

DROPLET EVAPORATION AND COMBUSTION:

- quasi-steady analysis review of assumptions and main results**
- preliminary analysis of droplet dynamics**

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2. QUASI-STEADY REGIMES

3. TRANSIENT REGIMES

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1. INTRODUCTION

1.1 OBJECTIVES

- **Present some cases of droplet evaporation and combustion**
- **Recall usual modeling assumptions**
- **Summarize some models**
- **Find simplifications which allow analytical considerations and elementary calculations**
- **Introduce the frequency response problem**

1.2 REACTIVE SCHEMES IN DROPLET EVAPORATION AND COMBUSTION PROCESSES

Sirignano

- Evaporation $L \rightarrow F$
- Combustion $F + O \rightarrow P$

Mauriot and Scherrer

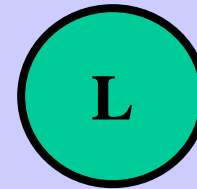
- Evaporation $L \rightarrow F$
- Decomposition $v_F F \rightarrow D$
- Combustion $v_O O + v_D D \rightarrow v_P P$

Chin and Lefebvre

- Evaporation $L + A \rightarrow F + A$

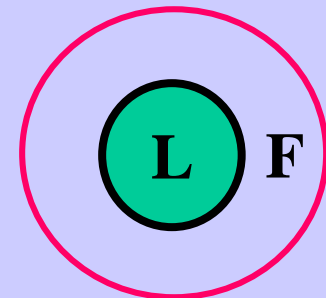
1.3 Application to spherical isolated droplets

Evaporation



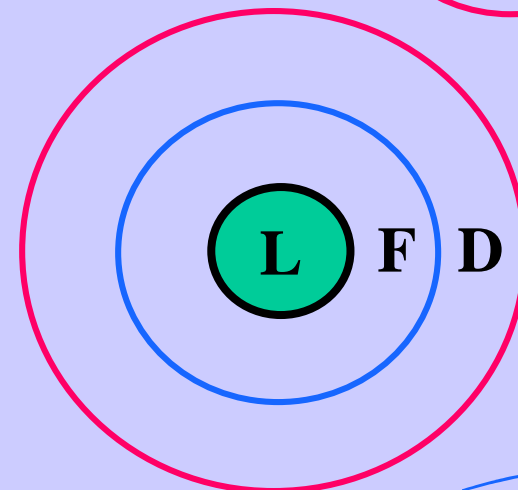
F or
F+A

Combustion without decomposition



O

Combustion with decomposition

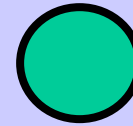


O

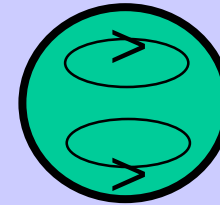
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1.4 EVAPORATION AND COMBUSTION REGIMES FOR SPHERICAL DROPLETS

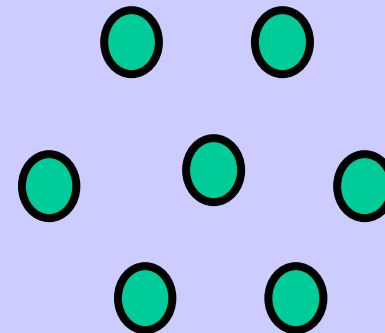
- Quasi-steady regime in a quiescent atmosphere (spherical isolated droplet)



- Transient regime with heat and moment exchanges (spherical isolated droplet with external flow)



- Collective effects : array models 1D, 2D ou 3D



1.5 ASSUMPTIONS

- Equivalence or non-equivalence of mass and thermal diffusions

$$L_e = 1 \quad \text{or} \quad L_e \neq 1$$

- With or without Stefan flow

Stefan flow (blowing effect due to evaporation) makes the laminar boundary layers thicker. One refers to the case of boundary layer above a flat plate.

Without evaporation : $\delta_{T0} = \frac{2r_s}{N_{U0} - 2}$, $\delta_{M0} = \frac{2r_s}{S_{h0} - 2}$

With evaporation : $\delta_T = F_T \delta_{T0}$, $\delta_{M0} = F_M \delta_{M0}$

$$F(B) = (1 + B)^{0,7} \frac{\text{Log}(1 + B)}{B}$$

Correlations recommended by Sirignano (in the absence of evaporation):

$$Nu_0 = 1 + (1 + Re Pr)^{1/3} f(Re)$$

$$Sh_0 = 1 + (1 + Re Sc)^{1/3} f(Re)$$

$$Re \leq 1 \Rightarrow f(Re) = 1; \quad Re \leq 400 \Rightarrow f(Re) = (Re)^{.077}$$

Evaporation in a quiescent atmosphere (Chin and Lefebvre):

$$Nu = \frac{2 \text{Log}(1 + B_T)}{B_T}$$

(is simply deduced from the definition of Nusselt number:

$$Nu = -\frac{2r_s}{T_s - T_\infty} \left(\frac{dT}{dr} \right)_s$$

and applying the classical theory of evaporation)

2. QUASI-STEADY REGIMES

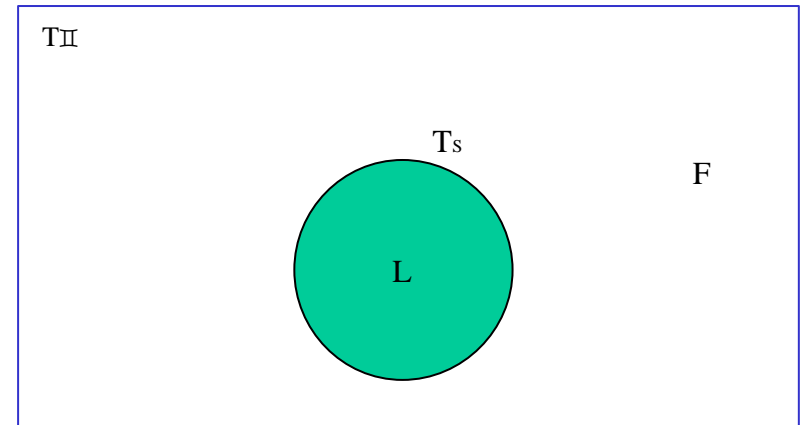
2.1 PURE FLUID :

**Uniform droplet
temperature**

**$T_s = T_L$; conduction and
evaporation.**

**Interesting example: critical
droplet**

**(T_C, p_C) in a supercritical
surrounding ($T_\infty > T_C, p_C$)
(Arias-Zugasti et al. 1999)**



2.2 MIXTURE

Conduction, diffusion and evaporation.

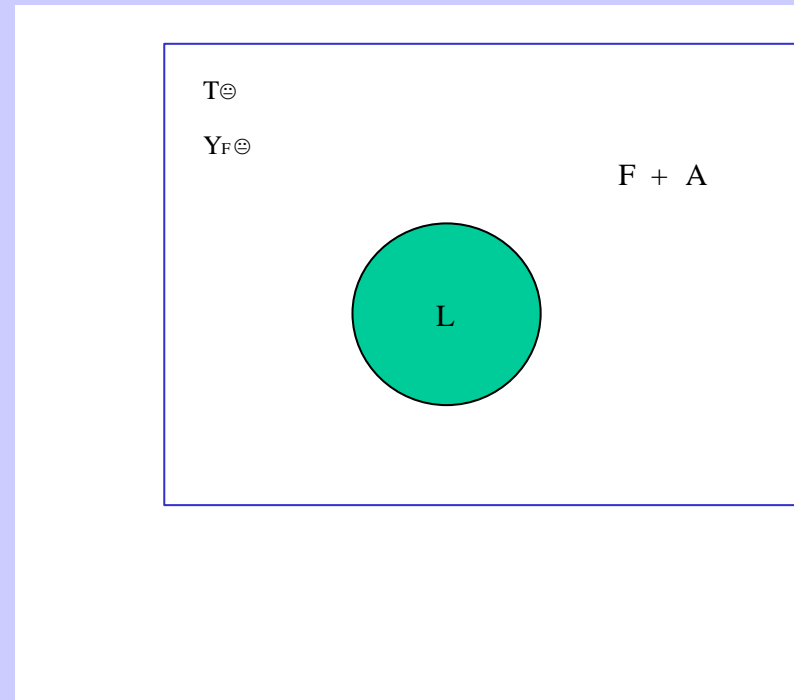
Basic equations:

Mass conservation: $\dot{M} = Cte$

And two other equations that can be solved separately using two boundary conditions (droplet surface and infinity)

$$\dot{M} \frac{dY_j}{dr} - 4\pi \rho D r \frac{d^2(rY_j)}{dr^2} = 0$$

$$\dot{M} c_p \frac{dT}{dr} - 4\pi k r \frac{d^2(rT)}{dr^2} = 0$$



2.2.1 Evaporation in a mixture: resolution of the mass diffusion equation

$$\dot{M} = 4\pi r^2 \dot{m}, \quad \dot{m} = \rho u$$

$$\dot{M} \frac{dY_j}{dr} - 4\pi \rho D r \frac{d^2(rY_j)}{dr^2} = 0$$

$$\xi_M = \frac{\dot{M}}{4\pi \rho D r} = \frac{ru}{D} \quad \Rightarrow \quad Y_j = a_j + b_j e^{-\xi_M}$$

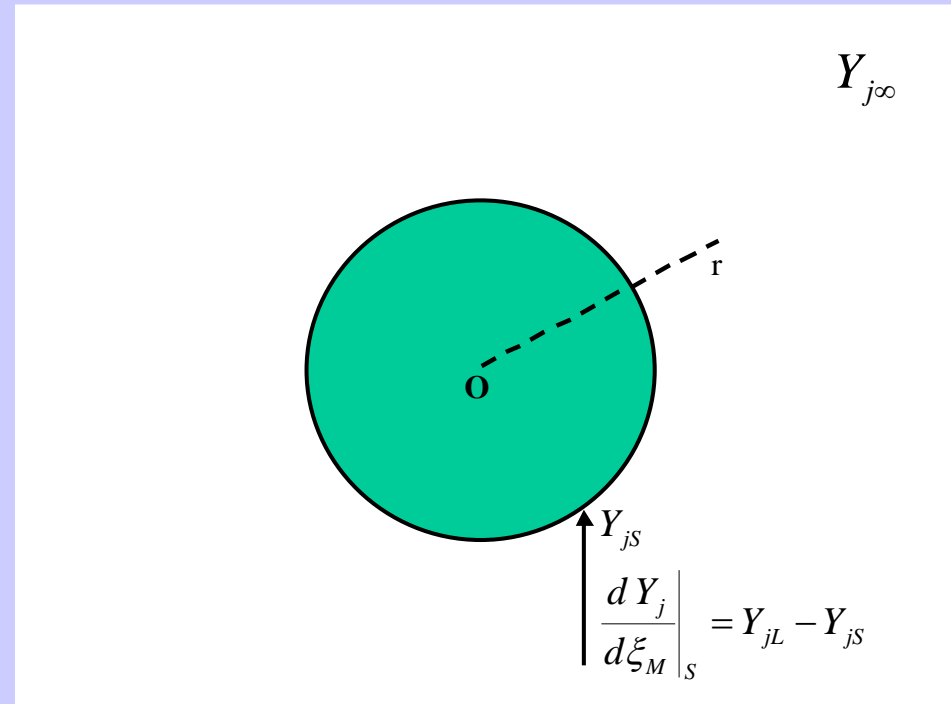
Two conditions are sufficient to determine a_j and b_j

We have in fact 3 conditions:

at infinity $r = \infty, \xi_M = 0: Y_j = Y_{j\infty}$,

and two conditions at the droplet surface r_S, ξ_{MS} :

$$Y_j = Y_{jS}, \quad \left. \frac{dY_j}{d\xi_M} \right|_S = Y_{jL} - Y_{jS}$$



Setting

$$B_M = \frac{Y_{jS} - Y_{j^\infty}}{Y_{jL} - Y_{jS}} \quad \left(B_M = \frac{Y_{FS} - Y_{F^\infty}}{1 - Y_{FS}} = \frac{Y_{A^\infty} - Y_{AS}}{Y_{AS}} \right)$$

Then

$$\xi_{MS} = \text{Log}(1 + B_M)$$

$$\Rightarrow \dot{M} = 4 \pi \rho D r_s \text{Log}(1 + B_M)$$

and one obtains the well-known law

$$D^2 = D_0^2 - K t$$

with

$$D = 2 r_s, \quad \dot{M} = - \frac{d M}{d t} = -4 \pi r_s^2 \frac{d r_s}{d t}$$

2.2.2 Evaporation in a mixture: resolution of the thermal conduction equation

$$\dot{M} = 4\pi r^2 \dot{m}, \quad \dot{m} = \rho u$$

$$\dot{M} c_p \frac{dT}{dr} - 4\pi k r \frac{d^2(rT)}{dr^2} = 0$$

$$\xi_T = \frac{\dot{M} c_p}{4\pi k r} \quad \Rightarrow \quad T = a_T + b_T e^{-\xi_T}$$

Two conditions are sufficient to determine a_T and b_T

We have in fact 3 conditions:

at infinity $r = \infty, \xi = 0$: $T = T_\infty$

and two conditions at the droplet surface r_S, ξ_S :

$$T = T_S, \quad \left. \frac{dT}{d\xi_T} \right|_S = -\frac{\ell}{c_p}$$

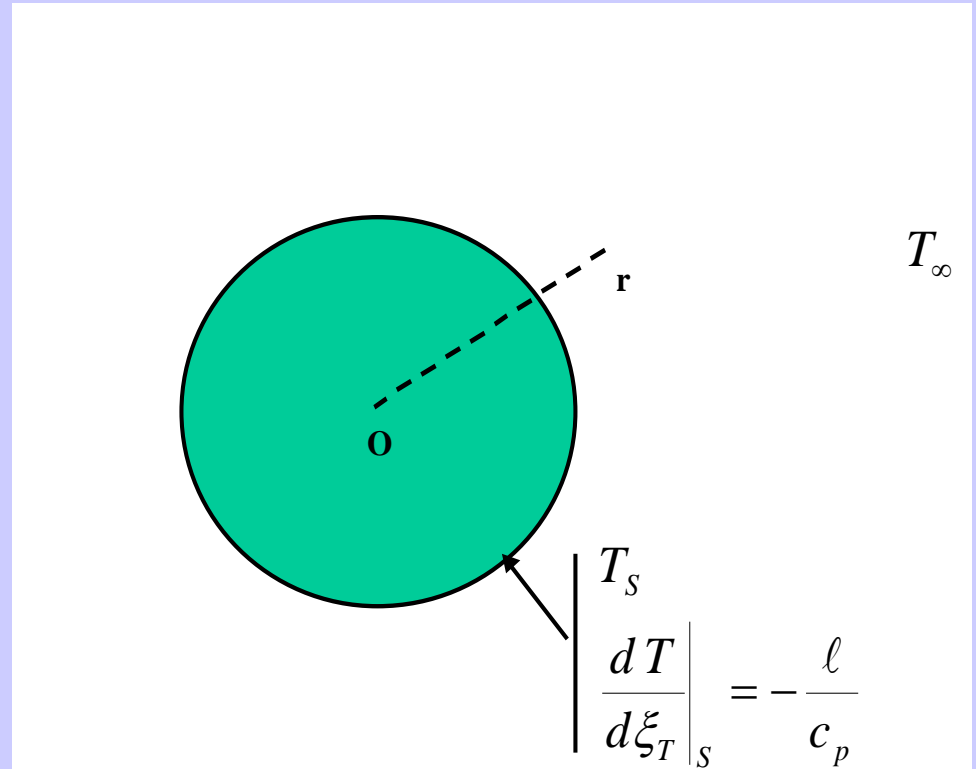
Setting

$$B_T = \frac{c_p (T_\infty - T_S)}{\ell}$$

Leads to:

$$\xi_{TS} = \text{Log}(1 + B_T)$$

$$\Rightarrow \dot{M} = 4\pi \frac{k}{c_p} r_S \text{Log}(1 + B_T)$$



The two calculations must give the same result, then

$$\dot{M} = 4\pi \rho D r_s \ln(1 + B_M) = 4\pi \frac{k}{c_p} r_s \ln(1 + B_T)$$
$$\text{Log}(1 + B_M) = Le \ln(1 + B_T)$$

Parameters B_M and B_T depend on boundary conditions which are not all known. One has then to *determine surface temperature and chemical composition*. They can be deduced from two specific relations.

Diffusion/Conduction relation

Let us consider the case of a Lewis number equal to unity: $Le=1$.
 One has $B_T = B_M = B$ or

$$B = \frac{Y_{FS} - Y_{F\infty}}{1 - Y_{FS}} = \frac{c_p (T_\infty - T_S)}{\ell}$$

with unknown T_S and Y_{FS} .

Finally one can also write

$$\frac{M_F Y_{A\infty} X_{FS}}{M_A X_{A\infty} (1 - X_{FS})} = \frac{c_p (T_\infty - T_S)}{\ell}$$

Liquid-vapor equilibrium relation

One has $\mu_L = \mu_F$. For a perfect gas mixture and if μ_L is only function of T , one has

$$p X_{FS} = p_{sat}(T_S) \text{ with, for ex. } p_{sat}(T_S) = e^{\left(a - \frac{b}{T_S - 43}\right)} \quad (\text{Chin \& Lefebvre})$$

$$\text{One obtains then } X_{FS} = \frac{M_A X_{A\infty} c_p (T_\infty - T_S)}{M_F Y_{A\infty} \ell + M_A X_{A\infty} c_p (T_\infty - T_S)} = \frac{1}{p} p_{sat}(T_S)$$

from which surface temperature and concentrations are deduced.

Physical properties generally vary in time and space as functions of temperature and of species concentrations.

Simplification:

One takes mean values as defined by

$$\bar{T} = T_S + A_r (T_\infty - T_S), \quad \bar{Y}_F = Y_{FS} + A_r (Y_{F\infty} - Y_{FS}) \quad \text{with } A_r = 1/3$$

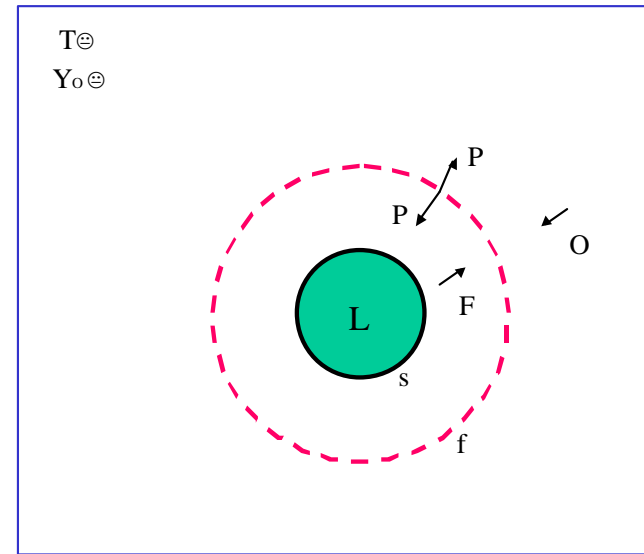
2.3 COMBUSTION

If $Le=1$,

$$\frac{k}{c_{pf}} = \rho D$$

one can assume the validity of

Shvab-Zeldovich approximation



$$\beta_j = Y_j / \nu_j M_j, \quad \beta_T = \sum_j Y_j \int_{T_0}^T c_{p,j} dT / \Delta H, \quad \Delta H = - \sum_j \nu_j M_j (q_f^0)_j$$

here: $\nu_F = \nu_O = -1, \quad \nu_P = 1 \Rightarrow \Delta H = M_F (q_f^0)_F + M_O (q_f^0)_O - M_P (q_f^0)_P$

One has to solve the following system:

$$\dot{M} \frac{d\beta_F}{dr} - \frac{d}{dr} \left(4\pi r^2 \rho D \frac{d\beta_F}{dr} \right) = 4\pi r^2 \dot{\zeta}$$

$$\dot{M} \frac{d\beta_O}{dr} - \frac{d}{dr} \left(4\pi r^2 \rho D \frac{d\beta_O}{dr} \right) = 4\pi r^2 \dot{\zeta}$$

$$\dot{M} \frac{d\beta_T}{dr} - \frac{d}{dr} \left(4\pi r^2 \frac{k}{c_{pf}} \frac{d\beta_T}{dr} \right) = 4\pi r^2 \dot{\zeta}$$

Let us consider the parameters

$$\alpha_M = \beta_F - \beta_O, \quad \alpha_T = \beta_T - \beta_O$$

These parameters verify

$$\dot{M} \frac{d\alpha_M}{dr} - \frac{d}{dr} \left(4\pi r^2 a \frac{d\alpha_M}{dr} \right) = 0$$

$$\dot{M} \frac{d\alpha_T}{dr} - \frac{d}{dr} \left(4\pi r^2 a \frac{d\alpha_T}{dr} \right) = 0$$

Resolution of the system for

$$Y_{OS} = Y_{F\infty} = 0$$

$$\xi = \dot{M} / 4\pi ar \Rightarrow \dot{M} = 4\pi ar_s \text{Log}(1 + B_M) = 4\pi ar_s \text{Log}(1 + B_T)$$

$$B_M = \frac{M_F}{1 - Y_{FS}} \left(\frac{Y_{O\infty}}{M_O} + \frac{Y_{FS}}{M_S} \right)$$

$$B_T = \frac{\Delta H}{\ell} \frac{Y_{O\infty}}{M_O} + c_p \frac{T_\infty - T_S}{\ell}$$

Finally $D^2 = D_0^2 - K t$

It is more complicated when $Le \neq 1$ is not unity.

As for evaporation, one has to determine surface temperature and concentrations.

Relation
diffusion/conduction

One has

$$B_T = B_M = B$$

$$B = \frac{Y_{FS}}{1 - Y_{FS}} + \frac{M_F}{M_O} \frac{Y_{O\infty}}{1 - Y_{FS}} =$$

$$\frac{c_p (T_\infty - T_S)}{\ell} + \frac{\Delta H}{\ell} \frac{Y_{O\infty}}{M_O}$$

Liquid-vapor equilibrium

One has $\mu_L = \mu_F$. For a perfect gas mixture and if μ_L is only function of T , one has

$$p X_{FS} = p_{sat}(T_S) \quad , \text{ then}$$

$$X_{FS} = \frac{M_O c_p (T_\infty - T_S) + \Delta H Y_{O\infty} - M_F \ell Y_{O\infty}}{M_O c_p (T_\infty - T_S) + \Delta H Y_{O\infty} + \frac{M_O M_F}{M_O + M_F} \ell (1 - Y_{O\infty})}$$

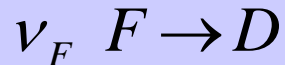
$$= \frac{1}{p} p_{sat}(T_S)$$

2.3 COMBUSTION + DECOMPOSITION

Between the droplet surface and the diffusion flame where

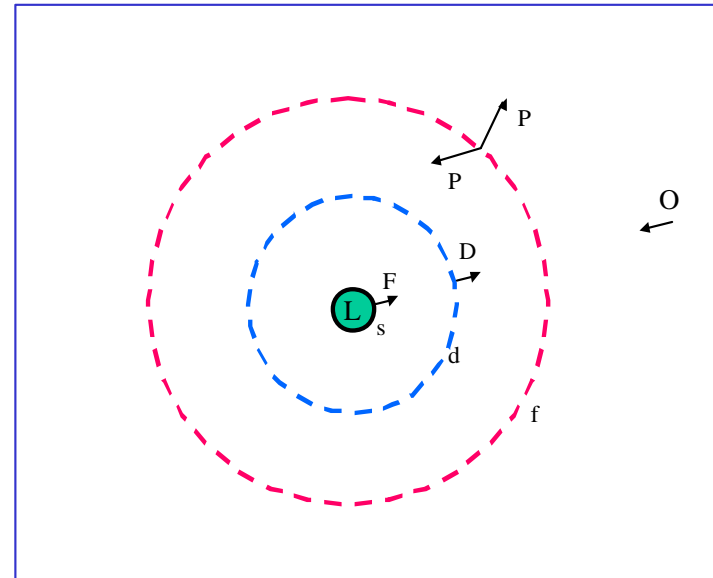


there is a decomposition flame



analogous to a premixed flame (Scherrer 1986, Mauriot 1992).

One result is an increasing vaporized mass flow rate in the case of an exothermal decomposition.



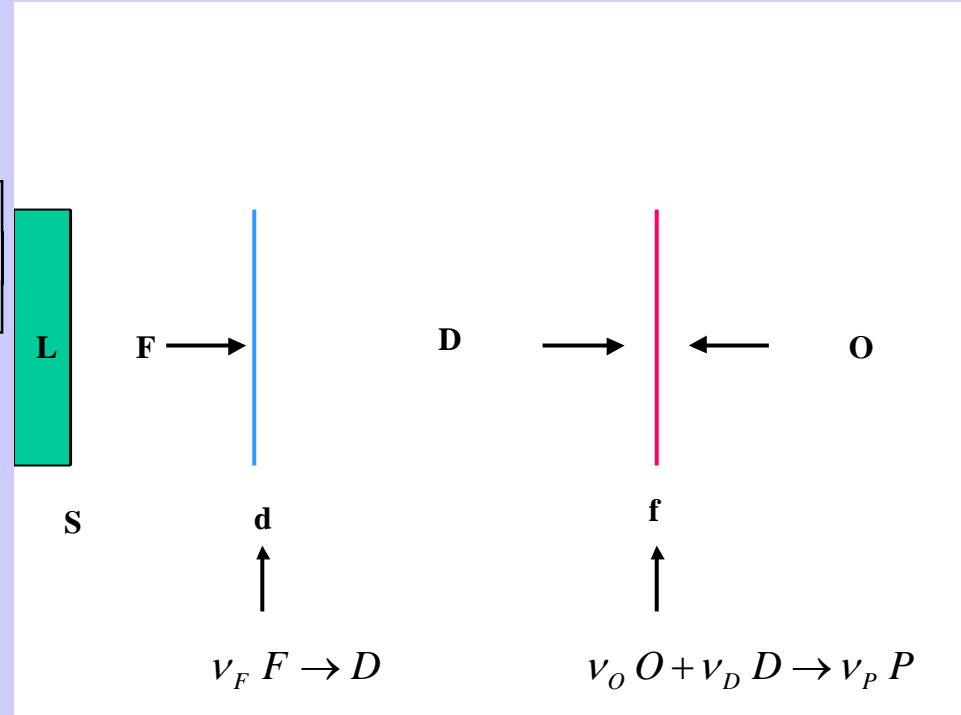
Analytical model (Mauriot)

Simplified case: mean c_p and ρD

$$\dot{M} = 4\pi r_s \overline{\rho D} \left[\text{Log} \left(1 + \frac{c_p (T_d - T_s)}{\ell} \right) + \xi_d \right]$$

$$1 = \frac{\dot{M}^2 c_p^2 B \rho_d}{8\pi^2 \overline{\rho D}^3 \xi_d^4 T_a^4} T_d^4 e^{-\frac{T_a}{T_d}}$$

$$\xi_d = \text{Log} \left[1 + \frac{c_p (T_\infty - T_d) + \Delta H_f^s \frac{Y_{O_\infty}}{\nu_O M_O}}{\ell + c_p (T_d - T_s) - \frac{\Delta H_d^s}{\nu_F M_F}} \right]$$



One deduces \dot{M} as function of r_s after elimination of T_d and ξ_d .

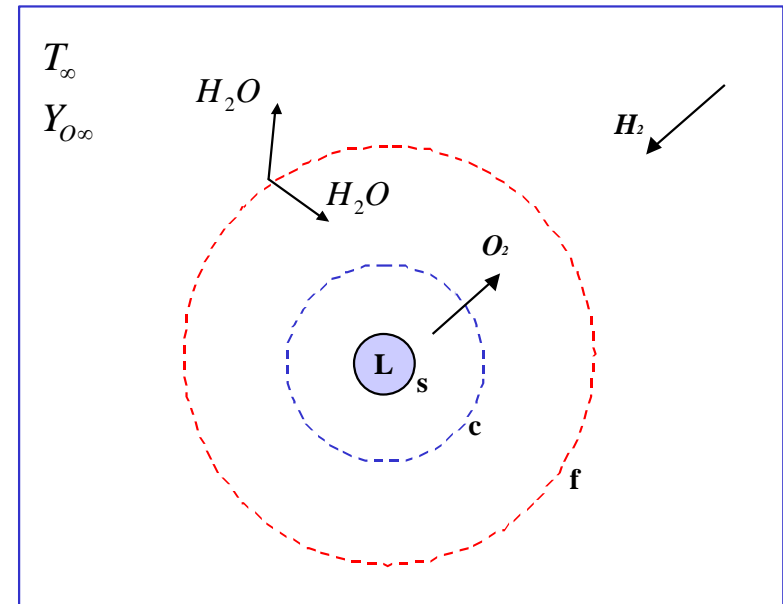
In the previous relations $\xi = \frac{\dot{M}}{4\pi \rho D r}$, ΔH_d is the decomposition reaction heat per mole, ΔH_f is the combustion heat, and ℓ is the latent heat of evaporation per unit mass.

2.3 COMBUSTION + CONDENSATION

Analytical model: Lafon & Prud'homme 1994. Main assumptions:

- Condensed vapor stays in a negligible volume and do not interfere with gaseous mixture
- Condensation acts as a mass sink and a local energy source
- Water do not dissolve into the oxygen droplet

Conclusion: condensation stays in the vicinity of the droplet and tends to increase the evaporation mass flow rate.



3. TRANSIENT REGIMES - Spherical droplet in quiet conditions at infinity - Heat and/or momentum exchanges

3.1 EVAPORATION WITH TEMPERATURE JUMP BUT WITHOUT EXTERNAL FLOW

Nearly the same problem that in 2.1 but with heat flux Q_L between gas and liquid droplet. Then the solution of diffusion equation is the same

$$B_M = \frac{Y_{FS} - Y_{F\infty}}{1 - Y_{FS}}$$

Heat-conduction equation

$$\xi_T = \frac{\dot{M} c_p}{4\pi k r} \quad \Rightarrow \quad T = a_T + b_T e^{-\xi_T}$$

Boundary conditions

$$T_\infty, \quad T_S, \quad \left. \frac{dT}{d\xi_T} \right|_S = - \frac{\ell + Q_L/\dot{M}}{c_p}$$

One puts $B_T = - \frac{c_p(T_\infty - T_S)}{\ell + Q_L/\dot{M}}$

One has to define the thermal exchanges inside the droplet. If they are fast, the droplet temperature is uniform, and one has

$$C_L \frac{dT_S}{dt} = Q_L$$

In non-stabilized regimes, characteristic time for liquid heating tends to be higher than the time for boundary-layer heating.

Heating time Δt_{hu} and droplet lifetime Δt_e

- are of the same order for kerosene droplets

- $\Delta t_{hu} > \Delta t_e$ for LOX droplets

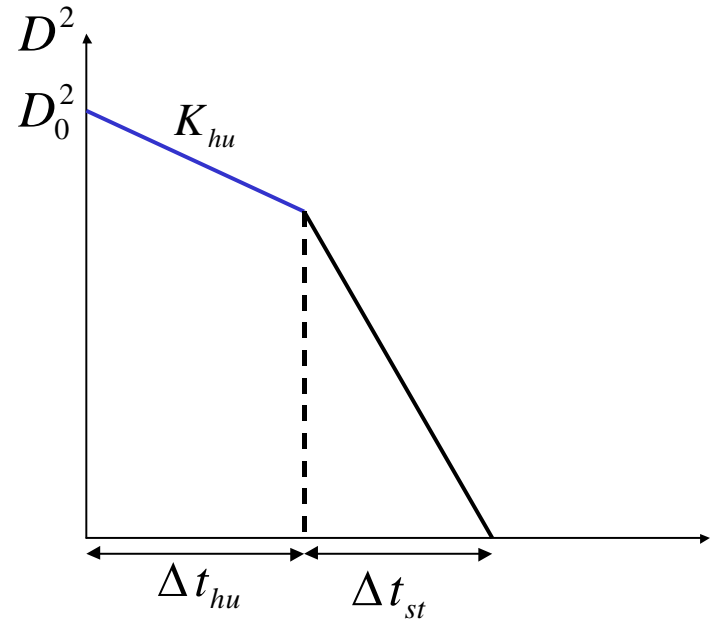
Simplification (Chin & Lefebvre)

Two D^2 laws: one for the heat-up period Δt_{hu} , the other for the steady-state phase Δt_{st} .

Question

Is this method valid

- in the presence of an external flow ?
- for combustion ?



$$\Delta t_{hu} = \frac{c_p \rho_L c_L D_{hu}^2 (T_{Se} - T_0)}{12k \text{Log}(1 + B_{Mhu}) \ell_{hu} (B'_{Thu} / B_{Mhu} - 1)^{-1}}$$

$$B'_T = \frac{c_p (T_\infty - T_{Shu})}{\ell_{hu}}, \quad B_{Mhu} = \frac{M_A}{M_F} \left(\frac{p}{e^{\frac{a-b}{T_{hu}-43}}} - 1 \right)^{-1}$$

$$D^2 = D_0^2 - K_{hu} t, \quad K_{hu} = \frac{8k}{c_p \rho_L} \text{Log}(1 + B_{Mhu})$$

$$D_{hu} = D_0 \left(1 + \frac{c_L (T_{Se} - T_0)}{2 \ell_{hu} \left(\frac{B'_T}{B_M} - 1 \right)} \right)^{-0,5}$$

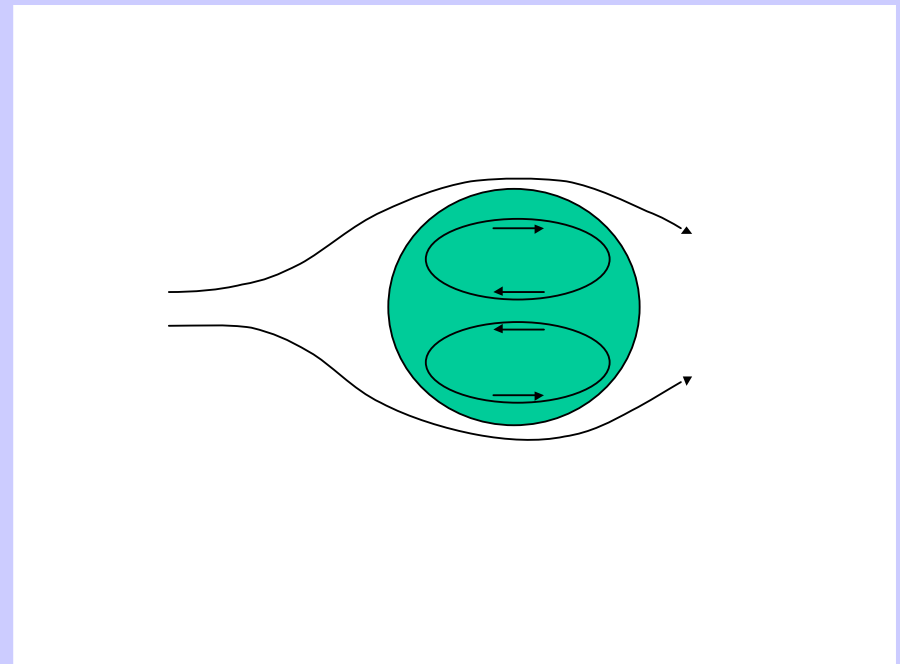
3.1 EVAPORATION WITH AN EXTERNAL FLOW

3.2.1 Droplet motion

$$\frac{d\vec{U}}{dt} = \frac{3}{2} \frac{C_D}{r_s} \frac{\rho_\infty}{\rho_L} |\vec{U}_\infty - \vec{U}| \left(|\vec{U}_\infty - \vec{U}| \right)$$

$$C_D = \frac{24}{\text{Re}} \left(1 + \frac{\text{Re}^{2/3}}{6} \right), \quad \text{Re} = \frac{2 \rho_\infty |\vec{U}_\infty - \vec{U}| r_s}{\bar{\mu}}$$

3.2.2 Motion inside the droplet



3.2.3 Models

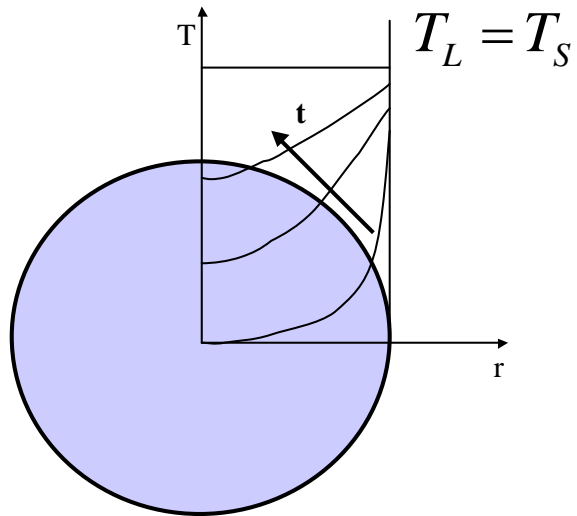
- Hill vortex
- Effective heat conduction coefficient $k_{eff} = \chi k_L$

$$\frac{d r_s}{d t} = - \frac{\dot{M}}{4 \pi \rho_L r_s^2}$$

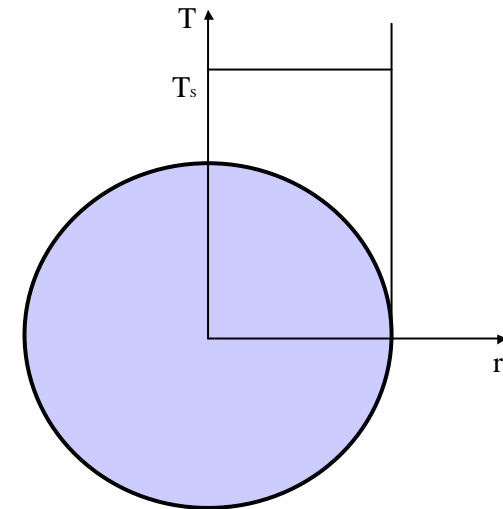
$$\chi = 1,86 + 0,86 th \left[2,245 \log \left(\frac{Pe_L}{30} \right) \right]$$

$$Pe_L = 2 U_{sr} / \alpha_L, \quad U_s = \frac{1}{32} \Delta U_\infty \frac{\mu}{\mu_L} Re_\infty C_f$$

$$C_f = 12,69 Re^{-2/3} / (1 + B_M)$$



Conductive heat exchange (Sirignano)

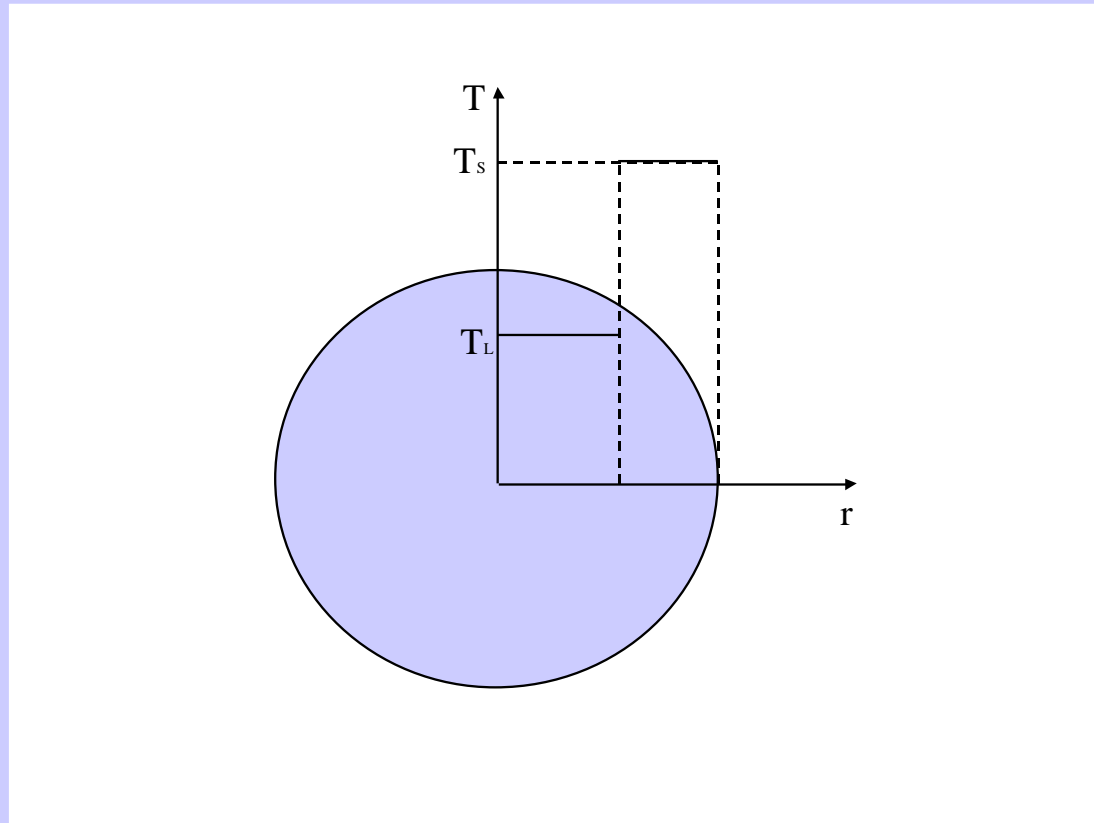


Infinite conductivity (cf. Chin & Lefebvre)

3.2.4 Simplifications

- Infinite conductivity
(uniform droplet temperature)

- Two-layers model (Present work)

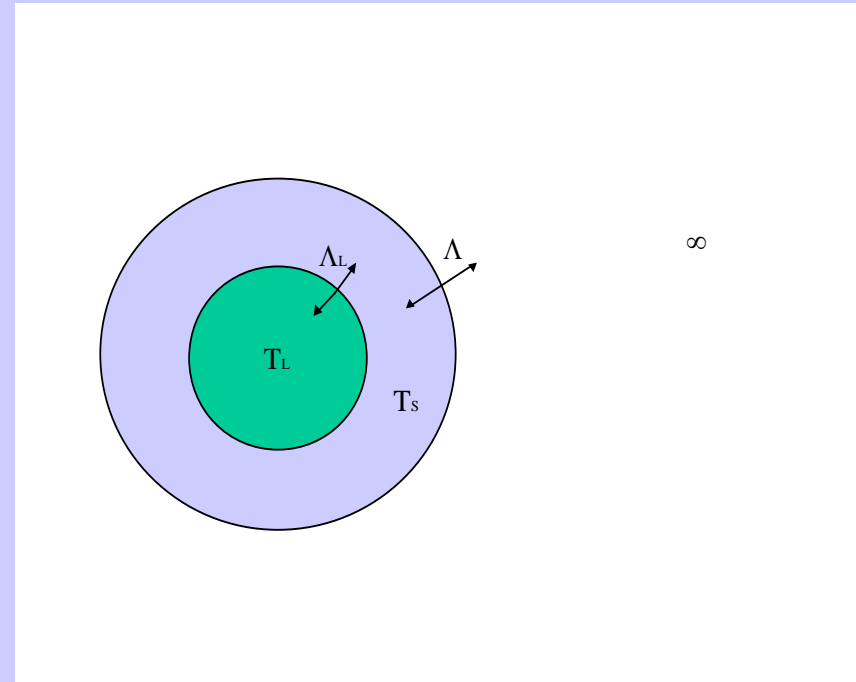


The case of rigid sphere without vaporization (Conduction)

$$\begin{cases} \frac{dT_L}{dt} = \beta_L (T_S - T_L) \\ \frac{dT_S}{dt} = -\beta (T_S - T_L) + \beta_S (T_\infty - T_S) \end{cases}$$

$$\beta_L = \frac{3\Lambda_L}{\rho_L c_L r_L}, \quad \beta = \frac{3\Lambda_L r_L^2}{\rho_L c_L (r_S^3 - r_L^3)}, \quad \beta_S = \frac{3\Lambda_L r_S^2}{\rho_L c_L (r_S^3 - r_L^3)}$$

$$\Rightarrow \frac{d^2 T_S}{dt^2} + (\beta + \beta_S + \beta_L) \frac{dT_S}{dt} + \beta_L \beta_S (T_S - T_\infty) = 0$$

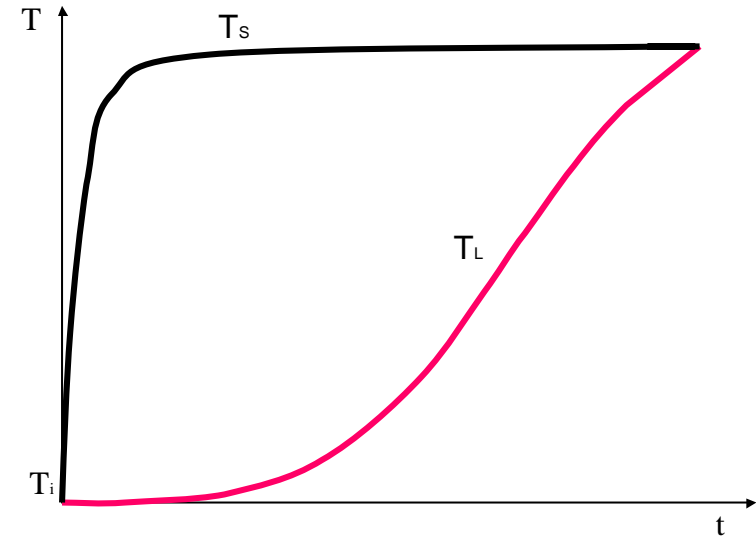


Very simple case: $\beta_L = \beta_S = 2\beta$, then

$$r_L = r_S / 3^{1/3}, \quad \Lambda_L = 3^{1/3} \Lambda / 2, \quad \text{and}$$

$$T_S = T_\infty + \frac{T_0 - T_\infty}{3} (2e^{-\beta t} + e^{-4\beta t})$$

$$T_L = T_\infty + \frac{T_0 - T_\infty}{3} (4e^{-\beta t} - e^{-4\beta t})$$



T_L Temperature at the droplet centre,
 T_s surface temperature

3.2.5 More sophisticated models

Chiang, Raju & Sirignano (1992) developed an advanced exact droplet vaporization model with few assumptions. This axisymmetric single-droplet model included effects of surface blowing, transient heating, internal circulation, and relative velocity variations from droplet injection.

This "complete" model is very useful as reference for validation of more simpler models could be measured. Main characteristics of the model are given here'after.

Gas phase

Gas-hydrodynamic times are introduced $\tau_{Hg} = \frac{t' \mu'_{g,\infty}}{a_0'^2 \rho'_{g,\infty}}, \tau_{El} = \frac{t' \kappa'_{g,0}}{a_0'^2 \rho'_{g,0} C'_{pg}}$.

One write continuity, species, momentum and energy equations in r, z, t and equation of state of the mixture.

Liquid phase

A liquid-hydrodynamic-diffusion time is introduced

a' droplet radius, 0 initial conditions

$$\tau_{HI} = \frac{t' \mu'_{l,0}}{a'^2_0 \rho'_{l,0}}$$

as well as a stream function ψ so that

$$V_r = \frac{1}{r} \psi_{,z}, V_{,z} = \frac{1}{r} \psi_{,r}$$

and vorticity

$$\omega = \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{1}{r} \frac{\partial \psi}{\partial z} \right)$$

One has

$$\begin{aligned} \frac{\partial(r\omega)}{\partial \tau_{HI}} + \frac{\partial}{\partial r} \left(\text{Re}_l \frac{\partial \psi}{\partial z} \omega \right) + \frac{\partial}{\partial z} \left(-\text{Re}_l \frac{\partial \psi}{\partial r} \omega \right) = \\ \frac{\partial}{\partial r} \left(r \frac{\partial(\mu_l \omega)}{\partial r} \right) + \frac{\partial}{\partial z} \left(r \frac{\partial(\mu_l \omega)}{\partial z} \right) - \mu_l \frac{\omega}{r} + \text{Re}_l \frac{\omega}{r} \frac{\partial \psi}{\partial z} \end{aligned}$$

A liquid thermal-diffusion-time is introduced

$$\tau_{El} = \frac{t' \kappa'_{l,0}}{a_0'^2 \rho'_{l,0} C'_{p,l,0}}, \kappa'_l \text{ conductivity of liquid phase (assumed to be constant)}$$

Energy equation

$$\frac{\partial(rT)}{\partial \tau_{El}} + \frac{\partial}{\partial r} \left(Pe_l T \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial z} \left(-Pe_l T \frac{\partial \psi}{\partial r} \right) = \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(r \frac{\partial T}{\partial z} \right)$$

Initial conditions

Gas/liquid interfacial conditions

Other boundary conditions: external flow, internal flow, symmetry axis.

Stokes equation

Total drag coefficient $C_D = C_{D,p} + C_{D,f} + C_{D,t}$

Pressure $C_{D,p}$

Friction C_{Df}

Thrust $C_{D,t}$

Droplet deceleration $\frac{dU_d}{d\tau_{Hg}} = -\frac{3}{8} \frac{\rho'_\infty}{\rho'_l} \frac{U_\infty^2}{a} \text{Re}_g C_D$

Nusselt number, Sherwood number.

Main results

Transient regime is important

Re decreases because droplet slowing-down

4. COLLECTIVE EFFECTS

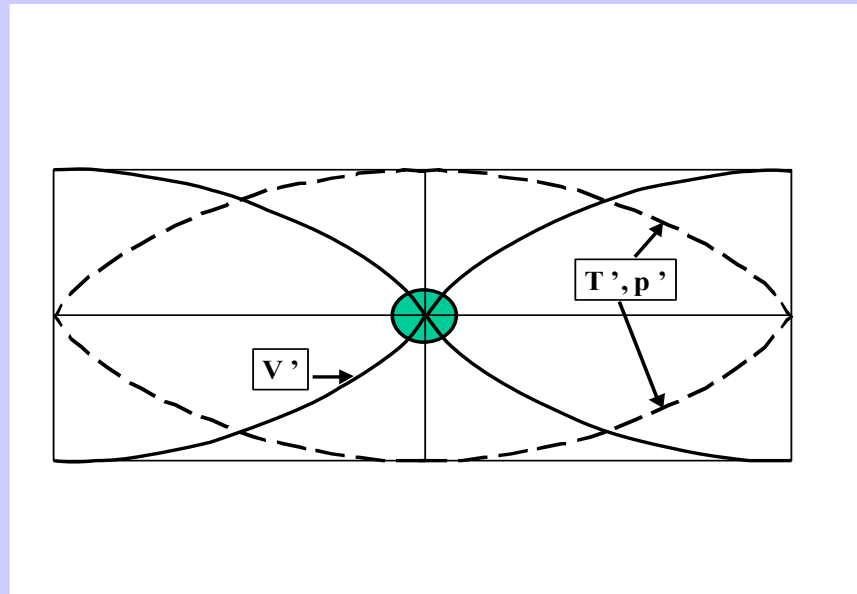
- **Figures showing droplets proximity influence**
 - **Borghgi & Lacas (1995)**
 - **Dietrich et al. (1999)**
- **Theories**
- **Microgravity experiments**

5. ANALYSIS OF FREQUENCY RESPONSE CHARACTERISTICS

Motivations: instabilities, coupling phenomena, non-stationary effects

Rayleigh criterion (1894): “ A mechanism can drive the instability if the burning rate associated with it oscillates with a large enough amplitude and with a small enough phase lag with the pressure oscillation”

Example of a droplet submitted to periodical oscillations



5.1 RESPONSE FACTOR ACCORDING TO HEIDMANN & WIEBER (1966)

- reduced pressure perturbation (entrance) $p' = \frac{p - \bar{p}}{\bar{p}}$

- reduced heat or mass resulting perturbation $q' = \frac{q - \bar{q}}{\bar{q}}$

- response factor
$$N = \frac{\iint_{V,t} q'(V,t) p'(V,t) dt dV}{\iint_{V,t} (p'(V,t))^2 dt dV}$$

- sinusoidal oscillations with the same periodicity $N = \frac{\hat{q}}{\hat{p}} \cos \theta$

5.2 LINEARIZED EQUATIONS FOR PROPELLANT VAPORIZATION

$$\bar{\tau}_v \frac{dM'}{dt} = -\dot{M}' \quad (\text{mass})$$

$$\dot{M}' = \frac{1}{2} M' + \beta p_L' - \left(\beta - \frac{1}{2} \right) p_C' \quad (\text{vaporization rate})$$

$$p_L' = b T_L' \quad (\text{vapor pressure})$$

$$\frac{\bar{c}_p \bar{T}_L}{\bar{k}} \bar{\tau}_v \frac{dT_L'}{dt} = Q_{in}' - Q_{out}' \quad (\text{thermal balance of droplet})$$

$$Q_{in}' = \frac{1}{2} M' + \frac{1}{2} p_C' \quad (\text{heat transfer to propellant surface})$$

$$Q_{out}' = \dot{M}' \quad (\text{heat transfer from propellant surface : associated to heat of vaporization})$$

$$\beta = \frac{\frac{\bar{p}_L}{\bar{p}_C - \bar{p}_L}}{\text{Log}\left(\frac{\bar{p}_C}{\bar{p}_C - \bar{p}_L}\right)} \quad \begin{cases} \bar{p}_C & \text{combustion chamber pressure} \\ \bar{p}_L & \text{vapor pressure at propellant surface} \end{cases}$$

Resulting equation

$$\left(1 + 2\bar{\tau}_v \frac{d}{dt}\right) \left(1 + \phi \bar{\tau}_v \frac{d}{dt}\right) \dot{M}' - \bar{\tau}_v \frac{d}{dt} \left[1 + (1 - 2\beta)\phi \bar{\tau}_v \frac{d}{dt}\right] \dot{p}_C = 0$$

One put

$$p'_C = \hat{p} e^{i\omega t}$$

$$\dot{M}' = \hat{M} e^{i(\omega t + \theta)}$$

Then

$$\frac{1}{\omega \bar{\tau}_v} \frac{\hat{M}}{\hat{p}} e^{i\left(\theta - \frac{\pi}{2}\right)} = \frac{1 + i(1 - 2\beta)\varphi \omega \bar{\tau}_v}{(1 + 2i\omega \bar{\tau}_v)(1 + i\varphi \omega \bar{\tau}_v)} \quad , \text{ and}$$

$$\theta = \frac{\pi}{2} + \text{Arctg}((1 - 2\beta)\varphi \omega \bar{\tau}_v) - \text{Arctg}(2\omega \bar{\tau}_v) - \text{Arctg}(\varphi \omega \bar{\tau}_v)$$

$$\frac{\hat{M}}{\hat{p}} = \frac{\omega \bar{\tau}_v}{\left[1 + (2\omega \bar{\tau}_v)^2\right]^{1/2}} \left[\frac{1 + [(1 - 2\beta)\varphi \omega \bar{\tau}_v]^2}{1 + (\varphi \omega \bar{\tau}_v)^2} \right]^{1/2}$$

$$N = 2(\omega \bar{\tau}_v)^2 \frac{1 + \beta\varphi - (1 - 2\beta)\varphi^2 \omega^2 \bar{\tau}_v^2}{(1 - 2\varphi \omega^2 \bar{\tau}_v^2)^2 + (2 + \varphi)^2 \omega^2 \bar{\tau}_v^2}$$

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