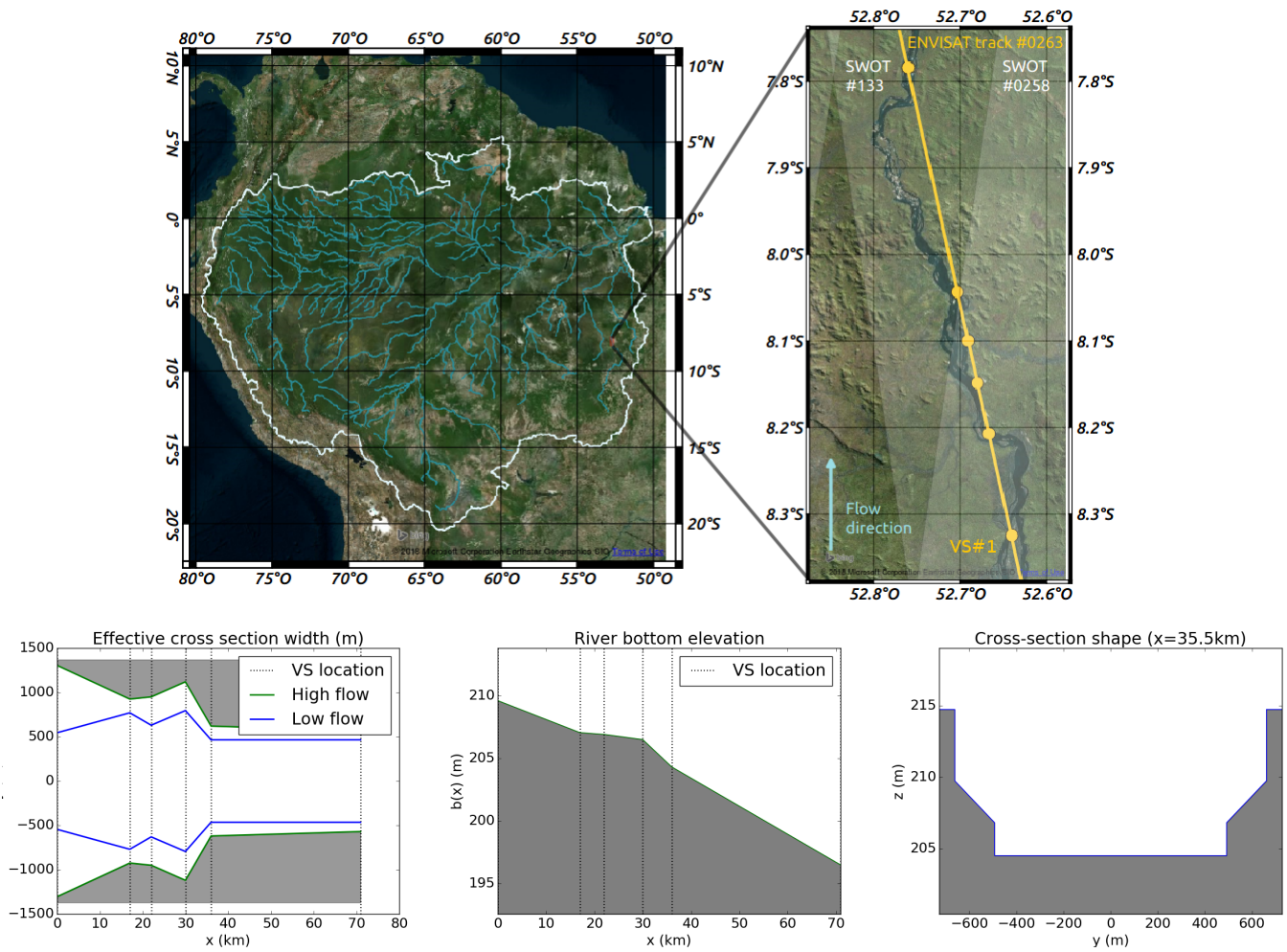


Figure 1: Study zone (top) with ENVISAT track #263 and virtual stations (orange dots); simulated SWOT tracks #133 and #258 on the 1st and 6th day every 21 days repeat cycle (transparent white). Effective river bathymetry derived from altimetric rating curves (Paris et al. [49]) and water surface width from satellite images.

194 7 for details. The control vector c contains the unknown “input parameters” of the 1D Saint-Venant shallow water
 195 flow model (eq. 1) considering effective cross sections (see figure 1). In the present study, c reads as:

$$196 \quad c = (Q_{in,0}, \dots, Q_{in,P}; b_1, \dots, b_R; \alpha_1, \dots, \alpha_N, \beta_1, \dots, \beta_N)^T \quad (3)$$

197 where temporally and spatially distributed controls are the upstream discharge $Q_{in,p}$, the river bed elevation b_r and
 198 the distributed friction parameters α_n and β_n .

199 The subscript p denotes the observation time $p \in [0..P]$ and r denotes the reach number, $r \in [1..R]$.

200 α_n and β_n are the parameters of the friction law depending on the model state h (2) for each patch $n \in [1..N]$
 201 with $N \leq R$.

202 The inversion consists to solve the following minimization problem: $c^* = \text{argmin } j(c)$ (eq. 8).

203 This minimization, optimization problem is solved using a first order gradient-based algorithm, more precisely
 204 the classical L-BFGS quasi-Newton algorithm.

205 3. Calibration of the effective hydraulic model on historical satellite altimetry

206 This section presents the calibration of the effective hydraulic model based on the reference effective geometry
 207 defined above (cf. section 2.2). The observed water elevation time series $\{Z_{s,p}^{obs}\}_{S,P}^{env}$ at $S = 5$ ENVISAT virtual
 208 stations are used to calibrate the friction law of the 1D Saint-Venant flow model (1). Since friction has a local
 209 and upstream influence on the flow line (low Froude fluvial flows, figure 9) the remaining ENVISAT time series at
 210 VS#6 downstream of the river domain will be used for inferring the full control vector c in next section - recall that
 211 a normal depth is used as downstream BC (cf. section 2.1).

212 A “reduced” control vector $c_{cal} = (\alpha_1, \dots, \alpha_N, \beta_1, \dots, \beta_N)$ consisting in spatially distributed friction parameters
 213 only is considered here. In order to avoid a spatial “overparameterization” regarding the 5 water height timeseries
 214 available at VS, the choice is made to spatialize friction on $N = 5$ patches, on each reach downstream an altimetric
 215 VS. The inverse method presented in Larnier et al. [42] and described in appendix (section 7) is used here with no
 216 regularization nor variable change for this “simple” calibration problem.

217 An optimal friction distribution c_{cal}^* is found with the inverse method and the calibrated values of $\alpha_{n=1..5}$ and
 218 $\beta_{n=1..5}$ are summed up in table 1. The resulting water height time series are compared to altimetric observations
 219 for each virtual station (cf. figure 2). The spatially distributed friction law 2 enables a fairly good reproduction of
 220 the observed water level variations on this braided river, across a wide range of flow regimes, even with an effective
 221 1D model built on multi-satellite data (fig. 2).

222 A constant friction in time would lead to systematical errors for a large range of flows as shown by grey curves
 223 on figure 2. The calibrated friction exponents β_n range between 0.482 and 1.133 except for the second reach (SV2-3)
 224 where a small β_n is found, that is a barely constant friction across flow regimes for this small reach (cf. fig. 2). The

Virtual station name	VS#1	VS#2	VS#3	VS#4	VS#5	VS#6
Flow distance to mouth [km]	1146	1129	1124	1116	1110	1075
Flow distance from the upstream [km]	0	17	22	30	36	71
Drainage area [km ²] (MGB model)	193.255	193.255	194.148	194.148	195.882	197.862
Z_0 [m] (reference : EGM2008) (Paris et al. 2016)	209.6	207.1	206.9	206.5	204.3	196.5
$W_{low}(x)$ Total low flow width [m] (derived from JERS)	1090	1540	1260	1590	930	930
$W_{high}(x)$ Total high flow width [m] (derived from JERS)	2610	1850	1900	2240	1240	1140
Calibrated friction factor $\alpha^{cal}(x)$ (downstream reach)	12.785	19.574	9.869	4.252	7.425	-
Calibrated friction exponent $\beta^{cal}(x)$ (downstream reach)	0.482	0.071	0.624	1.133	0.718	-

Table 1: Summary of the effective hydraulic model parameters including calibrated friction parameters $\alpha^{cal}(x)$ and $\beta^{cal}(x)$ (recall $K(x, h) = \alpha(x)h^{\beta(x)}$) using 8 years of WS elevation variations (ENVISAT data) given effective channel bathymetry and upstream discharge from the MGB hydrological model (de Paiva et al. [18]).

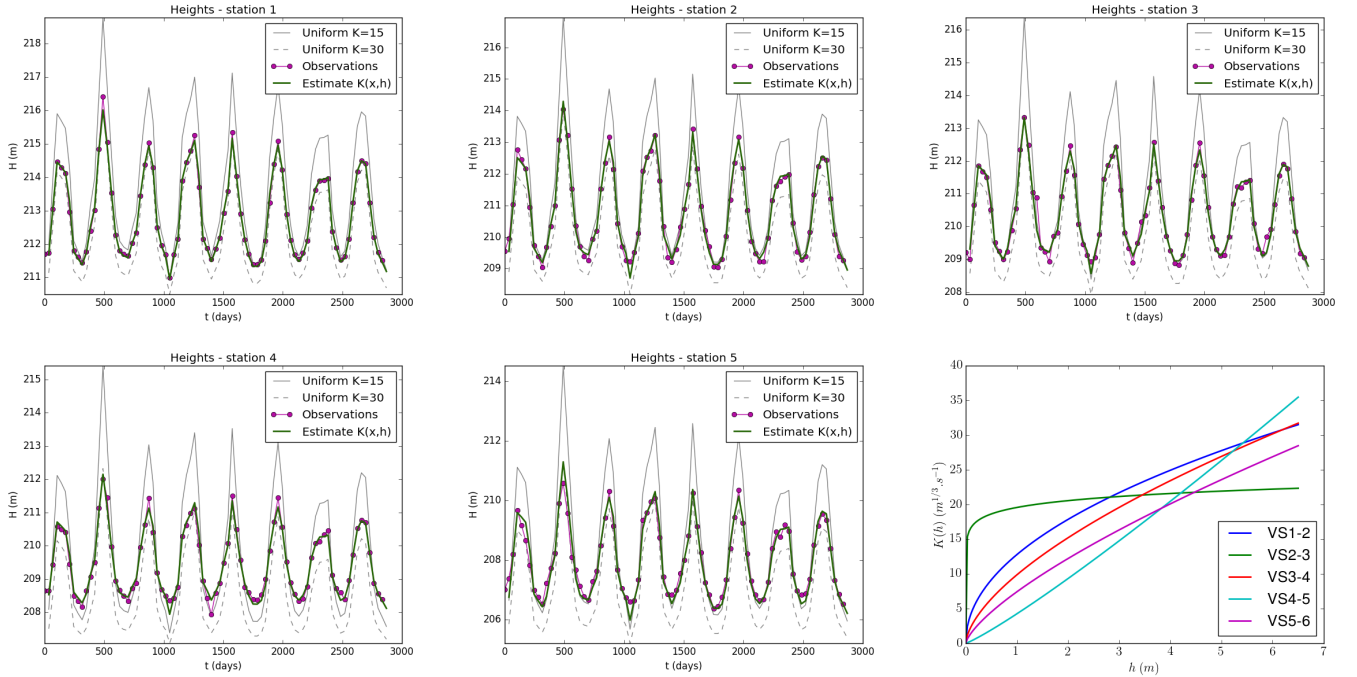


Figure 2: Calibration of variable friction $K(x, h)$ with 8 years of ENVISAT measurements at 6 VS using the variational method with $c = (\alpha_1, \dots, \alpha_5, \beta_1, \dots, \beta_5)$; $j_{obs} = 0.07$. (Bottom right) Effective friction law in function of water depth for each VS.

225 spatial pattern of α_n values calibrated here correspond to significant friction effects, varying across flow regimes,
226 and necessary to effectively represent braided reaches using a 1D effective cross section. Indeed the latest leads
227 to an underestimation of the hydraulic radius $R_h = A/P_h$ hence of the friction term $S_f = |Q|Q/\kappa^2 A^2 R_h^{4/3}$ in the 1D
228 Saint-Venant model (see section 2.1) for braided reaches.

229 4. Investigations on the inference from WS observations of distributed flow controls on braided river 230 flows

231 This section studies the challenging inference of the hydraulic triplet (discharge, bathymetry, friction) from multi-
232 satellite WS observations. The braided Xingu River morphology represents a supplementary difficulty for inversions
233 regarding the variability of local hydraulic behaviors across flow regimes as evidenced above by the calibrated

234 friction laws ($\beta^{cal} \neq 0$). The impact of spatial controls density and bathymetry representation is assessed in what
 235 follows regarding the spatial sparsity of observations. First is presented the numerical experiment framework, then
 236 the inferences with relatively “sparse” ENVISAT measurements and finally those with SWOT synthetic observations.

237 4.1. Inverse hydraulic modeling method with WS elevations gained from nadir altimetry and SWOT

238 The effective hydraulic model described in section 2.2 and calibrated in section 3 is used as a reference (“target”)
 239 in the following numerical experiments. The control vector (eq. 3) containing discharge, bathymetry and friction
 240 is sought with the inverse method described in section 2.3 (see also appendix, section 7). It is tested first with real
 241 ENVISAT time series representing a relatively sparse spatial sampling of WS signatures with 6 VS on this 71km
 242 long river, and next with synthetic SWOT observations sampling the flow line at $\Delta x = 200m$ (RiverObs product,
 243 see Frasson et al. [26]).

244 The Xingu River is observed either by a single along-stream ENVISAT track at 6 observation points (virtual
 245 stations) of flow lines every 35 days, or two SWOT tracks providing dense WS observations in space twice per
 246 21 days repeat cycle (5 days delay, cf. section 2.2). Note that the temporal sparsity of observations (35 days
 247 for ENVISAT or 5 days between the two SWOT passes every 21 days) only enables to identify low hydrograph
 248 frequencies, at observation times (see Brisset et al. [10] for a detailed analysis and identifiability maps). Indeed the
 249 hydraulic wave propagation time is around $T_{wave} \sim 9h$ which is much smaller than the lowest satellite revisit time
 250 of 5 days. This propagation time is calculated using the kinematic wave velocity for rectangular channels $c_k = 5/3U$
 251 and maximal high flow velocity $U = 2,17m/s$ from calibrated model outputs $c_k = 2.2m/s$ (second hydrograph peak
 252 at $t = 490 days$, see flow variables on figure 9). Let $I_{indent} = T_{wave}/\Delta t_{obs}$ be the identifiability index defined in
 253 Brisset et al. [10] as the ratio between flood wave propagation time and observation time step. This leads to a
 254 very low temporal identifiability index for this 71km river: $I_{indent} = 7.5 \times 10^{-2}$ for SWOT and $I_{indent} = 10^{-2}$ for
 255 ENVISAT. Consequently, only low temporal dynamics and discharge at observation times are inferable as shown in
 256 Brisset et al. [10]; SWOT and ENVISAT observations are thus considered separately in the present study.

257 The starting point of the VDA process in the parameter space, the so-called prior c_{prior} (cf. section 7), consists
 258 in a rough hydrological prior: $Q^{(0)} = \bar{Q}_{MGB}$ the mean discharge estimated from the MGB hydrological model, a
 259 spatially constant $\alpha^{(0)}$ friction defined a priori from classical hydraulic ranges (e.g. Chow [14]) and $\beta^{(0)} = 1$, the
 260 bathymetry $b^{(0)}$ is defined as a simple straight line over the whole domain for hydraulic analysis first. Note that
 261 the sensitivity of the inference to the prior definition is investigated in section 5.

262 In a noised observation context, we denote by δ the noise level such that $\|Z_{obs} - Z_{true}\|^2 \leq \delta$ for all spatial
 263 locations r with Z_r^{obs} the observed and Z_r^{true} the true WS elevation. A common technique to avoid overfitting noisy
 264 data, in the context of Tykhonov’s regularization of ill-posed problems, is Morozov’s discrepancy principle, (see e.g.
 265 Kaltenbacher et al. 40 and references therein): the regularization parameter γ (see eq. 6) is chosen *a-posteriori* such
 266 that j does not decrease below the noise level. In the present numerical experiments, the convergence is stopped if
 267 $j_{obs}(c) \leq 10^{-1}$ or if j_{obs} is not decreased anymore for higher discrepancies.

268 4.2. Inference of distributed hydraulic controls ($Q(t), K(x, h), b(x)$) with spatially sparse WS observations: real
 269 ENVISAT altimetric snapshots

270 In this section the assimilation is based on WS elevations $\{Z_{s,p}^{env}\}_{S,P}$ at $S = 6$ virtual stations observed simul-
 271 taneously by ENVISAT during 8 years every 35 days, i.e. $P = 77$. In this spatially sparse observation context, the
 272 impact of spatial controls density is investigated.

273 First, we consider a “full” control vector c (cf. eq. 3) including $P = 77$ inflow discharges, all 1D model
 274 bathymetry points $R = 1420$ and $N = 5$ friction patches between ENVISAT virtual stations (cf. section 2.2). The
 275 inferred inflow discharge, bathymetry and friction are presented in figure (3) (case Env.a). Despite the satisfying
 276 value of the hydraulic controls reached at iteration 35, the descent is still possible as shown by j_{obs} decreasing of
 277 about 20% at iteration 96. Although it enables to fit the observations according to the a priori convergence criteria
 278 defined in section 4.1, the solution found after the VDA process is not very accurate nor realistic as shown by peak
 279 flow underestimations and significant oscillations of the identified friction and bathymetry. The spatial sparsity of
 280 observations prevents to infer these relatively dense bathymetry controls; in this case the considered inverse problem
 281 is underconstrained.

282 In order to better constrain the inverse problem in case of sparse spatial observability, a bathymetry represen-
 283 tation is consistently introduced at the scale of the observation grid and applied to the finer flow modeling grid.
 284 Based on the physical analysis of the SW model (1) behaviour and the WS signature of bathymetry/friction con-
 285 trols (see Montazem et al. [47], Montazem et al. [46], Montazem [45]), a linear bathymetry interpolation is used
 286 between successive couples of bathymetry controls defined at observation points only. The resulting bathymetry
 287 $\tilde{b}(x) \in \mathcal{C}^0(\mathbb{R}), \forall x \in [0, L]$ is piecewise linear and strongly constrains the bathymetry profile between the sought
 288 bathymetry points - instead of using only a weak constrains $j_{reg}(c) = \frac{1}{2} \|b''(x)\|_2^2$ in the optimization process (cf.
 289 appendix 7) as done in the next section 4.3 with spatially dense SWOT observations. Using this bathymetry con-
 290 strain with $R = 6$ bathymetry controls defined at each ENVISAT virtual station results in 5 reaches and $N = 5$
 291 friction patches are consistently applied to each. This leads to a more robust and accurate inference as shown in
 292 Figure 4 (case Env.b). The discharge inferred for 8 years is fairly correct (RMSE = 520 m³/s, Nash = 0.95) and rel-
 293 atively realistic bathymetry/friction patterns are found, with some compensations between spatial controls locally
 294 in space, which is further analyzed in what follows.

295 The impact on the inferred parameters of searching a spatially uniform friction law is tested with the piecewise
 296 linear bathymetry representation used above. The resulting discharge inference is fairly correct (RMSE = 608 m³/s,
 297 Nash = 0.93) and interestingly the bathymetry spatial pattern is well retrieved but shifted above the reference one
 298 (cf. figure 5) (case Env.c). The inferred friction coefficients are $\alpha = 22.621$, $\beta = 0.217$, which represents a lower
 299 friction effect on most flow regimes regarding the calibrated ones (cf. table 1). This inferred effective friction law and
 300 bathymetry pattern, leading to somehow effective stage-discharge relationships locally given the inferred hydrograph,
 301 enable to approximate the observed WS variations ($j_{obs} = 1.269$) but with a less accurate fit than with spatially

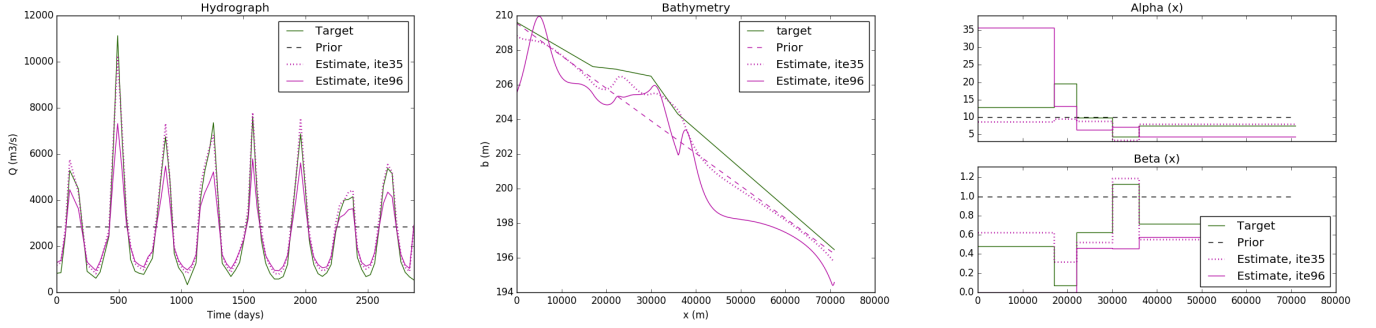


Figure 3: Identification of $(Q(t), K(x, h), b(x))$ with ENVISAT observations and overparameterized $c = (Q_{in,0}, \dots, Q_{in,P}; b_1, \dots, b_R; \alpha_1, \dots, \alpha_N, \beta_1, \dots, \beta_N)^T$ with $P = 77$, $R = 1420$, $N = 5$, bathymetry regularization weight $\gamma = 10^{-3}$; $j_{obs} = 0.098$ at iteration 35 (top) and $j_{obs} = 0.077$ at iteration 96 (bottom) (Env.a)

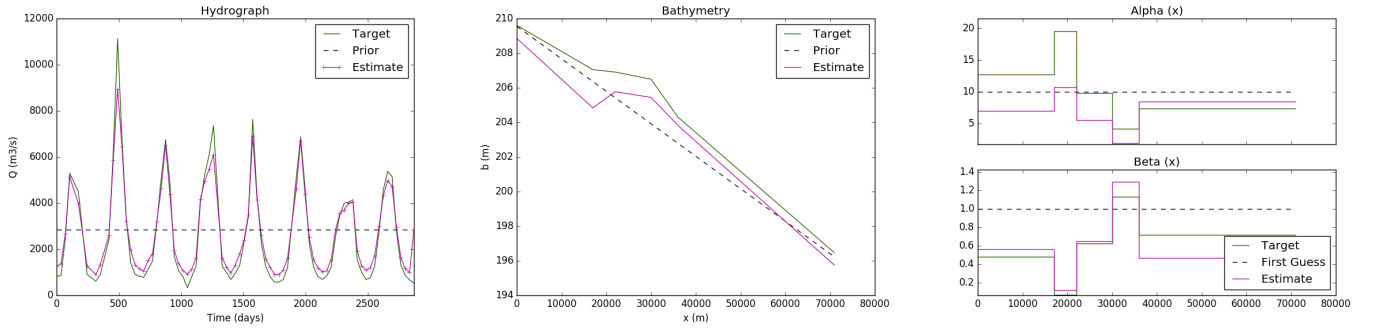


Figure 4: Identification of $(Q(t), K(x, h), b(x))$ with ENVISAT observations and effective $c = (Q_{in,0}, \dots, Q_{in,P}; b_1, \dots, b_R; \alpha_1, \dots, \alpha_N, \beta_1, \dots, \beta_N)^T$ with $P = 77$, $R = 6$, $N = 5$ with a piecewise linear bathymetry $b(x)$ reconstruction, $\gamma = 0$; $j_{obs} = 0.118$ at iteration 51. (Env.b)

302 distributed friction ($j_{obs} = 0.118$). Note in that case of a lower model complexity an underestimation of the low
 303 flow discharges.

304 These inferred friction laws and bathymetry patterns - simultaneously inferred with the discharge hydrograph -
 305 correspond to “effective rivers” enabling to fit the observed variability of flow lines. Recall that the observations
 306 consist in real measurements of WS elevations gained by nadir altimetry on multichannel reaches of the Xingu River.
 307 The complexity of the forward-inverse modeling approach, in coherence with the spatial sparsity of observation grid,
 308 enables to approximate satisfactorily the one of the observed multichannel flow. The additional constrain provided
 309 by spatially dense flow lines observations is investigated in the next section with SWOT synthetic data.

310

311

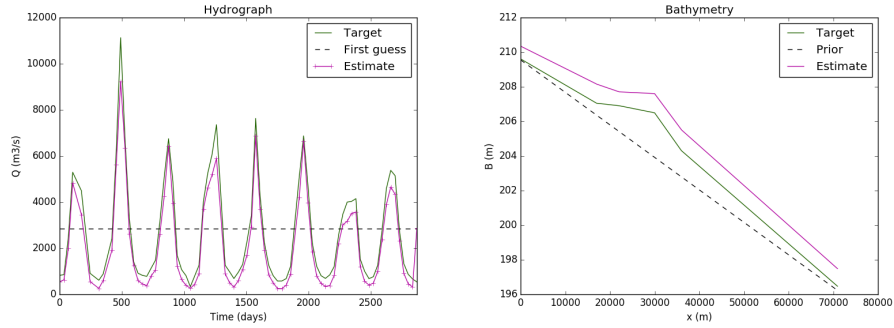


Figure 5: Inference of $Q(t)$, $b(x)$ and spatially uniform $K(h) = \alpha h^\beta$ with ENVISAT WS observations and effective $c = (Q_{in,0}, \dots, Q_{in,P}; b_1, \dots, b_R; \alpha, \beta)^T$, $P = 77$, $R = 6$, no bathy $\gamma = 0$; $j_{obs} = 1.269$ at iteration 54. The identified friction coefficients are $\alpha = 22.621$, $\beta = 0.217$. (Env.c)

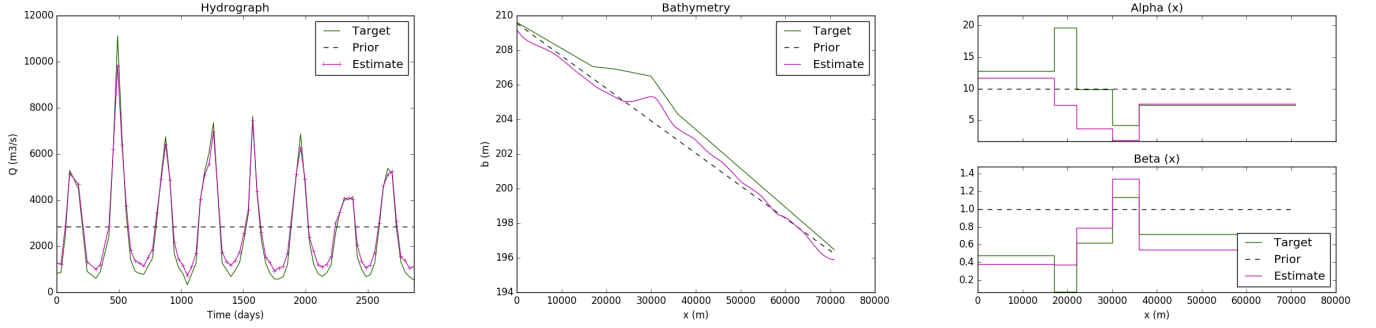


Figure 6: Identification of $(Q(t), K(x, h(x, t)), b(x))$ with SWOT-sge observations and effective $c = (Q_{in,0}, \dots, Q_{in,P}; b_1, \dots, b_R; \alpha_1, \dots, \alpha_N, \beta_1, \dots, \beta_N)^T$ with $P = 276$, $R = 1420$, $N = 1419$, $\gamma = 10^{-3}$; $j_{obs} = 0.099$ at iteration 41. (SWOT.a)

312 *4.3. Inference of distributed hydraulic controls $(Q(t), K(x, h), b(x))$ with spatially dense WS observations: SWOT*
 313 *synthetic observations*

314

315 In this section the full hydraulic control c (cf. eq. 3) is inferred by assimilating SWOT-like observations. Those
 316 noisy data are computed using the SWOT hydrology simulator applied to flow lines from the effective hydraulic
 317 model calibrated above (cf. section 3). The SWOT spatio-temporal pattern over the studied river is obtained by
 318 overlapping the river centerline and the expected SWOT orbit and swaths (cf. figure 1). Finally the synthetic
 319 SWOT-like observables consist in WS elevations $\{Z_{obs}^{SWOT}\}_{r,p}$ with $p \in [1..P]$ and $P = 276$ generated on the fine
 320 scale model grid i.e. $r \in [1..1420]$.

321 The inflow discharge, bathymetry and friction are inferred by assimilating SWOT WS observations $\{Z_{obs}^{SWOT}\}_{r,p}$
 322 on the same spatial grid as that of the numerical hydraulic model with c_{prior1} . The estimates are presented on figure
 323 (6). The inferred discharge hydrograph is accurate (RMSE = 391 m³/s, Nash = 0.97) and bathymetry/friction pat-
 324 terns are relatively well retrieved. Using SWOT spatially distributed observations and piecewise constant roughness
 325 enable to constrain the inference of bathymetry controls at a fine spatial resolution (model grid); the inverse method
 326 including covariance matrices acting as spatial or temporal smoothers/regularizations (cf. eq. 11 in appendix). The
 327 inferred discharge and the spatially distributed controls are slightly more accurate than previously in a comparable
 328 inversion scenario with sparse ENVISAT observations in space and piecewise linear bathymetry constrain (case
 329 Env.b, cf. table 2 and figure 4). Note that the friction is sought by reaches which enables to consider more dense
 330 bathymetry controls. Again, the compensation between spatial controls appears locally in space but enables the
 331 best fit to distributed measurements of WS elevations given the inferred discharge ($j_{obs} = 0.099$).

332

333

334

335 5. Numerical investigation of the bathymetry-friction equifinality

336 The hydrograph is responsible for flow variability in time, hence enabling to retrieve the temporal dynamics of
337 the observed flow lines (Brisset et al. [10], Larnier et al. [42]). The friction and bathymetry controls have a correlated
338 influence on the modeled flow lines therefore leading to an ill-posed inverse problem (cf. Garambois and Monnier
339 [28], Larnier et al. [42] for investigations on this “*bathymetry-friction equifinality*” in a comparable data-inversion
340 context). In this section the influence of the prior value on the quality of the inferences with spatially distributed
341 controls is investigated. Next, is proposed a numerical analysis of the sensitivity of the friction source term S_f in
342 the Saint-Venant equations (1) to the flow controls (triplet) that are embeded in it.

343 5.1. Sensitivity to the prior of the hydraulic inference from altimetric observations of WS signature

344 Given altimetric measurements of WS variabilities and the first guess c_{prior1} , the inverse method enables to
345 infer a complex control vector composed of temporally and spatially distributed controls of the 1D SW model (1).
346 In the numerical experiments above, the discharge hydrograph $Q(t)$ is accurately infered at observation times but
347 because of the ill-posedness of the inverse problem, compensations can occur between the sought parameters and
348 especially between the spatial controls - the bathymetry $b(x)$ and the distributed friction parameters $\alpha(x)$ and
349 $\beta(x)$. As already pointed out in the VDA inferences performed with the DassFlow model using SWOT like data
350 in (Brisset et al. [10], Larnier et al. [42]) and AirSWOT data (Tuozzolo et al. [55]), the accuracy of the inferred
351 discharge depends on the quality of the prior.

352 The sensitivity of the inference to the quality of the prior control vector is investigated here for the most
353 challenging inverse problem with spatially distributed controls and sparse ENVISAT data. First the inflow prior is
354 varied of $\pm 30\%$ around the mean true discharge; the river bottom elevation and friction priors are set as previously
355 in c_{prior1} . The infered hydraulic controls are presented in 7 and various inference scores are sumed up in table 2.
356 For each inflow prior, the temporal variations of the inflow hydrograph are very well retrieved as shown on figure
357 7 - runs Env.b2 and Env.b3. However a biased inflow prior results in a biased hydrograph estimate (with correct
358 temporal variations) which is coherent with results of Larnier et al. [42], Tuozzolo et al. [55]).

359 Next, the sensitivity to the prior bathymetry and friction is tested. The prior bathymetry is infered with the
360 low-complexity system proposed in the hierarchichal HiVDI model chain (Larnier et al. [42]) for ungauged rivers.
361 It consists in estimating an effective prior bathymetry from WS observables using the low Froude model and prior
362 discharge from a hydrological model ($\overline{Q_{MGB}}$ here) and prior friction ($\alpha^{(0)}, \beta^{(0)}$). Two prior c_{man1} and c_{man2} are
363 considered with prior friction under/over-estimations compared to calibrated ones (cf. 8). As shown on figure 8,
364 the inference in case Env.b31 (blue) results in an accurate estimation of discharge, very similar to Env.b (purple).
365 It is started from a prior c_{man1} that underestimates river bottom elevation and overestimates the spatially averaged
366 friction effect compared to calibrated values (cf. figure 8, bottom). In that case, fitting WS elevations enables
367 to infer an effective river channel (bathymetry and friction) but also to infer a fairly realistic upstream temporal

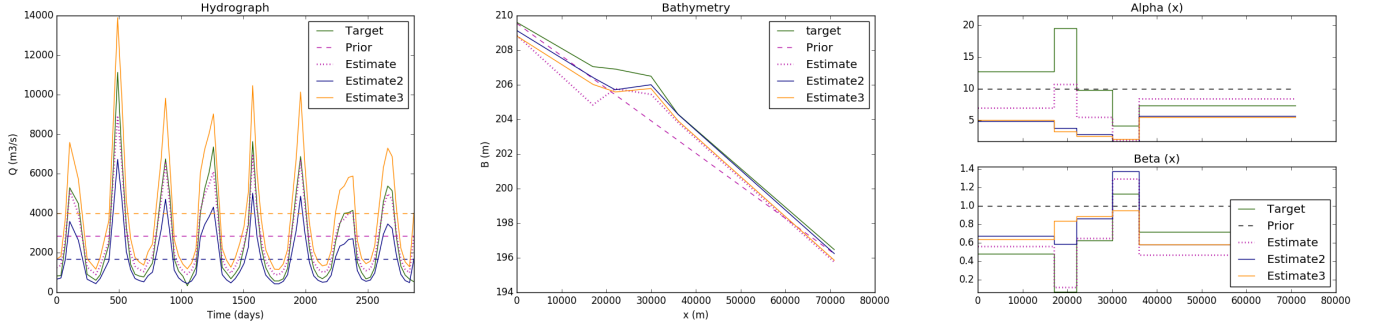


Figure 7: Sensitivity test to prior discharge $\overline{Q_{MGB}} \pm 30\%$; identification (var change) of $(Q(t), K(x, h), b(x))$ with ENVISAT observations $c = (Q_{in,0}, \dots, Q_{in,P}; b_1, \dots, b_R; \alpha_1, \dots, \alpha_S, \beta_1, \dots, \beta_S)^T$ with $P = 77$, $R = 6$, $N = 5$ and with a piecewise linear $b(x)$ and $S = R = 5$. “Estimate” (case Env.b) $j_{obs} = 0.118$ at iteration 51, “Estimate2” (case Env.b21) $j_{obs} = 0.125$ at iteration 41, “Estimate3” (case Env.b21) $j_{obs} = 0.125$ at iteration 25.

368 control (discharge hydrograph). Using the prior c_{man2} that overestimates both river bottom elevation and spatially
 369 averaged friction effect results in a comparable fit to the observed WS elevations. However this correct fit stems from
 370 the compensation between an inferred effective channel of reduced conveyance capacity (comparable friction effects
 371 but overestimated bed levels) and consequently an inferred hydrograph with underestimated low-flow discharges (in
 372 yellow).

373

374 5.2. Spatio-temporal sensitivity of the friction term

375

376 The considered flow controls $(Q(t), K(x, h), b(x))$ of the 1D Saint-Venant shallow water equations (1) have a
 377 complex non linear influence on the modeled flow line and consequently on the fit to the observed flow lines. The
 378 variation of momentum expressed by the second flow equation is due to a pressure source term $-gA \partial_x Z$ (including
 379 the longitudinal variation of fluid-to-fluid pressure, the longitudinal variation of lateral and bottom wall-to-fluid
 380 pressure) and a dissipation term $-gAS_f$. Discharge and bathymetry appear in the momentum and pressure terms
 381 while all flow controls are embedded in the friction source term S_f . Note that for a locally steady uniform flow
 382 $S_f = -\partial_x Z$ and an infinity of friction and bathymetry values can correspond to a single value of discharge (cf.
 383 Garambois and Monnier [28], Larnier et al. [42]).

384 We propose a simple calculation in order to make appear the sensitivity of the friction term to a change on
 385 controls; let us express the differential of S_f assuming $Q > 0$:

$$\begin{aligned}
 dS_f &= d \left(\frac{1}{K^2} \frac{Q^2}{A^2 R_h^{4/3}} \right) \\
 &= -\frac{2}{K^3} \frac{Q^2}{A^2 R_h^{4/3}} dK - \frac{2}{A^3} \frac{Q^2}{K^2 R_h^{4/3}} dA - \frac{4}{3R_h^{7/3}} \frac{Q^2}{K^2 A^2} dR_h + \frac{1}{K^2} \frac{2Q}{A^2 R_h^{4/3}} dQ
 \end{aligned} \tag{4}$$

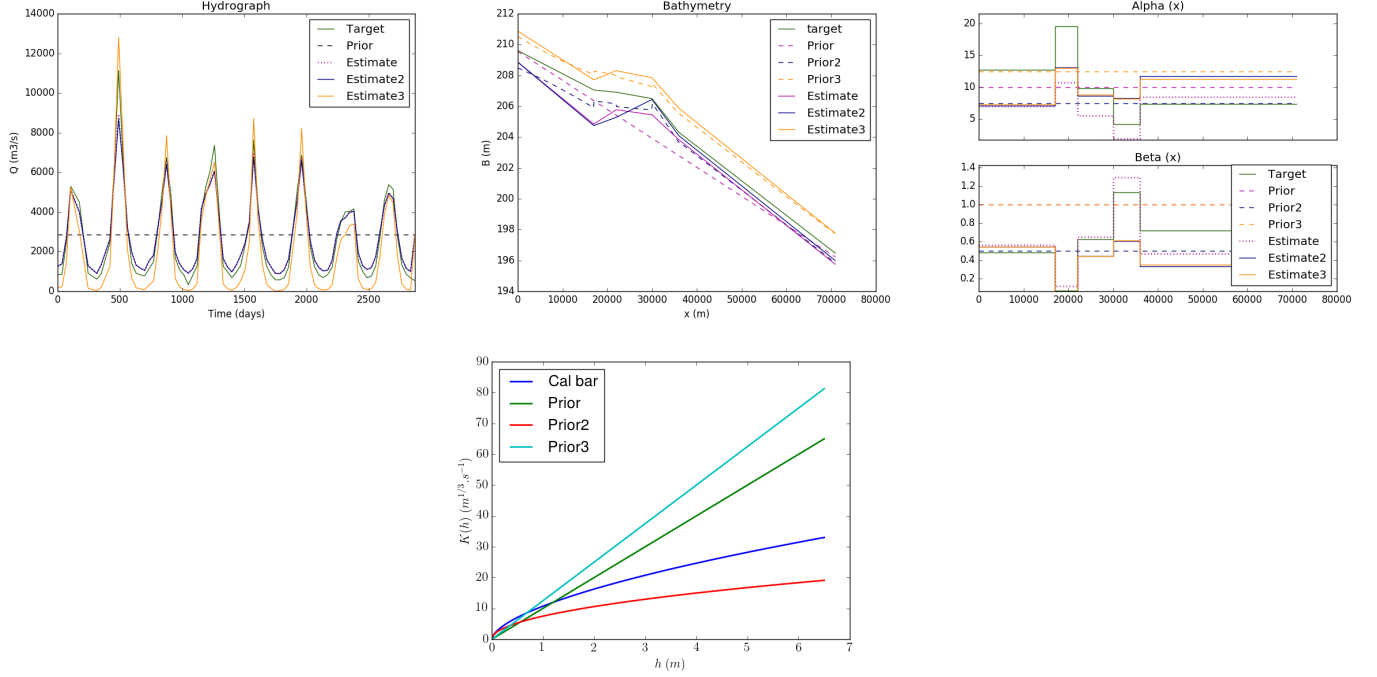


Figure 8: Sensitivity test to prior friction and bathymetry estimated using the “Manning” method from Larnier et al. [42] (c_{man1} ($\alpha^{(0)} = 7.5$; $\beta^{(0)} = 0.5$) and c_{man2} ($\alpha^{(0)} = 12.5$; $\beta^{(0)} = 1$)); identification (var change) of $(Q(t), K(x, h), b(x))$ with ENVISAT observations $c = (Q_{in,0}, \dots, Q_{in,P}; b_1, \dots, b_R; \alpha_1, \dots, \alpha_S, \beta_1, \dots, \beta_S)^T$ with $P = 77$, $R = 6$, $N = 5$ and with a piecewise linear $b(x)$ and $S = R = 5$. “Estimate” (case Env.b) $j_{obs} = 0.118$ at iteration 51, “Estimate2” (case Env.b31) $j_{obs} = 0.116$ at iteration 46, “Estimate3” (case Env.b32) $j_{obs} = 0.122$ at iteration 41. (Bottom) prior effective friction laws and spatially averaged calibrated friction law ($\bar{\alpha}_{cal} = 10.74$ and $\bar{\beta}_{cal} = 0.6$, “Cal bar”).

Case	Control	Prior	RMSE $_{Q^{(0)}}$ (m^3/s)	rRMSE $_{Q^{(0)}}$ (%)	Nash $_{Q^{(0)}}$ (-)	RMSE $_{b^{(0)}}$ (m)	RMSE $_{\alpha^{(0)}}$ ($m^{1/3-\beta}/s$)	RMSE $_{\beta^{(0)}}$ (-)
Env.a	Dense $b(x)$	c_{prior1}	2254	194	-0.01	1.19	4.93	0.49
Env.b	Piec. $b(x)$	c_{prior1}	”	”	”	”	”	”
Env.c	Piec. $b(x), K(h)$	c_{prior1}	”	”	”	”	”	”
SWOT.a	Dense $b(x)$	c_{prior1}	”	”	”	”	”	”
Env.b21	Piec. $b(x)$	$Q_{prior1}^{(0)} - 30\%$	2433	97	0.18	1.19	4.93	0.49
Env.b22	Piec. $b(x)$	$Q_{prior1}^{(0)} + 30\%$	2626	297	-0.37	”	”	”
Env.b31	Piec. $b(x)$	c_{man1} ($\alpha^{(0)} = 7.5$; $\beta^{(0)} = 0.5$)	2254	194	-0.01	0.77	5.63	0.34
Env.d32	Piec. $b(x)$	c_{man2} ($\alpha^{(0)} = 12.5$; $\beta^{(0)} = 1$)	2254	194	-0.01	1.13	5.43	0.49

Case	Control	Prior	RMSE $_Q$ (m^3/s)	rRMSE $_Q$ (%)	Nash $_Q$ (-)	RMSE $_b$ (m)	RMSE $_{\alpha}$ ($m^{1/3-\beta}/s$)	RMSE $_{\beta}$ (-)
Env.a	Dense $b(x)$	c_{prior1}	830	57	0.86	1.97	10	0.46
Env.b	Piec. $b(x)$	c_{prior1}	520	61	0.95	1.07	4.8	0.37
Env.c	Piec. $b(x), K(h)$	c_{prior1}	608	58	0.93	1.05	-	-
SWOT.a	Dense $b(x)$	c_{prior1}	391	38	0.97	0.91	5.67	0.2
Env.b2	Piec. $b(x)$	$Q_{prior1}^{(0)} - 30\%$	1229	39	0.7	0.48	7.83	0.28
Env.b3	Piec. $b(x)$	$Q_{prior1}^{(0)} + 30\%$	1473	104	0.57	0.75	5.09	0.22
Env.bm2	Piec. $b(x)$	c_{man1} ($\alpha^{(0)} = 7.5$; $\beta^{(0)} = 0.5$)	550	61	0.94	1.22	4.64	0.32
Env.bm3	Piec. $b(x)$	c_{man2} ($\alpha^{(0)} = 12.5$; $\beta^{(0)} = 1$)	885	78	0.84	1.30	5.50	0.35

Table 2: Scores of the inferences (bottom) performed with various priors (top), ENVISAT (“Env”) or SWOT (“SWOT”) observations.

386 Since $dR_h = d(A/P) = \frac{1}{P}dA - \frac{A}{P^2}dP = \frac{1}{P}(dA - R_h dP) = \frac{1}{P}(dA_0 - R_h dP_0) + df(h)$ with $A_0 = W_0 h_0$ and
387 $P_0 = W_0 + 2h_0$ respectively the unobserved low flow area and perimeter under our modeling hypothesis (cf. section
388 2.2 and figure 1, see also Larnier et al. [42] for details on cross section representation). It follows that $f(h)$ is a
389 function depending on the modeled water depth h and of the observed cross-section variation δA above low flow
390 (h_0), W_0 being defined from observables. We get $dR_h = \frac{1}{P} \left(1 - \frac{2R_h}{W_0}\right) dA_0 + df(h)$ and finally:

$$dS_f = \frac{1}{K^2} \frac{Q}{A^2 R_h^{4/3}} \left(-2 \frac{Q}{K} dK - \frac{Q}{A} \left\{ 2 + \frac{4}{3} \left(1 - \frac{2R_h}{W_0} \right) \right\} dA_0 + 2dQ \right) - d\phi(h) \quad (5)$$

391 with $\phi(h) = \frac{4}{3R_h^{7/3}} \frac{Q^2}{K^2 A^2} df(h)$ a function depending on the observed geometry of a cross section above low flow and
392 of the simulated flow (A, Q hence $h(A)$ given a channel geometry). We rewrite equation 5 as $dS_f = \partial_K S_f dK +$
393 $\partial_{A_0} S_f dA_0 + \partial_Q S_f dQ - d\phi(h)$ and under our modeling hypothesis we have $\partial_K S_f < 0$, $\partial_{A_0} S_f < 0$, $\partial_Q S_f > 0 \forall x, t$, i.e.
394 opposite effects of local values of friction K , low flow area A_0 and simulated local discharge Q values on S_f . Those
395 terms are plotted on figure 9 along the Xingu River, on model grid, from hydraulic variables simulated (forward
396 run) with calibrated parameters (cf. table 1). Note that $d\phi(h)$ is not studied with this simple method.

397 Interestingly, $|\partial_K S_f|$ is about 100 times greater than $|\partial_{A_0} S_f|$ or $|\partial_Q S_f|$ at high flow and about 10 times greater
398 at low flow. This is consistent with the singular value of friction that is found 1000 times greater than the one of
399 reach averaged discharges by Garambois and Monnier [28] through a singular value decomposition of the normal
400 equations of reach averaged Manning equations - applied to 70km of the Garonne River downstream of Toulouse
401 (France). In other words, the friction term in the present modeling context must be more sensitive to a change in
402 friction than unknown low-flow bathymetry or discharge.

403 Remark that for low-flow, S_f is more sensitive to discharge than unknown cross sectional area ($|\partial_Q S_f| > |\partial_{A_0} S_f|$)
404 and conversely for high-flow. Moreover the spatial variability of the three sensitivities is more pronounced at low flow.
405 Abrupt changes are found at locations corresponding to bottom slope or channel width changes. The influences
406 of the bottom slope break at $x = 30\text{km}$ is clearly visible at low-flow and the influence of the width contraction at
407 $x = 17\text{km}$ at high flow, which is fully consistent with the findings of Montazem et al. [46]. Further investigations
408 on the sensitivity of the full Saint-Venant equations in space and time could be of interest to better tailor and
409 constrain methods for tackling hydraulic inverse problems.

410

411 6. Conclusion

412 This paper investigates the challenging inference of the hydraulic triplet (discharge, bathymetry, friction) from
413 real or synthetic altimetric WS observations only on an ungauged multichannel river.

414 The HiVDI inverse method presented in Larnier et al. [42] is adapted for reproducing a multichannel flow by
415 introducing a *spatially distributed* friction law depending on modeled water depth h and by using multi-satellite

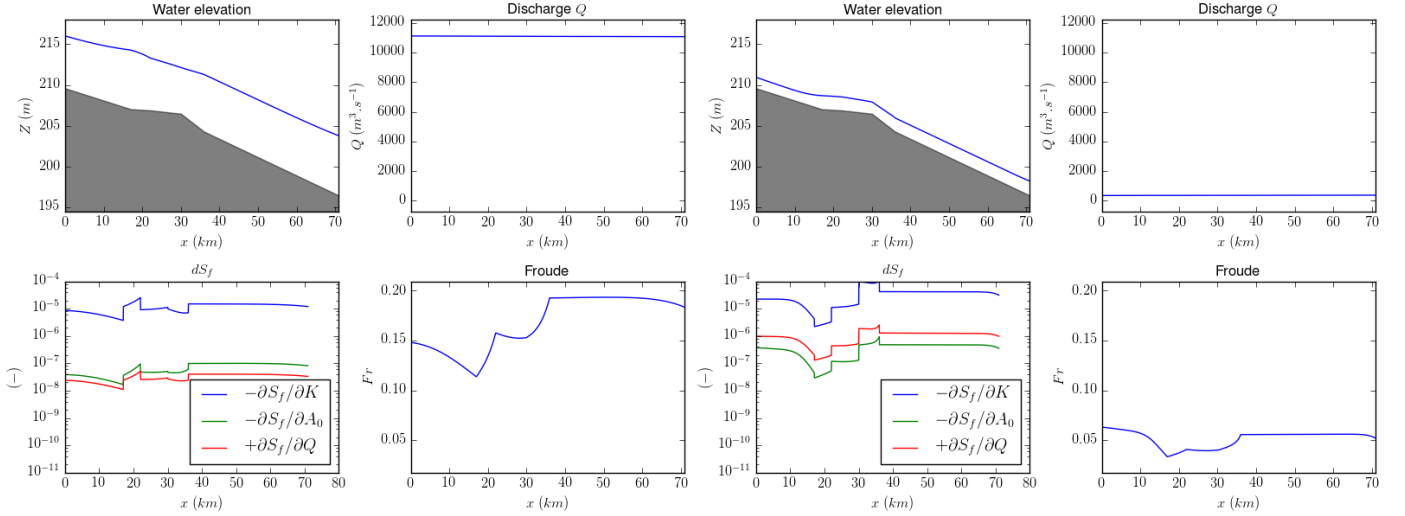


Figure 9: Evaluation of the partial derivatives of the friction source term S_f ; forward run with the calibrated parameter set (cf. table 1) and true inflow discharge.

416 data.

417 The friction law coefficients are spatialized by reach to be coherent with the observation grid and with the (rather
 418 large) meaningful scale of these parameters in the 1D Manning-Strickler equation (see e.g. Guinot and Cappelaere
 419 [34]). This effective modeling approach enables a fairly accurate reproduction of the multichannel flows observed
 420 during 8 years by nadir altimetry (ENVISAT) on this 71km braided river.

421 The inference capabilities of hydraulic parameters patterns from real altimetric observations along a single
 422 ENVISAT track or from the future spatially dense SWOT observations are demonstrated. For the present observed
 423 multichannel river complexity, the inverse method enables to infer a fairly realistic upstream discharge hydrograph
 424 along with an effective river channel. The estimated bathymetry and friction patterns somehow result in local
 425 and effective stage-discharge relationships. In case of spatially sparse observations, the coherence between the
 426 sparse observation grid and the dense model grid is ensured using a piecewise linear bathymetry representation
 427 along with a friction power law with piecewise constant parameters. This constrain on the VDA process provided
 428 by the above defined effective bathymetry-friction representation by reach is highlighted with spatially sparse
 429 ENVISAT observations. Moreover the additional constrain provided by the forthcoming SWOT observations to
 430 infer a discharge hydrograph and densely distributed spatial controls is assessed on this effective multichannel river
 431 representation; the definition of friction by reaches enabling to consider more dense bathymetry controls.

432 SWOT observations would represent unprecedented measurements of hydraulic processes signatures from the
 433 local to the hydrographic network scales, including complex flow zones such as braided ones. On-going researches
 434 focus on the detection and use of various hydraulic signatures in WS as highlighted here for bottom slope (resp.
 435 channel width) breaks in low (resp. high) flows (see WS curvature analysis and SW model behavior in Montazem
 436 et al. [46]), on the estimation of reliable priors and inverse problems at the scale of larger river network portions
 437 including complex flow zones.

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439 The contributions of the respective authors are as follows. Pierre-André Garambois designed the research
440 plan and performed the numerical investigations and analysis. Pierre-André Garambois, Pascal Finaud-Guyot,
441 Kevin Larnier and Amanda Montazem contributed to the hydraulic understanding and sensitivity analysis. Jérôme
442 Monnier is the principal designer of the inverse computational method and its analysis. Jonas Verley has started
443 the present study during the beginning of his PhD. This study is warmly dedicated to him.

444 The computational software DassFlow1D and satellite data curation toolbox were adapted from their previous
445 versions (Larnier et al. [42]) by Jonas Verley, Pierre-André Garambois and Kevin Larnier, this last generated the
446 SWOT synthetic data using the large scale simulator and computational resources of CNES (“Centre National
447 d’Etudes Spatiales”, French space agency); Amanda Montazem processed and analyzed the SWOT data. Stephane
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453 7. Appendix: the computational inverse method

454 As already briefly summarized in Section 2.3, the computational inverse method is based on Variational Data
455 Assimilation (VDA) applied to the Saint-Venant flow model (1). The computational inverse method is those
456 presented in Brisset et al. [10], Larnier et al. [42] with an augmented composite control vector c , see (3): c contains
457 a spatially distributed friction coefficient enabling to model complex flow zones (while it is an uniform friction law
458 $K(h)$ in Larnier et al. [42]). This definition of $K(x, h)$ enables to consider more heterogeneous bathymetry controls.

459 It is important to point out that the imposed downstream boundary condition is an unknown of the inverse
460 problem. It is constrained with the observed water elevations and inferred river bottom slope using a locally uniform
461 flow hypothesis (i.e. Manning equation, cf. section 2.1).

462 The cost function $j(c)$ is defined as:

$$j(c) = j_{obs}(c) + \gamma j_{reg}(c) \quad (6)$$

463 where $\gamma > 0$ is a weighting coefficient of the so-called “regularization term” $j_{reg}(c)$. The term $j_{obs}(c)$ measures the
464 misfit between observed and modeled WS elevations such that:

$$j_{obs}(c) = \frac{1}{2} \|Z(c) - Z_{obs}\|_{\mathcal{O}}^2 \quad (7)$$

465 The norm $\|\cdot\|_{\mathcal{O}} = \|\mathcal{O}^{1/2}\cdot\|_2$ is defined from an a-priori positive definite covariance matrix \mathcal{O} . Assuming uncorrelated
 466 observations $\mathcal{O} = \text{diag}(\sigma_Z)$ with σ_Z the a-priori observation error on Z_{obs} - $\sigma_Z = 15\text{cm}$ in this study.

467 The modeled WS elevations Z depend on c through the hydrodynamic model (1) and the inverse problem reads
 468 as

$$c^* = \operatorname{argmin}_c j(c) \quad (8)$$

469 This optimal control problem is solved using a Quasi-Newton descent algorithm: the L-BFGS algorithm version
 470 presented in 31. The cost gradient $\nabla j(c)$ is computed by solving the adjoint model; the latter is obtained by
 471 automatic differentiation using Tapenade software [37]. Detailed know-hows on VDA may be found e.g. in the
 472 online courses Bouttier and Courtier [9], Monnier [44].

473 To be solved efficiently this optimization problem needs to be “regularized”. Indeed the friction and the
 474 bathymetry may trigger indiscernible surface signatures therefore leading to an ill-posed inverse problem; we refer
 475 e.g. to Kaltenbacher et al. [40] for the theory of regularization of such inverse problems and to Larnier et al. [42]
 476 for a discussion focused on the present inverse flow problem.

477 Following Larnier et al. [42], the optimization problem (8) is regularized as follows. First the regularization term
 478 j_{reg} is added to the cost function, see (6). We simply set: $j_{reg}(c) = \frac{1}{2} \|b''(x)\|_2^2$. Therefore this term imposes (as
 479 weak constrains) the inferred bathymetry profile $b(x)$ to be an elastic interpolating the values of b at the control
 480 points (i.e. a cubic spline).

481 A specificity of the present context is the large inconsistency between the large observation grid (altimetry
 482 points) and the finer finer model grid. Between the sparse observations points (equivalently the control points),
 483 the bathymetry profile $b(x)$ is reconstructed as a piecewise linear function. It is worth to point out that the
 484 resulting reconstruction is consistent with the physical analysis presented in Montazem et al. [47], Montazem
 485 et al. [46], Montazem [45]. (This study analyses the adequation between the SW model (1) behavior and the WS
 486 signature).

487 Next and following Lorenc et al. [43], Weaver and Courtier [57], Larnier et al. [42], the following change of control
 488 variable is made:

$$k = B^{-1/2}(c - c_{prior}) \quad (9)$$

489 where c is the original control vector, c_{prior} is a prior value of c and B is a covariance matrix. The choice of B is
 490 crucial in the VDA formulation; its expression is detailed below. After this change of variable the new optimization
 491 problem reads:

$$\min_k J(k) \text{ with } J(k) = j(c) \quad (10)$$

492 It is easy to show that this leads to the following new optimality condition: $B^{1/2}\nabla j(c) = 0$; somehow a

493 preconditioned optimality condition. For more details and explanations we refer to 35, 36 and Larnier et al. [42] in
 494 the present inversion context.

495 Assuming uncorrelated controls B is defined as a block-diagonal matrix:

$$B = \begin{pmatrix} B_Q & 0 & 0 & & \\ 0 & B_b & 0 & & \\ 0 & 0 & B_\alpha & & \\ 0 & 0 & 0 & B_\beta & \end{pmatrix} \quad (11)$$

496
 497 Still following Larnier et al. [42], the matrices B_Q and B_b are set as the classical second order auto-regressive
 498 correlation matrices :

$$(B_Q)_{i,j} = (\sigma_Q)^2 \exp\left(-\frac{|t_j - t_i|}{\Delta t_Q}\right) \text{ and } (B_b)_{i,j} = (\sigma_b)^2 \exp\left(-\frac{|x_j - x_i|}{L_b}\right) \quad (12)$$

499 The VDA parameters Δt_Q and L_b represent prior hydraulic scales and act as correlation lengths. Given the
 500 frequency (few days) and spatial resolution of observations (200m long “pixels” for SWOT), the low Froude braided
 501 river flows of interest, adequate values for those parameters are: $\Delta t_Q = 24$ h and $L_b = 3km$ km We refer to Brisset
 502 et al. [10] for a thorough analysis of the discharge inference in terms of frequencies and wave lengths and Section
 503 4.1 in the present river-observation context. In the present study, the friction parameters applied to deca-kilometric
 504 patches are assumed to be uncorrelated thus the matrices B_α and B_β are diagonal:

$$(B_\alpha)_{i,i} = (\sigma_\alpha)^2, (B_\beta)_{i,i} = (\sigma_\beta)^2 \quad (13)$$

505 The scalar values σ_\square may be viewed as variances ; their values are given in the numerical results section.

506 8. References

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