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# Sensitivity Analysis of Beams Controlled by Shunted Piezoelectric Transducers

G. Matten, M. Collet, S. Cogan, and E. Sadoulet-Reboul

**Abstract** This paper aims at quantifying the robustness of a cantilever beam behavior, controlled with a shunted piezoelectric patch, with respect to uncertainties in the system parameters. Vibration control based on piezoelectric transducers has proven to be particularly efficient, in particular the use of thin piezoelectric patches shunted by negative capacitance circuits allowing broadband applications. However, the optimal tuning of the control can lead to instability and the influence of a variation in the physical properties or in geometry is not well known. Through multiphysical finite element simulations, a sensitivity analysis of the beam coupled with a piezoelectric transducer is performed. The effects of a variability in the material characteristics, the geometry of the system and the parameters of the shunt circuit on the stability and the damping performances are investigated.

**Keywords** Vibration control • Sensitivity • Robust design • Negative capacitance controller • Morris

## 24.1 Introduction

Appearing in the late 1970s, vibroacoustic control has been extensively studied [1–3]. In this context, a technique based on shunted piezoelectric transducers has been proven to be efficient. It consists in adding thin piezoelectric patches on a structure and connecting electrical shunts between the two electrodes of the transducers. With the piezoelectric effect, they are both sensors and actuators and the shunt behaves as a feedback controller. One particular shunt is a negative capacitance circuit. It is designed to counteract the capacitive effect of the transducer in order to realize a broadband resistive dissipator. The experiments performed so far have shown that the tuning of such a shunt is delicate. Indeed, the damping of the structure is very sensitive to the variation of the shunt capacitance. This paper intends to better understand the sensitivity of such systems to variations or uncertainties in the system characteristics. The goal is to determine which parameters are critical for the efficiency and for the stability of the controlled beam so as to propose a more robust design.

## 24.2 The System

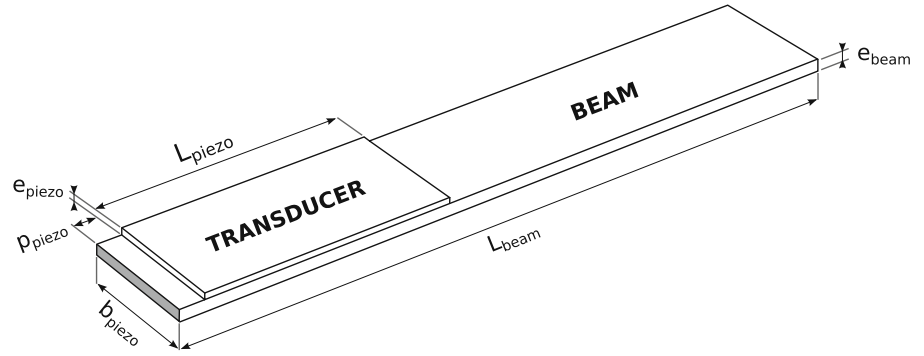
The system studied here is a steel fixed-free cantilever beam with a PZT-5A piezoelectric transducer fixed on it with epoxy glue (see Fig. 24.1). The control of this system with a negative capacitance shunt has been achieved in [4]. A three dimensional finite element model is presented here to study the sensitivity of the damping efficiency towards the material properties (density, Young modulus and Poisson ratio), the geometric properties (position and dimensions of the transducer and dimensions of the beam) and the shunt properties (resistance  $R$  and capacitance  $C$ ).

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**Fig. 24.1** Geometry of the system



An eigenvalue study of the system is performed. Specifically,  $\lambda_{\text{bend}}$ , the eigenvalue associated with the first bending mode of the beam is computed. Following the method presented in [4], the optimal shunt has a resistance of  $1463\Omega$  and a capacitance of  $-4.918\text{nF}$ . The lower face of the transducer is electrically grounded. The shunt is modeled by a weak contribution of a voltage, calculated as a function of the electric charge, and is applied on the upper face of the transducer. The following mathematical equation is verified:

$$u = -q \frac{1 - \lambda RC}{C} \quad (24.1)$$

with  $R$  and  $C$  the resistance and capacitance of the shunt,  $q$  the electric charge on the upper face of the transducer and  $\lambda$  the eigenvalue variable. The steel substrate is modeled by a linear elastic material, the transducer by a piezoelectric material, both with a  $0.1\%$  isotropic loss factor. The glue between these two parts is modeled with shell elements and a linear elastic material. All the three domains share the same displacement field  $u$ . The end of the substrate (see the grey surface on Fig. 24.1) is fixed ( $u = 0$ ). The electrodynamical problem is solved by computations of a multiphysical finite element model. The sensitivity analysis is performed using an in-house analysis software developed in matlab which controls finite element computations performed with Comsol<sup>©</sup>. After a convergence validation, a 3,338 elements with displacement and voltage degrees of freedom mesh has been chosen. The computation time is about half a minute.

## 24.3 Sensitivity Analysis

### 24.3.1 Model Parameters

The objective of this study is to analyse the sensitivity of the model presented in the previous section. Especially the geometry, the material properties and the shunt parameters have been chosen to have a sufficient knowledge of the system behavior to design an experiment.

The beam parameters are:

Parameter	L_beam	b_beam	e_beam	Nu_beam	Rho_beam	E_beam
Description	Length	Width	Thickness	Poisson ratio	Density	Young modulus
Nominal value	5.5 cm	1 cm	1 mm	0.3	$1,500 \text{ kg.m}^{-3}$	200 Gpa
Parameter deviation	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$

The shunted transducer parameters are:

Parameter	e_piezo	L_piezo	p_piezo	R_shunt	C_shunt
Description	Thickness	Length	Position	Shunt resistance	Shunt capacitance
Nominal value	$500 \mu\text{m}$	2 cm	2 mm	$1,463 \Omega$	$-4.918 \text{ nF}$
Parameter deviation	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$

The glue parameters are:

Parameter	e_glue	Nu_glue	Rho_glue	E_glue
Description	Thickness	Poisson ratio	Density	Young modulus
Nominal value	100 $\mu\text{m}$	0.3	1,300 $\text{kg.m}^{-3}$	150 Gpa
Parameter deviation	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$

The sensitivity is done using the Morris method [5] and all the previously described parameters are considered in the study.

### 24.3.2 Design Objectives

Two objectives have been considered. Firstly, the efficiency on the vibration damping is evaluated by measuring the damping of the first mode response. This is given by the quantity  $\arctan \frac{\text{imag}(\lambda_{\text{bend}})}{\text{real}(\lambda_{\text{bend}})}$ , where  $\lambda_{\text{bend}}$  is the eigenvalue associated to the first bending mode of the beam, “real” and “imag” are the real and the imaginary parts. Secondly, the stability is evaluated by the sign of the real part of the previously defined  $\lambda_{\text{bend}}$  and has to be negative to insure the stability of the system.

### 24.3.3 Morris Method

The Morris method [5, 6] is a sensitivity analysis that allows the more sensitive parameters to be determined. It runs a one-at-a-time input parameter variation following a preset number of trajectories. Therefore it measures the elementary effects  $d_i$  of the input vector of  $n$  parameters  $\mathbf{x} = (x_1, \dots, x_n)$  by locally evaluating the model output  $y$ . It gives:

$$d_i(\mathbf{x}) = \frac{[y(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_k) - y(\mathbf{x})]}{\Delta} \quad (24.2)$$

where  $\Delta$  is the step in the input parameter between two evaluations. Figure 24.2 shows the example of a 2-input/4-levels problem where  $\Delta = \frac{1}{3}$ . The arrows highlight the set of parameters needed to evaluate the elementary effects. Afterwards, two indicators are returned. The mean  $\mu$  estimates the global effect on the output and the standard deviation  $\sigma$  evaluates second order effects or more (couplings between parameters for instance).

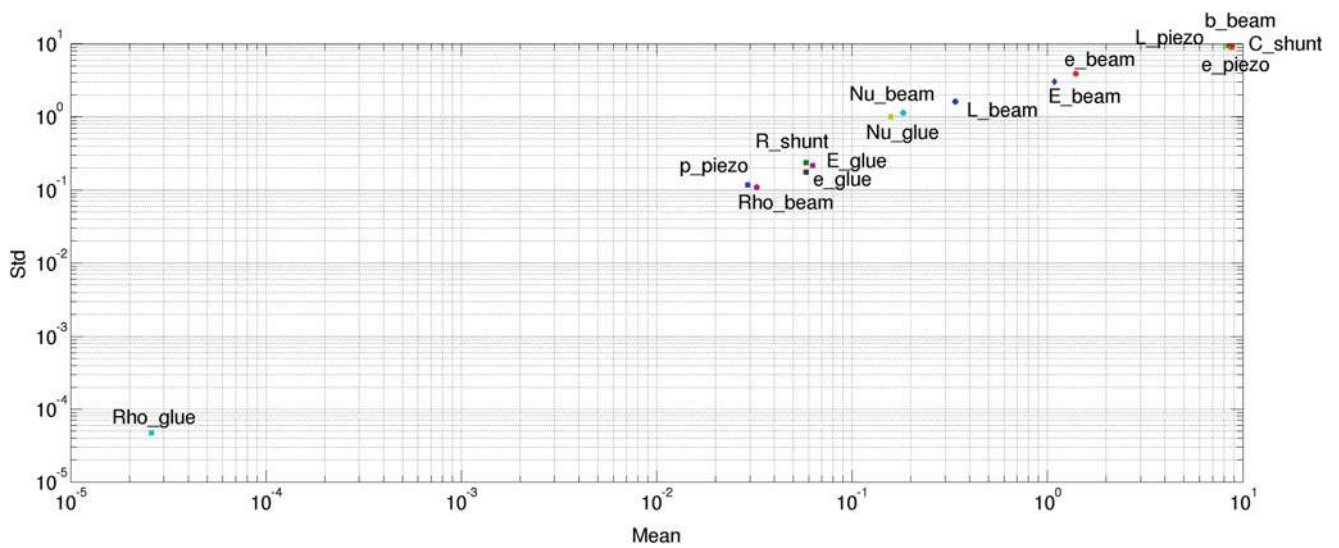
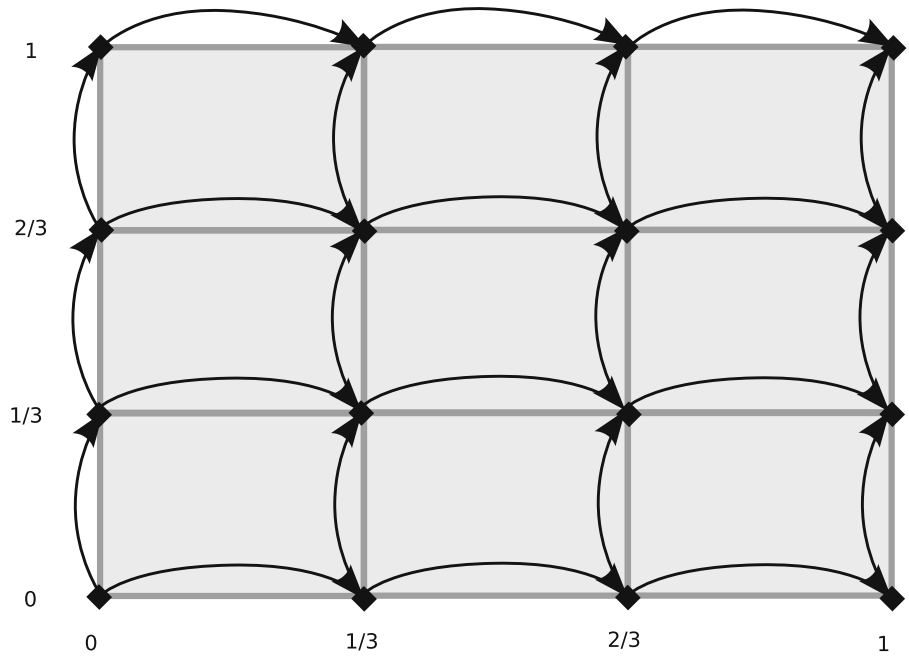
## 24.4 Results

Figure 24.3 shows the variance/mean plot of the variables for the damping objective after a 100-levels/100-trajectories simulation. Four parameters are by far more sensitive than the others, namely the dimensions of the transducer and the capacitance of the shunt. The ranking of the variables, based on the mean is also a relevant tool to evaluate the sensitivity of the output, as shown on Fig. 24.4. The variance/mean plot shows that there are some non linear effects. The more sensitive parameters are the ones concerning the transducer and its shunt capacitance. It can be noted that the glue layer between the transducer and the beam has no significant effect on the damping. The stability, based on the real part of the first eigenvalue is mostly affected by the geometry of the transducer and the capacitance of the shunt (see Fig. 24.5).

## 24.5 Conclusion and Perspectives

The sensitivity analysis shows that the most sensitive parameters are the geometry of the transducer and the capacitance of the shunt. The glue layer between the beam and the transducer has no significant effect on the system damping efficiency nor on its stability. Previous experiments have already shown such behaviour and it is now confirmed by the simulations presented in this paper. If the geometry of the transducer is important, its fabrication is well mastered. Nevertheless, the capacitance of

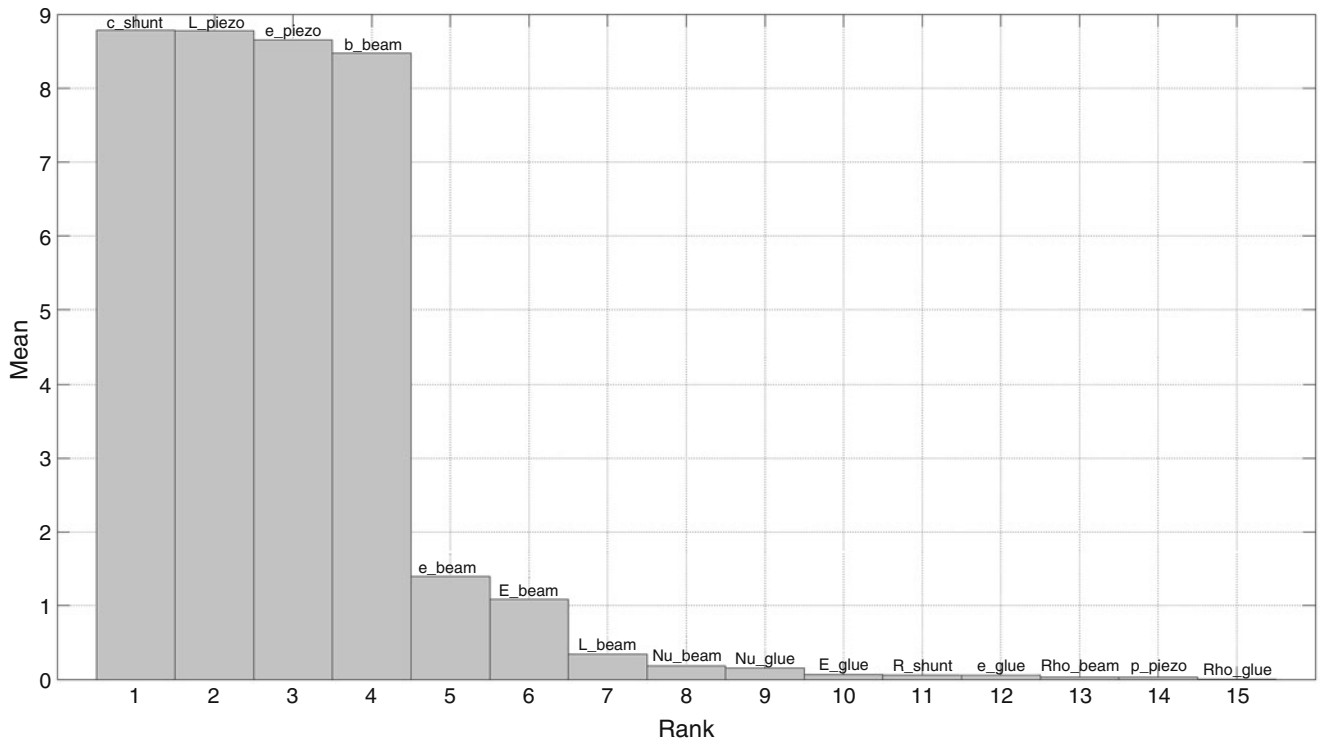
**Fig. 24.2** Example of 2-input/4-levels problem where the arrows highlight the set of parameters needed to evaluated each elementary effect



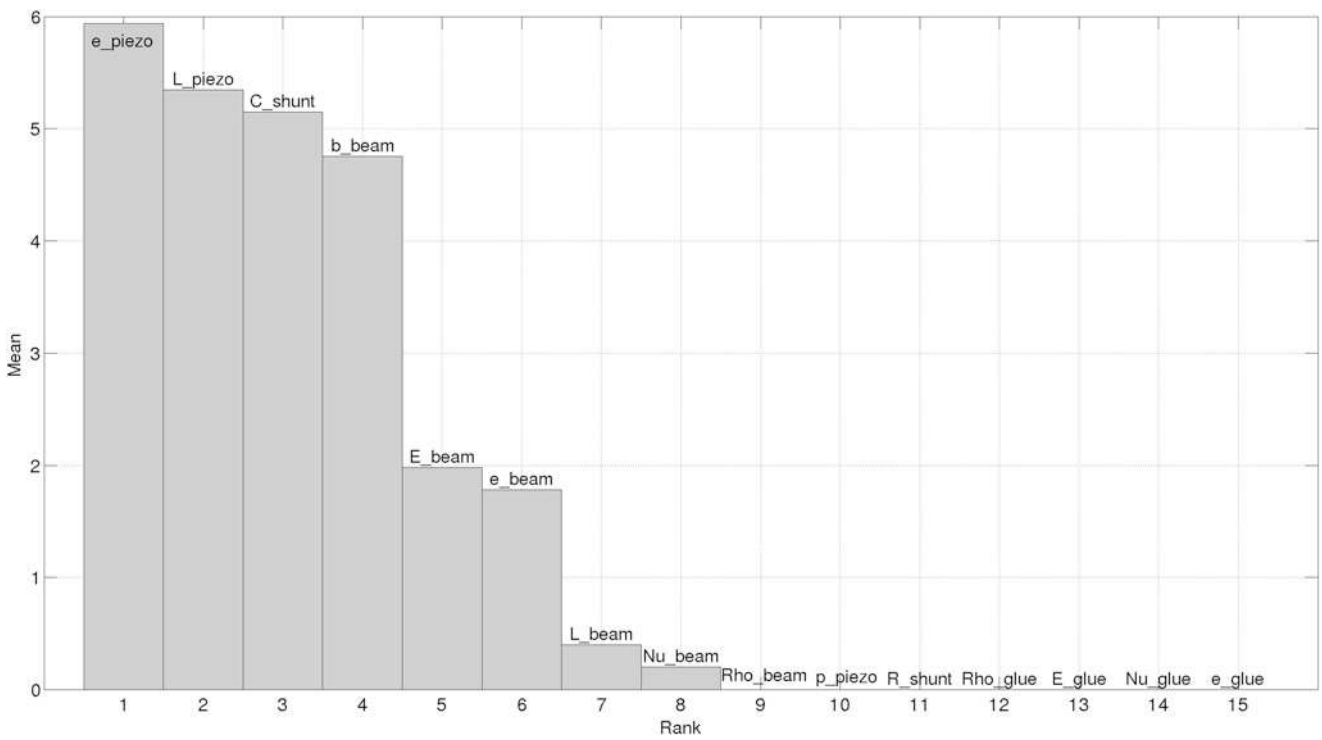
**Fig. 24.3** Standard deviation of the damping objective variable towards its mean value, sensitivity analysis on the geometric and piezoelectric parameters performed using the Morris method

the shunt is more difficult to master because it is implemented by a digital electrical circuit whose resolution has to be fine enough to insure the stability and the performances of the system. Now that the sensible parameters are identified, a robust design study is on-going to evaluate an optimal design for such a system and further on distributed applications.

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**Fig. 24.4** Ranking of the design parameters according to their influence on the damping variable objective variable



**Fig. 24.5** Ranking of the design parameters according to their influence on the stability variable objective variable

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