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# When ‘knowing whether’ is better than ‘knowing that’

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## Abstract

We study a simple epistemic logic with a restricted language where formulas are boolean combinations of (epistemic) atoms: sequences of ‘knowing whether’ operators followed by propositional variables. Our language is strictly more expressive than existing restricted languages, where atoms are sequences of epistemic operators and negations followed by propositional variables (in other words: atoms are epistemic formulas without conjunctions and disjunctions). Going further beyond existing approaches, we also introduce a ‘common knowledge of a group whether’ operator. We give an axiomatization for this logic and show that the model checking and satisfiability problems can be reduced to their classical counterparts.

## 1 Introduction

Suppose there are  $n$  agents, each of which knows some secret: a piece of information that is not known to the others. They communicate by phone calls, and whenever one person calls another they tell each other all they know at that time. How many calls are required before each item of gossip is known to everyone? This is known as the gossip problem. It is of great interest in the conception of communication networks [15] and in parallel and distributed computing, but there are other less obvious applications like the management of data on storage devices [20], or the computation of the syntenic distance between two genomes (minimum number of fusions, fissions, and translocations required to transform one into the other) [24]. Several variants have been studied in the literature, and a survey of these alternatives and the associ-

ated results has been published [15]. The gossip problem can be viewed as a multiagent epistemic planning task where the goal is shared knowledge: everybody knows all secrets. It is a purely epistemic planning task because it is only the agents’ knowledge that evolves, while the facts of the world remain unchanged. There are numerous variations of such a planning problem; see e.g. [10] for an overview. The goal may in particular be to achieve higher-order shared knowledge: everybody knows that everybody knows all secrets, and so on. Due to its simplicity and its variations, the gossip problem can be viewed as a paradigmatic epistemic planning task, much in the same way as the blocksworld is a paradigmatic classical planning task.

The aim of the present paper is to follow up on [8], in which we introduce a simple logical approach within which we can account for epistemic planning tasks such as the above. We could have chosen Dynamic Epistemic Logic (DEL) for that purpose: its event models allow us to model telephone calls as private announcements. However, we take inspiration from a simpler framework: the Dynamic Epistemic Logic of Propositional Assignment and Observation (DEL-PAO), introduced in [16] and further developed in [6, 17, 25]. It is based on visibility atoms, that are recursively defined as either propositional variables or atomic pieces of information about whether or not an agent sees the truth value of a visibility atom. DEL-PAO was applied to epistemic planning in [9]. We here concentrate on the epistemic logic underlying DEL-PAO, disregarding the dynamic aspects. The logic is consequently called the Epistemic Logic of Observation, abbre-

viated to EL-O. We improve in [8] over [9] by integrating a form of common knowledge into planning, and polynomially embed the EL-O-based planning we define into classical planning. It follows that deciding the solvability of a simple epistemic planning task is PSPACE complete.

However, the form of common knowledge we have in [8] only accounts for knowledge shared by *all* of the agents. In particular, in the gossip problem, we cannot model the common knowledge between only two agents that we would expect to occur after a phone call between those agents. In many situations we can imagine knowledge becoming common within a subgroup of agents, such as agents in a call or agents present in a room (while some other agents may be outside of that room). In this paper we therefore study a generalization of EL-O which accounts for joint vision within subgroups of agents.

The paper is organized as follows. In Section 2 we provide a detailed motivation of our approach. In Section 3 we introduce our generalization of EL-O. In Section 4 we study its properties. We conclude in Section 5.

## 2 Background and motivation

Reasoning in epistemic logic as introduced by Hintikka [18] and popularized in AI by Halpern and colleagues [12] is strictly more complex than in classical logic: the satisfiability problem is at least in PSPACE, and is EXPTIME-complete if common knowledge is involved [14]. The complexity gap widens for planning tasks: while the solvability problem is PSPACE complete for classical planning [5], it is undecidable for planning in DEL [3]. Such DEL-based accounts of epistemic planning provide rather expressive models for the agents' perception of actions. These models parallel the standard epistemic logic modelling of uncertainty in terms of indistinguishability relations between possible states: uncertainty in the perception of actions is modelled by so-called event models in terms of indistinguishability relations between possible events. As explored in a series of papers, undecidability is already the case under very weak hypotheses about these event models [1, 4, 7]. Basically, DEL planning tasks are only decidable when the event model is a singleton, i.e., when all actions are public. However, actions in the gossip problem as well as in many real world applications are not public.

We propose in [8] to base epistemic planning on an epistemic logic that is simpler than Hin-

tikka's. Basically, what we do is restrict epistemic information to atomic formulas. We are not the first to do this: several previous approaches have proposed languages where the scope of the epistemic operator  $K_i$  is restricted to literals, or literals that are preceded by a sequence of epistemic operators [11, 28, 21], possibly with negations [27]. Such restrictions however exclude formulas such as  $K_i(K_j p \vee K_j \neg p)$  expressing that agent  $i$  knows that agent  $j$  knows whether the propositional variable  $p$  is true. This is a major drawback because such formulas are fundamental in communication and more generally in any forms of interaction: a situation where agent  $i$  does not know whether  $p$  is the case or not ( $\neg K_i p \wedge \neg K_i \neg p$ ) but knows that  $j$  knows ( $K_i(K_j p \vee K_j \neg p)$ ) may lead agent  $i$  to ask  $j$  about  $p$ . For example, in the modelling of the gossip problem 'knowing-whether' information is required to describe the initial situation

$$\bigwedge_i ((K_i s_i \vee K_i \neg s_i) \wedge \bigwedge_{j \neq i} (\neg K_i s_j \wedge \neg K_i \neg s_j))$$

where  $s_i$  is the secret of agent  $i$ , as well as the goal  $\bigwedge_{i,j} (K_i s_j \vee K_i \neg s_j)$ .

We here make a similar restriction, but move from the primitive 'knowing-that' operator  $K_i$  to the less standard 'knowing-whether' or 'knowing-if' operator  $Kif_i$ , also considered in [26]. In the unrestricted language these two operators are interdefinable: we have  $Kif_i \varphi \leftrightarrow K_i \varphi \vee K_i \neg \varphi$  and  $K_i \varphi \leftrightarrow \varphi \wedge Kif_i \varphi$ , for arbitrary formulas  $\varphi$  [13]. However, the situation changes when we restrict the language to sequences of 'knowing-whether' operators followed by propositional variables: we can not only define  $K_i p$  as  $p \wedge Kif_i p$  and  $K_i \neg p$  as  $\neg p \wedge Kif_i p$ , but we can also express formulas such as the above  $K_i(K_j p \vee K_j \neg p)$ , namely by  $Kif_j p \wedge Kif_i Kif_j p$ .

We present in [8] the logic EL-O, where  $Kif_i$  is written as  $S_i$ . We read  $S_i p$  as "agent  $i$  sees whether  $p$  is true or not". When we define  $K_i p$  as  $p \wedge S_i p$  we therefore consider that  $i$ 's knowledge comes from what  $i$  sees. Such a primitive was introduced in the model checking literature in order to represent epistemic models in a compact way [29, 19, 2]. All these approaches were based on the hypothesis that who sees what is common knowledge. This is a very strong hypothesis: it follows that whenever  $i$  knows that  $p$  is true then  $i$  knows exactly who else knows that  $p$  is true. In terms of the visibility operator, these papers consider  $S_i S_j p$  to be valid for arbitrary  $i$  and  $j$ . As first proposed in [16], this restriction can be overcome by abandoning the validity of  $S_i S_j p$  and giving the status of first-class citizens to such sequences.

Formally, in EL-O, it is considered that every visibility atom  $S_{i_1} \dots S_{i_n} p$  is an atomic formula. Com-

plex formulas are then boolean combinations of such atomic formulas. The reader should keep in mind that  $S_i$  is not a modal operator, in opposition to  $Kif_i$  (and our move in notation also aims at stressing that). This comes with a move away from Kripke models with accessibility relations: a model is simply a valuation over the set of visibility atoms, alias a state. We identify such valuations with sets of visibility atoms. As argued in [29, 2], such compact models are more attractive than Kripke models when it comes to model checking.

Simplifications of epistemic logic typically lack a modal operator of common knowledge. In [8] we introduce the operator  $\mathcal{J}$ , reading  $\mathcal{J}p$  as “all agents jointly see the value of  $p$ ”, or “all agents jointly see whether  $p$  is true or not”. Metaphorically, joint attention about a propositional variable  $p$  can be understood as eye contact between the agents when observing  $p$ .  $\mathcal{J}$  is a powerful operator that represents common knowledge obtained through co-presence. To understand the power of  $\mathcal{J}$ , one may think of  $\mathcal{J}p$  as implying the infinite set of propositions of the form  $S_{i_1} \dots S_{i_n} p$  for all  $i_1, \dots, i_n$  and all  $n \geq 1$ . Suppose  $p$  stands for ‘the door is open’. We can imagine that  $\mathcal{J}p$  is true when all agents are present in the same room and not only observe the door, but also mutually observe each other. In this concrete example, if an agent leaves the room and closes the door behind her,  $\mathcal{J}p$  becomes false because the agents no longer mutually observe each other (even if the agent who has left can see the closed door from the outside). As joint visibility of  $p$  should imply individual visibility of  $p$ , we require that states containing  $\mathcal{J}p$  should also contain every  $S_i p$ , and more generally every  $p$  preceded by any sequence of  $S_i$  and  $\mathcal{J}$ . Defining  $CKp$  as  $p \wedge \mathcal{J}p$ , we obtain a common knowledge operator that satisfies all the standard properties except the induction axiom  $CK(p \rightarrow \bigwedge_{i \in \text{Agt}} K_i p) \rightarrow (p \rightarrow CKp)$ ; actually the antecedent cannot be directly expressed by means of  $\mathcal{J}$  operators.

This ‘joint vision’ operator still has expressive limits. In particular, it does not account for joint vision between groups of agents, when those groups are strictly smaller than the set of all agent. We therefore generalize the notion of joint vision to ‘joint vision within a group’, and introduce the corresponding operator  $\mathcal{J}_G$ , where  $G$  is a group of agents. The operators  $\mathcal{J}$  and  $S_i$  can be expressed as  $\mathcal{J}_{\text{Agt}}$  and  $\mathcal{J}_{\{i\}}$  respectively. We can now express by  $\mathcal{J}_{\{1,2\}} p$  the fact that agents 1 and 2 jointly see  $p$ , for example after 1 has called 2 to share information about  $p$ , excluding 3 from the conversation.

It is generally considered in the AI literature that a reasonable epistemic logic should satisfy introspection: the formulas  $K_i \varphi \rightarrow K_i K_i \varphi$  and  $\neg K_i \varphi \rightarrow K_i \neg K_i \varphi$  should both be valid.<sup>1</sup> In terms of visibility this means that states should contain every  $\mathcal{J}_{G'} \mathcal{J}_G p$  for  $G' \subseteq G$ , and more generally  $\mathcal{J}_{G'} \mathcal{J}_G p$  preceded by any sequence  $\mathcal{J}_{H_1} \dots \mathcal{J}_{H_n}$ .

In the rest of the paper we will work out the details of what we have sketched up to now.

### 3 EL-O: Epistemic Logic of Observation

We introduce our generalization of the Epistemic Logic of Observation, abbreviated to EL-O.

#### 3.1 Atoms, introspective atoms, and atomic consequence

Let  $Prop$  be a countable set of propositional variables and let  $\text{Agt}$  be a finite set of agents. We call finite set of agents *groups*, and denote them by  $G$ ,  $H$ , etc.

We use  $\sigma, \sigma'$ , etc. to denote elements of  $(2^{\text{Agt}})^*$ , i.e., words on the set of groups. The set of non-empty words built from elements of  $G$  is  $(2^G)^+$ .

An *atom* is a word of groups followed by a propositional variable. We use  $\alpha, \alpha'$ , etc. to denote elements of the set of atoms  $ATM = (2^{\text{Agt}})^* \times Prop$ . We treat atoms as words, too, allowing the notation  $\sigma\alpha$  for atoms, and identify in atoms any group  $G$  with the operator  $\mathcal{J}_G$ ; for example, we write  $\mathcal{J}_{G_1} \mathcal{J}_{G_2} p$  instead of the couple  $\langle G_1 G_2, p \rangle$ . We read  $\mathcal{J}_G \alpha$  as “group  $G$  jointly sees  $\alpha$ ”. Atoms with an empty sequence of groups are nothing but propositional variables.

Here are some examples:  $\mathcal{J}_{\{1\}} p$  reads “1 sees the value of  $p$ ”. It means that 1 knows whether  $p$  is true or false.  $\mathcal{J}_{\{1,3\}} \mathcal{J}_{\{2\}} q$  reads “1 and 3 jointly see whether agent 2 sees the value of  $q$ ”. In other words, there is *joint attention* between 1 and 3 concerning 2’s observation of  $q$ : agent 2 may or may not see the value of  $q$ , and in both cases this is jointly observed.

We are going to design the semantics in such a way that principles of introspection are valid. An atom is *introspective* if it is of the form  $\sigma \mathcal{J}_H \mathcal{J}_G \alpha$  with  $H \subseteq G$ . We call *I-ATM* the set of all introspective atoms.

1. We are aware that negative introspection was criticised in the literature as being too strong [22, 23]. One may however argue that these criticisms do not apply to visibility-based knowledge.

*Atomic consequence* is a relation on *ATM* that is defined inductively by:

$$\begin{aligned} p \Rightarrow \beta &\text{ iff } p = \beta \\ \mathcal{J}_G \alpha \Rightarrow \beta &\text{ iff } \beta = \sigma \alpha \text{ and } \sigma \in (2^G)^+ \end{aligned}$$

For example,  $\mathcal{J}_{\{1,2,3\}} p \Rightarrow \mathcal{J}_{\{3\}} \mathcal{J}_{\{1,2\}} p$ . The relation  $\Rightarrow$  is clearly reflexive and transitive. When  $\alpha \Rightarrow \beta$ , we say that  $\alpha$  is a *cause* of  $\beta$  and that  $\beta$  is a *consequence* of  $\alpha$ . We will, of course, ensure that atomic consequences are valid implications.

The *set of atomic consequences* of an atom  $\alpha$  is  $\alpha^\Rightarrow = \{\beta : \alpha \Rightarrow \beta\}$ . The *set of atomic causes* of an atom  $\alpha$  is  $\alpha^\Leftarrow = \{\beta : \beta \Rightarrow \alpha\}$ . The set of consequences of an atom other than a propositional variables is infinite. The set of causes of an atom  $\alpha$  is finite and if we define the length of a visibility operator  $\mathcal{J}_G$  to be one, then each cause is no longer than  $\alpha$ .

We generalise the definition of atomic consequence to sets of atoms  $s \subseteq ATM$  in the obvious way:  $s^\Rightarrow = \bigcup_{\alpha \in s} \alpha^\Rightarrow$ .

### 3.2 Language

The language of EL-O is defined by the following grammar:

$$\varphi ::= \alpha \mid \neg\varphi \mid (\varphi \wedge \varphi)$$

where  $\alpha$  ranges over *ATM*. The set of EL-O *formulas* is noted  $Fml_{EL-O}$ .

The boolean operators  $\top$ ,  $\perp$ ,  $\vee$ ,  $\rightarrow$  and  $\leftrightarrow$  are defined in the standard way. The set  $ATM(\varphi)$  of atoms of formula  $\varphi$  is defined by:

$$\begin{aligned} ATM(\alpha) &= \{\alpha\} \\ ATM(\neg\varphi) &= ATM(\varphi) \\ ATM(\varphi \wedge \varphi') &= ATM(\varphi) \cup ATM(\varphi') \end{aligned}$$

For example,  $ATM(\mathcal{J}_{\{1,3\}} q \wedge \mathcal{J}_{\{2\}} p) = \{\mathcal{J}_{\{1,3\}} q, \mathcal{J}_{\{2\}} p\}$  and  $ATM(\mathcal{J}_{\{1\}} \mathcal{J}_{\{2,3\}} p) = \{\mathcal{J}_{\{1\}} \mathcal{J}_{\{2,3\}} p\}$ . Note that neither  $p$  nor  $\mathcal{J}_{\{2,3\}} p$  are atoms of  $\mathcal{J}_{\{1\}} \mathcal{J}_{\{2,3\}} p$ .

### 3.3 Interpretation of formulas

A state is a subset of the set of atoms *ATM*. We denote states by  $s$ ,  $s'$ , etc. The set of all states is  $2^{ATM}$ .

An obvious way of guaranteeing introspection is to interpret formulas exclusively in *introspectively closed states*: states that contain all introspective atoms and are closed under  $\Rightarrow$ , i.e., sets of atoms that equal  $s^\Rightarrow \cup I-ATM$  for some state  $s \subseteq ATM$ . This is what was done in [16]. The drawback of such a semantics is that introspective states are infinite,

$s \models \alpha$	iff $\alpha \in s^\Rightarrow \cup I-ATM$
$s \models \neg\varphi$	iff not $(s \models \varphi)$
$s \models \varphi \wedge \varphi'$	iff $s \models \varphi$ and $s \models \varphi'$

Table 1 – Interpretation of formulas

while model checking requires a finite state. This problem can be overcome by defining states that are ‘sufficiently introspective’, as done in [25]. We here take an easier road by directly working with finite models, interpreting formulas in such a way that introspection is simulated.

The truth conditions for EL-O formulas are given in Table 1. The non-standard one is the one for atoms  $\alpha$ :  $\alpha$  is true in state  $s$  if and only if  $\alpha$  is introspective or  $\beta \Rightarrow \alpha$  for some  $\beta \in s$ .

**Example 1.** In the initial state of the gossip problem every agent only knows her own secret. For the sake of simplicity we moreover suppose that all secrets are initially true. Therefore  $s_0 = \{\mathcal{J}_{\{i\}} s_i : i \in Agt\} \cup \{s_i : i \in Agt\}$ . Then  $s_0 \models \mathcal{J}_{\{i\}} s_i$  and  $s_0 \models \bigwedge_{j \neq i} \neg \mathcal{J}_{\{i\}} s_j$  for every  $i \in Agt$ .

When  $s \models \varphi$  for every  $s \subseteq ATM$  then we say that  $\varphi$  is *valid*; when  $s \models \varphi$  for some  $s \subseteq ATM$  then we say that  $\varphi$  is *satisfiable*.

**Proposition 1.** *For every  $\alpha \subseteq ATM$ ,  $\alpha$  is valid iff  $\alpha$  is introspective.*

*Proof.* For the left-to-right direction, suppose  $\alpha$  is valid. Then in particular  $\emptyset \models \alpha$ . As  $\emptyset^\Rightarrow$  is still empty,  $\alpha$  must be introspective.  $\square$

If  $\alpha \Rightarrow \beta$  then  $\alpha \rightarrow \beta$  is EL-O valid. The converse is not always the case. However:

**Proposition 2.**  *$\alpha \rightarrow \beta$  is EL-O valid iff  $\beta$  is introspective or  $\alpha \Rightarrow \beta$ .*

*Proof.* The right-to-left direction is obvious. For the left-to-right direction, suppose  $\alpha \rightarrow \beta$  is EL-O valid. Then in particular  $\{\alpha\} \models \beta$ . Hence  $\beta$  is either an introspective atom or an atomic consequence of  $\alpha$ .  $\square$

## 4 Properties

We now reduce EL-O to Classical Propositional Calculus, then give an axiomatization for it.

## 4.1 Relation with Classical Propositional Calculus

In our framework there are two options when it comes to defining Classical Propositional Calculus (CPC). The first is to restrict the language to formulas without operators of visibility; classical logic is then the set of validities of that language. The second option is more interesting here: we consider the full language built from  $ATM$ , but we modify the interpretation of visibility atoms to

$$s \models^{\text{CPC}} \alpha \text{ iff } \alpha \in s$$

and keep the same interpretation as in EL-O otherwise. Hence we have visibility atoms but do not care about introspection; for example, the formulas  $\neg \mathcal{J}_{[i]} \mathcal{J}_{[i]} p$  and  $\mathcal{J}_{[i,j]} p \wedge \neg \mathcal{J}_{[i]} p$  are classically satisfiable: for the former,  $\emptyset \models^{\text{CPC}} \neg \mathcal{J}_{[i]} \mathcal{J}_{[i]} p$ ; for the latter,  $\{\mathcal{J}_{[i,j]} p\} \models^{\text{CPC}} \neg \mathcal{J}_{[i]} p$ .

In the rest of the section we provide two embeddings from EL-O into CPC: one where we expand states, and one where we expand formulas.

**Proposition 3.** *For every state  $s \subseteq ATM$  and formula  $\varphi \in Fml_{\text{EL-O}}$ ,  $s \models \varphi$  if and only if  $s^{\Rightarrow} \cup I-ATM \models^{\text{CPC}} \varphi$ .*

*Proof.* The proof is by induction on the form of  $\varphi$ . The base case is immediate from the interpretation of atoms in EL-O.  $\square$

The above proposition confirms that we simulate introspectively closed states with our interpretation of formulas. It also entails the following properties:

**Lemma 1.** *For all states  $s, s' \subseteq ATM$  and for every formula  $\varphi \in Fml_{\text{EL-O}}$ , if  $s$  and  $s'$  classically agree on atoms of  $\varphi$  then they classically agree on  $\varphi$ . That is, if  $s \cap ATM(\varphi) = s' \cap ATM(\varphi)$  then  $s \models^{\text{CPC}} \varphi$  iff  $s' \models^{\text{CPC}} \varphi$ .*

**Proposition 4.** *For every state  $s \subseteq ATM$  and formula  $\varphi \in Fml_{\text{EL-O}}$ ,  $s \models \varphi$  if and only if  $(s^{\Rightarrow} \cup I-ATM) \cap ATM(\varphi) \models^{\text{CPC}} \varphi$ .*

**Corollary 1.** *For every state  $s \subseteq ATM$  and formula  $\varphi \in Fml_{\text{EL-O}}$ ,  $s \models \varphi$  if and only if  $(s^{\Rightarrow} \cup I-ATM) \cap ATM(\varphi) \models \varphi$ .*

*Proof.* This stems from the fact that if  $s_\varphi = (s^{\Rightarrow} \cup I-ATM) \cap ATM(\varphi)$ , then  $(s_\varphi)_\varphi = s_\varphi$ . Therefore  $s_\varphi \models \varphi$  iff  $s_\varphi \models^{\text{CPC}} \varphi$ .  $\square$

As  $ATM(\varphi)$  is finite, we hence have the finite model property both in EL-O and in the classical translation. In particular, the model checking problem and the satisfiability problem have the same complexities as in CPC.

The second embedding is more interesting from a planning point of view, as it does not modify the states. We define the *expansion* of an atom as follows:

$$\text{Exp}(\alpha) = \begin{cases} \top & \text{if } \alpha \in I-ATM \\ (\bigvee \alpha^{\Leftarrow}) & \text{otherwise} \end{cases}$$

We extend this definition homomorphically to formulas. For example, if  $Ag_t = \{i, j\}$ , the expansion of the EL-O unsatisfiable  $\mathcal{J}_{[i,j]} p \wedge \neg \mathcal{J}_{[i]} p$  is  $\mathcal{J}_{[i,j]} p \wedge \neg(\mathcal{J}_{[i]} p \vee \mathcal{J}_{[i,j]} p)$ , which is classically unsatisfiable; and the expansion of the EL-O satisfiable  $\mathcal{J}_{[i,j]} \mathcal{J}_{[i]} p \wedge \neg \mathcal{J}_{[i]} p$  is the classically satisfiable  $(\mathcal{J}_{[i,j]} \mathcal{J}_{[i]} p \vee \mathcal{J}_{[i,j]} p) \wedge \neg(\mathcal{J}_{[i]} p \vee \mathcal{J}_{[i,j]} p)$ . This can be generalised:

**Proposition 5.** *For every state  $s \subseteq ATM$  and formula  $\varphi \in Fml_{\text{EL-O}}$ ,  $s \models \varphi$  iff  $s \models^{\text{CPC}} \text{Exp}(\varphi)$ .*

*Proof.* The proof is by induction on the form of  $\varphi$ . For the base case we distinguish two subcases. First, if  $\alpha \in I-ATM$  then  $\text{Exp}(\alpha) = \top$  and therefore  $s \models \alpha$  and  $s \models^{\text{CPC}} \text{Exp}(\alpha)$ . Second, if  $\alpha \notin I-ATM$  then  $s \models \alpha$  iff  $\beta \Rightarrow \alpha$  for some  $\beta \in s$ . The latter is equivalent to  $s \models^{\text{CPC}} \bigvee \alpha^{\Leftarrow}$ .  $\square$

In the version of EL-O with only the  $\mathcal{J}$  and  $S_i$  operators, the expansion was polynomial and allowed us to go from epistemic to classical planning with no increase in complexity. However, in this more general case, this expansion causes an explosion of the length of the formula, as the number of causes of an atom  $\mathcal{J}_G \alpha$  is exponential in the size of  $G$ . We will come back to this issue in the conclusion.

## 4.2 Axiomatization

The valid EL-O formulas are axiomatized by the schemas of Table 2.

**Proposition 6.** *For every formula  $\varphi \in Fml_{\text{EL-O}}$ ,  $\varphi$  is EL-O valid iff  $\varphi$  is provable in CPC from the four axiom schemas  $Vis_1$ – $Vis_4$  of Table 2.*

*Proof.* We take advantage of Proposition 3 and show that the schemas  $Vis_1$ – $Vis_4$  characterise the set of introspectively closed states  $\{s^{\Rightarrow} \cup I-ATM : s \subseteq ATM\}$ .

The right-to-left direction is clear: each of the four axiom schemas is valid in introspectively closed states.

For the left-to-right direction, we show that every  $s$  satisfying  $Vis_1$ – $Vis_4$  is introspectively closed. We begin with closure under atomic consequence. Let  $s \models \alpha$ . The interesting case is when  $\alpha = \mathcal{J}_G \alpha'$ . Then  $s \models \mathcal{J}_G \mathcal{J}_{H_1} \alpha'$  for any  $H_1 \subseteq G$  by axiom ( $Vis_4$ ), and

$\mathcal{J}_{S_H} \mathcal{J}_{S_G} \alpha$	for $H \subseteq G$	( $Vis_1$ )
$\mathcal{J}_{Agt} \mathcal{J}_{S_H} \mathcal{J}_{S_G} \alpha$	for $H \subseteq G$	( $Vis_2$ )
$\mathcal{J}_{S_G} \alpha \rightarrow \mathcal{J}_{S_H} \alpha$	for $H \subseteq G$	( $Vis_3$ )
$\mathcal{J}_{S_G} \alpha \rightarrow \mathcal{J}_{S_G} \mathcal{J}_{S_H} \alpha$	for $H \subseteq G$	( $Vis_4$ )

Table 2 – Axioms for introspection

also  $s \models \mathcal{J}_{S_G} \mathcal{J}_{S_{H_2}} \mathcal{J}_{S_{H_1}} \alpha'$  for any  $H_1, H_2 \subseteq G$ , and so on: we can generate any  $s \models \mathcal{J}_{S_G} \sigma \alpha'$  for any  $\sigma \in (2^G)^*$  and then, by ( $Vis_3$ ), we can obtain  $s \models \mathcal{J}_{S_H} \sigma \alpha'$  for any  $H \subseteq G$  and any  $\sigma \in (2^G)^*$ . We can therefore obtain that  $s$  satisfies any sequence whose strict postfix is  $\alpha'$ , that is, every  $\sigma \alpha'$  for any  $\sigma \in (2^G)^+$ .

We use the same technique to show that a state  $s$  satisfying  $Vis_1$ – $Vis_4$  satisfies every  $\sigma \mathcal{J}_{S_H} \mathcal{J}_{S_G} \alpha$  for  $\sigma \in (2^{2^{Agt}})^*$  and  $H \subseteq G$ , using ( $Vis_1$ ) if  $\sigma$  is empty, and ( $Vis_2$ ), ( $Vis_4$ ) and ( $Vis_3$ ) otherwise.  $\square$

## 5 Conclusion

In this paper, we have generalized the logic EL-O, subsuming its observability operators by an operator of joint vision within an arbitrary group. We have defined and axiomatized the corresponding logic, and shown that it can be reduced to classical propositional logic.

Based on the visibility information that is contained in the states, an accessibility relation can be defined for every group of agents to in order to interpret a common knowledge operator  $CK_G$ , which becomes the standard knowledge operator  $K_i$  when  $G = \{i\}$  and the common knowledge operator  $CK$  when  $G = Agt$ . The properties of the latter and the relation with standard epistemic logic are discussed in [8] and can be extended to this generalized definition. In particular,  $CK_G \alpha$  is equivalent to  $\alpha \wedge \mathcal{J}_{S_G} \alpha$ .

We give in [8] a definition of planning in EL-O as well as a reduction to classical planning. The decidability of EL-O-based epistemic planning contrasts with the undecidability of DEL-based epistemic planning, which is the case even for simple fragments.

It is proved in [8] that the complexity of solvability in EL-O-based planning is PSPACE complete. The proof relies on the fact that the length and number of causes of a given atom are not greater than the length of that atom, resulting in polynomial translations to CPC and classical planning. In the more general setting where joint vision within

arbitrary groups is allowed, the number of causes of an atom becomes exponential, and the property no longer holds. We can however retain this low complexity while keeping our framework relevant for a number of applications by allowing joint vision only for relevant groups. For example, in the gossip problem, we only need joint vision between pairs of agents. Similarly, it is natural to imagine that there may be a limit on the number of agents that can be present in a room at the same time, or that can join in on a conversation.

We have as of now only presented the generalization of EL-O. The next step is of course to extend this generalization to epistemic planning, and further refine the limits in complexity mentioned above. More generally, it will be interesting to study how our epistemic actions compare to DEL event models.

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