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Extension of Modal Reduction Methods to Non-Linear Coupled Structure-Acoustic Problems

Youssef GERGES — Emeline SADOULET-REBOUL — Morvan OUISSE — Noureddine BOUHADDI

Institut FEMTO-ST, UMR 6174, Département de Mécanique Appliquée
Université de Franche-Comté,
24 rue de l'Epitaphe,
25000 Besançon France

youssef.gerges@edu.univ-fcomte.fr

RÉSUMÉ. *Cet article propose une méthode de réduction robuste dédiée aux problèmes vibroacoustiques non-linéaires. Le type de non-linéarité envisagé dans cette étude est géométrique localisée. La méthode de réduction introduite consiste à enrichir la base modale tronquée du système découplé sans non-linéarité par des réponses statiques de la structure dues à des efforts unitaires sur les degrés de liberté non-linéaires ainsi que par la réponse statique du fluide induite par le couplage avec la structure. Les réponses temporelles d'une plaque appuyée sur une cavité acoustique et d'un tuyau sonore suspendu sont simulées pour montrer l'efficacité de la méthode proposée.*

ABSTRACT. *This paper propose a robust reduced method dedicated to non-linear vibroacoustic problems in the context of localized geometrical non-linearities. The method consists in enriching the truncated uncoupled modal basis of the linear model by a static response due to unit forces on the non-linear degrees of freedom and by the static response of the fluid due to the interaction with the structure. To show the effectiveness of the proposed method, numerical simulations of responses of an elastic plate closing an acoustic cavity and a hang-on exhaust are performed.*

MOTS-CLÉS : *Réduction de modèle, Interaction fluide structure, Vibration non-linéaire, Couplage vibroacoustique, Intégration temporelle.*

KEYWORDS: *Reduced model, Fluid structure interaction, Nonlinear vibration, Vibro-acoustic coupling, Time integration.*

1. Introduction

This paper presents a reduced model method dedicated to non-linear vibroacoustic problems. Modeling this problem still remains a heavy exercise for industrial applications. The starting point is the formulation choice ; structural displacement and acoustic pressure (u, p) are used in this paper. Modeling the problem using the finite element method (FEM) leads to an unsymmetrical matrix system with large dimensions. Model reduction seems to be a required tool for optimisation and robustness studies. Most of the applications do not consider the non-linear behaviour which is found in many examples. The few of them treating non-linear effect use specific methods dedicated to non-linear dynamics. Problems are solved through the proper orthogonal decomposition or the non-linear normal modes (Amabili *et al.*, 2007; Amabili, 2008). Modeling the problem using an interaction between the finite element method (structure) and the boundary element method (BEM) for fluid is also possible (Soares-Jr *et al.*, 2005). This study is based on the development of a reduced basis dedicated for non-linear vibroacoustic problem where geometrical non-linearities are localized. The formulation of the non-linear vibroacoustic problem with structural displacements and acoustic pressure formulation is first carried out. Reduced order method of the studied problem is presented in section 3. Newmark algorithm dedicated to temporal integrations is studied in section 4 (Géradin *et al.*, 1997; Bathe, 1982). Section 5 presents the predictor indicators used for comparison with the full model.

2. Formulation of non-linear vibroacoustic problem

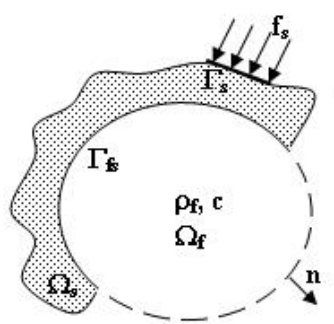

Figure 1. *Vibroacoustic problem*

The studied vibroacoustic problem is presented in figure 1. Let us consider a fluid-filled domain Ω_f coupled with a structure Ω_s presenting large displacements. Γ_{fs} presents the coupled surface. The structure is submitted to volume and surface loads $f_v(t)$ and $f_s(t)$.

2.1. *Structural problem formulation*

The structural equation of motion is deduced from the virtual work principle written as :

$$\delta \int_{t_1}^{t_2} (U + T) dt = 0, \qquad [1]$$

where :

– T is the kinetic energy :

$$T = \frac{1}{2} \int_{\Omega_s} \rho_s \dot{u}\dot{u} d\Omega, \qquad [2]$$

– U is the potential energy defined as the sum of the strain energy U_{strain} and the potential energy due to the applied load U_{pot} :

$$U_{pot} = -\int_{\Omega_s} f_v(t) u d\Omega - \int_{\Gamma_s} f_s(t) u d\Gamma, \qquad [3]$$

$$U_{strain} = \frac{1}{2} \int_{\Omega_s} \tau^T D \tau d\Omega. \qquad [4]$$

D is the material stiffness matrix and τ is the 2nd order Green Lagrange strain tensor related to the displacement field. Large displacement theory with small strain is considered. This leads to a strain tensor written as follow :

$$\tau(u) = \underbrace{\frac{1}{2}(\nabla u + \nabla u^t)}_{\tau^l} + \underbrace{\frac{1}{2} \nabla u^t \nabla u}_{\tau^{nl}} \qquad [5]$$

where τ^l and τ^{nl} respectively represent the linear and non-linear parts of the tensor.

Linear-elastic material leads to a constitutive equation written as :

$$S = D\tau, \qquad [6]$$

where S is the 2nd Piola-Kirchhoff stress tensor.

The equation of motion becomes (Morand *et al.*, 1992; Pérignon, 2004) :

$$\begin{aligned} \int_{\Omega_s} S^T \tau^l(\delta u) d\Omega + \int_{\Omega_s} S^T \tau^{nl}(u, \delta u) d\Omega + \int_{\Omega_s} \rho_s \frac{\partial^2 u}{\partial t^2} \delta u d\Omega = \\ \int_{\Omega_s} f_v \delta u d\Omega + \int_{\Gamma_s} f_s \delta u d\Gamma + \int_{\Gamma_{fs}} p n \delta u d\Gamma. \end{aligned} \qquad [7]$$

The term $\int_{\Gamma_{fs}} p n \delta u d\Gamma$ represents the action of the fluid on the structure.

2.2. *Fluid problem formulation*

This section recalls the formulation associated to the fluid domain Ω_f.

The equilibrium state in term of pressure formulation is expressed by equation :

$$\Delta p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}. \qquad [8]$$

The boundary condition applied on the coupling area Γ_{fs} is expressed by the sliding condition :

$$-\frac{\partial p}{\partial n} = \rho_f \frac{\partial^2 u}{\partial t^2}. \qquad [9]$$

By applying the Green formula, the variational formulation of the fluid is written as (Morand *et al.*, 1992) :

$$\frac{1}{\rho_f}\int_{\Omega_f} \nabla p \nabla \delta p d\Omega + \frac{1}{\rho_f c^2}\int_{\Omega_f} \frac{\partial^2 p}{\partial t^2}\delta p d\Omega + \int_{\Gamma_{fs}} \frac{\partial^2 u}{\partial t^2} n \delta p d\Gamma = 0 \qquad [10]$$

2.3. *Coupled formulation*

Coupling both problem leads to :

$$\begin{cases} \int_{\Omega_s} S^T \tau^l(\delta u) d\Omega + \int_{\Omega_s} S^T \tau^{nl}(u, \delta u) d\Omega + \int_{\Omega_s} \rho_s \frac{\partial^2 u}{\partial t^2}\delta u d\Omega - \\ \qquad\qquad \int_{\Gamma_{fs}} p n \delta u d\Gamma = \int_{\Gamma_s} f_s \delta u d\Gamma \\ \int_{\Omega_f} \nabla p \nabla \delta p d\Omega + \frac{1}{c^2}\int_{\Omega_f} \frac{\partial^2 p}{\partial t^2}\delta p d\Omega + \rho_f \int_{\Gamma_{fs}} \frac{\partial^2 u}{\partial t^2} n \delta p d\Gamma = 0 \end{cases} \qquad [11]$$

Using the finite element method, the problem can be written in a matrix form as :

$$\begin{bmatrix} M_s & 0 \\ \rho_f C^T & M_f \end{bmatrix} \begin{bmatrix} \ddot{U} \\ \ddot{P} \end{bmatrix} + \begin{bmatrix} K_s(U) & -C \\ 0 & K_f \end{bmatrix} \begin{bmatrix} U \\ P \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \qquad [12]$$

– M_s and K_s are the mass and stiffness matrices of the structure. K_s is a function of U ; in geometrical non-linearities it is the sum of a linear term arising from the linear problem K_s^l and a non-linear term. In localized non-linearity case, K_s can be written as :

$$K_s(U) = K_s^l + K_s^{nl} diag(U)^{n-1}, \qquad [13]$$

where K_s^{nl} is the hardening coefficient of the non-linearity and n is the degree of non-linearity.

– M_f and K_f are respectively the matrices corresponding to the discretization of kinematic energy and compressibility matrix of fluid.

– C is the coupling matrix corresponding to the action of the structure on the fluid or vice versa.

– F is the structure excitation function of the time t.

The matrix system can be expressed as a differential equation written as :

$$M\ddot{X} + K(X)X = f(t). \qquad [14]$$

Solving this kind of equation depends on the nature of $f(t)$. Using the modal decomposition method to bring the problem diagonal is not possible. This is due to the non-linear character of the problem.

If $f(t)$ is harmonic, one of the efficient techniques to solve the problem is the harmonic balance method (Nayfeh *et al.*, 1995).

Using an arbitrary excitation requires the implementation of time integration method (Géradin *et al.*, 1997; Bathe, 1982).

In both cases, solving the problem is time consuming. A reduced order method dedicated to this kind of problem is required. The model basis should take into account both non-linear and coupling effects. The reduced order method in this paper is presented in the following paragraph.

3. Reduced order method

3.1. *Uncoupled modal basis*

As it was mentioned in the previous paragraph, a reduced order method is required for modeling non-linear vibroacoustic problem. The proposed basis should be robust and easy to implement. A first approximation model is to use the uncoupled modal basis issued from the in vacuo linear structural problem $(K_s^{nl} = 0)$ and the rigid wall cavity problem. This leads to a finite element approximation written as follow :

$$\begin{bmatrix} U \\ P \end{bmatrix} \approx \begin{bmatrix} T_{smb} & 0 \\ 0 & T_{fmb} \end{bmatrix} \begin{bmatrix} q_s \\ q_f \end{bmatrix}, \qquad [15]$$

where T_{smb} and T_{fmb} respectively represent the truncated structure and fluid modal bases.

3.2. *Non-linear enriching*

Localized non-linear behaviour is considered as a perturbation modifying the linear response. This perturbation is assimilated to an excitation force. To take into account this force in the non-linear reduced model, a linear static response of the in vacuo structure due to a unit load on each non-linear degree of freedom is considered :

$$\Delta T_{snl}^i = (K_s^l)^{-1} f^i, \qquad [16]$$

where i is the ith non-linear degree of freedom and

$$f^i = [0...1...0]^T \,. \qquad [17]$$

Orthogonalization is necessary to ensure good conditioning of the problem. This is realized with a singular value decomposition (SVD).

$$T_s = [T_{smb}|\Delta T_{snl}]_{SVD} \,. \qquad [18]$$

This leads to a new finite element approximation written as follows :

$$\begin{bmatrix} U \\ P \end{bmatrix} \approx \begin{bmatrix} T_s & 0 \\ 0 & T_{fmb} \end{bmatrix} \begin{bmatrix} q_s \\ q_f \end{bmatrix} . \qquad [19]$$

3.3. *Coupling enriching*

The main goal of this section is to propose a modal synthesis method that can take into account coupling effects. The homogeneous matrix formulation associated to the equation [12] in the frequency domain is written as :

$$\left(\begin{bmatrix} K_s(U) & -C \\ 0 & K_f \end{bmatrix} - \omega^2 \begin{bmatrix} M_s & 0 \\ \rho_f C^T & M_f \end{bmatrix} \right) \begin{bmatrix} U \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \qquad [20]$$

The term $\rho_f C^T U$ can be interpreted as an excitation of the fluid due to the structure that modifies the fluid behaviour. For the fluid part, we have :

$$\left(K_f - \omega^2 M_f\right) P = \omega^2 \rho_f C^T U, \qquad [21]$$

This excitation is not known but it is approximated by projecting the displacement on the structural basis figured out in the previous section [18]. Updating the fluid basis by including this response leads to a new reduced basis. This new residual basis is written as (Tran, 2009; Tran *et al.*, 2010) :

$$\Delta T_{fs} = \left(K_f - \omega_c^2 M_f\right)^{-1} C^T T_s, \qquad [22]$$

it should be decomposed in singular value to ensure orthogonality :

$$\begin{bmatrix} U \\ P \end{bmatrix} \approx \begin{bmatrix} T_s & 0 \\ 0 & T_f \end{bmatrix} \begin{bmatrix} q_s \\ q_f \end{bmatrix} , \qquad [23]$$

where,

$$T_f = [T_{fmb}|\Delta T_{fs}]_{SVD} \,. \qquad [24]$$

3.4. *Coupling with heavy fluid*

The reduced order method presented above is dedicated to the non-linear vibroacoustic problem with light coupling. In the case where the fluid density is not neglected compared to the structure, the coupling is considered heavy. The fluid behaviour impacts the structure and should be considered in the modal synthesis. Linear static response of the structure due to the fluid effect is written as :

$$U = K_s^{l-1} C P. \qquad [25]$$

This excitation is not known but it is approximated by projecting the pressure P on the fluid basis figured out in the section bellow [24]. Enriching the structural basis by the static residual response due to the heavy fluid impact leads to a new basis written as follows (Tran, 2009; Tran *et al.*, 2010) :

$$\begin{bmatrix} U \\ P \end{bmatrix} \approx \begin{bmatrix} T_{shf} & 0 \\ 0 & T_f \end{bmatrix} \begin{bmatrix} q_s \\ q_f \end{bmatrix}, \qquad [26]$$

where

$$T_{shf} = [T_s | \Delta T_{sf}]_{SVD} \,. \qquad [27]$$

and

$$\Delta T_{sf} = K_s^{l-1} C T_f. \qquad [28]$$

Once the reduced basis has been established, the model reduction is performed as follows :

$$X = \begin{bmatrix} U \\ P \end{bmatrix} = T \begin{bmatrix} q_s \\ q_f \end{bmatrix} = T q \qquad [29]$$

T is the reduced basis defined in the sections above. Dynamics equilibrium defined in equation [14] becomes :

$$M_r \ddot{q} + K_r\left(q\right) q = f_r\left(t\right), \qquad [30]$$

where :

– M_r is the reduced mass matrix ; $M_r = T^T M T$
– K_r is the reduced stiffness matrix ; $K_r = T^T K T$
– f_r is the reduced force vector ; $f_r = T^T f$

4. Non-linear temporal response

As it was mentioned before, the strategy to solve the dynamic equations depends on the excitation. Equilibrium equation in presence of dissipative energy modeled by a damping matrix D is written as :

$$r(X) = M\ddot{X}(t) + D\dot{X}(t) + K(X)X(t) - f(t) = 0, \qquad [31]$$

In the case of an arbitrary excitation, numerical integration in the time domain is required. It consists in calculating the iterative state of the system -displacement, velocity, acceleration- as a function of time progress. A widely used technique is the Newmark algorithm ; compared to other numerical integration, the Newmark algorithm is relatively stable. It uses the following state description :

$$A_n = \begin{bmatrix} X_n \\ \dot{X}_n \\ \ddot{X}_n \end{bmatrix} = \begin{bmatrix} X(t_n) \\ \dot{X}(t_n) \\ \ddot{X}(t_n) \end{bmatrix}, \qquad [32]$$

where A_n represents the system state calculated at time t_n. A_{n+1} represents the system state at one time step later, it is written as (Géradin *et al.*, 1997; Bathe, 1982) :

$$\begin{cases} X_{n+1} = X_n + h\dot{X}_n + h^2(\frac{1}{2} - \beta)\ddot{X}_n + h^2\beta\ddot{X}_{n+1} \\ \dot{X}_{n+1} = \dot{X}_n + (1-\gamma)h\ddot{X}_n + \gamma h\ddot{X}_{n+1} \\ \ddot{X}_{n+1} \end{cases} \qquad [33]$$

where h is the chosen time step, γ and β are the parameters of the Newmark algorithm. $\ddot{X}_{n+1}$ is calculated through the dynamic equation [31].

Non-linear behaviour can lead to bad predictions. A residue evaluation is considered to ensure dynamic equilibrium. The residual equation is evaluated at each time step through the linearized first order equation :

$$r_{n+1} + S^i_{n+1}\Delta X = 0, \qquad [34]$$

where $S^i_{n+1} = \left[\frac{\partial r}{\partial X}\right]_{X^i_{n+1}} = K^t + \frac{\gamma}{\beta h}D^t + \frac{1}{\beta h^2}M$ is the Jacobian matrix of r ; K^t and D^t are the Jacobian matrices of K and D respectively and $\Delta X = -(S^i_{n+1})^{-1}r_{n+1}$ the displacement correction. i is the ith iteration of the residue evaluation (correction). Experience shows that for null acceleration at the beginning of every iteration, and by correcting the approximation during iterations, this can provide stable and fast process. For the same reason, a constant Jacobian matrix can be used in the correction algorithm. The advantage compared to the evaluation of the Jacobian matrix at each step is the inversion procedure. Otherwise, convergence to the equilibrium state will be longer. Choosing one of these techniques will depend on the application.

The same algorithm is used for the integration of the reduced model [30]. Matrices M K and D and vectors X and f are replaced by their corresponding reduction (M_r, K_r, D_r, q and f_r respectively). Once a reduced state is evaluated, physical response is performed using the equation [29].

The numerical integration algorithm is resumed as follows :

Prediction

$$\boxed{\begin{array}{rcl} X_{n+1} & = & X_n + h\dot{X}_n + h^2(\frac{1}{2}-\beta)\ddot{X}_n \\ \dot{X}_{n+1} & = & \dot{X}_n + (1-\gamma)h\ddot{X}_n \\ \ddot{X}_{n+1} & = & 0 \end{array}}$$

$$\downarrow$$

Residues evaluation

$$\boxed{\begin{array}{rcl} \epsilon, \quad r_{n+1} & & \\ \textbf{while } |r_{n+1}| & > & \epsilon \\ r_{n+1} & = & M\ddot{X}_{n+1} + D\dot{X}_{n+1} + K(X_{n+1})X_{n+1} - f_{n+1} \end{array}}$$

$$\downarrow$$

Correction

$$\boxed{\begin{array}{rcl} \Delta X & = & -(S^i_{n+1})^{-1} r_{n+1} \\ X^{i+1}_{n+1} & = & X^i_{n+1} + \Delta X \\ \dot{X}^{i+1}_{n+1} & = & \dot{X}^i_{n+1} + \frac{\gamma}{\beta h}\Delta X \\ \ddot{X}^{i+1}_{n+1} & = & \ddot{X}^i_{n+1} + \frac{1}{\beta h^2}\Delta X \\ \textbf{end while} & & \end{array}}$$

5. Prediction indicators

Results comparison tools are based on statistic indicators associated to the structure and fluid responses in addition to the energetic indicators that result from acoustic and kinetic energies.

5.1. *Temporal moments*

Temporal moments are used in transient responses in order to quantify the comparison between different models. It is used to qualify the model response. The $i^{\,th}$ order of the temporal moment of a response $y(t)$ is defined as (Masson *et al.*, 2006) :

$$M_i = \int_{-\infty}^{+\infty} (t - t_s)^i \left(y(t)\right)^2 dt, \qquad [35]$$

where t_s represents the temporal shift and i the moment index order.
In this case, the temporal moment M_i is defined for $t_s = 0$ and normalized as follows :

$$\begin{cases} E = M_0, \text{ Energy } (m^2 s) \\ T = \frac{M_1}{M_0}, \text{ Central time (centroid) } (s) \\ D^2 = \frac{M_2}{M_0} - \left(\frac{M_1}{M_0}\right)^2, \text{ Root mean square duration } (s^2) \end{cases} \qquad [36]$$

5.2. *Energetic indicators*

The indicators that are used are the acoustic energy and the mean square velocity. The acoustic energy is defined by the sum of the kinetic and potential energies in the fluid domain. The discretized form is expressed as follow :

$$E_a = \frac{1}{2\rho_f} P K_f P + \frac{1}{2\rho_f} \dot{P} M_f \dot{P}. \qquad [37]$$

The discretized form of the mean square normal velocity is defined as :

$$\bar{V_n}^2 = \frac{1}{|S_s|} V M_{v_n} V, \qquad [38]$$

where M_{v_n} comes from the discretization of $\int v_n^2 dS$.

6. Application

In order to illustrate the proposed idea, two applications are presented in this paragraph. A parallelepipedic acoustic cavity fixed with a thin plate presenting non-linear localized behaviour and an exhaust filled of air with non-linear links. Two types of excitation are studied. For the acoustic cavity, the response of the system due to an impact of $500N$ is considered. For the exhaust, a sweep sine in the frequency band of the model is exciting the system. The full model response is compared to the results obtained changing the reduction basis. The reduced order methods presented in this application are the following :

– reduced model using the uncoupled modal basis of the system **(Modal basis)** ;

– enrichment of the uncoupled structural modal basis by the static response of the structure due to the unit forces on the non-linear degrees of freedom **(NL residues)** ;

– enrichment of the uncoupled fluid basis only by the static response of the fluid due to the presence of the structure **(Coupling residues)** ;

– enrichment of the uncoupled structural modal basis by the static response of the structure due to the unit forces on the non-linear degrees of freedom and enrichment of the uncoupled fluid basis by the static response of the fluid due to the presence of the structure and by taking into account the non-linear behaviour **(Coupling + NL residues)**.

6.1. *Acoustic cavity*

The first example is an academic application to illustrate the non-linear coupling effects. Let us consider a thin plate $(0.654 \times 0.527 \times 0.003m^3)$ with localized non-linearities (17 at all) fixed on an acoustic cavity $(0.654 \times 0.527 \times 0.6m^3)$ filled with air. Figure 2 shows the finite element model of the system. The model size is about 11000

dofs (3000 structural 8000 fluid dofs). The model frequency band is $[0 - 300Hz]$; reduced model size is about 150 dofs against 11000 for the full model. Dissipation energy is introduced by modeling a proportional damping deduced from the first four structural mode with a damping ratio of 0.1%. Structure is excited using an impact excitation of $500N$ in a period of $10ms$; figure 3 shows the temporal and spectral representation of the impact.

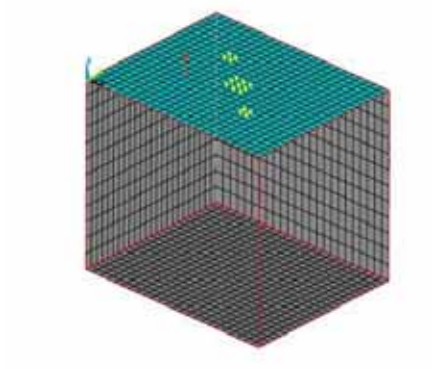

Figure 2. *Acoustic cavity*

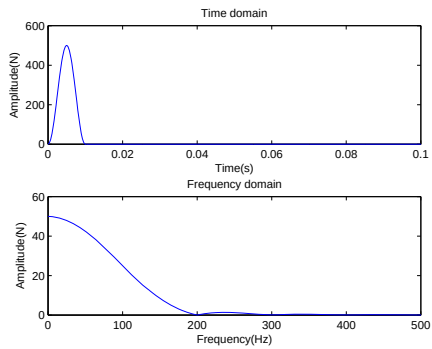

Figure 3. *Impact spectrum*

The temporal moments for a period of $0.1s$ of the structural displacement and the acoustic pressure for the considered model reduction strategies are presented in tables 1 and 2. Structural displacement at non-linear dofs and prediction indicators are presented in figures 4 and 5. The linear model response is also presented in the same figures to have an idea of the impact of the non-linear effects.

	T	E	D
Full model	0.0439	6.3637e-06	8.48869e-04
Modal basis	0.0438	6.1156e-06	8.4772e-04
Error / Full model (%)	-0.2	-3.9	-0.1
Coupling residues	0.0438	6.0208e-06	8.4647e-04
Error / Full model (%)	-0.2	-5.4	-0.3
NL residues	0.0439	6.3331e-06	8.4906e-04
Error / Full model (%)	0	-0.5	0.02
Coupling + NL residues	0.0439	6.3633e-06	8.4882e-04
Error / Full model (%)	0	-0.00	-0.00

Tableau 1. *Temporel moments for structural displacements*

In this application, results show the need to enrich the bases. In fact, temporal moments for structural displacements show the need to take into account non-linear effects and temporal moments for acoustic pressure show the need to take into account

	T	E	D
Full model	0.0529	2.7754e+06	8.1539e-04
Modal basis	0.0538	2.8350e+06	8.0777e-04
Error / Full model (%)	1.7	2.1	-0.9
Coupling residues	0.0532	2.7550e+06	8.0692e-04
Error / Full model (%)	0.6	-0.7	-1.03
NL residues	0.0540	2.8928e+06	8.1152e-04
Error / Full model (%)	2.1	4.2	-0.5
Coupling + NL residues	0.0529	2.7745e+06	8.1512e-04
Error / Full model (%)	0	-0.03	-0.03

Tableau 2. *Temporel moments for acoustic pressure*

the coupling effects. In both cases modal bases seems to be insufficient as a model reducing tool.

Let us consider the same application as above. Replacing the air by a heavy fluid as water, leads to a new model. As it was mentioned before, reduced order method should take into account heavy fluid behaviour. Figure 6 presents the predictive indicators of the full model compared to the reduced model taking account and not the heavy fluid. Results show the need to take into account heavy fluid effects. Figures show the non-convergence of the model that do not consider the heavy fluid effect compared to the full model.

6.2. *Exhaust*

Let us consider an air cavity in a fold exhaust suspended with 18 non-linear springs. The exhaust dimensions are $L1 + L2 + L3 = 0.2 + 0.25 + 0.2m$, $R1 = R3 = 0.05m$ and $R2 = 0.125m$ (figure 7). The finite element model contains 8500 dofs (5000 for the structure and 3500 for the fluid). The model is valid in the frequency band $[0 - 450Hz]$. A sweep sine in the frequency band of interest is used to excite the structure with a $1N$ amplitude. Figure 8 shows the temporal and spectral representations of the excitation. Proportional damping is used to introduce dissipation in the model. It is deducted from the first four structural damped modes with a damping ratio of 0.1%. Reduced order methods presented previously are compared to the full model. The reduced model size is 200 dofs. The temporal moments of structural displacements and acoustic pressure are presented in tables 3 and 4. The acoustic energy and the mean square velocity are presented in the figures 9 and 10.

In this application, results are not the same as the application before. It shows the need of enriching ; but it shows also that enriching can diverge the response compared to the full model. It is the case with the reduced model using the coupling effects.

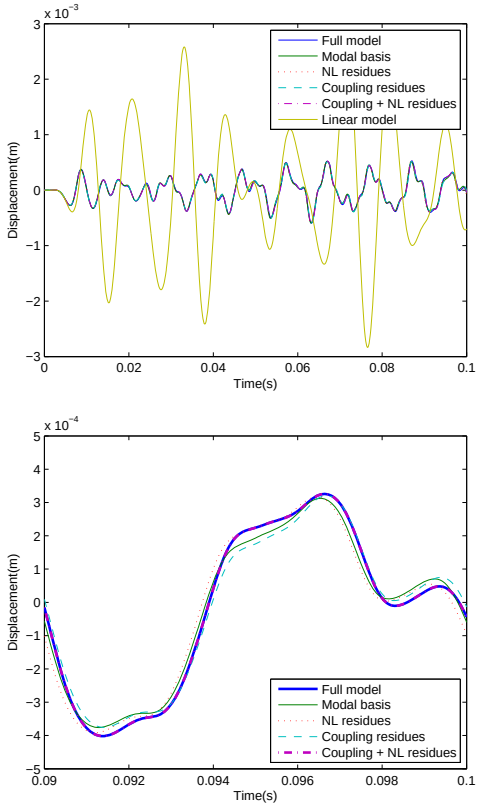

Figure 4. *Displacement as a function of time. (up) in a period of 0.1s with the linear model, (down) in the period [0.09-0.1]s*

In the same time, enriching using only non-linear effects seems to be insufficient. It predict the same behaviour as the reduced model using the modal basis.

Both applications show the need for the reduced model tool. It shows also the necessity of enriching to ensure the good converging. Using the reduced model tool provides a benefit in time consuming. In these applications the calculating time was divided by 10 compared to the full model. Concerning the result precision, the temporal moments show the necessity on enriching the basis. In fact, according to the response, enriching should take into account non-linear and coupling effects to converge to the solution of the full model. In the case of heavy fluid, numerical results shows the limit of the proposed method and the needs to consider the heavy fluid in the reduced basis to ensure good predicting.

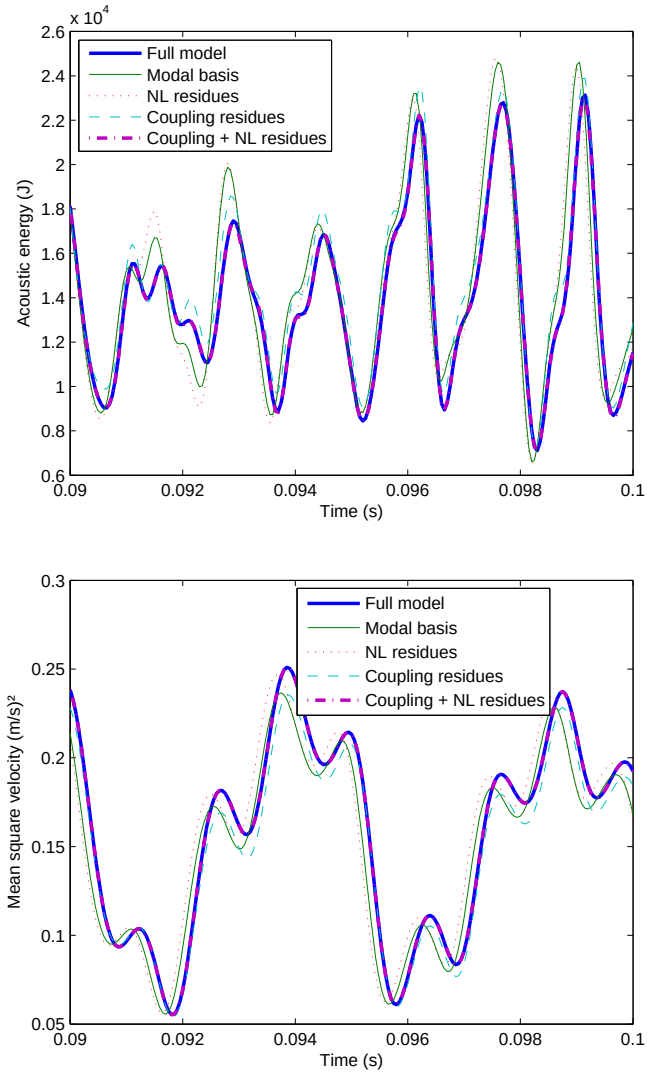

Figure 5. *Prediction indicators - light coupling*

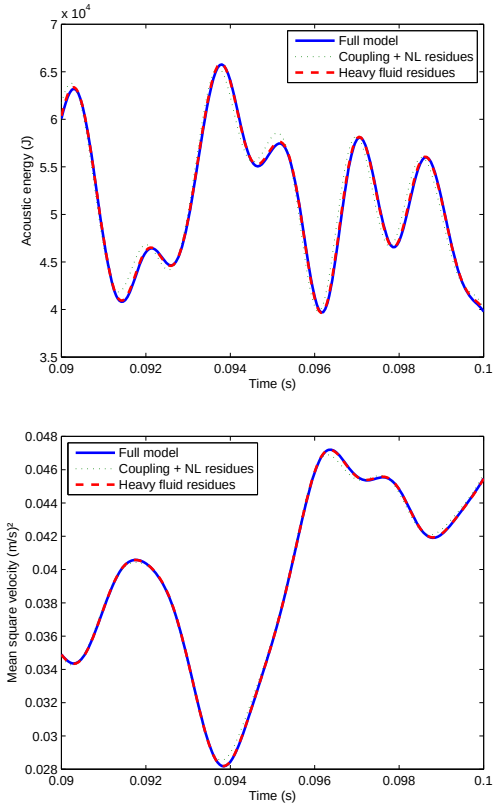

Figure 6. *Prediction indicators - heavy coupling*

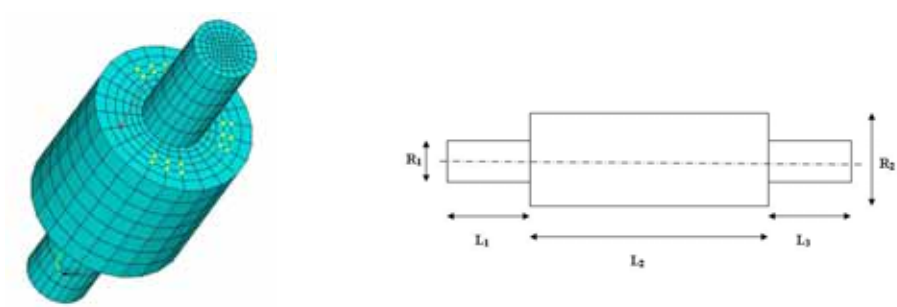

Figure 7. *Exhaust*

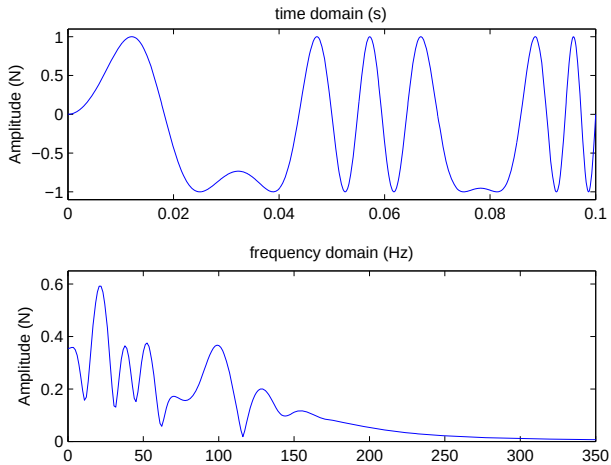

Figure 8. *Excitation sine sweep*

	D	E	T
Full model	4.1211E-04	1.1772E-04	0.0650
Modal basis	4.1025E-04	1.1633E-04	0.0646
Error / Full model (%)	-0.5	-1.2	-0.6
Coupling residues	4.1026E-04	1.2074E-04	0.0651
Error / Full model (%)	-0.4	2.6	0.2
NL residues	4.1018E-04	1.1627E-04	0.0646
Error / Full model (%)	-0.5	-1.2	-0.6
Coupling + NL residues	4.1197E-04	1.1739E-04	0.0649
Error / Full model (%)	-0.03	-0.3	-0.2

Tableau 3. *Temporal moments for structural displacements*

7. Conclusion

A reduced order method adapted to non-linear coupling problems has been presented. It is required for localized geometrical non-linear problems and is in the process of being extended to other types of non-linearities. It consists in enriching the uncoupled modal basis by residues taking into account non-linearity and coupling conditions. An example in the temporal space has been studied. An application of the approach in the frequency domain is conceivable using the harmonic balance method adapted for geometric non-linearities.

	D	E	T
Full model	4.3529E-04	1.2554E+05	0.0670
Modal basis	4.2964E-04	1.2277E+05	0.0663
Error / Full model (%)	-1.3	-2.2	-1.0
Coupling residues	4.2312E-04	1.4276E+05	0.0676
Error / Full model (%)	-2.8	13.8	0.9
NL residues	4.2961E-04	1.2276E+05	0.0663
Error / Full model (%)	-1.3	-2.2	-1.0
Coupling + NL residues	4.3179E-04	1.2583E+05	0.0669
Error / Full model (%)	-0.8	0.2	-0.1

Tableau 4. *Temporal moments for acoustic pressure*

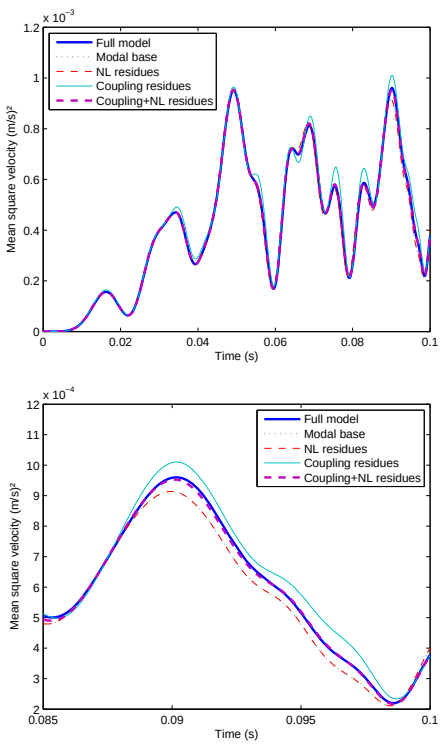

Figure 9. *Mean square velocity - (up)* $t = [0 - 0.1s]$, *(down)* $t = [0.085 - 0.1s]$

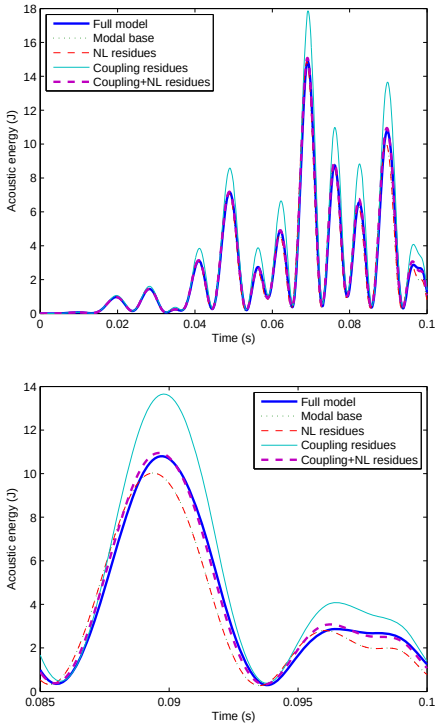

Figure 10. *Acoustic energy - (up)* $t = [0 - 0.1s]$, *(down)* $t = [0.085 - 0.1s]$

8. Bibliographie

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