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# T-matrix methods for electromagnetic structured beams: A commented reference database for the period 2014-2018

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## Abstract

T-matrix methods for electromagnetic structured beams mainly encompass Generalized Lorenz-Mie Theories (GLMTs) which form a set of analytical approaches dealing with the interaction between electromagnetic structured beams and a class of particles possessing enough symmetries to allow one to use the method of separation of variables, and Extended Boundary Condition Method (EBCM) which is a semi-analytical method dealing in principle with arbitrary shaped particles. Both approaches, when dealing with structured beams (in principle in the case of GLMTs and occasionally in the case of EBCM) rely on the description of the illuminating beam in terms of Beam Shape Coefficients (BSCs). The present paper provides a commented reference database concerning GLMTs and EBCM for structured beams, and related topics for the period 2014-2018.

Keywords : T-matrix methods, generalized Lorenz-Mie theories, Extended Boundary Condition Method, structured beams, mechanical effects of light, optical particle characterization.

## 1 Introduction.

The main intended topics considered in this paper are Generalized Lorenz-Mie theories (GLMTs) which deal with analytical solutions of scattering problems, e.g. [1], [2], [3] and references therein dating back to [4], and Extended

Boundary Condition Method (EBCM) which deals with semi-analytical solutions of scattering problems, e.g. [5], [6]. GLMTs deal with arbitrary shaped beams (structured beams) but are limited to a class of particles which possess enough degrees of symmetry to allow one to use the method of separation of variables, while EBCM may deal with arbitrary shaped particles, although most work concerning EBCM up-to-now concerned the case when the scatterers are illuminated by plane waves. The GLMT *stricto sensu* concerns the case when the scatterer is a homogeneous spherical particle defined by its diameter and its (complex) refractive index, while "GLMTs" denotes a larger class of scatterers. The EBCM, also denoted Null-Field Method (NFM), although tailored for arbitrary shaped particles, is most often *in practice* better restricted to isotropic and homogeneous particles, and becomes possibly ill-conditioned when dealing with highly elongated or flattened particles, e.g. [7], [8], [9], [10].

In Gouesbet and Lock [11] devoted to a list of problems for future research in generalized Lorenz-Mie theories and related topics, two problems concerned EBCM, namely Problem 13 ("Implement a localized approximation in the EBCM") and Problem 14 ("Extend the EBCM to arbitrary-shaped beams with the use of Beam Shape Coefficients (BSCs)"). The fact that these problems indeed received solutions, i.e. that EBCM has indeed been used in the case of structured beam illumination, is one motivation to explicitly incorporate it in the present review, see subsection 8.1. Let us nevertheless note that EBCM has originally been used by relying on an Angular Spectrum Decomposition (ASD), i.e. on a decomposition in terms of plane waves, see [12] for a review devoted to the electromagnetic scattering of arbitrary shaped beams by arbitrary shaped particles and [13] for the relationship between BSCs and plane wave spectra.

Furthermore, although the expression "T-matrix method" has been recurrently used as synonymous with the EBCM (a traditional but nowadays confusing practice, particularly unfortunate for newcomers), it happens that GLMTs actually may be viewed as T-matrix methods as well [12], [14], according to the following definition, slightly adapted from a series of T-matrix reference databases, e.g. [15]: "in "a" T-matrix method, the incident and scattered fields are expanded in terms of suitable vector spherical wave functions, VSWFs (*or in an equivalent formulation*), and the relation between the columns of the respective expansion coefficients is established by means of a transition matrix (or T-matrix)". The expression "*or in an equivalent formulation*" refers to the fact that the formulation may be possibly described in such a way that the T-matrix is not *explicitly* displayed, but can be made easily available. This is the case in the GLMT expressed in terms of Bromwich or Hertz potentials instead of in terms of VSWFs, in which the T-matrix occurs only in an implicit way, e.g. [16], [17] for examples of conversions between Bromwich and VSWFs formulations. As stated in [15], the definition above is more inclusive than the original too restricted notion of the EBCM. The extended point of view so introduced is also shared by Doicu *et al.* [9] stating that "the null-field method is only one among many methods that can be used to compute the transition matrix", or by Nieminen *et al.* [10], [18]. Therefore, any scattering theory (approach) relying on "a" T-matrix method (not only GLMTs or EBCM) may be said to pertain to "the"

T-matrix formulation. Extended points of view may invoke vector spheroidal wave functions [19], [20], vector cylindrical wave functions [18], or any kind of vector wave functions [21], instead of solely VSWFs. Clearly, an expansion of a given type of vector wave functions in terms of VSWFs can always be used to meet the aforementioned restricted definition in which VSWFs only are considered (relationships between various kinds of vector wave functions will form the basis of "extrinsic" methods for the evaluation of BSCs, to be discussed in subsection 2.2).

The fact that "T-matrix" should not be considered any more as synonymous with EBCM is illustrated by the following quote extracted from Alonso *et al.* [22] who wrote : "Scattering of arbitrary fields is described using the generalized Lorenz-Mie theory (GLMT) which relies on the decomposition of the incident field in terms of vector multipoles and the fulfillment of boundary conditions through the use of the appropriate T-matrix". Indeed the definition provided above from [15] does not tell us anything on the way to be used to compute the T-matrix and any method allowing one to do it is a T-matrix method, entering in the framework of a T-matrix formulation of the scattering problem under study. In other words, any scattering theory or approach relying on "a" T-matrix method may be said to pertain to "the" T-matrix formulation. Beside GLMTs and EBCM, other examples of T-matrix methods are the Discrete Dipole Approximation (DDA), also called the Coupled Dipole Method (CDM), the Finite-Difference-Time-Domain method (FDTD), or the point-matching method [10], among others. See subsection 8.5 for DDA and FDTD, the former method representing the scatterer as a collection of interacting electric dipoles, the latter one relying on the discretization of time-dependent Maxwell's equations on a grid leading to a system of finite-difference equations which are solved using a "leapfrog procedure", similar to the use of finite-difference equations in fluid mechanics when dealing with Navier-Stokes equations with a "marching procedure" or an implicit scheme, e.g. [23] and references therein.

A typical statement concerning the interest of a T-matrix method is as follows : "The elements of the T-matrix are independent of the incident and scattered fields and depend only on the shape, size parameter and refractive index of the scattering particle and on its orientation with respect to the reference frame, so that the T-matrix needs to be computed only once and then can be used in computations for any directions of light incidence and scattering" ([7], p.41).

An important common point between GLMTs and EBCM is that both of them rely on the expansions of the incident wave and of the scattered wave in terms of VSWFs, with the incident beam encoded in a set of coefficients named expansion coefficients, or better BSCs, see [14] for the relationship between expansion coefficients and BSCs. Therefore, in the present review, a particular attention and insistence will be paid to the methods used to evaluate BSCs. Various methods dealing with this issue of BSC evaluations will be discussed when appropriate. Let us just recall that the most classical techniques for such evaluations are by using quadratures, e.g. [24], localized approximations, e.g. [25], [26] or finite series, e.g. [27].

The present review pertains to a series of regular reviews commenting new advances in GLMTs and applications, and follows previous reference databases published in 1994 [28], 2000 [29], 2009 [30], [31] and 2014 [32]. To build the bibliography quoted in this review, I used alerts from Google Scholar Citations (GSC) and gathered the papers which quoted papers devoted to GLMTs and related topics. This process ensures a very exhaustive character to this review as far as GLMTs are concerned. It may happen that papers retrieved from GSC were not always closely related to GLMTs. Some of them have therefore been rejected while others have been preserved, allowing one to extend the review, although much less exhaustively, to trends in light scattering theories and applications. Other related papers may also be possibly quoted when they came to my knowledge from other sources than GSC. Furthermore, papers prior to the fiducial period 2014-2018 (named the fiducial period) may also be quoted when they serve the purpose of enlightening the perspective of the comments of the present database. However, as a consequence of the above strategy, the articles cited in this review are not necessarily the first articles that were published on each individual topic considered. Prior works may be reached from the recent second edition of a textbook [3] and from previous reviews already quoted above. With such a strategy, I believe that the present review may be useful to experts in scattering, but provides as well a fairly comprehensive inroad to be used by the newcomer.

The intention of this paper is not to provide an extended analysis of each article quoted but it provides a coarse-grained map of the landscape, hopefully sufficiently accurate to help the reader to orient himself and access to the inroads he needs. In other words, the paper may be used as a guide to fly over the landscape before possibly land on and zoom on a particular location (a quoted paper) if the reader is interested by it. Comments attached to a quoted paper are not to be taken as clues to assess the quality of the paper. Rather, they provide a testimony of the biases induced by the particular interest of the author, at the moment of the writing. Sometimes, comments have been secured thanks to a private communication with authors. In such cases, the private communication is acknowledged and I take here the opportunity to kindly thank the authors who replied to my inquiries.

It may furthermore happen that a given paper might deserve to be quoted in several sections (or subsections) of this paper. In such a case, comments are provided at a certain location of the paper from a certain point of view while it is mentioned as well in the different sections (or subsections) in which it could have been located from different points of view. This process, however, is not always exhaustive particularly for spherical particles and Gaussian beams, because it would require too many extra-mentions for the comfort of the reading. Most of such papers can be obtained with a find button of a pdf.file and the use of keywords such as "spheres", "spherical", "beads", "droplets", "bubbles" and "Gaussian". Also, the process has not always been successfully implemented due to possible absentmindedness of the author. However, as an example, Kiselev and Plutenko [33] is mentioned in subsection 2.1 devoted to spherical particles and to the use of spherical coordinates while it is commented in subsection 3.4

devoted to helical beams.

The paper is organized as follows. Section 2 considers GLMTs for various shapes and morphologies (discussing spheres and spherical coordinates, spheroids and spheroidal coordinates, cylinders and cylindrical coordinates, metamaterials). Section 3 considers GLMTs for various beam shapes (Gaussian beams, laser sheets, non-diffracting beams, helical beams, pulses and laser pulses, acoustical beams, photonic nanojets, miscellaneous beams, with a subsection devoted to warnings concerning the use of localized approximations). Section 4 deals with the mechanical effects of light, while section 5 deals with internal fields (whispering-gallery modes and/or morphology-dependent resonances). Section 6 discusses the optical theorem. Section 7 is devoted to the issue of Optical Particle Characterization (OPC), discussing phase-Doppler technique, rainbow refractometry, holography, imaging and microscopy, as well as miscellaneous measurement techniques. Section 8 is devoted to alternative light scattering approaches (EBCM for structured beams, ray optics, Debye series, dipole approximations, as well as numerical techniques such as based on surface integrals, volume equations, or finite elements). Section 9 deals with miscellaneous papers which, although relevant to the present review, did not find their natural places in the table of contents above while section 10 deals with reviews and tutorials. Section 11 is a conclusion.

## **2 Around generalized Lorenz-Mie theories for various particle shapes and morphologies.**

Let us note that although this section and the next one are *in principle* devoted to GLMTs, I shall incorporate a few papers dealing with incident plane waves (i) because of the strong similarity between plane wave and structured beam scattering due to the fact that LMT is simply a special case of GLMT obtained by using appropriate plane wave BSCs and (ii) because, in the general case of oblique illumination, plane wave scattering must be treated in a GLMT framework, with BSCs more complicated than the ones used in LMT e.g. [34] for oblique illumination, and [4] for BSCs in the LMT framework.

### **2.1 Spheres and spherical coordinates.**

Spherical particles, whether they are beads, bubbles or drops (when tension forces are dominant), possess the highest degree of symmetry possible and, although many particles in nature are not spherical, dealing with spherical particles is one of the most common practice in light scattering. Let us however note that spherical coordinates to be used for homogeneous spherical particles defined by their diameter and their refractive index, e.g. in the framework of

the GLMT *stricto sensu*, are relevant as well to the study of scattering by many other kinds of particles, such as multilayered spheres, e.g. [35], [36], [37], spheres with an eccentrically located spherical inclusion (sometimes referred to as the eccentric stratified sphere model), e.g. [17], [38], [39], and assemblies of spheres and aggregates, e.g. [16], [40].

Following a chronological order, we then have Wang *et al.* [41] who used an eccentric stratified dielectric sphere model in a GLMT framework to study the shaped beam scattering from a single nucleated lymphocyte cell illuminated by an arbitrarily incident Gaussian beam. Zhang and Li [42] dealt with the electromagnetic scattering by a partially charged multilayered sphere, using what can be viewed as a GLMT, without however carrying out numerical simulations. Selmke [43] studied the electromagnetic scattering of plane waves by an infinite Coulomb-like scatterer, i.e. in a special  $1/r$  inhomogeneously refracting but non-absorbing transparent medium. Divergences spoiling the classical scattering approaches for unscreened Coulomb scattering were due to the infinite range of the Coulomb interaction. The introduction of a shaped beam with a finite lateral extent within the GLMT framework removed the divergence of the cross-sections.

Tang and Lin [44] dealt with the scattering of randomly distributed spherical particles illuminated with a Gaussian beam, in the framework of a GLMT for assemblies of spheres. Lock [45] discussed the scattering of the evanescent components in the angular spectrum of a tightly focused electromagnetic beam by a spherical particle. The angular spectrum of a tightly focused beam in general contains both oscillatory and evanescent components. This paper focused on the contribution of the evanescent components to the scattering by a spherical particle placed in the beam. The contribution of these components was explicitly calculated and the validity of their neglect in various situations was assessed. When required, the BSCs of each plane wave component in the ASD were analytically used in closed-forms. The total BSCs of the incident beam could then be evaluated from the BSCs of each plane wave component weighted by their respective amplitude and phases, and then summed over all the components.

Shang *et al.* [46] dealt with the scattering from a multilayered chiral sphere using an iterative method. Electromagnetic fields in each region were expanded in terms of VSWFs, taking into account for the boundary conditions on each layer. BSCs were obtained in a GLMT framework. Zambrana-Puyalto and Bonod [47] investigated the interaction between a chiral dipolar emitter and a spherical optical antenna, with a particular attention paid to spherical nanoparticles since they can be fabricated at a large scale via colloidal synthesis or laser ablation with excellent surface and optical properties. The addressed question was whether individual spherical nanoparticles could be used to produce strong chiral effects when interacting with chiral emitters. In other words, the aim was to study how the antenna modified the chirality of the emitted field in the far-field region. For this purpose, a simple formalism to analyze the chirality of the scattered light was developed using the Mie theory in the helicity basis (which can be used in a GLMT-framework as well, although it does not correspond to the traditional formulation). The formalism was applied to plasmonic antennas

composed of gold spherical nanoparticles and to dielectric spherical resonators made of silicon. Höller *et al.* [48] dealt with a bottom-up protein-assisted assembly of modular 3D plasmonic raspberry-like core/satellite nanoclusters, and examined the correlations between structures and optical properties. GMM (generalized multi-particle method) simulations were employed to describe the optical responses of individual nanoparticles as well as assemblies of nanoparticles, i.e. to examine the electromagnetic coupling within the nanoclusters. Let us note that GMM is an extension of LMT to the multi-sphere case, and may be viewed as a special case of the GLMT for assemblies of spheres and aggregates [16] in the same way that LMT is a special case of GLMT. Frezza and Mangini [49] studied the electromagnetic scattering of an inhomogeneous elliptically polarized plane wave by a multilayered sphere, the solution of the scattering problem being achieved using the Legendre functions generalized via hypergeometric and gamma functions. Using this representation (allowing one to deal with inhomogeneous waves), it has been possible to determine the expansion coefficients of the impinging electric field as a superposition of vector spherical harmonics. Ding *et al.* [50] dealt with the scattering of a cylindrical vector beam by a sphere within the framework of the GLMT, with the BSCs being calculated using analytical expressions obtained by the Integral Localized Approximation (ILA, e.g. [26]), with the scattering coefficients being expanded using Debye series (DSE). There was an emphasis on the rainbows produced by the interaction of cylindrical vector beams and spherical particles. Bai *et al.* [51] dealt with the electromagnetic scattering for multilayered spheres illuminated by laser sheets, in a GLMT framework. Comparisons among the scattering properties of laser sheets, Gaussian beams and plane waves are carried out. The formalism relied on the use of an algorithm based on logarithmic derivatives of the Riccati-Bessel functions. See also Jia [52] for a discussion of recursive logarithmic derivative algorithms, together with the use of the Lentz algorithm for the computations of Bessel functions by using a continuous fraction representation which was already efficient to deal with homogeneous spheres, e.g. [53] and [3], pp.249-250. For logarithmic derivative algorithms in a GLMT formulation, see [37] and [35]. Logarithmic derivatives have also been used by Wang *et al.* [54] in a study devoted to the scattering characteristics for metallic spheres at terahertz frequencies, taking the frequency dispersion characteristics into account. See also Ladutenko *et al.* [55] who developed an algorithm to calculate electric and magnetic fields inside and around a multilayered sphere.

Lock [56] discussed the electromagnetic scattering of a plane wave by a radially inhomogeneous sphere in the short wavelength limit, using a WKB solution of the radial differential equation, and a Debye series decomposition of the scattered wave, with some emphasis on two-rainbow coalescences and longitudinal cusp caustics. Wang *et al.* [57] studied the internal field distribution of a radially inhomogeneous droplet illuminated by an arbitrary shaped beam within the framework of GLMT, with particular attention paid to a new derivation associated with a numerically stable algorithm. The Modified Localized Approximation (MLA) method was applied to evaluate the BSCs of Gaussian beams due to its well known high efficiency in terms of computational times and accuracy.



See [58] for the terminology "modified localized approximation", together with the complementary terminology "Original Localized Approximation (OLA)" to be used in the sequel. Frisvad [59] discussed the scattering of an inhomogeneous electromagnetic plane wave by a spherical particle, in relevance with the situation when the scatterers are embedded in an absorbing host medium. The author mentioned that his solution could possibly be combined with GLMT to deal with arbitrary positions of the scattering sphere surrounded by an absorbing host medium in an incident beam of arbitrary shape. Zambrana-Puyalto *et al.* [60] experimentally studied the interaction of a micron-sized single sphere with beams having a well-defined angular momentum and helicity, over a certain spectral range by using a tunable fiber diode laser, together with a spatial light modulator to create a vortex beam. They overall demonstrated that the helicity and angular momentum of light are two crucial properties in order to understand, predict and control the light scattering of particles. Experimental results were quantitatively discussed by relying on the properties of the Mie multipolar decomposition of the incident beams.

For examples of other papers devoted to the issue of the present subsection, see Mitri [61] in subsection 3.2 devoted to laser sheets, Chen *et al.* [62], Chen *et al.* [63], Preston and Reid [64], Li *et al.* [65], Wen *et al.* [66], Wang *et al.* [67], Qu *et al.* [68], Gong *et al.* [69] in subsection 3.3 devoted to non-diffracting beams, Kiselev and Plutenko [33], Qu *et al.* [70], [71], Al-Jarro *et al.* [72] in subsection 3.4 devoted to helical beams, Li *et al.* [73], Yu and She [74], Li *et al.* [75], Leartprapun *et al.* [76], [77] in section 4 devoted to mechanical effects of light, Li *et al.* [78] in subsection 8.3 devoted to Debye series, and Guo *et al.* [79] in subsection 7.5 devoted to miscellaneous measurement techniques.

## 2.2 Spheroids and spheroidal coordinates.

Before entering the present subsection devoted to spheroids (and ellipsoids) and spheroidal coordinates, we need to be aware of the distinction between intrinsic and extrinsic methods to the evaluation of BSCs. The dichotomy between intrinsic and extrinsic methods may be defined as follows [11]: intrinsic methods evaluate BSCs of a GLMT posed in a certain coordinate system in terms of quantities pertaining to the same coordinate system while, conversely, extrinsic methods evaluate BSCs of a GLMT posed in a certain coordinate system in terms of BSCs in a different coordinate system. For example, the GLMT *stricto sensu* evaluates the BSCs by an intrinsic method, the derivation of the expressions of the BSCs being made by using spherical coordinates which are used as well to derive the formalism of the GLMT. Conversely, the GLMT for spheroids (which will be discussed in the present subsection) originally dealt with an extrinsic method in which the spheroidal BSCs (BSCs in spheroidal coordinates) were expressed in terms of spherical BSCs (BSCs in spherical coordinates). The distinction between intrinsic and extrinsic methods is useful for

both this section in spheroidal coordinates and the next one in cylindrical coordinates. For a review on intrinsic and extrinsic methods, see the introduction of [80]. The evaluation of spheroidal BSCs using either intrinsic or extrinsic methods is discussed as well in [81].

The introduction of the extrinsic method is due to specific difficulties encountered when dealing with spheroidal coordinates and spheroidal vector wave functions, e.g. [82]. There exists however a (not yet fully achieved) line of research to develop intrinsic techniques in spheroidal coordinates. A general solution to the intrinsic method for the evaluation of BSCs in spheroidal coordinates has been introduced in [83]. Numerical implementations have been published in the case of an on-axis standard (parallel) configuration [84] and in the case of an on-axis oblique illumination [80]. The case of off-axis configurations is still to be implemented to complete this line of research.

Furthermore, there exist several variants of the GLMT for spheroids, with different conventions and notations which may make difficult for the newcomer to enter the field, but see review in [85] which compared different approaches and stressed the ways with which one approach can be converted to the other.

This being introduced, we begin to review the fiducial period with Zhao *et al.* [86] who dealt with the scattering of an oblique incident focused Gaussian beam by a prolate spheroidal particle in the GLMT framework. Results from GLMT were compared with those from the surface integral equation method (SIEM), and very good agreements were observed. BSCs were evaluated by using an extrinsic method which expresses the spheroidal BSCs in terms of the spherical BSCs, the latter being evaluated using the MLA. For SIEM and more papers on SIEM, see subsection 8.5.

Next, Sun and Wang [87] provided the general formulas that can be used to compute the amplitude matrix for on-axis polarized Gaussian light beam scattering by a spheroidal particle with an inclusion which can be spherical, confocal spheroid or non-confocal spheroid, within the GLMT framework. An extrinsic approach with spherical BSCs expressed by a localized approximation has been used by Sun *et al.* [88] dealing with the scattering of polarized Gaussian light by a spheroidal particle, in the GLMT framework.

Tang and Zheng [89] dealt with the scattering of spheroids within a Gaussian beam, in a GLMT framework, and used the sphere of the same volume of the spheroid for calculating the scattered intensity of the spheroid within the Gaussian beam. For a comparison, the scattering intensity of the spheroid illuminated by a plane wave was also calculated. This paper may be completed by Zheng and Tang [90] who dealt with the scattering cross-sections of mineral aerosols illuminated by Gaussian beams. Spheroidal shapes were used to model non-spherical mineral aerosol particles, and a model of equal surface area to represent spheroids was discussed.

For other papers devoted to the issue of the present subsection, see Zhang *et al.* [91], Yuan *et al.* [92] in the next subsection devoted to cylinders and cylindrical coordinates, Mitri [93], [94] in subsection 3.2 devoted to laser sheets, Chen *et al.* [95], Han *et al.* [96], [97], Han [98] in subsection 3.3 devoted to non-diffracting beams, Gong *et al.* [99] in subsection 3.7 devoted to acoustical beams,

Yang *et al.* [100], Arzola *et al.* [101], Liaw *et al.* [102], Li *et al.* [103], Simpson *et al.* [104], Xu *et al.* [105], Mitri [106], [107] in section 4 devoted to mechanical effects of light, Seo *et al.* [108] in subsection 7.3 devoted to holography, Yi *et al.* [109] in subsection 7.5 devoted to miscellaneous measurement techniques, Chang *et al.* [110] in subsection 8.2 devoted to ray optics, Lin *et al.* [111] in subsection 8.3 devoted to Debye series, Brzobohaty *et al.* [112], Yang *et al.* [113], Zhang [114], Gao and Zhang [115], Wang *et al.* [116], [117], Cui *et al.* [118], Wu *et al.* [119], Chen *et al.* [120] in subsection 8.5 devoted to numerical techniques, and Han *et al.* [121] in section 10 devoted to reviews and tutorials.

### 2.3 Cylinders and cylindrical coordinates.

Wang *et al.* [122] studied the reflection and the transmission of Gaussian beams by a uniaxial anisotropic slab. They expanded the incident Gaussian beam in terms of cylindrical vector wave functions and used an extrinsic method to express the cylindrical BSCs in terms of on-axis spherical BSCs  $g_n$  in a localized approximation. Chen *et al.* [123] dealt with the transmission of a Gaussian beam through a gyrotropic cylinder, in the framework of a GLMT formulated in terms of cylindrical vector wave functions, using an extrinsic method for the evaluation of BSCs, with on-axis spherical BSCs  $g_n$  expressed by using a localized approximation. Yan *et al.* [124] dealt with the transmission of a Gaussian beam through a coated chiral cylinder, in a GLMT-framework, using expansions over cylindrical vector wave functions, and an extrinsic method in terms of on-axis spherical BSCs  $g_n$  expressed by using a localized approximation. Huang *et al.* [125] dealt with the propagation (reflection and transmission) of Gaussian beam through a uniaxial anisotropic slab, but using expansions of the incident Gaussian beam in terms of a continuous spectrum of rectangular vector wave functions. BSCs were expressed by using a localized approximation in terms of the on-axis spherical BSCs  $g_n$ .

Li *et al.* [126] investigated the characteristics of reflected and transmitted fields of a vector Bessel vortex beam through multilayered isotropic media (layer of slabs) on the basis of the vector angular spectrum expansion and presents the effects of media on intensity, phase, and polarization. Li *et al.* [127] studied the deformations of obliquely incident circularly polarized Bessel vortex beam reflected and transmitted by a uniaxial anisotropic slab, by use of cylindrical vector wave functions. For interaction with a slab, see also Li *et al.* [128] in subsection 3.4 devoted to helical beams.

Zhang *et al.* [91] studied the Gaussian beam scattering by a particle above a plane surface. BSCs were expressed from expansions in terms of vector cylindrical wave functions and evaluated by using an extrinsic method in terms of on-axis spherical BSCs  $g_n$  of GLMT within the framework of the OLA. Numerical simulations of differential scattering cross-sections were carried out for spheres, prolate and oblate spheroidal particles, and also circular cylindrical particles.

The proposed theory was to a certain extent verified by comparing the numerical results yielded from the new formulae for the case of a polystyrene sphere illuminated by a plane wave with those from the EBCM, see Yuan *et al.* [92] for a similar report. Iqbal *et al.* [129] studied the scattering of plane waves by elliptical/circular homogeneous, isotropic, and non-magnetic dielectric cylinders coated with chiral medium and surrounded by a perfectly electrical conducting sheath helix structure, allowing one to use the ASD method for studying the scattering of Gaussian shaped beam. Mitri [130] aimed to a complete analytical modeling for the local scattering, absorption and extinction cross-sections (or efficiencies) for a circular electrically conducting cylinder in a homogeneous non-magnetic medium, illuminated by plane waves of different polarizations and arbitrary incidence angle located near a perfect-conducting corner space. The analysis used the classical method of images, and the electromagnetic scattering from the cylindrical particle located near the corner could be formulated as a multiple scattering phenomenon on four cylinders with four incident waves based on the multipole expansion method in cylindrical coordinates as well as on the translation addition theorem of cylindrical wave functions. Zhu *et al.* [131] dealt with the transmission of arbitrary electromagnetic beam through uniaxial anisotropic cylinder using surface integral equations and computed normalized internal and near-surface field intensity distributions in the cases of on-axis Gaussian beam and Hertzian electric dipole illuminations.

For other papers devoted to the issue of the present subsection, see Mitri [132], [133], [134] in subsection 3.2 devoted to laser sheets, Kozlova *et al.* [135] in subsection 3.6 devoted to pulses and laser pulses, Mitri [136], [137], Zhang *et al.* [138], [139], Gong *et al.* [140] in subsection 3.7 devoted to acoustical beams, Mitri [141], [142] in section 4 devoted to mechanical effects of light, Gagnon and Dubé [143] in section 5 devoted to internal fields, Mitri [144] in section 6 devoted to the optical theorem, Swirniak and Mroczka [145], [146], Swirniak and Glomb [147], Duan *et al.* [148] in subsection 7.2 devoted to rainbow refractometry, Laurendeau *et al.* [149] in subsection 7.4 devoted to imaging and microscopy, Wang *et al.* [116], Yu *et al.* [150], Cui *et al.* [118], Zhu *et al.* [131] in subsection 8.5 devoted to numerical techniques, Gagnon *et al.* [151], Shiloah [152] in section 9 devoted to miscellaneous issues, and Han *et al.* [121] in section 10 devoted to reviews and tutorials.

## 2.4 Metamaterials.

Ambrosio [153] studied the extraction of metamaterial properties of negative-index and plasmonic scatterers from the Mie scattering coefficients, breaking up these coefficients into two terms, the first one representing the expected fields of a corresponding positive-index sphere, and the second one emerging from the intrinsic metamaterial nature of the scatterer. This work was devoted to the task of providing a theoretical formalism for the understanding of the scattering properties of NRI (negative refractive index) spheres in terms of additional

contributions which arose exclusively from the metamaterial nature of the scatterers. The results obtained are useful both in Lorenz-Mie Theory (LMT) and GLMT insofar as Mie coefficients are shared by both theories (with, let us recall, LMT being a limit case of GLMT). On a similar issue, Ambrosio [154] dealt with symmetry relations in the GLMT for lossless NRI spherical scatterers, particularly symmetries exhibited by NRI BSCs when compared to the conventional PRI (positive refractive index) case, see [155] for PRI symmetries. The theoretical analysis could be simplified for Rayleigh scatterers where only the dipolar coefficients are retained, all other coefficients being discarded as long as resonances were neglected. The knowledge of such symmetries allows one to speed up numerical evaluation of physical parameters such as optical forces and scattering intensities.

Wang *et al.* [156] analytically and numerically investigated the internal optical forces exerted by an electromagnetic wave inside an amorphous metamaterial medium, in relation with stretching and deforming effects. The paper also provided a comparison between the results obtained by using various expressions of the stress tensor, namely Helmholtz, Einstein-Laub and Minkowski stress tensors. It was concluded that the use of the Helmholtz stress tensor was appropriate in the framework of the study expounded in the paper. Some comments concerned the use of GLMT and of the Maxwell stress tensor for the evaluation of optical forces.

### **3 Around generalized Lorenz-Mie theories for various beam shapes.**

#### **3.1 Gaussian beams.**

Gaussian beams have been, for a long time, the most common kind of beams considered in various fields, particularly OPC, and the first concrete applications of the GLMT *stricto sensu* have indeed been devoted to Gaussian beam applications, see the introduction to section 7 devoted to OPC. Gaussian beams are still of current use and, accordingly, many papers in this review are concerned with the use of Gaussian beams. In principle, they can be reached by using the keyword "Gaussian" with the find button of a pdf.file. However, for a few typical examples, see Chrissoulidis and Richalot [157] dealing with Gaussian beam scattering by an off-axis sphere, relying on the decomposition of the incident beam into the spectrum of plane waves (ASD), and also Li *et al.* [73] in section 4 devoted to mechanical effects of light, Yu *et al.* [158] in subsection 7.2 devoted to rainbow refractometry, Wu *et al.* [159] in subsection 7.3 devoted to holography, Selmke and Cichos [160] in subsection 7.4 devoted to imaging and microscopy, Mackowski and Mischenko [161] in subsection 8.1

devoted to EBCM for structured beams, and Li *et al.* [78] in subsection 8.3 devoted to Debye series.

### 3.2 Laser sheets.

After applications of the GLMT to the case of Gaussian beam illumination, early applications of GLMT dealt with the case of laser sheet illumination as well, e.g. [162], [163], [164], and [165]. During the fiducial period, the issue of laser sheets has been vigorously investigated by Mitri in several papers which have been devoted to various kinds of light sheets. I start with Mitri [133] who dealt with cylindrical particle manipulation and negative spinning using a nonparaxial Hermite-Gauss light-sheet beam, relying on the ASD, with BSCs evaluated using numerical integration. The analysis was extended to evaluate the longitudinal and transverse radiation forces as well as the spin torque on an absorptive dielectric cylindrical particle in 2D. The dynamics of the cylindrical particle were examined as well based on Newton's second law of motion. Laguerre-Gauss light-sheet beams were also considered. Mitri [166] dealt with nonparaxial fractional Bessel and Bessel-Gauss auto-focusing light-sheet pincers and their higher-order spatial derivatives which are synthesized stemming from the ASD. The analysis then presented a rigorous approach for the description of a novel type of light-sheets having asymmetric tight bending and curving characteristics in the form of pincers. Mitri [93] studied negative optical radiation force and spin torques on subwavelength prolate and oblate spheroids in fractional Bessel-Gauss pincers light-sheets, with arbitrary orientation in space, using the ASD. Numerical results were obtained in the framework of the dipole approximation. Mitri [132] studied nonparaxial scalar Airy light-sheets and their higher-order spatial derivatives, with BSCs being evaluated by using standard numerical integration. He used the ASD as well as the multipole expansion method in cylindrical coordinates. Mitri [61] studied pulling and spinning reversal of a subwavelength absorptive sphere in adjustable vector Airy light-sheets which have the property of being nondiffracting, self-accelerating and self-healing, in the framework of the electric dipole approximation. Mitri [134] studied radiation force and torque produced by light-sheets expressed using a GLMT framework expansion in terms of BSCs which were derived using an ASD. Numerical illustrative examples concerned dielectric absorptive cylinders in different wavefronts, ranging from plane waves to nonparaxial scalar Airy and Gaussian light-sheet beams. The general aim of this work was to provide a study of mechanical effects of light produced by Airy light-sheet beams of arbitrary character illuminating 2D objects of arbitrary cross section, using exact closed-form partial wave series expansion in cylindrical coordinates. Mitri [167] dealt with self-bending scalar and vector bottle sheets, demonstrating the generation of auto-bending cylindrical/tubular Bessel-Gauss bottle beams in homogeneous 2D space. Mitri [94] dealt with the computation of negative radiation forces on a subwavelength spheroid and spin torque reversal in adjustable vector Airy

light-sheet single optical tweezers, using a dipole approximation. This is to be completed by Li *et al.* [168] who used a GLMT-like approach for the scattering of structured beams in terms of Taylor expansions, with the first term of the expansion corresponding to a plane wave, with the formalism being applied to the case of a particle in an elliptical (light-sheet) Gaussian beam and, with a similar approach using Taylor series, by Li and Feng [169] who dealt with the scattering of an electromagnetic wave propagating into an anisotropic medium.

For other papers devoted to the issue of the present subsection, see Bai *et al.* [51] in subsection 2.1 devoted to spheres and spherical coordinates, Bai *et al.* [170] in section 4 devoted to mechanical effects of light, Wu *et al.* [171] in subsection 7.2 devoted to rainbow refractometry, Wu *et al.* [172], [173] in subsection 7.3 devoted to holography, Meng *et al.* [174] in subsection 7.4 devoted to imaging and microscopy, and Satinover *et al.* [175] in subsection 7.5 devoted to miscellaneous measurement techniques.

### 3.3 Non-diffracting beams.

Non-diffracting beams have been introduced by Durnin *et al.* [176], Durnin [177] and Mishra [178]. They may be defined as monochromatic beams preserving an unchanged transverse intensity profile during free-space propagation. In other words, they are "tube-like" and do not exhibit any waist, in contrast with the most familiar case of Gaussian beams. For reviews, the reader may refer to Bouchal [179], to a book edited by Hernandez-Figueroa *et al.* [180], entitled "localized waves" (another terminology for non-diffracting waves), and to Recami *et al.* [181] in a book edited again by Hernandez-Figueroa *et al.* [182]. The most famous example of non-diffracting beams is Bessel beams to which a huge literature is already dedicated. Because their ASD is made of plane waves propagating tilted with respect to the general direction of propagation of the beam ("direction of propagation" of the beam which is different from all directions of propagation of the plane waves encompassed in the ASD), they are self-healing, i.e. they efficiently regenerate themselves after being obstructed by a scattering particle, due to the replacement of plane wave disturbances by fresh elements of plane waves which have not been disturbed. They are characterized in particular by a propagation term of the form  $\exp(ikz \cos \alpha)$  in which  $\alpha$  is the half-cone angle (or axicon angle) which differs from the more usual propagation term of the form  $\exp(ikz)$  shared by many kinds of beams, such as Gaussian beams. The existence of the axicon angle implies that the velocity of the beam along its "direction of propagation" is  $c \cos \alpha$ , in which  $c$  is the speed of light, i.e. it is smaller than the speed of light. Although this property of having an average velocity smaller than the speed of light is well apparent for beams exhibiting an axicon angle, it is actually a general property for all kinds of beams, excepted for plane waves [183]. The existence of the axicon angle furthermore implies a deleterious effect for the validity of localized approximations, see subsection 3.5 discussing warnings concerning the use of localized approximations.

Chen *et al.* [62] dealt with the scattering of an on-axis zeroth-order Bessel beam by a concentric sphere (i.e. a sphere with a concentric spherical inclusion) in a GLMT framework. The BSCs were evaluated by using the ILA. GLMT results were compared with those of SIEM (see subsection 8.5 for papers devoted to SIEM) and good agreement was observed. Chen *et al.* again [95] dealt with the scattering of an on-axis zeroth-order Bessel beam by a conducting spheroidal particle with a dielectric coating, in a GLMT framework. They evaluated the spheroidal BSCs by using an extrinsic method in terms of on-axis spherical BSCs  $g_n$ , quoting Chen *et al.* [62] which used the ILA to evaluate the spherical BSCs. Han *et al.* [96] studied the expansion of a zeroth-order Bessel beam in spheroidal coordinates in the GLMT framework, using an intrinsic method to the evaluation of BSCs. From the same group [97], we also have a study of the internal and near-surface electromagnetic fields for a dielectric spheroid illuminated by an on-axis zeroth-order Bessel beam, in the framework of GLMT, using an intrinsic method to the evaluation of BSCs.

Chen *et al.* [63] dealt with a scattering analysis of an on-axis zeroth-order vector Bessel beam by a multilayered sphere, in a GLMT framework, with BSCs evaluated by using a localized approximation, and validation by using the SIEM. Preston and Reid [64] discussed the angular scattering of light by a homogeneous spherical particle in a zeroth-order Bessel beam and its relationship to plane wave scattering, in a GLMT framework. They investigated the dependence of the angular scattering on the semi-apex angle of the Bessel beam and discussed the major features of the resulting scattering plots. The BSCs were computed by the double quadrature technique. Mitri *et al.* [184] provided a study of the resonance scattering of a homogeneous dielectric sphere illuminated by electromagnetic Bessel beams with arbitrary incidence and selective polarizations, in the context of a modified version of the GLMT. BSCs were derived using the ILA and an addition theorem, while Debye series were used to deal with the scattering coefficients, therefore separating diffraction from the reflection and internal reflection terms (see subsection 8.3 for Debye series). The analysis was furthermore extended to include rainbow formation in Bessel beams, and a generalized formulation for the deviation angle of the two primary rainbows was obtained with further extension to any high-order rainbows. Li *et al.* [65] dealt with the scattering of a zeroth-order Bessel beam by uniaxial anisotropic bispheres, relying on the GMM, with BSCs evaluated by using quadratures.

Chafiq and Belafhal [185] dealt with Mathieu beams which are non-diffracting beams formed from a superposition of Bessel beams of various orders, and calculated radiation pressure cross sections exerted on homogeneous dielectric spherical particles by linearly polarized zeroth-order Mathieu beams (and zeroth-order Bessel beams which are particular cases of Mathieu beams), in the framework of GLMT, with BSCs computed by using ILA. The same authors, Chafiq and Belafhal, [186] furthermore also dealt with the scattering of Lommel beams (another variant of superposition of Bessel beams, relying on a discrete summation of first-kind Bessel beams with various orders) by homogeneous spherical particles in the GLMT framework, using the ILA for the evaluation of BSCs. See also Cui *et al.* [118] for another study of Lommel beams in subsection 8.5 devoted



to numerical techniques.

A thorough analysis of Bessel beams has been carried by Wang *et al.* [187], [188] and [189]. The first paper in this series [187] provided a general description of circularly symmetric Bessel beams of arbitrary order, bridging the gap between different descriptions of Bessel beams and leading to a general description of circularly symmetric Bessel beams, in such a way that the Davis Bessel beams and the aplanatic Bessel beams are merely the two simplest cases of an infinite number of possible circularly symmetric Bessel beams. Additionally, magnitude profiles of the electric and magnetic fields, the energy density and the Poynting vector, were displayed for Bessel beams in both paraxial and non-paraxial cases. Beside, various kinds of polarization were examined, which were not circularly symmetric, namely linearly x-polarized Bessel beam, and linearly y-polarized Bessel beam. Next, Wang *et al.* [188] provided a general description of transverse mode Bessel beams and the construction of basis Bessel fields, as a continuation and extension of the previous paper. A vector analysis of polarized Bessel beams in a transverse mode was implemented using the Hertz vector potentials and the ASD. Finally, Wang *et al.* [189] dealt with the multipole expansion of circularly symmetric Bessel beams of arbitrary order for scattering calculations, and provided analytical expressions of BSCs both for on-axis and off-axis cases.

Briard *et al.* [190] dealt with the scattering of aggregated particles illuminated by a zeroth-order Bessel beam in a GLMT framework with BSCs computed using analytical closed-form expressions. Wen *et al.* [66] studied the scattering of a vector Bessel-Gauss beam by a sphere in the framework of GLMT, using the ASD, and the analytic expressions of BSCs were derived utilizing a multipole expansion method using vector spherical harmonics. Various polarizations were considered. Wang *et al.* [67] studied the scattering of a Bessel beam by a nucleated biological cell, using an eccentric sphere model in the framework of GLMT. BSCs were evaluated in closed forms and compared with BSCs evaluated using the ILA. Differential scattering cross sections were calculated for on-axis and off-axis zeroth-order Bessel beams, with parallel and oblique illuminations. Han [98] studied the scattering of a high-order Bessel beam by a homogeneous spheroidal particle within the framework of GLMT. The spheroidal BSCs were computed using an intrinsic method. Numerical results concerning scattered field in the far zone were displayed for various parameters of the incident Bessel beam and of the scatterer. Qu *et al.* [68] studied the interactions of high-order Bessel vortex beams with a multilayered chiral sphere, focusing on scattering properties and on an orbital angular momentum spectrum analysis. They used what they called the vector wave theory which is actually a GLMT (private communication) with BSCs computed by using the quadrature technique. Let us however note and insist on the fact that closed form expressions were available to compute the BSCs of such beams, allowing one to avoid the time-consuming quadrature method, e.g. [189], [191], [192]. Gong *et al.* [69] dealt with the scattering of a vector Bessel beam by a charged sphere, with BSCs evaluated by using analytical expressions which were derived using the ASD. Numerical results were displayed and discussed for scattering,

extinction, and absorption efficiencies. The framework was explicitly the one of a GLMT for charged spheres, a fundamental difference with a classical GLMT concerning the modification of boundary conditions on the surface of the scatterer due to the presence of charges.

We now deal with another class of non-diffracting beams formed by a superposition of Bessel beams, denoted as frozen waves. Ambrosio [193] introduced particular classes of vector waves for light scattering calculations, viz. structured light fields composed of specific superpositions of circularly symmetric Bessel beams of arbitrary order. Also known as generalized frozen waves, such beams carry all the non-diffracting properties of their constituent with the additional feature of allowing for an arbitrary design of the longitudinal intensity pattern. This feature makes the generalized frozen waves especially suitable for optical confinements and manipulation and atom guiding and selection. The BSCs were evaluated in exact and analytic form in a GLMT framework and optical forces were calculated for specific dielectric Rayleigh particles. Frozen waves, specifically zeroth-order continuous vector frozen waves, were also considered by Angelis and Ambrosio [194] to evaluate optical forces in the Rayleigh regime, envisioning applications in optical trapping and micromanipulation, atom guiding, particle sorting and so on. Such microstructured light fields can be designed to provide a wide variety of longitudinal intensity patterns along the optical (longitudinal) axis and on the micrometer scale, being constructed from a continuous superposition of Bessel beams. Also, Ambrosio and Zamboni-Rached [195] dealt with an analytical approach of ordinary frozen waves for optical trapping and micromanipulation. The optical properties of frozen waves were theoretically and numerically investigated using the GLMT together with the ILA. Frozen waves were constructed from a suitable superposition of equal-frequency ordinary Bessel beams and were capable of providing almost any desired longitudinal intensity profile along their optical axis, thus being of potential interest in applications in which intensity localization may be used advantageously, such as in optical trapping and micromanipulating systems. Expressions for the BSCs were provided in the framework of the GLMT which was used to reconstruct the intensity profiles of the waves and to predict their optical properties for possible biomedical optics purposes. On a similar but extended issue, Ambrosio *et al.* [196] dealt with discrete vector frozen waves in a GLMT framework, under various polarizations (linear, azimuthal and radial polarizations).

A limit of Bessel beams is that they are not physical beams as far as they convey an infinite amount of energy, although good enough approximations of them can be constructed in the laboratory. In relation with this issue, Ambrosio [197] dealt with analytical descriptions of finite-energy Bessel beams, as a first attempt to introduce physical or finite-energy axial symmetric fields, under the paraxial approximation into the general framework of GLMT, relying on a discrete superposition of Bessel-Gauss beams. As an example of application, optical forces exerted on spherical nano and microparticles were calculated and compared with those evaluated with ideal scalar Bessel beams.

For other papers devoted to the issue of the present subsection, see Li *et al.* [126], [127] in subsection 2.3 devoted to cylinders and cylindrical coordinates,

Mitri [61] in subsection 3.2 devoted to laser sheets, Mitri and Silva [198], Gong *et al.* [99], Leao-Neto *et al.* [199], Jiang *et al.* [200], Gong *et al.* [140], Mitri [201] in subsection 3.7 devoted to acoustical beams, Ruffner and Grier [202], Song *et al.* [203], Preston *et al.* [204], Wang *et al.* [205], Ambrosio and Ferreira [206], Ambrosio and Zamboni-Rached [207], Mitri *et al.* [208], Yang *et al.* [209], Mitri *et al.* [210], Mitri [106], [107], Li *et al.* [75], [211], Lu *et al.* [212], Chen *et al.* [213], Mckell and Bonin [214], Bai *et al.* [215], [216], Li *et al.* [217] in section 4 devoted to mechanical effects of light, Chen *et al.* [218] in section 5 devoted to internal fields, Rondo-Ojeda and Soto-Eguibar [219] in section 6 devoted to the optical theorem, Li *et al.* [78], Qin *et al.* [220] in subsection 8.3 devoted to Debye series, and Cui *et al.* [221], [222], Sundaram and Wen [223], Yang *et al.* [224], [225], Zhang [114], Wang *et al.* [116], Yu *et al.* [150], Wu *et al.* [119], [226], Chen *et al.* [120] in subsection 8.5 devoted to numerical techniques. For a warning concerning the use of localized approximations, including ILA, in the case of non-diffracting beams, see subsection 3.5 below.

### 3.4 Helical beams.

Fields of helical beams possess a term reading as  $\exp(i\nu\varphi)$  in which  $\varphi$  is an azimuthal angle and  $\nu$  is best called the topological charge (or sometimes the order of the beam). Helical beams may possess as well an axicon angle term such as higher-order Bessel beams and higher-order beams based on the superposition of higher-order Bessel beams. Such "axiconic"-helical beams have been considered in the previous subsection. Let us consider them again for a while in the present paragraph to explain why such helical beams are also (most usually) named vortex beams. The total phase of the field is then  $\phi = (kz \cos \alpha + \nu\varphi)$ . The wavefront is a surface of constant phase. Therefore, as the beam evolves toward positive  $z$ 's, the angle  $\varphi$  has to simultaneously decrease to keep the total phase constant. As a result, the wavefront has a helical structure. Furthermore, the phase is indeterminate on the beam axis producing a phase singularity, which can be physically accommodated only if the amplitude (intensity) on the beam axis is zero, i.e. the beam is dark on the beam axis, exhibiting a central dark core. There is however a subtlety that is hidden behind the above argument, namely that a helical beam is not necessarily a vortex beam. This happens for example for higher-order Bessel beams having a topological charge equal to  $\pm 2$  in which all components contain optical vortex contributions with zero magnitude on the beam axis; while some components furthermore contain  $J_0$ -contributions which are not optical vortices, see a detail discussion in [227], and also [228] for a complementary discussion concerning the introduction of the darkness theorem. Other examples are obtained from localized models of Laguerre-Gauss beams freely propagating for topological charges equal to  $\pm 2$ . The paraxial approximations of such beams are vortex beams but they are not Maxwellian (they do not satisfy Maxwell's equations). Conversely, their localized models are Maxwellian but are no more vortex beams [229]. For these reasons,

due to the existence of counter-examples, beams considered in this subsection are named helical beams rather than vortex beams. Furthermore, higher-order Bessel beams and superpositions of such beams are no more considered, insofar as they have already been discussed in the previous subsection. We therefore now deal with helical beams without any axicon angle. The best example is formed by Laguerre-Gauss beams which carry angular momentum of two kinds, a spin angular momentum associated with the polarization state of the beam and an orbital angular momentum related to the spatial variations of the field.

We may now proceed with a review concerning the fiducial period, starting with Kiselev and Plutenko [33] who discussed the scattering of Laguerre-Gauss beams by spheres, with an emphasis on photonic nanojets (see subsection 3.8) and near-field optical vortices, with BSCs expressed from double quadratures quite similar to the ones used in the GLMT framework. Qu *et al.* [70] dealt with the analysis of the radiation force of a Laguerre-Gauss beam exerted on an uniaxial anisotropic sphere. Based on the expansion of Hermite-Gauss (HG) beam in terms of VSWFs and on the relation between Hermite-Gauss beams and Laguerre-Gauss modes, they derived the expansion of Laguerre-Gauss beams in terms of VSWFs. The radiation force exerted on a uniaxial anisotropic sphere by a Laguerre-Gauss beam was then calculated in a GLMT framework. Qu *et al.* [71] similarly dealt with the light scattering of a Laguerre-Gauss beam by a chiral sphere in a GLMT framework, with the BSCs evaluated by expressing the Laguerre-Gauss beam as a superposition of Hermite-Gauss modes, similarly as in [70].

Al-Jarro *et al.* [72] carried out the analysis of the resonant intermixing between optical orbital and spin angular momentum of Laguerre-Gauss beams, mediated by chiral clusters made of silicon nanospheres. They provided a comprehensive description of the scattering of Laguerre-Gauss beams from clusters of spherical nanoparticles with specific rotational symmetry properties, by analyzing both the physical properties of the scattered field as well as the nature of the near-field. They focused their study on  $LG_{0l}$  beams ( $p = 0$  being the radial index and  $l$  the topological charge). Scattering was analyzed using a multiple-scattering matrix method in which fields were expanded over VSWFs and the T-matrix was evaluated in a GLMT-like method. Mitri [230] developed the most general solution corresponding to a transition from a vortex to a nonvortex higher quasi-Gaussian beam. It was indeed shown that it is possible to create a coherent superposition of two propagating high-order quasi-Gaussian vortex beams of opposite handedness (i.e. opposite helicity or topological charge) and different amplitudes. Depending on the amplitude difference between the two beams, the resulting field could carry a total angular momentum (i.e. beam of vortex nature), or not (i.e. beam of nonvortex type). See also Mitri [231] for another paper dealing with vector spherical quasi-Gaussian beams.

Li *et al.* [128] dealt with the reflection and transmission of Laguerre-Gauss beams in a dielectric homogeneous slab, relying on a decomposition of the incident fields using an ASD. The interaction of each individual plane wave in the spectrum with the slab is a fairly classical problem, and the interaction produced by the Laguerre-Gauss beam was viewed as the combination of the

interaction of the elementary plane wave components of the ASD. Analytical expressions of total reflected and transmitted field intensities were simulated. A particular attention was paid to the effects of the beam waist radius and of the topological charge of the beam.

For other papers devoted to the issue of the present subsection, see Arzola *et al.* [101], Yu and She [74], Zhang and Yang [232] in section 4 devoted to mechanical effects of light, Guo *et al.* [79], [233] in subsection 7.5 devoted to miscellaneous measurement techniques, and Yu *et al.* [234] in the subsection 8.5 devoted to numerical techniques. For a warning concerning the use of localized approximations, including ILA, in the case of helical beams, see next subsection below.

### **3.5 Localized approximations and warnings for the use of localized approximations.**

Among the various methods used to evaluate BSCs, localized approximations (including variants) have been the most popular and, in particular, allowed one to deal efficiently with the GLMT for Gaussian beams and laser sheets, see [235] for a review and [58], [236], [237] for complements. This issue of localized approximation was extensively investigated by Jia *et al.* [238] who provided a very comprehensive study devoted to GLMT-computations based on localized beam models, taking account the most recent remarks concerning the status of the art in this topic [58], in particular the recommendation to use a new terminology, carefully distinguishing between what is called the Original Localized Approximation (OLA) and what is called the Modified Localized Approximation (MLA). In both cases, they provided an improved algorithm of GLMT calculations based on localized beam models, in order to overcome the difficulties which may be due, in certain cases, to the limit of computer resources. The essence of the algorithm was to calculate BSCs and involved angular functions by multiplying them with pre-factors so as to keep their values in a reasonable range, leading to an algorithm which is efficient, reliable and robust. They also compared the relative merits of OLA and MLA and, although MLA is in principle more accurate, they convincingly pointed out that OLA is more practical because it considerably simplifies and speeds up numerical calculations with respect to MLA. Whether an user would prefer to use OLA or MLA is still a matter of choice but the very detailed paper by Jia *et al.* is an irreplaceable source of information to help making one choice. There is only one quite extra-minor remark that I would like to do. The last sentence of Section 4 in [238] concerning the use of higher-order approximations of Gaussian beams should be read while having in mind the remarks made below concerning the use of higher-order Davis-Barton beams in relationship with the use of localized approximations, and the fact that localized approximations applied to first-order Davis beams anticipate the behavior of higher-order beams.

This issue of anticipation may be discussed as follows: we can indeed introduce a localized operator in higher-order Davis-Barton beams and this indeed provides genuine beams, but it is erroneous to believe that applying a localized approximation to higher-order beams provides better results than applying it to first-order beams. On the contrary, it deteriorates the situation. This happens indeed because applying a localized approximation to first-order Davis beams anticipates the behavior of higher-order beams. For more details, the reader may refer to [239], 12 lines below Eq.(75), in a sentence containing "anticipates the form", and 3 lines below Eq.(84), in a sentence containing "so closely related", and also Fig 1. See also [240], page 2139, bottom of column 1, [241], page 38, [242] applying a localization operator to higher-order beams, particularly conclusions page 2704, column 2, stating that such a process does not improve the quality of the Gaussian beam, and p. 2705, column 1, [243], p. 23, column 2 and quoted references (25), (27), (33), (36), and [244], section 4, pp.516-518, for a similar discussion in cylindrical coordinates.

A compact formulation of the BSCs for elliptical Gaussian beams (also called laser sheets) based on a localized approximation is discussed by Shen *et al.* [245], including a comparison with the ILA, to be completed by Wang and Shen [246] dealing again with BSCs for an elliptical Gaussian beam using a 1D quadrature and localized approximation methods, and by Shen *et al.* [247] who dealt again with elliptical Gaussian beams and evaluated BSCs by comparing the quadrature method, a localized approximation, and the implementation of a method based on an ASD leading to 1D-quadratures. The three methods were compared in terms of computing time efficiency and of accuracy. They confirmed that a localized approximation is the most efficient and that the original quadrature technique is the most time consuming. For the quadrature technique to the evaluation of BSCs, return to [24] and references therein. For laser sheets, return to [163], [164] and references therein. See also Qiu and Shen [248] in section 10 devoted to reviews and tutorials.

I end this subsection with a story (which has not yet reached its end) which deserves to be told in its own right. This story concerns the domains of validity of localized approximations in spherical coordinates (and could certainly be extended to the use of other kinds of coordinates, if only by relying on the use of extrinsic methods). After a justification of a localized approximation for Gaussian beams in 1994 [239], [249], which can however be extended to all existing variants including the variant known as ILA, a justification for "arbitrary" shaped beams was produced in 1999 using what is known as a N-procedure [250]. The definition of "arbitrary" shaped beams, fairly reasonable at this time, particularly in the field of OPC, assumed that the fields were expressed by a propagation factor of the form  $\exp(\pm ikz)$ , for a propagation along the axis  $z$ , multiplied by a non singular function expressed in terms of a triply infinite multivariate polynomial involving powers of the Cartesian coordinates  $x$ ,  $y$  and  $z$ . However, as seen in subsections 3.3 (non-diffracting beams) and 3.4 (helical beams), several papers used beams which exhibit a propagation factor of the form  $\exp(\pm ikz \cos \alpha)$  in which  $\alpha$  is the axicon angle. In the case of such beams (even with a topological charge equal to zero), it has recently been shown that

the N-procedure fails to justify the use of localized approximations [251], addressing the question to know to which extent localized approximations can be used for "axiconic" beams, in particular, in the case of Bessel beams.

There are some examples quoted above of papers having indeed used a localized approximation, more specifically ILA, for the evaluation of BSCs of beams exhibiting an axicon angle, although I have not necessarily pointed them out. Examples are now to be given in the framework of a fruitful and efficient collaboration between Rouen (France) and São Paulo (Brazil). In particular, Ambrosio and Hernandez-Figueroa described zeroth-order [252] and higher-order Bessel beams [253] by using ILA with applications to optical trapping. The same issue is to be stressed on the case of beams formed by a superposition of Bessel beams, e.g. frozen waves [195], [207]. By comparing BSCs of zeroth-order Bessel beams evaluated either by a localized approximation or by a rigorous quadrature method amenable to closed forms, Gouesbet *et al.* [254] confirmed that there is indeed a limited domain of validity of localized approximations for Bessel beams, although they perform well when the axicon angle is small enough (typically less than  $10^\circ$ ). Calculations of radiation forces under Bessel beam illumination, previously reported by Ambrosio and co-workers, were thereafter revisited [255]. It was concluded that localized approximations, in particular ILA, provide adequate BSCs for Bessel beams up to certain limits and that, even when they fail to do so, reliable information on physical optical properties of interest can still be inferred, depending on specific geometries and electromagnetic aspects of the scatterer. For instance, the determination of optical forces and torques is fairly immune to the limitations of localized approximations in the case of Rayleigh particles, even for axicon angles as large as  $20^\circ$  and even more. Another paper by Ambrosio *et al.* [256] discussed the validity of localized approximations for discrete superposition of Bessel beams. Similar considerations hold for Mathieu beams which are built from Bessel beams as well [185], [257].

Other cases when the assumptions used in [250] are not satisfied are the ones of helical beams (having a topological charge), possibly exhibiting singularities, such as discussed in [229], [258] and, worst, the one when both axicon angles and topological charges are present on the stage [259]. There may be a moral to this story, namely that a theoretical method (here localized approximation, including variants) may have been so successful that it presents the risk of being used outside of its original domain of validity, a case of undue generalization. Ideally, when using a theory, we should always carefully return to its domain of validity before applying it to what could unfortunately become, otherwise, undue situations.

### 3.6 Pulses and laser pulses.

GLMTs under laser pulse illuminations have been discussed by Gouesbet and Gréhan [260], Méès *et al.* [261], [262], [263], [264], [265], Han *et al.* [266], and Méès *et al.* [267]. Returning to the fiducial period, we have Li *et al.*

[268] who dealt with the scattering and propagation of plane wave/Gaussian terahertz pulses in random soot aggregate systems by using the GMM. Next, Li *et al.* [269] studied the scattering and propagation of a UV pulse in soot aerosols using again the GMM. Soot aerosols were generated by the diffusion-limited aggregation model. Scattering intensities versus scattering angles were provided and discussed.

Kozlova *et al.* [135] modeled the resonance focusing of a picosecond laser pulse using a dielectric microcylinder. Based on the FDTD, they studied temporal variations of focal spot parameters when focusing a plane wave by use of a dielectric microcylinder. It was shown that as a picosecond pulse travels through the microcylinder, the focal spot gets tighter, whereas its intensity and the mode energy stored in the microcylinder grow. Du Preez-Wilkinson *et al.* [270] presented a method for the precise calculation of optical forces due to a tightly-focused pulsed laser beam using GLMT. They calculated forces for femtosecond pulses of various widths, and compared with forces due to a continuous wave beam. They assumed that the pulse had a Gaussian envelope, with a specified full-width half-maximum (FWHM). The beam was focused with a convergence angle of  $50^\circ$  (which corresponds to an optimally-filled objective of numerical aperture 1.02), and trapping took place in water. Shakov *et al.* [271] dealt with femtosecond optical trap-assisted nanopatterning through microspheres by a single Ti: Sapphire oscillator. Experimental results were "said to be" reasonably matched by GLMT in which BSCs were calculated by a quadrature method.

Devi and De [272] used GLMT, with BSCs evaluated by using a localized approximation, for a study of optical Kerr effect in femtosecond laser trapping of dielectric nanoparticles, calculating trapping optical forces and potentials. Results obtained by GLMT were compared with those of a dipole approximation. They concluded that there was a quantitative disagreement between the results obtained using either GLMT and dipole approximation, although the qualitative nature of the force (or potential) curves remained the same. The quantitative disagreement was attributed to the obvious fact that calculations were carried out under a tight focusing condition for which the beam shape factor  $s$  was quite high (0.179). See [3], Chapter 4 for a discussion concerning the beam shape factor and implications. Another paper by Devi and De [273] dealt with the theoretical estimation of nonlinear optical forces on dielectric spherical particles of arbitrary size under femtosecond pulsed excitation, using on-axis GLMT and a localized approximation to evaluate the BSCs, with a comparison between GLMT results and results obtained by using a dipole approximation. Experimental evidence indicated that high-repetition ultrafast pulsed excitation is more efficient in optical trapping of dielectric nanoparticles as compared with continuous-wave excitation at the same average power.

Zensen *et al.* [274] dealt with a femtosecond resolved measurement of optical scattering forces. They demonstrated how the scattering force acting on a single gold nanoparticle in solution could be measured. An optically trapped 80 nm particle was repetitively pushed from the side with laser light resonant to the particle plasmon frequency. A lock-in analysis of the particle movement provided



a measured value for the scattering force. The authors obtained a resolution of less than 3 femtonewtons (fN) which is an order of magnitude smaller than any measurement of switchable forces performed on nanoparticles in solution with single beam optical tweezers to date. They compared the results of the force measurement with LMT simulations of the optical scattering force on a gold nanoparticle and found good agreement between experiment and theory within a few fN.

Dealing with laser pulses, we now come to a series of 3 papers by Lock and Laven [275], [276], [277], with laser pulses being explicitly considered in the last paper of the series. These papers discussed co-polarized and cross-polarized scattering of an off-axis focused Gaussian beam by a spherical particle in a GLMT framework. The issue discussed in this series relied on the fact that, in static and dynamic light scattering, it has frequently been claimed that cross-polarized scattering cannot occur for single-scattering by a homogeneous sphere, a statement which is true for both plane wave and on-axis Gaussian beam incidence but which, as discussed in the present series, is not correct when the beam is translated off-axis perpendicularly to the scattering plane. The first installment of the series [275] was devoted to the main features of GLMT relevant to the issue to be discussed, using a localized beam model, particularly concerning the evaluation of scattering quantities requiring sums over both partial waves (usually denoted by a subscript  $n$ ) and azimuthal modes (usually denoted by a subscript  $m$ ) to be analytically or numerically performed. It was demonstrated that cross-polarized intensities are indeed nonzero when the beam is displaced off-axis perpendicularly to the scattering plane. Examples were provided, the issue was discussed in the framework of ray optics, and it was emphasized that nonzero cross-polarized scattering is a consequence of wave theory via a symmetry breaking process. In Part II [276], the GLMT angular functions were accurately approximated for large partial waves, weakly focused Gaussian beams, and specific ranges of values of the scattering angle  $\theta$ , namely both for  $\theta \approx 0^\circ$  and  $\theta \approx 180^\circ$ , as well as for  $0^\circ \ll \theta \ll 180^\circ$ . This allowed the sum over azimuthal modes to be evaluated analytically, leaving only the sum over partial waves to be evaluated numerically, and therefore greatly decreasing the computer run time required for off-axis GLMT computations for particles having a large enough size parameter. The approximation provided a close fit to the exact GLMT polarization-resolved intensity for a number of off-axis locations of the incident beam. Part III [277] focused on a study of diffraction in the near-forward direction for the localized model of an off-axis focused Gaussian beam scattered by a sphere when the beam was translated off-axis either in the scattering plane or perpendicular to it, in the framework of the Fraunhofer diffraction approximation. Various Debye contributions to scattering amplitudes were obtained, and time-domain scattering was discussed in the case of pulsed illumination, both for exact GLMT scattering amplitudes and as well for the approximation in which the sum over azimuthal modes was evaluated analytically.

For other papers devoted to the issue of the present subsection (or to low-coherence scattering sharing with laser pulses the property of exhibiting an

extended spectral range), see Minin and Minin [278] in subsection 3.8 devoted to photonic nanojets, Swirniak [279] in subsection 7.2 devoted to rainbow refractometry, and Lock and Laven [280] in subsection 8.3 devoted to Debye series.

### 3.7 Acoustical beams.

Although the present paper is essentially devoted to electromagnetic beams, it is worth devoting a subsection to acoustical beams due to the many analogies relating electromagnetic and acoustical scattering (more generally scalar scattering, including quantum scattering, e.g. [281], [282]). Such analogies are exhibited in a review by Thomas *et al.* [283] who very comprehensively discussed acoustical and optical radiation pressures and the development of single beam acoustical tweezers, with a strong emphasis on the mutual enrichment which arose from a long common history between electromagnetic and acoustical scattering. Differences have been pointed out such as the fact that acoustical tweezers overcome a limitation of optical tweezers which is the weak force that can be applied without thermal damage due to optical absorption, while ultrasounds are proven to be innocuous for living cells and propagate without significant attenuation in many materials. Conversely, analogies occurred from both experimental (existence of an acoustic equivalent of the "all-optical light trap", use of acoustical and optical vortex beams) and theoretical (Maxwell's stress tensor versus Brillouin's stress tensor, use of the GLMT in optics versus its acoustical counterpart, and evaluations of electromagnetic or acoustical BSCs, long-wavelength simplified expressions in optics and acoustics) points of view. A prospective section was devoted to the future of the field of acoustical tweezers. Another review by Thomas *et al.* [284] discussed the similarities and differences between optical and acoustical radiation pressure, and reviewed the different experimental schemes available to generate acoustical tweezers. It was stressed that the optical GLMT was recently adapted to acoustics, leading to a general expression of the acoustical radiation pressure exerted by an arbitrary beam on an elastic sphere and to the notice that the expressions of the optical or acoustical radiation pressures acting on a particle were remarkably very similar. In particular, both can be derived from the flux of pseudo-momentum on a far field closed surface.

Mitri and Silva [198] proposed a generalization of the extended optical theorem for scalar arbitrary shape acoustical beams of arbitrary character with any angle of incidence interacting with an object of arbitrary geometric shape and size, and placed randomly in the beam's path with any scattering angle (see section 6 for a discussion of the optical theorem). Analytical expressions for the extinction, absorption and scattering cross sections were derived, and the connection with the axial (e.g. along the direction of wave propagation) torque and radiation force calculations was discussed. As examples to illustrate the analysis for a viscoelastic object, the extinction, absorption, and scattering

cross sections were provided for an infinite plane progressive wave, infinite non-diffracting Bessel beams, a zeroth-order spherical quasi-Gaussian beam, and a Bessel-Gauss vortex beam emanating from a finite circular aperture (which reduces to a finite high-order Bessel beam), a finite zeroth-order Bessel beam, and a finite piston radiator vibrating uniformly with appropriate selection of beam parameters. A similarity with asymptotic quantum inelastic cross sections was also mentioned, e.g. [285] and references therein. Still on the optical theorem, Mitri [137] proposed a generalization in cylindrical coordinates for on-axis and off-axis scalar monochromatic acoustical beams of arbitrary wavefront interacting with a scatterer of arbitrary geometry and size, located arbitrarily in space.

Silva *et al.* [286] calculated the acoustic radiation force produced by an arbitrary shaped beam exerted on a sphere suspended in an inviscid fluid, using the translational addition theorem for spherical functions and the partial-wave expansion method ("acoustical GLMT"). BSCs were first evaluated for an acoustic beam relative to a reference frame, and the translational addition theorem could afterward be used to obtain them relative to the sphere positioned anywhere in the medium. Mitri [136] dealt with the interaction of an acoustical 2D arbitrarily-shaped beam with an elastic cylinder with arbitrary location that is immersed in a non-viscous fluid, generalizing the classical Resonance Scattering Theory (RST) for plane wave in acoustics. The analysis relied on the use of partial wave series expansion method in cylindrical coordinates which would be akin again to an "acoustical GLMT". Zhang *et al.* [138] studied the interaction between an acoustical Gaussian beam and cylindrical particles in water. Based on a finite series method (similar to the one used for vector beams, e.g. [27]), the Gaussian beam was expanded as cylindrical functions and the acoustical BSCs of the Gaussian beam was obtained. An expression for the acoustic radiation force function that is the radiation force per unit energy density and unit cross-sectional surface area for a cylindrical particle placed along the axis of a Gaussian beam and immersed in an ideal fluid was presented. Numerical results for the radiation force function of a Gaussian beam were presented for rigid cylinders, liquid cylinders, elastic cylinders, and viscoelastic cylinders to illustrate the theory. This paper is to be completed by Zhang *et al.* [139] who computed the acoustic radiation force exerted on a rigid cylinder in off-axial Gaussian beam using the translational addition theorem, still relying on a finite series method for the evaluation of BSCs. Gong *et al.* [99] dealt with the acoustical scattering of an acoustical Bessel beam by a rigid spheroid.

Leao-Neto *et al.* [199] dealt with the scattering of a longitudinal Bessel beam of arbitrary order by a spherical inclusion embedded in an isotropic elastic solid matrix. Absorbing, scattering and extinction efficiency factors were obtained and an extended optical theorem was derived. Several features of zeroth- and first-order Bessel beams scattering were illustrated. Beam shape coefficients were evaluated in closed forms. Jiang *et al.* [200] provided a general expression for the acoustic radiation force on a sphere immersed in a non-viscous fluid, placed in a progressive and standing zeroth-order quasi-Bessel-Gauss beam. Let us note that the quasi-Bessel-Gauss wave possesses the non-diffracting properties of the Bessel wave and the energy-focusing properties of the Gauss wave.

BSCs were evaluated using finite series expansions which were originally used to calculate the coefficients of Gauss beams in electromagnetic fields. Gong *et al.* [140] dealt with a "T-matrix" method for acoustical Bessel beam scattering from a rigid finite cylinder with spheroidal endcaps. Mitri [201] used a GLMT-equivalent approach using the ASD to investigate scattering, radiation force and torque of fractional Bessel beams and Airy beams. Wang *et al.* [287] studied the modulation of acoustic radiation forces on three-layered nucleate cells in a focused Gaussian beam, in an acoustical GLMT framework with BSCs evaluated by using finite series.

### 3.8 Photonic nanojets.

While the subsections above in this section were concerned with various *incident* beam shapes, we here consider a specific class of *scattered* beam shapes, namely photonic jets (or nanojets) which may be defined as narrow, high-intensity, beams emerging from the shadow-side of an illuminated particle and which propagate over a distance longer than the wavelength, simultaneously maintaining a sub-wavelength FWHM.

Han *et al.* [288] discussed photonic jets generated by spheroidal particles under Gaussian beam illumination, in the framework of a GLMT. Simulations demonstrated that, by simply varying the ellipticity of the spheroid, it is possible to obtain localized photon fluxes having different power characteristics and spatial dimensions. This paper is to be completed by Han *et al.* [289] who studied controllable and enhanced photonic jets generated by an optical fiber combined with dielectric spheroidal particles, achieving strong intensity confinement below the standard diffraction limit. This study was again carried out in a GLMT framework. Neves [290] studied photonic nanojets generated by microspheres in trapping equilibrium employing double optical tweezers involving two collinear and co-propagating beams. Optical forces were evaluated in the framework of GLMT, under an on-axis situation which simplifies the evaluation of the BSCs, under plane-wave incidence and highly focused Gaussian beam incidences. Mao *et al.* [291] studied a tunable photonic nanojet formed by a generalized Luneberg lens, using both ray optics and FDTD.

FDTD was also used by Liu and Lin [292] in relationship with an experimental study of geometric effect on photonic nanojet generated by dielectric microcylinders with non-cylindrical cross-sections, the non-cylindrical cross-sections cylinders including truncated microcylinders and elliptical microcylinders. Instead of FDTD, Degtyarev *et al.* [293] used a FEM (finite-element-method, see subsection 8.5) to demonstrate the possibility of generating a photonic nanojet with a helix shape, named a photonic nanohelix, with the diffraction of a Gaussian beam on a spiral axicon. Darafsheh and Bollinger [294] carried out a systematic study of the characteristics of photonic nanojets formed by dielectric microcylinders, in the visible spectrum, in terms of different tuning parameters, such as refractive index and index contrast, size and shape of the particle, state

of polarization and wavelength of the incident plane-wave light. Another study of photonic nanojet involving FEM has been carried out by Xing *et al.* [295] who studied side lobes controlling of photonic nanojets using a graded-index microcylinder which was divided by multilayers parallel to the direction of light propagation. They achieved super narrow photonic nanojets with FWHM of approximately 120 nm with a 500 nm excitation wavelength. Kozlov *et al.* [296] dealt with the analysis of dielectric circular cylinder light narrowing by WGMs and influence of material absorption, under plane wave illumination, relying on a numerical analysis. They observed two near-surface maxima of intensity generated at the cylinder output, one focus being formed by one of the WGM lobes with an intensity being 50 times the incident light intensity and a FWHM equal to 0.155 of the incident wavelength, the second focus being a photonic nanojet stretched toward the optical axis with a width equal to 0.44 of the wavelength and a length equal to two wavelengths. It is furthermore interesting to note that we can face acoustical jets similar to photonic jets, as discussed by Mitri [297].

For other papers devoted to the issue of the present subsection, see Kiselev and Plutenko [33] in subsection 3.4 devoted to helical beams, Sentis *et al.* [298] in subsection 7.3 devoted to holography, Nayak *et al.* [299], Patel *et al.* [300] in subsection 8.5 devoted to numerical techniques, Luk'yanchuk *et al.* [301] in section 10 devoted to reviews and tutorials. For a review, see Minin and Minin [278] discussing the formation of photonic nanojets by nonspherical axially and spatially asymmetric 3D dielectric particles, including cuboid dielectric particles, mesoscale regular hexaedron-aided terajets, curved photonic jet formation (photonic hook) with a cuboid with a triangle bar, dielectric particles of arbitrary 3D shapes, not forgetting the more classical case of spherical-based dielectric particles, and the less classical case of femtosecond pulse illumination.

## 4 Mechanical effects of light.

A thorough review on GLMTs and mechanical effects of laser light, for the period 1985-2018 has been provided in [81], on the occasion of Arthur Ashkin's receipt of the 2018 Nobel prize in physics for his pioneering work in optical levitation and manipulation. For the sake of completeness, the present section deals with the same issue, but is restricted to the fiducial period. We may then start with Li *et al.* [73] who studied radiation torques exerted on uniaxial anisotropic spheres in a GLMT-like framework. Incident fields were expressed using the BSCs of an off-axis Gaussian beam, together with the rotation theorem of the VSWFs. Song *et al.* [203] studied the optical forces on a large sphere illuminated by Bessel beams, and compared the results obtained by using either ray optics or GLMT. Particles with positive and negative refractive indices were examined. BSCs were computed in closed forms using the method given in Chen *et al.* [191], [192]. An interesting by-product of this work was that,

for Bessel beams, optical forces can be decomposed into gradient and scattering forces irrespective of the particle size, then providing a partial solution and a clue to the general solution of Problem 23 in [11], namely: "Derive various physically motivated contributions to radiation forces and torques for arbitrary-shaped beams", a definition which should be completed by mentioning that, in the mind of the authors, these derivations should be explained and obtained in terms of BSCs. Other insights into Problem 23 may be obtained from Wang *et al.* [302] who dealt with gradient and scattering forces of anti-reflection-coated spheres in an aplanatic beam using GLMT, with some focus on the backward scattering intensity, and from Zheng *et al.* [303] who explicitly dealt with the issue of decomposition of optical forces into gradient and scattering parts. See also Wang *et al.* [287] for a discussion of gradient and scattering forces in an acoustical context.

Preston *et al.* [204] achieved an experimental and theoretical study on the equilibrium position of an aerosol droplet trapped in a counter-propagating Bessel beam and experiencing a gas flow, providing an aerosol analogue to the separation of particles of different size and refractive index in counter-propagating laser beam and liquid flow, referred as to optical chromatography. The position of the droplet was greatly affected by the excitation of whispering gallery modes, leading to sharp peaks in the equilibrium position curve. Calculations of optical forces were carried out in a GLMT framework (Barton's version) using the double quadrature method to evaluate the BSCs. Wang *et al.* [205] dealt with a tractor effect for core-shell nanoparticles using the GLMT and Maxwell stress tensor formulation. They demonstrated how optical pulling forces can be induced and optimized by using first-order Bessel beams with appropriate polarization. The transverse stability of the core-shell nanoparticle under ambient damping was also verified by linear stability analysis and dynamical simulation. BSCs were derived using an analytical approach provided by Chen *et al.* [191]. Tractor effects were also considered by Mitri *et al.* [208] who dealt with optical pulling force on a magneto-dielectric Rayleigh sphere in zeroth- and first-order Bessel tractor polarized beams, with particular emphasis on the beam polarization and topological charge. The analysis was focused on identifying the regions and some of the conditions allowing one to achieve retrograde motion of the sphere centered on the axis of propagation of the incident beam or shifted off-axially. Yang *et al.* [100] dealt with the computation of stress on the surface of a soft homogeneous arbitrarily shaped particle, using a numerical technique based on SIEM. The comparison of the computed stress profile with that predicted by the GLMT for a water droplet of diameter equal to 51 times the wavelength in a focused Gaussian beam demonstrated that the precision of the numerical technique used was very good. Stress profiles on spheroids with different aspect ratios were afterward computed, with illumination by a Gaussian beam of different waist radii at different incidences. Physical analysis of the mechanism of optical stress was given with the help of a recently developed Vectorial Complex Ray Model (VCRM). Computational study of stress on surface of a bi-concave cell-like particle, which is a typical application in life science, was also undertaken. Phillips *et al.* [304] dealt with

shape-induced force fields on extended dielectric particles of general shape in optical trapping, using 3D T-matrix calculations (presumably EBCM). To illustrate the approach used, the authors described the design and fabrication of a passive force clamp based on a tapered cylinder and capable of applying a constant force displacement of several micrometers. They demonstrated its use in an optically controlled scanning probe microscope for ultralow force imaging. Potential future applications of this device may include the imaging of sensitive biological membranes. They calculated optical forces via surface integrals of the Minkowski stress-energy tensor, and showed the effect of particle shape on optical force in a Gaussian beam. With the sake of an experimental verification, they measured force-displacement profiles of horizontally trapped cylinders and tapers. Tapp *et al.* [305] proposed a theoretical framework to predict the 3D steady-state shapes of optically deformed micron-sized emulsion droplets with ultra-low interfacial tension, under the influence of one or more optical traps. The resulting shape and size of the droplet arose out from a balance between the interfacial tension and optical forces. Using an approximation of the laser field as a Gaussian beam, working with the Rayleigh-Gans regime and assuming isotropic surface energy at the oil-water interface, the authors numerically solved the resulting shape equations to elucidate the 3D droplet geometry. They used both a localized ray-optics and FDTD simulations. Pan *et al.* [306] dealt with the prediction of radiation pressure force exerted by structured beams (in the present case Gaussian beams) on moving particles by using a fast full-wave method in which the problem of evaluating optical forces exerted on a moving particle by a single-excitation beam was converted into that of computing optical forces exerted on a static particle by multiple beams. In other words, from the simulation point of view, the authors could either simulate the moving particle in the fixed optical tweezers directly or changed the state of the optical tweezers by varying parameters of the light beam while assuming the particle is static. To ease the implementation, the latter was employed in their study. Surface integral equation method (SIEM) was used. Li *et al.* [307] dealt with ultrasensitive size-selection (sorting) of plasmonic nanoparticles by Fano interference optical forces under a Gaussian beam excitation (plane wave illumination was considered as well), relying on the fact that the scattering of a plasmonic nanoparticle at its Fano resonance becomes strongly asymmetric in the lateral direction and consequently results in a net transverse scattering force, that is, Fano interference-induced force. The magnitude of this transverse scattering force was comparable with the gradient force in conventional optical manipulation experiments. More interestingly, the Fano scattering force was ultrasensitive to the particle size and excitation frequency due to phase sensitivity of the interference between adjacent plasmon modes in the particle. Computations of optical forces used a GLMT-like method with Gaussian beam BSCs evaluated in this framework. Gloppe *et al.* [308] dealt with bidimensional nano-optomechanics and topological backaction in a non-conservative radiation force field. This study was in the filiation of works devoted to cavity optomechanics which successfully addressed the experimental challenge of coupling a mechanical oscillator to a high-finesse optical cavity, which serves as a noise-free

optical amplifier that enables quantum-limited optomechanical interaction. The authors then presented a versatile optomechanical approach that goes beyond the standard scalar description of cavity optomechanics. Its principle relied on coupling a suspended subwavelength-sized crystalline nanowire to a strongly focused beam. This cavity-free and optically broadband approach gave rise to large optomechanical coupling in both axial and transverse vibration directions, enabling an ultrasensitive readout of the nanomechanical motion for incident optical power in the 100  $\mu\text{W}$  range only. Burgin *et al.* [309] reported experimentally and theoretically on the significant exaltation of optical forces on microparticles when they are partially coated by metallic nanodots, silica-gold raspberry-like particles, flowing in a microfluidic channel, and shined with laser light within a surface plasmon resonance. For pure silica particles, calculations were performed with LMT and ray optics calculations. For metal-dielectric particles, calculations were carried out using a Matlab code implemented for spherical core-shell particles. As the metal nanodots adsorbed on the silica microparticle surface were very small compared to the optical wavelength and since they were isotropically distributed onto this surface with a volume fraction  $\phi$  around the percent, the authors confidently assumed a description of the shell as a continuous composite medium and use the Maxwell-Garnett approach to determine its dielectric constant. Measurements were carried out using an optical chromatography setup. Arzola *et al.* [101] experimentally studied the rotation, oscillation and hydrodynamic synchronization of optically trapped oblate spheroidal microparticles, in focused elliptically polarized vortex beams, generating several optical traps with independently controlled parameters using a spatial light modulator. They demonstrated that such particles can be trapped in 3D, with the minor axis oriented perpendicular to the (linear) beam polarization and the beam propagation, can spin in a circularly polarized beam and an optical vortex beam around the axis parallel with the beam propagation, and can exhibit a periodic motion in the plane transversal to the beam propagation. They measured that the transfer of the orbital angular momentum from the vortex beam to the spheroid gave rise to torques one order of magnitude stronger compared to that of the circularly polarized Gaussian beam. Cao *et al.* [310] dealt with the optical collection of multiple spheres in single tightly focused beams, and proposed a model of optical trapping of multiple spheres and multiple trapping in a single beam based on optical forces. They dynamically simulated the process of self-arrangements. Ruffner and Grier [202] dealt with universal, strong and long-ranged trapping by optical conveyors which are active tractor beams that selectively transport illuminated objects either upstream or downstream along their axes. Formed by the coherent superposition of coaxial Bessel beams, an optical conveyor featured an axial array of equally spaced intensity maxima that acted as optical traps for small objects. The authors demonstrated through measurements on colloidal spheres and numerical calculations based on the GLMT that optical conveyors interferometric structure endows them with trapping characteristics far superior to those of conventional optical tweezers, and that optical conveyors form substantially stiffer traps and can transport a wider variety of materials over a much longer axial range. Wang *et al.* [311]



dealt with laser induced surface stress on water droplets under plane wave or Gaussian beam illumination. Siler and Zemanek [312] considered that the high intensity focal area of a focused laser beam is accompanied by several secondary maxima on the optical axis as well as by a structure of rings away of the optical axis and accordingly dealt with optical trapping in secondary maxima of focused laser beam, using GLMT (in Barton's approach).

Dutra *et al.* [313] gives me the opportunity to comment of a frequently made mistake concerning GLMT. The authors provided a study allowing one to proceed to an absolute calibration in optical tweezers. In particular, they investigated in detail the effects of all primary aberrations on the optical trapping force. They developed a model that takes into account the presence of primary aberrations on the focused trapping beam in addition to the interface spherical aberration. They demonstrated that one additional optical aberration, namely astigmatism, is the main effect responsible for the transverse stiffness degradation in the range  $a \lesssim \lambda/2$  (with  $a$  being the radius of the trapped particle). They found an agreement with the experimental data within error bars with no fitting procedure. They could then demonstrate that absolute calibration of the trap stiffness could be achieved, provided that all relevant experimental parameters, including astigmatism were carefully characterized. Apart of the scientific interest of this paper, I indeed have now to mention an "epistemological" interest, namely that it conveys a common mistake according to which the GLMT would be restricted to the use of paraxial Gaussian model which provides an incorrect representation of a strongly focused beam. This mistake is already present in a synthetic account of Ashkin's works concerning optical trapping and manipulations of neutral particles using lasers, including a collection of reprints [314]. In this last reference, GLMT was discussed on pp.196-198 with indeed a misleading statement according to which GLMT would not be valid for the high numerical aperture incident beams typically used for tweezer traps. The misunderstanding likely arises from the initial focus to OPC-issues in the earlier developments of GLMT. In particular, in the "pivot"-paper of [1], some emphasis was put on the case of a Gaussian beam described by using a first-order Davis approximation [315] which indeed is inadequate for the case of optical tweezers using highly focused beams. However, GLMT is not restricted to such first-order Davis beams. It is actually a genuine Arbitrary Beam Theory (ABT) as already mentioned in the conclusion of [316], reiterated page 1430 of [1], concisely exposed in another 1988-paper [317], and later more extensively discussed in a synthetic paper devoted to GLMT and applications to optical sizing [2]. Implementation of a given arbitrary shaped beam in GLMT "simply" requires the evaluation of the BSCs which encode the properties of the beam. Indeed, in a two-part paper [318], [319], Lock calculated radiation trapping forces in the GLMT framework for a beam tightly focused and truncated by a high-numerical aperture, furthermore aberrated by its transmission through the wall of a sample cell. The first part was devoted to the calculations of BSCs achieved by using a localized beam model while Part II was devoted to the use of the localized beam model to calculate the on-axis trapping force on a spherical particle whose size could range from the Rayleigh scattering regime to the ray

optics regime. Radiation trapping forces were calculated for two different beam profiles, experimental data were provided and discussed, and it was concluded that GLMT wave theory bridges the gap between the small (Rayleigh) and the large (geometrical optics) size parameter regimes. See also Lock *et al.* [320] for the same issue (scattering of a tightly focused beam in a GLMT framework), and [81] for another common mistake according to which GLMT would not be able to deal with internal fields.

Next, Yu and She [74] computed the radiation force exerted on a sphere by focused Laguerre-Gauss beams, in a GLMT framework. BSCs were evaluated from an ASD in terms of some auxiliary coefficients for which recurrence formulas were provided. Bai and Suo [321] dealt with the optomechanics of soft materials, deriving a dimensionless number to estimate when light can induce large deformation, for a structure made of soft dielectrics with size comparable to the wavelength of light. They studied the two-way interaction between light and structure by combining the electrodynamics of light and the nonlinear mechanics of elasticity, using a Maxwell stress tensor analysis. It was noticed that the Maxwell stress due to a laser can cause small strains in stiff materials, but large strains in soft materials. Datsyuk and Pavlyniuk [322] compared and discussed the validity of various stress tensors to evaluate induced normal pressure and tangential stress at the surface of a small dielectric sphere (or cavity) in the framework of an associated controversial issue, namely Minkowski, Einstein-Laub, Abraham, and Lorentz forms of the Maxwell stress tensor. The aim of the paper was to compare predictions of these different forms of the stress tensor with experiments. Ranjit *et al.* [323] dealt with attonewton force detection using laser-cooled silica microspheres in a dual-beam optical trap in high vacuum. They demonstrated trap lifetimes exceeding several days, attonewton force detection capability, and wide tunability in trapping and cooling parameters. Measurements have been performed with charged and neutral beads to calibrate the sensitivity of the detector. The theoretical analysis relied on a full wave analysis (LMT or GLMT). Zhang and Yang [232] dealt theoretically and experimentally with radiation forces on Rayleigh particles using a focused anomalous vortex beam under paraxial approximation in the framework of Rayleigh scattering theory. Chvatal *et al.* [324] theoretically studied the binding of a pair of Au nanoparticles in a wide Gaussian standing wave, in the framework of GLMT for assemblies of spherical particles with BSCs evaluated by using a localized approximation. Binding may be defined as a self-arrangement of particles based upon the equilibrium between the optical forces resulting both from the incident beam illumination and from the illumination by the light re-scattered by other objects. Brzobohaty *et al.* [325] dealt with non-spherical gold nanoparticles trapped in optical tweezers, and compared theoretical results based on the coupled-dipole method with published theoretical and experimental data. Ambrosio and Ferreira [206] studied time-average forces exerted over dielectric, magnetodielectric and negative-index Rayleigh particles by frozen waves (let us recall: superposition of equal-frequency arbitrary-order Bessel beams which are capable of providing virtually any desired longitudinal intensity pattern, serving as potential beams in optical trapping, atom guiding, optical bistabilities,

and so on). Analytical expressions for gradient and scattering forces experienced by very small particles were deduced. A related work is by Ambrosio and Zamboni-Rached [207] who dealt with optical forces experienced by arbitrary-sized spherical scatterers exerted by frozen waves, in the framework of GLMT with BSCs computed with the aid of ILA. Brzobohaty *et al.* [112] studied the complex rotational dynamics of multiple spheroidal particles in a circularly polarized dual beam trap comprising counter-propagating Gaussian beams of opposing helicity. Isolated spheroids underwent continuous rotation with frequencies determined by their size and aspect ratio, whilst pairs of spheroids displayed phase locking behavior. The introduction of additional particles has been leading to yet more complex behavior. Experimental results were supported by numerical calculations using a T-matrix theory (actually EBCM, with BSCs evaluated by using a localized approximation, as checked by a private communication) and numerical simulations based on finite element method. De Coster *et al.* [326] dealt with a mass-manufacturable polymer microfluidic device for dual fiber optical trapping, and calculated the optical forces that acted on particles flowing in the microchannel using wave optics in combination with ray tracing. Ray tracing results were compared with the results obtained with the T-matrix method (actually a GLMT, as confirmed by a private communication). BSCs were calculated analytically around the beam waist using a localized approximation method. Let us remark that the exact meaning of T-matrix in [112] and [326] were secured by private communications, illustrating the confusing effect of the traditional use of T-matrix as synonymous with EBCM. But it happens that the exact meaning of the expression "T-matrix" cannot always be secured.

Finally, as a final paper for the year 2015, let us quote Lee *et al.* [327] who studied the particle size dependence of axial trapping efficiency. An optical tweezers system using laser beams with a Gaussian intensity profile and doughnut intensity profiles made by hollow core optical fiber and axicon lenses, respectively, was constructed. The axial trapping efficiencies for three intensity profiles were measured and compared with each other. Theoretical analysis was carried out by using a modified ray optics model.

Yang *et al.* [209] studied the direction reversal of the optical spin torque (OST) on a Rayleigh absorptive sphere in vector Bessel polarized beams, using the dipole approximation method, with a particular emphasis on the polarization of the plane wave components forming the beam, and on the sensitivity of vortex-like behaviors and negative spin torque generation with respect to the polarization type, topological charge and half-cone angle of the Bessel beams. Liaw *et al.* [102] theoretically examined a spinning gold nanoparticle (sphere, prolate and oblate spheroids) driven by circularly polarized plane waves. Li *et al.* [103] investigated the spatial orientation dependence of optical trapping force and intrinsic torques exerted on spheroidal Rayleigh particles under irradiation of highly focused linearly and circularly polarized beams. Zemanek *et al.* [328] proposed a feasible experimental set-up for a proof-of-principle experiment that uses methods of optical trapping in shaped laser beams which provide cubic and quadratic potentials, in order to study noise-to-signal transition of a Brownian particle, considering nanoparticles of sizes close to the Rayleigh scattering limit,

in the potential of an optical trap that uses two pairs of counter-propagating Gaussian beams. Theoretical estimates and results from the numerical simulations indicated that the noise-to-signal transition could be observed under realistic experimental conditions. Li *et al.* [329] theoretically established an optical trap force model and calculated the optical trap force of trapped microspheres whose size is in the beam wavelength scale, based on ray optics and T-matrix method, under Gaussian beam illumination. Calculations results of axial and transverse trapping efficiencies based on the two models used agreed qualitatively, but differed quantitatively. Simulation results showed that best agreement between ray optics model and T-matrix calculation method was met when a strongly focused laser beam traps a large microsphere in and near the beam waist plane. Girot *et al.* [330] reported experimental observation concerning the motion of a light-absorbing particle heated by a vertical laser beam at the water-air interface. They have shown that, at small laser power, the particle is trapped in on-axis configuration, similarly to 2-dimensional trapping of a transparent sphere by optical forces. Conversely, on-axis trapping became unstable at higher power. The particle then escaped off the laser axis and started orbiting around the axis. It was found that the laser-heated particle behaved as a microswimmer with velocities on the order of several  $100\mu\text{m/s}$  with just a few milliwatts of laser power. Radii of the orbits were stable and reproducible, but orbital velocities seemed random. It was not rare that a particle orbit would reverse in direction in the course of a single experiment. Although a theory of the observed phenomena is still lacking, optical forces were evaluated using a GLMT based on the core-shell model. Zhou *et al.* [331] theoretically and experimentally studied optical trapping using radially polarized beams, using a ray optics model and taking into account for the pupil apodization functions on trapping efficiencies. Pilat *et al.* [332] remarked that it was recently suggested that the electrostatic double-layer force between colloidal particles might weaken at high hydrostatic pressure encountered, for example, in deep sea or during oil recovery. They addressed this issue by means of a specially designed optical trapping setup that allowed one to explore the interaction of a micrometer-sized glass bead and a solid glass wall in water at hydrostatic pressures of up to 1 kbar. They then reported on the measurements of surface forces in aqueous electrolytes under such a configuration and concluded that, within the statistical error, the force curve at high pressures showed no difference compared to those forces recorded at ambient pressure. The experimental configuration has been designed by using GLMT calculations. Chen *et al.* [333] dealt with chirality sorting of nanoparticles immersed in a two-plane-wave-interference system which induced lateral optical forces. Calculations were carried out by using the GLMT although the particles were illuminated by plane waves. This is an opportunity to recall that the scattering by tilted (obliquely incident) plane waves must be handled indeed in a GLMT framework, not simply in the LMT one, e.g. [34]. Simpson *et al.* [104] presented and analyzed a mechanism for the synchronization at low Reynolds number of colloidal rotors (oblate spheroids) through angular optical binding generated by optical coupling torques in counter-propagating polarized Gaussian beams of opposite helicity. The effect was

qualitatively captured by a coupled dipole model and quantitatively reproduced by T-matrix calculations, specifically EBCM with BSCs evaluated by using a localized approximation.

Yu *et al.* [334] proposed a simple recurrence algorithm for partial wave expansion of plasmonic and evanescent fields which was efficient up to arbitrary order of expansion of VSWFs, allowing one to bypass the limitations of the time-consuming traditional quadrature method. The algorithm was applied to the calculation of optical forces and torques acting on a spherical particle immersed in a plasmonic field, up to particle having a size parameter of about 200 and a cut-off order of VSWFs close to 500. Mitri *et al.* [210] theoretically investigated axial and transverse radiation force cross-sections of optical tractor Bessel polarized beams illuminating dielectric spheres, with particular emphasis on the influence of the topological charge, axicon angle and polarization, using a multipole expansion method in terms of VSWFs with an evaluation of BSCs in the framework of an ASD. Numerical computations were carried out in the case of zeroth-order and first-order (with positive or negative helicity) beams. The aim of the study has been to identify some of the tractor beam conditions allowing one to achieve retrograde motion (tractor motion) of a dielectric sphere arbitrarily located in space. Xu *et al.* [105] proposed a fast and accurate algorithm for repeated optical trapping simulations on arbitrarily shaped particles using SIEM. A motivation behind this numerical work was the requirement, in optical trapping applications, of investigating optical forces over a wide range of parameters in the configuration space modeling the experimental set-up, a process which is time costing because it involves many repetitions of extensive computations. The algorithm proposed exploited a previously developed skeletonization framework which succeeds in reducing the number of repetitions. Numerical results were obtained under Gaussian beam illumination using Davis-Barton fifth order approximation. Particles considered were an oil spheroidal droplet in air, an Au sphere between two identical Au five-pointed star shaped Au particles in air, and a red blood cell in water. Li *et al.* [211] studied the optical torque exerted on an absorptive magneto-dielectric sphere by an axicon-generated vector Bessel (vortex) beam with selected polarizations in the framework of the dipole approximation. The total optical torque was expressed as the sum of orbital and spin torques. Herranen *et al.* [335] established a theoretical framework for solving the equations of motion, both for translational and rotational dynamics, for an arbitrarily shaped, inhomogeneous dust particle in the presence of radiation pressure, under plane wave illumination. The repeated scattering problem involved was solved using a state-of-the-art volume integral equation based on T-matrix method. Mitri [106] studied the optical radiation forces, spin and orbital torques exerted on a subwavelength prolate gold spheroid coated by a layer of plasmonic material with negative permittivity and illuminated by either a zeroth-order (non vortex) or a first-order vector Bessel (vortex) beam, in the framework of the electric dipole approximation method. Li *et al.* [75] studied the optical spin torque (OST) induced by vector Bessel beams with selective polarizations on a light-absorptive sphere of arbitrary size, in a GLMT framework, with particular emphasis on the topological charge, the polarization of the plane

wave components forming the beam, and the half-cone angle. BSCs were calculated using the ASD. Lu *et al.* [212] dealt with rigorous full-wave calculation of optical forces on dielectric and metallic spherical microparticles immersed in a vector Airy beam beyond the paraxial approximation, based on the GLMT and the Maxwell stress tensor. Dielectric and gold microparticles were shown to be trapped within the main lobe or the nearby side-lobes of the beam mostly by the transverse gradient optical force while driven forward along the parabolic trajectory of the Airy beam by the longitudinal scattering force. It was thus demonstrated theoretically that the vector Airy beam has the capability of precisely transporting both dielectric and metallic microparticles along prespecified curved paths. The non-diffracting Airy beams discussed in the paper were introduced by using phase-modulation of Gaussian beams. Mitri [107] dealt with reverse orbiting and spinning of Rayleigh dielectric lossless subwavelength particles (prolate and oblate spheroids, spheres) with arbitrary orientation in a zeroth-order Bessel optical beam, relying on the electric dipole approximation. Chen *et al.* [213] derived an analytic expression of the optical torque on chiral particles in generic monochromatic optical fields within the dipole approximation. The analysis incorporated recoil terms which have a significant contribution to the optical torque in many situations. It was furthermore proven that the net optical torque vanished for a lossless isotropic chiral particle of any size illuminated by arbitrary monochromatic optical fields. The special case of a zeroth-order vector Bessel beam was considered as an example with results corroborated by the precise full wave simulations based on the GLMT. Gargiulo *et al.* [336] studied the accuracy and mechanistic details of optical printing of single Au and Ag nanoparticles, using different laser powers and wavelengths. For quantitative comparisons to the experiments, GLMT was employed to deal with optical forces on spherical nanoparticles illuminated by Gaussian beams. Yu *et al.* [334] discussed a simple algorithm for partial wave expansion of plasmonic and evanescent fields. More specifically, they dealt with the evaluation of BSCs (or PWECS, partial wave expansion coefficients, using the terminology of the paper) for general incident fields which are generally computed by taking the inner product of the incident field with the VSWFs, termed projection method (i.e. quadrature method in a more traditional language) which requires numerically computing integrals of various oscillating functions. They then presented a recurrence algorithm to evaluate the PWECS numerically up to arbitrary order of expansion in VSWFs. The approach is especially useful for large particles where numerical integration in the conventional projection method becomes more and more numerically challenging and time consuming. As applications to their approach, they calculated the optical force and torque acting on a spherical particle immersed in the plasmonic field, up to particle radius as large as over 30 times of the incident wavelength and with the cut-off order of VSWF expansions close to 500. This approach remained stable while the conventional projection method was challenged due to the numerical integration of highly oscillating integrands. The recurrence algorithm was corroborated by comparisons with the previous conventional, but more cumbersome, method, and was proven to be efficient and powerful for the scattering of optical fields

that can be modeled by a class of fields which includes evanescent waves and simple plasmonic fields as two special cases.

McKell and Bonin [214] used standing Bessel beam waves to form traps in 1D (along the optical axis) and corals in 2D, that is to say in planes perpendicular to the optical axis at the antinodal region of the standing waves. They therefore created optical conditions allowing particles to freely move within a limited range in a planar region perpendicular to the optical axis, justifying the name of "corrals" assigned to such traps. The interest of using Bessel beams to build such traps was emphasized. The aim of building them was to investigate Brownian motion of microparticles in order to measure particle diffusion coefficients in semi-dilute polymer solutions of both spheres and asymmetric particles. Experimental results were favorably compared with theoretical investigations carried out by calculating optical forces in the framework of GLMT, with the BSCs evaluated by using closed form expressions published by Wang *et al.* [189]. Bai *et al.* [215] dealt with an analysis of lateral binding forces exerted on multilayered spheres induced by high-order Bessel beams with arbitrary polarization angles. The analysis was carried out using an analytical method, namely GMM (akin to GLMT for assemblies of spheres and aggregates [16] as acknowledged by the authors). In such a framework, the description of the illuminating beam is a key fundamental topic which amounts to the evaluation of BSCs encoding the properties of the illuminating structured beam. In the present paper, BSCs were evaluated by using closed form expressions previously published by Wang *et al.* [189]. Extensive numerical results were given in the case of the assembly of spheres being made of only two particles which is sufficient to demonstrate the binding effect. Discussions incorporated both particle parameters (number, geometrical properties, and refractive indices of the layers) and beam parameters (axicon angle, polarization state, wavelength). Mitri [141] discussed pushing, pulling and electromagnetic radiation force cloaking by a pair of conducting cylindrical particles, showing that two conducting cylindrical particles illuminated by an axially-polarized electric field of plane progressive waves at arbitrary incidence will attract, repel or become totally cloaked (i.e. invisible to the transfer of linear momentum carried by the incident waves), depending on their sizes, the interparticle distance as well as the angle of incidence of the incident field. Based on the rigorous multipole expansion method and the translational addition theorem of cylindrical wave functions, the electromagnetic radiation forces arising from multiple scattering effects between a pair of perfectly conducting cylindrical particles of circular cross-sections were derived and computed. On the same issue, the one of cloaking, see also Mitri [142] dealing with active electromagnetic invisibility cloaking and radiation force cancellation and showing that an active emitting electromagnetic Dirichlet source, i.e. with axial polarization of the electric field, in a homogeneous non-dissipative/non-absorptive medium placed near a perfectly conducting boundary can render total invisibility (i.e. zero extinction cross-section or efficiency) in addition to a radiation force cancellation on its surface. The analysis used the partial-wave series expansion method in cylindrical coordinates. Leartprapum *et al.* [76] presented a method for continuous measurements of axial radiation-pressure

forces from a weakly focused beam on polystyrene micro-beads suspended in viscous fluids over a depth range of 400  $\mu m$ , based on real-time monitoring of particle dynamics. Experimental results were compared to theoretical predictions of radiation-pressure forces based on GLMT, and it was found that measurements were consistent with theoretical trends. Also, Leartprapun *et al.* [77] relied on modulated radiation pressure forces from a low numerical aperture optical beam to track sub-nanometre oscillations of beads embedded in viscoelastic hydrogels. Evaluations of radiation pressure forces under focused Gaussian beam illumination were carried out by using a Matlab implementation of GLMT. Ge *et al.* [337] studied the effect of nanoscale surface roughness on the optical trapping properties of nanoparticles in a vicinity of a composite gold film with periodic structures, using a numerical method, namely FDTD associated with the Maxwell stress tensor equation. Periodic structures were made of circular holes and a Monte-Carlo method was used to implement the surface roughness characteristics. The scatterer was illuminated by a Gaussian beam described in the well established Davis scheme of approximation at first order [315]. Accuracy of the model was verified by comparing numerical results concerning optical trapping forces with results obtained by using GLMT in the case of single nanoparticles. It was demonstrated that trapping can be robust even in the presence of roughness although, depending on the parameters of the roughness (correlation length and RMS height), the optical trapping forces may be mitigated. Li *et al.* [217] dealt with optical Bessel tractor polarized beams on a charged sphere of arbitrary size. The axial and the transverse radiation pressure cross-sections for a charged sphere illuminated by a vector Bessel beam were computed in the framework of GLMT, with BSCs provided in closed forms. Bai *et al.* [170] dealt with the analysis of lateral binding force exerted on a bi-sphere induced by an elliptic Gaussian beam. The analysis relied on the use of GLMT, with BSCs evaluated by using a localized approximation, including a multiple scattering theory approach to deal with the interactional scattering between two isotropic spheres induced by an elliptic Gaussian beam. A similar study by Bai *et al.* [216] was again devoted to the analytical description of lateral binding force exerted on bi-sphere but under higher-order Bessel beam illumination. The authors used GMM which is actually, again as acknowledged by the authors, a special case of the GLMT for assemblies of spheres and aggregates [16], with closed form expressions for the BSCs following the work by Wang *et al.* [189]. Rafferty and Preston [338] described a dual-beam optical trap that can simultaneously determine  $n$  and  $k$  (for a complex refractive index written as  $m = n + ik$ ) and the radius  $s$  of weakly absorbing aerosol particles. They used a cavity-enhanced Raman scattering which monitors the positions of WGM resonances to determine  $n$  and  $s$  and electromagnetic heating from the trapping laser itself to determine  $k$ . The size and the real part of the refractive index could be extracted with an accuracy of up to 1 nm and  $10^{-4}$ , respectively. GLMT was used for the evaluation of absorption cross-sections  $C_{abs}$  using the BSCs of a Gaussian beam.

For reviews devoted to mechanical effects of light, particularly to optical tweezers, see Nieminen *et al.* [339] and Ciminelli *et al.* [340]. For other papers



devoted to the issue of the present section, see Wang *et al.* [156] in subsection 2.4 devoted to metamaterials, Mitri [61], [93], [94], [133], [134] in subsection 3.2 devoted to laser sheets, Ambrosio and Zamboni-Rached [195], Chafiq and Belafhal [185], Ambrosio [193], [197], Angelis and Ambrosio [194] in subsection 3.3 devoted to non-diffracting beams, Qu *et al.* [70], [71] in subsection 3.4 devoted to helical beams, Ambrosio and Hernandez-Figueroa [252], [253], Ambrosio *et al.* [255], [256] in subsection 3.5 devoted to localized approximations, Shakov *et al.* [271], Zensen *et al.* [274], Devi and De [273] in subsection 3.6 devoted to pulses and laser pulses, Bain *et al.* [341] in subsection 7.5 devoted to miscellaneous measurement techniques, Loudet *et al.* [342], Chang *et al.* [110], Haghshenas-Jaryani *et al.* [343], Delabre *et al.* [344] in subsection 8.2 devoted to ray optics, Li *et al.* [78] in subsection 8.3 devoted to Debye series, Yang *et al.* [113], [224], Pan *et al.* [345], Reid and Johnson [346], Yang *et al.* [225] in subsection 8.5 devoted to numerical techniques, Kalume *et al.* [347], Alonso *et al.* [22], Gutiérrez-Cuevas *et al.* [348] in section 9 devoted to miscellaneous issues, Simpson [349], Stangner *et al.* [350], Conteduca *et al.* [351], Bui *et al.* [352], Sukhov and Dogariu [353], Gong *et al.* [354], Polimeno *et al.* [355] in section 10 devoted to reviews and tutorials.

## 5 Internal fields, resonances and WGMs/MDRS.

Usually Whispering-Gallery Modes (WGMs) and Morphology-Dependent Resonances (MDRs) are taken as synonymous and denote modes which are creeping below the surface of a scatterer and produce high intensity modes by satisfying a phase matching condition. Let us however note that there are cases in which WGMs and MDRs must be distinguished, for instance in the case of a spherical particle hosting an internal spherical particle. We must then distinguish between modes below the surface of the host sphere which may be viewed both as WGMs and MDRs, and modes which interact with the internal sphere which then are MDRs without being WGMs. This case has been investigated both in the framework of Hamiltonian chaotic phenomena using ray optics [356], [357] and within the framework of a GLMT [38], [39].

With this warning in mind, let us return to the fiducial period and start with Gagnon and Dubé [143] who used a 2D GLMT for a study of 2D scattering with arrays of cylindrical scatterers and resonance calculations, involving an addition theorem for cylindrical functions, with an emphasis being placed on the derivation of BSCs. The authors insisted on the fact that their 2D-GLMT is not restricted to disconnected cylinders embedded in an infinite medium, and that cylinders embedded in other cylinders are also within the reach of the method. Chen *et al.* [218] dealt with Fano resonance-induced negative optical scattering force on plasmonic nanoparticles, and demonstrated theoretically that Fano resonances can induce a negative optical scattering force acting on plasmonic nanoparticles in the visible light spectrum when an appropriate manipulating

laser beam is adopted. Specifically, under the illumination of a zeroth-order Bessel beam, the plasmonic nanoparticle at its Fano resonance exhibits a much stronger forward scattering than backward scattering, and consequently leads to a net longitudinal backward optical scattering force, termed Fano resonance-induced negative optical scattering force. The analysis was carried out in a GLMT framework with BSCs evaluated using closed-forms. The plasmonic particles considered could be core-shell, homogeneous, or hollow nanoparticles.

Buker [358] carried out an analysis of the ability of colloidal silver to act as a biosensor for the steroid testosterone by observing changes in the properties of localized surface plasmon, i.e. collective

oscillation of electrons along the surface of the metal, particularly those of resonances. Such an analysis relied on the fact that silver nanomaterials scatter light within the visible spectrum allowing scientists to measure their optical characteristics using standard UV-visible light spectroscopy, which can be predicted using GLMT. Experimental results presented in the paper demonstrated the ability of colloidal silver to act as a biosensor for the concentration of steroid testosterone, by relating resonance shifts with the aggregation of silver nanoparticles due to host-guest chemistry. Hoang *et al.* [359] dealt with Fano resonances from coupled WGMs in photonic molecules (optically coupled microspheres in which a single sphere is called an atom), and presented a rigorous investigation of resonant coupling between microspheres whose diameters are in the range of several micrometers, based on multipole expansion. The microspheres could be used to realize various photonic molecule configurations. The interaction between the WGMs inside individual microspheres and the propagation modes of the entire photonic molecule structure was quantified. Suryadharma *et al.* [360] studied plasmonic resonance modes of hierarchical self-assembled meta-atoms (made from a large number of suitably arranged metallic nanoparticles) based on their transfer matrix, either in a global coordinate system expressing the response of the entire meta-atom or in a local coordinate system expressing the response of each individual metallic nanoparticle that makes up the meta-atom. It was shown that the diagonalization of the transfer matrix is a convenient tool to study the modes of isolated and coupled meta-atoms.

Foreman *et al.* [361] provided a review concerning whispering gallery mode sensors. For other papers devoted to the issue of the present section, see Mitri *et al.* [184] in subsection 3.3 devoted to non-diffracting beams, Al-Jarro *et al.* [72] in subsection 3.4 devoted to helical beams, Kozlova *et al.* [135] in subsection 3.6 devoted to pulses and laser pulses, Kozlov *et al.* [296] in subsection 3.8 devoted to photonic jets, Preston *et al.* [204], Rafferty and Preston [338] in section 4 devoted to mechanical effects of light, Murib *et al.* [362], Bain *et al.* [341] in subsection 7.5 devoted to miscellaneous measurement techniques, Kalume *et al.* [347], Luo *et al.* [363] in section 9 devoted to miscellaneous issues, Santos *et al.* [364] and Conteduca *et al.* [351] in section 10 devoted to reviews and tutorials.

## 6 Optical theorem.

The optical theorem considered in this section (not to be confused with the Schelkunoff optical/equivalence theorem used in EBCM) relates the forward scattering amplitude to the total extinction cross section of the scatterer. It soon appeared, at the end of the 1970's already, that the classical version of the electromagnetic optical theorem was not valid in the GLMT framework. Nevertheless, a big number of years has been necessary before the publication of reports devoted to this issue in the archival literature, namely by Lock *et al.* [365] who announced the failure of the optical theorem for Gaussian beam scattering by a spherical particle and by Gouesbet *et al.* [366] who proposed a generalized optical theorem for on-axis Gaussian beams. These papers relied on analytical derivations without however explaining the essence of the issue which remained misunderstood. Another big number of years has been necessary, after a story which I hope will be told elsewhere, to understand the gist of it. As published in 2016 [183], the explanation of the failure of the optical theorem for structured beams relies in a very simple way on the ASD which decomposes a structured beam into a set of plane waves. The optical theorem is valid for each plane wave in the spectrum, but is not valid for the whole beam because the forward directions of the plane wave components do not correspond to the forward direction of the beam. By the way, the same argument allows one to understand that the average velocity of a structured beam is smaller, even in vacuum, than the speed of light. It is again the same argument which allows one to understand the self-healing (self-reconstruction) character of a family of structured beams, such as Bessel, Mathieu, Lommel beams and frozen waves, e.g. [367]. Such a failure of the optical theorem is not restricted to vectorial beams such as electromagnetic beams, but occurs as well for scalar beams, e.g. [282] in a quantum mechanical context and Mitri and Silva [198], Mitri [137], Leao-Neto *et al.* [199] in an acoustical context.

Next, we have Mitri [144] providing a generalization of the optical theorem for monochromatic electromagnetic beams of arbitrary wavefront in cylindrical coordinates, Eremin [368] dealing with the generalization of the optical theorem for an arbitrary multipole in the presence of a local inhomogeneity introduced in a transparent half-space, Eremin and Sveshnikov [369] dealing with a generalized optical theorem for the case of a penetrable obstacle excited by a multipole of arbitrary order in the presence of a transparent substrate, Rondo-Ojeda and Soto-Eguibar [219] which derived an expression for the optical theorem based on electromagnetic theory, for probe sources given in terms of propagation invariant beams (also known as non-diffracting beams) with an example for Rayleigh scattering, and Krasavin *et al.* [370] who experimentally demonstrated a complete violation of the commonly used form of the optical theorem for radially polarized beams at both visible and microwave frequencies, showing that a plasmonic particle illuminated by such a beam exhibits strong extinction, while the scattering in the forward direction is zero. A numerical analysis was performed using a finite element software.

## 7 Optical Particle Characterization.

Optical particle characterization has been the original motivation to build the original GLMT *stricto sensu*, as described in [3], see also [371]. It has been as well the motivation for the creation of the OPS (Optical Particle Sizing) conference in Rouen, France in 1987, e.g. [372], thereafter reformatted to OPC (Optical Particle Characterization) and eventually to LIP (Laser-light and Interactions with Particles) as an OPC follow-up, again in Rouen, France in 2012, e.g. [373] afterward in Marseille, France, in 2014 [374], Xi'An, China, in 2016 [375], College Station, U.S.A. in 2018 [376]. See [377] for a review devoted to the state-of-the-art in optical particle velocimetry and sizing before the completion of GLMT and also [378] for a recent review. For other reviews on OPC, see Xu [379], Barbosa *et al.* [380], Tu *et al.* [381] and Gong *et al.* [354] in section 10 devoted to reviews and tutorials.

### 7.1 Phase-Doppler technique.

More specifically, the first genuine applications of GLMT have been devoted to the phase-Doppler technique [36], [382], [383], [384], [385]. The issue is less prolific today insofar as the phase-Doppler technique became very mature, but see Yokoi and Aizu [386] who dealt with the measurement of size and absorption coefficient of a single moving particle by using dual burst Doppler signal, e.g. [384], and Xie *et al.* [387] who dealt with the sizing of charged particles by phase-Doppler anemometry, with a theoretical analysis which can be carried out in a LMT, or in a GLMT framework. The Mie coefficients contained a surface conductivity which was a phenomenological description of the net surface charge carried by the particle. See also Verdier *et al.* [388] in next subsection devoted to rainbow refractometry, and Wu *et al.* [389] in subsection 7.3 devoted to holography.

### 7.2 Rainbow and rainbow refractometry.

Swirniak and Mroczka [145] dealt with an approximate solution for optical measurements of the diameter and refractive index of a small and transparent axisymmetric and dielectric fiber under plane wave illumination, in the framework of a wave theory (LMT-like) for cylinders, relying on some rainbow features and using a beam of light exhibiting low temporal coherence enabling to reduce the detrimental sensitivity of the rainbow features to variations of the fiber properties, therefore allowing high-precision estimates. See Swirniak and Mroczka [146], Swirniak and Glomb [147], as well for a similar issue. Also, Swirniak and Glomb [390] reported on the application of a rainbow technique

to characterize both the core and cladding diameter of a single, silica-based, step-index optical fiber, with numerical predictions carried out in the LMT framework, using an incident beam of low temporal coherence. Rosebrock *et al.* [391] dealt with time-resolved detection of diffusion limited temperature gradients inside single isolated burning droplets using rainbow refractometry. They used different droplet heating models, in particular a model incorporating diffusion limited heat transport, and a theoretical analysis which relied on scattering by a "LMT" for multilayered spheres. There was a good agreement between theoretical approaches and experimental results.

Duan *et al.* [148] dealt with a study devoted to the influence of spatial curvature of a liquid jet on the rainbow position, in contrast with more usual studies in which liquid jets are viewed as cylinders with a constant section. More specifically, the paper reported experimental measurements of the shifts of the rainbow position in the horizontal and vertical directions of a liquid jet whose section depends on the distance from the exit, illuminated by a laser diode beam whose coherence length is at least of the order of the diameter of the jet. It was demonstrated that the shifts of the rainbow position were very sensitive to the optical curvature of the jets. Also, rainbows of different orders of the liquid jet were well separated when the jet was illuminated by a thin laser sheet. Experimental results were supplemented with a theoretical approach by an improved ray tracing method, known as Vectorial Complex Ray Model (VCRM), in which the wavefront curvature becomes an intrinsic property of the rays and is governed by a wavefront equation. One of the interests of this work concerned the characterization of the structure of jets in relationship with the development of instabilities, for instance in liquid atomization, as already approached in a study devoted to the investigation of initial disturbances in a liquid jet by rainbow sizing [392].

Promvongsa *et al.* [393] dealt with an innovative optical technique, namely One-Dimensional Global Rainbow Technique (1D-GRT), to characterize locally the evaporation of gasoline droplets by measuring the droplet evaporation rate and refractive index (i.e. temperature and composition). The information on the refractive index and the droplet evaporation rate was extracted from the ripple structure angular shift. LMT was used to numerically investigate rainbow behavior, particularly to determine the correlation between the ripple-phase change and the changes in droplet size and refractive index. By processing recorded rainbow signals with a numerical code based on Nussenzweig's theory, the refractive index and particle size were instantaneously extracted, enabling the quantification of the droplet size change at a nanometric scale. Verdier *et al.* [388] used GRT again, in association with Phase Doppler Anemometry, to carry out an experimental study of local flame structures and fuel droplet properties of an n-heptane spray jet flame. Li *et al.* [394] reported on the use of rainbow refractometry to investigate temperature distributions inside burning single-component n-alkane droplets, together with the measurement of droplet diameters using ILIDS interferometric laser imaging (which takes advantage of the interference pattern of light scattered in the forward direction), relevant to the investigation of spray combustion. Droplets with different initial diameters

were investigated during a combustion process, and the effect of initial diameter on rainbow position and temperature distribution of the droplets has been investigated. Experimental and calculated rainbow and interference patterns (from LMT), used to calibrate the scattering angle ranges of two CCD cameras recording the fringe patterns, were in good agreement. ILIDS was selected for droplet diameter measurements to get rid of the sensitivity of rainbow droplet sizing to refractive index (temperature) variations inside the burning droplet. A droplet combustion model, relying on a time-varying heat conduction equation of a sphere having a time-varying radius, was applied to describe the heat conduction process, and to obtain temperature distributions inside the burning droplet depending on the evolution of the combustion process. The simulated temperature distributions were in very high agreement with the experimental ones obtained from rainbow refractometry. Ouattara *et al.* [395] dealt with large droplet sizing and mixture fraction measurement in liquid-liquid flows with rainbow-angle diffractometry. A particular feature was that, for the droplets involved in this study, essentially characterized by a low relative refractive index, the first-order rainbow was localized in the near-forward to sideways regions. Luo *et al.* [396] applied global rainbow refractometry to the measurement of temperature and size of cryogenic propellants spray droplets (i.e. liquid oxygen and liquid methane). The refractive index and the probability distribution of the droplet sizes were retrieved from the global rainbow pattern with an inversion method using the complex angular momentum (CAM) scattering theory. Swirniak [279] dealt with a numerical analysis of elastic light scattering for refractive-index detection of liquids in a microcapillary with low-coherence rainbow diffractometry, for the purpose of capillary electrophoresis. The paper offered both an explanation of the scattering mechanisms contributing to the far-field intensity pattern based on numerical simulations as well as an analysis of the sensitivity of the measurement data to changes in the refractive index. Simulations were made by using LMT, and the time-domain results reported in this paper allowed to distinguish various scattering mechanisms that are not detectable in the scattering-angle domain, this separation of various scattering mechanisms being similar to what is obtained by using laser pulses, e.g. papers in subsection 7.6 devoted to pulses and laser pulses. Li *et al.* [397] dealt with the application of rainbow refractometry to the measurement of droplets with solid inclusions, identifying the effect of solid inclusions on the rainbow signal and on the measured parameters.

Yu *et al.* [158] dealt with the measurement of refractive index and size of a spherical drop from Gaussian beam scattering in the primary rainbow region. The scattering was computed using GLMT and Debye series decomposition of the Gaussian beam scattering. Size and refractive index were retrieved using a GLMT inversion of the primary rainbow pattern. The results of these simulations demonstrated that the measurement accuracy was dependent on both the beam waist radius and on the position of the drop in the beam waist. Wu *et al.* [171] carried out simultaneous measurement of monocomponent droplet temperature/refractive index, size and evaporation rate with phase rainbow refractometry. Spherical and homogeneous droplets were investigated, and the

light scattering of droplets under laser sheet illumination was rigorously modeled with GLMT. Experiments were carried out on single droplets and on droplet streams. Simulations were in agreement with the theoretical predictions.

### 7.3 Holography.

Seo *et al.* [108] dealt with the measurement of the light scattering properties and orientation of a spheroidal particle using digital in-line holography, under laser illumination. As a feasibility test, the 3D orientation of a transparent spheroidal particle suspended in a microscale pipe flow was successfully reconstructed. The hologram image was numerically reconstructed by using an angular spectrum algorithm. Consecutive holograms of the spheroidal particle suspended in the microscale pipe flow were recorded and analyzed to investigate the motions of particles in a flow. It was concluded that, using the proposed technique, it was effectively possible to investigate the dynamic behavior of spheroidal particles in complex flows. Coetmellec *et al.* [398] proposed to describe a particle transmission function in a digital holography setup, allowing both opaque and phase particles to be determined. They then demonstrated that it is possible to estimate the digital in-line hologram produced by a spherical particle. The experimental intensity distribution due to an opaque micro-inclusion was compared to the theoretical one obtained by the proposed transmission model whose results were furthermore compared with the results obtained by simulating holograms by LMT. See also Coetmellec *et al.* [399] for digital in-line holography in a droplet with cavitation air bubbles. They elaborated a model in which the inclusion of an air bubble in a droplet was treated from a scalar theory point of view in the Fresnel's theory framework. The elaborated model was compared with rigorous near-field Lorenz-Mie scattering theory and with experimental results. Latychevskaia and Fink [400] dealt with holographic particle image velocimetry allowing one to track particle trajectories in time and space. They demonstrated a three-dimensional volumetric deconvolution applied to reconstructed wavefront which resulted in resolving all particles simultaneously in three-dimension. They applied the 3D volumetric deconvolution to the reconstruction of a time-dependent sequence of holograms of an ensemble of polystyrene spheres moving in water. From each hologram, they simultaneously resolved all particles in the ensemble in 3D and from the sequence of holograms they obtained the time-resolved trajectories of individual polystyrene spheres. Marie *et al.* [401] dealt with Lagrangian measurements of the fast evaporation of falling diethyl ether droplets using in-line digital holography and a high-speed camera, with the evaporation of falling droplets being measured by following them along their trajectories. It happens that, once evaporation starts, the interfaces of the droplets were surrounded by air/vapor mixtures with refractive index gradients that modify the holograms. In particular, the central part of the droplet holograms was unusually bright compared to what was expected and observed for non-evaporating droplets. Simulations

using GLMT for a multilayered sphere were found to reproduce well the brightness excess of the experimental holograms. Ling and Katz [402] dealt with the location of the center of a microparticle in dense suspensions using correlations among reconstructed fields of two parallel holograms. They then provided an application of digital holographic microscopy for measuring the spatial distribution of particles in a dense suspension.

Wu *et al.* [389] dealt with the measurement of slurry droplets (micro-sized particles in the range 1-5  $\mu\text{m}$ ) by Digital Holographic Microscopy (DHM), together with phase-Doppler anemometry for the measurement of concentration. A numerical simulation of DHM capabilities was carried out by using LMT. Hattay *et al.* [403] dealt with a new wavelet-based reconstruction algorithm for twin image removal in digital in-line holography, see also Hattay *et al.* [404]. Yao *et al.* [405] dealt with the characterization of atomization and breakup of acoustically levitated drops with digital holography while Yao *et al.* [406], relying on a numerical study, proposed a method for simultaneous 3D temperature and velocity field measurements of micro-flow with laser-induced fluorescence and micro-digital holographic particle tracking velocimetry. The 3D temperature field was characterized with two-color laser-induced fluorescence particles which were tracked with micro-digital holographic particle tracking velocimetry. GLMT was used to compute holograms.

Wu *et al.* [172] aimed to explain some typical features related to the characterization of transparent spherical particles (droplets and bubbles) in both in-line and off-axis holography in near-field, under elliptical Gaussian (light sheet) illumination, using a LMT (possibly a GLMT) framework and Debye series. Wang *et al.* [407] dealt with in-line holographic characterization of colloidal micrometer-scale fractal aggregates. They used an effective sphere model to obtain each aggregate's size and the population-averaged fractal dimension, and demonstrated this technique experimentally using model fractal clusters of polystyrene nanoparticles and fractal protein aggregates. In the effective sphere model, each irregularly-shaped-protein was treated as an effective sphere composed of the aggregate itself and of the fluid medium filling its pores. From the theoretical point of view, they developed an effective- sphere formalism for Lorenz-Mie microscopy of fractal aggregates and demonstrated its efficiency through measurement on model systems.

Wu *et al.* [159] were concerned with the 3D characterization of inclusions in a host droplet. In the framework of the generalized Huygens-Fresnel integral using a tensor ABCD matrix, a model was therefore proposed to describe in-line holography using a Gaussian beam on inclusion inside a host droplet with misalignment caused by a tilt incidence of a laser beam. This model took account of droplet morphology, inclusion position, as well as refractive index difference between a droplet and the surrounding medium. Sentis *et al.* [298] proposed a new computationally efficient approach for determining the refractive index of spherical transparent particles, in addition to their size and 3D position, using digital in-line holography. The method was based on the localization of the maximum intensity position of the photonic jet with respect to the particle center retrieved from the back propagation of recorded holograms. Rigorous electro-



magnetic calculations (actually using an Opaque and Phase Disk models) and experimental results demonstrated that, for liquid-liquid systems and droplets with radius  $> 30\mu\text{m}$ , a refractive index measurement with a resolution inferior to  $4.10^{-3}$  is achievable. Wu *et al.* [173] dealt with simultaneous amplitude and phase contrast imaging of burning fuel particle and flame with digital in-line holography model and verification, aiming to the investigation of the formation of holograms of composite burning gas-fuel particles as well as to the characteristics of their reconstructed images, relying in particular on a GLMT-based model, on Debye series, and on diffraction-based model, under illumination by an elliptical Gaussian beam (laser sheet). Burning fuel particles were viewed as spherical particles concentrically located in a spherical flame i.e. as multilayered particles. Jolivet *et al.* [408] dealt with a regularized reconstruction, as an inverse problem, of absorbing and phase objects from a single in-line hologram, with applications to fluid mechanics and micro-biology.

## 7.4 Imaging and microscopy.

Selmke and Cichos [160] dealt with energy-redistribution signatures in transmission microscopy of Rayleigh and Mie particles. They described the transmission characteristics associated with the interaction of a structured beam with spherical particles, possibly multilayered, of arbitrary size and electric permeability in the framework of GLMT. They evaluated on-axis spherical BSCs  $g_n$  using a localized approximation for on-axis Gaussian beams, and also general BSCs  $g_n^m$  for off-axis situations. De Cock *et al.* [409] dealt with the measurements of reference ISO nozzles (droplet sizes and velocities) by high-speed imaging with a digital acquisition technique, viewed as an alternative to well-established techniques such as the phase Doppler particle analyzer or laser diffraction spectrometry. The analysis was performed by an algorithm which required double exposed shadow images acquired in a back-lighted arrangement with a Particle Image Velocimetry (PIV) camera and a pulsed light emitting diode (LED). Klaiber *et al.* [410] used a real-time process analysis system of imaging for the simultaneous acquisition of spray characteristics, providing reliable measurement results for properties not only of (almost) spherical but also of arbitrarily shaped objects in spray and atomization processes. The proposed real-time process analysis system evaluated a spray or atomization process for arbitrary time period by processing each image frame in real time and transmitting the extracted measurement data to a PC.

Waduge *et al.* [411] dealt with the prediction of concentration of fine particles in enclosed vessels using a camera based system and CFD (Computational Fluid Dynamics) simulations, aiming to the exploration of the possibility of verifying and validating CFD models using a bench scale dustiness tester and a single camera system with a laser source to illuminate particles. This study was relevant to the issue of dust explosions during the handling and storing of biomass such as wood pellets presenting a high risk to plant operators and

possibly causing serious structural damages. Berisha *et al.* [412] dealt with an interactive simulation of broadband imaging using Mie theory, allowing one to obtain a framework for interactively computing and visualizing electromagnetic fields scattered by micro- and nano-particles. Fields could be evaluated both inside and outside of the particles. Monte-Carlo sampling was used to reconstruct and visualize the three-dimensional structure of fields, spectral profiles at individual points, structure of fields at the surface of the particle, and the resulting image produced by an optical system. The incident focused field was represented as a superposition of plane waves. Laurendeau *et al.* [149] dealt with particle image velocimetry (PIV) experiment analysis using large-eddy simulation, with application to plasma actuators. Let us recall that, in PIV, particles are illuminated by a laser sheet. A DELPI software (application with Windows interface calculating scattering diagrams and extinction of homogeneous spheres and cylinders, as well as multilayered particles, according to a private communication) was used to carry out required calculations. Zhou *et al.* [413] dealt with super-resolution, i.e. with contrast enhancement of microsphere-assisted super-resolution imaging in dark-field microscopy, achieving subwavelength imaging at visible frequencies. They achieved experiments which demonstrated in particular that fused silica microspheres of  $\sim 10 \mu\text{m}$  diameter under no and partial immersion in ethyl alcohol are effective in achieving subwavelength imaging with a minimum discernible feature size of  $\sim \lambda/5$ . The imaging property of microsphere-assisted subwavelength microscopy was studied using FDTD numerical simulations, with a linearly polarized propagating plane wave as a light source. The authors noticed that it has been suggested that the super-resolution capability of microspheres was linked to the development of the "photonic nanojet" which is emitted from such microparticles and can enable sharp focus, see subsection 3.8 on photonic nanojets.

Meng *et al.* [174] dealt with the determination of the refractive index of particles using glare-point Interferometric Particle Imaging (IPI) technique, in which the refractive index of a particle could be extracted from the glare point separation, having to account for the influence of the particle diameter on the separation. The method could be applied to simultaneously measure the particle size and refractive index when combined with an extended IPI system with opposite two-sheet illumination at a  $90^\circ$  scattering angle. Hollerbach *et al.* [414] dealt with super-resolution imaging for sizing sub-diffraction limit electro-sprayed droplets by structured illumination microscopy. In contrast with usual methods which fail to measure droplets smaller than about 400 nm in diameter due to constraints imposed by the diffraction limit of light, the authors used a super-resolution fluorescence microscopy based method for determining the sizes of electro-sprayed droplets. The smallest detectable droplets confidently measured with the described method were 200 nm in diameter. Kang *et al.* [415] dealt with the measurement of dispersity of fine dusts by using an in-situ and noninvasive shadowgraph method with an image analysis processing technique. Gragston *et al.* [416] dealt with the characterization of spray flames by using shadowgraph imaging for analysis of spray geometry and droplet levels in cold-flow and ignited sprays, quoting Gouesbet and Gréhan [372], i.e. the Pro-

ceedings of the first OPS conference in Rouen, 1987, to which the reader might like to refer for an overview with a historical flavour. To end this subsection, see Luk'yanchuk *et al.* [301] in section10 devoted to reviews and tutorials.

## 7.5 Miscellaneous measurement techniques.

Xu *et al.* [417] proposed a robust modified Landweber algorithm to retrieve the particle size distributions from Fraunhofer diffraction. Satinover *et al.* [175] dealt with single-particle optical sizing of microbubbles by using two techniques, namely light obscuration and flow cytometry, in order to determine the size distributions of microbubble ultrasound contrast agents and to study the dynamics of individual microbubbles during ultrasound stimulation. Experimental data were compared with predictions from GLMT, with BSCs computed using a localized approximation under laser sheet illumination. It was in particular found that GLMT accurately predicts the characteristic Lissajous-like serpentine scattering plot measured by flow cytometry for microbubbles. It was concluded that GLMT calculations can be combined with optical forward and side scatter measurements to accurately determine microbubble sizes. Murib *et al.* [362] discussed an optical biosensor concept, based on elastic light scattering from sapphire microspheres, in which transmitted and elastic scattering intensity of a microsphere (radius 500  $\mu\text{m}$ ) on an optical fiber half coupler was analyzed at 1510 nm. Shift of WGMs, after an add-on poly-dopamine layer to the sphere, was performed and the system was modeled using GLMT.

Bosch and Sancho-Parramon [418] numerically investigated procedures for the measurement of the extinction cross section of one spherical particle (in particular 50 nm radius gold particle, 50 nm radius water particle, and 500 nm radius gold and also water particle), using a linearly polarized slightly focused Gaussian beam (divergence angles up to a maximum of  $10^\circ$ ). El Bassri *et al.* [419] dealt with nanosecond coherent anti-Stokes Raman scattering (CARS) for particle size characterization, by measuring the macroscopic Raman spectral response of the whole set of particles excited by a laser beam. Using samples of latex beads in water with different diameters, the authors experimentally and theoretically demonstrated the ability of their device to discriminate objects having the size from micrometric to nanometric dimensions by using the change of Raman spectra versus particle size. Rosenkranz *et al.* [420] dealt with the modeling of photon transport in turbid media for measuring colloidal concentration in drops, as well as sizes and velocities, using the time-shift technique relying on a ray tracing model, incorporating interactions with randomly placed monodispersed scattering centers within the spherical drop. The model created a random walk propagation trajectory, known from radiative transfer problems. The model approximated Mie scattering from each internal particle with a Gaussian distribution. Experiments were performed using a conventional instrument, first with water as a reference and for validation, and then with different concentrations of a milk/water emulsion. Comparisons of the modeled

and received signals exhibited very good agreement, confirming the possibility of measuring the colloidal concentration in drops using the time-shift technique.

Huang *et al.* [421] dealt with the scattering properties and polarization of aerosol particles, in different humidity conditions, by a Hermite-Gauss beam in marine atmosphere, relying on the complex-source-point method and on the GLMT and aiming to applications in the fields of remote sensing, atmospheric sounding, and environmental monitoring. It was concluded that Hermite-Gauss beams have better capability of detecting the changes in relative humidity than the plane wave and the Gaussian beam. Wang and Muth [422] described an optical-fiber-based airborne particle counter where the air flow with particles was guided through the coupling region between two single mode fibers. They avoided to use particle light scattering as a method since the desire was to minimize the volume of the sensor package and the amount of energy consumed by the light source. The resulting sensor measured the drop in intensity as the particle disrupts the electromagnetic mode of the light beam. The implementation described was best suited for measuring the coarse fraction since it has the best performance for particles in the 1 to  $10\mu\text{m}$  size range. Rather than simple geometric occlusion of light passing through the beam, the passing particles disrupted the electromagnetic mode, changing the coupling between the two fibers. This disruption caused a drop in transmission that could be used to count particles. D'Amore *et al.* [423] performed a glass particle size analysis using a laser granulometer in the context of a study devoted to an innovative thermal and acoustical insulation foam from recycled waste glass powder, with data processing using LMT. Guo *et al.* [233] dealt theoretically with the remote sensing of aerosol particles in marine atmosphere using the scattering of optical vortex Laguerre-Gauss beams. Relying on GLMT to study the relationship between relative humidity and aerosol scattering properties, it was found that the scattering properties of aerosol particles by Laguerre-Gauss beam were more sensitive to the change of relative humidity, compared to those of a plane wave and Gaussian beam.

Xie *et al.* [424] dealt with particle sizing from Fraunhofer diffraction pattern using a digital micro-mirror device used to reflect the intensity at different angles to a single photodiode detector, therefore capturing the scattered intensity at different angles. Songfang and Jianqi [425] (in Chinese) experimentally and theoretically dealt with particle size analysis based on backward-scattered light, relying on the computations of the angular distribution of backward-scattered light with different particle sizes and refractive indices based on GLMT. Bain *et al.* [341] dealt with the determination of size and refractive index of single aerosol particles using angular light scattering and Mie resonances. Particles were investigated in an optical trapping set-up made from two focused counter-propagating beams, within a relative humidity-controlled cell. Both one- and two- dimensional phase functions were recorded and Mie resonances were characterized using cavity-enhanced Raman scattering. The accuracy of the methods used, relying on best-fits, were discussed and compared. Theoretical evaluations were carried out using LMT rather than GLMT. The authors, relying on the comparisons of 1D phase functions using either LMT or GLMT, commented

that the plane wave approximation they used was satisfactory to their purpose, although the comparisons were made for a particle having a radius of  $3 \mu\text{m}$  in a beam waist radius of  $4.3 \mu\text{m}$ , i.e. for a tightly focused situation. That the plane wave approximation was satisfactory relied on the fact that only the shape of the angular scattering spectrum mattered in the analysis. Another comment is that location of resonances is insensitive to the degree of focus of the Gaussian beam (private communication and see [426]). Yi *et al.* [109] studied the influence of spheroidal particle shape on particle size characterization by multi-wavelength light extinction method. Extinction coefficient and extinction spectrum were calculated under the configuration of the GLMT. Guo *et al.* [79] numerically dealt with the backscattering remote sensing for aerosol particles, regarded as coated-spherical particle, illuminated by a Laguerre-Gauss vortex beam, in different atmosphere environment, relying on GLMT. De Rond *et al.* [427] described a protocol to derive the size of extracellular vesicles (which can be biomarkers of disease) and other nanoparticles from the scatter intensities measured by a flow cytometer, using a scatter intensity-to-size relationship, relying on Mie theory although, as noticed by the authors, GLMT may be required for highly focused beams.

For other papers devoted to the issue of the present subsection, see Hanske *et al.* [428] in section 9 devoted to miscellaneous issues, and Simpson [349] in section 10 devoted to reviews and tutorials.

## 8 Alternative light scattering approaches.

### 8.1 Extended Boundary Condition Method for structured beams.

As well apparent from the discussion on EBCM in the introduction of the present paper, EBCM for structured beams deserves a special mention. This issue was mentioned in Problems 13 and 14 of [11] in 2013: "Implement a localized approximation in the EBCM" and "Extend the EBCM to arbitrary-shaped beams with the use of BSCs". The key to solve these problems is that EBCM relies on exactly the same description of incident beam than the one used in GLMT, relying on expansions over VSWFs, and therefore requiring the evaluation of BSCs. See sections 3 and 4 of the supplemental materials for the second edition of [3]. Outside of the fiducial period here considered, we may then mention Mackowski and Mishchenko [161] who carried out EBCM calculations for discrete random media illuminated by an on-axis Gaussian beam, using a localized approximation to the evaluation of BSCs.

Returning to the fiducial period, we have Chen *et al.* [429] dealing with the scattering of on-axis Gaussian beams by uniaxial anisotropic objects (spheres, prolate and oblate spheroids, circular cylinders of finite length) based on EBCM,

with on-axis spherical BSCs  $g_n$  evaluated by using a localized approximation, and EBCM for spheroids being compared with the ones obtained by using the GLMT for spheroids. Next, Wang *et al.* [430] studied light scattering from an optically anisotropic particle illuminated by an arbitrary shaped beam, in practice a Gaussian beam. Zheng *et al.* [431] studied the scattering of on-axis Gaussian beam by an arbitrarily shaped chiral object and viewed the method used as a combination of the EBCM and of the GLMT. Zheng *et al.* again [432] dealt with an object with a chiral inclusion (spheroid with a chiral spheroid inclusion, spheroid with a dielectric spheroid inclusion, spheroid with a chiral cylinder inclusion), with BSCs evaluated by using a localized approximation. Briard *et al.* [40] dealt with shaped beam scattering by an aggregate of particles using both the GLMT for assemblies of spheres and aggregates [16] and a publicly available "T-matrix" code. Chen *et al.* [433] studied on-axis Gaussian beam scattering by gyrotropic anisotropic objects (spheres, spheroids, circular cylinders) relying on a localized approximation for the evaluation of BSCs. Mishchenko and Dlugach [434] dealt with the scattering of Gaussian beams by disordered particulate media using the superposition "T-matrix" method.

For EBCM in which the incident field is expressed in terms of plane wave spectra, see [12]. For EBCM under plane wave illumination, see Bi *et al.* [435] in Section 9 devoted to miscellaneous issues. For structured beams, see also Phillips [304], Brzobohaty *et al.* [112], Simpson *et al.* [104], Li *et al.* [329] in section 4 devoted to mechanical effects of light. I end this subsection with a paper by Wu *et al.* [436] who studied plasmon-induced strong interaction between chiral molecules in a cluster of nanoparticles and orbital angular momentum of light, using a T-matrix approach. This paper, beside its scientific interest, is interesting as well from the point of view of the issue of terminology addressed in the present review concerning the meaning of "T-matrix". From the contents of the paper, I inferred that, in the present case, "T-matrix" was to be understood as "GLMT" not as "EBCM". Indeed, as confirmed by a private communication, "all the T-matrices are indeed evaluated by the GLMT approach". "However, it is worth to mention that the present theory can be easily extended to deal with systems consisting of nanoparticles with arbitrary shapes by using the EBCM".

## 8.2 Ray optics.

Ray optics is valid when the size(s) parameter(s) of the scatterer is large enough. For a discussion on this issue, the reader may refer to Ungut *et al.* [437] dealing with a quantitative comparison between geometrical (ray) optics and wave optics. Returning to the fiducial period, we have Haghshenas-Jaryani *et al.* [343] who presented new experimental observations and numerical simulations to study the dynamic behavior of micro-nano-sized objects under the influence of optical tweezers. They carried out an experimental determination of underdamped regime and a numerical simulation using a multiscale analysis. The motion of three polystyrene microspheres of different diameters (1950, 990

and 500 nm) were examined. The results showed a transition from the overdamped motion of the largest bead to the underdamped motion of the smallest bead. The experiments were verified using a dynamical model of a microbead under the influence of Gaussian beam in an optical tweezers with a ray optics model. Chang *et al.* [110] studied the behavior of an ellipsoidal particle subjected to a vertical force by a loosely focused laser beam in a uniform flow, in the configuration of a continuous separation set-up based on cross-type optical separation. Optical force calculations were conducted using the ray optics method. Loudet *et al.* [342] dealt with a study of optically driven oscillations of ellipsoidal particles using ray optics calculations.

Delabre *et al.* [344] investigated the deformation of phospholipid vesicles in a dual-beam laser trap, also called an optical stretcher. They computed the optical stress using ray optics. Yang *et al.* [438] carried out a comparison of scattering diagrams of large non-spherical particles (triaxial ellipsoid, prolate and oblate spheroids) calculated either by VCRM or SIEM. Yu *et al.* [439] dealt with the application of the ray tracing to the computation of Möbius shifts (from spherical to spheroidal shapes) for the primary and secondary rainbows.

Stegmann *et al.* [440] dealt with a comparison of measured and computed phase functions of individual tropospheric ice crystals. Experiments were carried out by using a Particle Habit Imaging and Polar Scattering system which is capable of simultaneously measuring the polar phase function and capturing the generating particle shape via stereoscopic imaging. Calculations were carried out by using a geometrical optics code. Haussmann [441] dealt with polarization-resolved simulations of multiple-order (primary-quinary) rainbows from realistic size-dependent shape falling raindrops, taking into account that the drops' bottom part is flattened to higher degree than the dome-like top part. Theoretical approaches relied on geometrical optics and on Debye-Möbius-Können method, including Möbius-like shifts.

See also Haussmann [442] for another study on a similar issue, dealing not only on theoretical approaches but on experiments as well using (i) sessile, i.e. sitting, drops on ultrahydrophobic surfaces and (ii) glass beads in the shape of falling raindrops. Experimental results were compared to polarization-resolved Monte-Carlo ray-tracing simulations. Lock and Können [443] relied on ray theory to study the Möbius shift for higher-order rainbows, particularly the Möbius shift of the  $(p - 1)$ -order rainbow angle for a particle having an elliptical cross section, in a situation which is adapted to the geometry of scattering of light rays from the sun by a falling water drop as a function of sun height angle. Sun *et al.* [444] dealt with physical-geometric optics method for large size faceted particles.

For other papers devoted to the issue of the present subsection, see Yang *et al.* [100], Tapp *et al.* [305], Burgin *et al.* [309] Lee *et al.* [327] in section 4 devoted to mechanical effects of light, Duan *et al.* [148] in subsection 7.2 devoted to rainbow refractometry, Rosenkranz *et al.* [420] in subsection 7.5 devoted to miscellaneous measurement techniques, Bui *et al.* [352], Polimeno *et al.* [355] in section10 devoted to reviews and tutorials.

### 8.3 Debye series.

Ray optics acts by relying on the use of various ray paths from a geometrical point of view. The Debye series method relies on wave optics but succeeds to select different waves corresponding to various ray paths. In other words, Debye series is the wavy analog of the geometrical ray optics. It has been introduced in the GLMT context in [445].

Bi *et al.* [446] dealt with optical tunneling by arbitrary macroscopic three-dimensional objects (e.g. spheres, incomplete spheres, spheroids, faceted hexagonal particles), using the invariant embedding principle and the Debye series expansion technique. Li *et al.* [447] used a laser source to measure the refractive index of glass beads relying on a Debye theory analysis and on the analysis of rainbows. Li *et al.* [78] provided a Debye series analysis of optical forces induced by an axicon-generated Bessel beam exerted on a spherical particle. The BSCs for Bessel beams were calculated in closed forms using the ILA.

Qin *et al.* [220] dealt with a Debye series analysis of internal and near-surface fields for a homogeneous sphere illuminated by an axicon-generated vector Bessel beam, in the framework of GLMT. An analytical expression of BSCs was derived using the ASD method, and the scattering coefficients were expanded using Debye series in order to isolate the contribution of single scattering processes.

Debye series were used by Lin *et al.* [111] to determine the optimal edge-effect terms for computing the extinction efficiencies of spheroids, invoking the use of Debye series to improve the accuracy of high-frequency extinction formulas for spheroids in the context of Complex Angular Momentum (CAM) theory and of the extension of Debye series to non-spherical particles.

Chou [448] dealt with coherent backscatter enhancement in single scattering, by relying on an expansion into iterative series in an order-of-scattering form, where the interference between conjugate terms representing reversible sequences of elementary scatterers is constructive at the backscattering direction, resulting in a coherent backscatter enhancement (CBE). Such CBE is reminiscent of a process similar to what is known as coherent backscattering for disordered media. It is found that CBE applies to a wide range of particles, including non-absorptive spheres, spheroids, smooth hexagonal particles, and roughened hexagonal particles. The author convincingly expected that Debye series should be useful for future studies, due to its ability to identify conjugate surface paths of spheres and its applicability to non-spherical particles. Bi *et al.* [449] used Debye series to study the depolarization of nearly spherical particles.

For a review devoted to Debye series and its use in time-domain scattering, see Lock and Laven [280]. For other papers devoted to the issue of the present subsection, see Ding *et al.* [50], Lock [56] in subsection 2.1 devoted to spheres and spherical coordinates, Mitri *et al.* [184] in subsection 3.3 devoted to non-diffracting beams, Yu *et al.* [158] in subsection 7.2 devoted to rainbow refractometry, Wu *et al.* [172], [173] in subsection 7.3 devoted to holography.



## 8.4 Dipole approximation and DDA.

While ray optics is valid in the case of large particles (with respect to the wavelength), the dipole approximation is conversely valid for small particles (with respect to the wavelength), also named Rayleigh particles viewed as elementary dipoles. DDA is a numerical technique in which large particles are viewed as a collection of elementary particles. For the dipole approximation, we may quote Li *et al.* [450] who calculated the intrinsic optical torque of cylindrical vector beams (focused radially polarized beams) on Rayleigh absorptive spherical particles via a corrected dipole approximation. For DDA, we may quote Skorupski [451] who used it to determine the extinction cross section of black carbon. The GMM was used as a reference algorithm, and Li and Feng [452], after a discussion of the limitations of DDA, dealt with the modeling of electric multipole moment in target scattering in the framework of a study devoted to spherical targets and ballistic missile flat-bottomed warheads.

For other papers devoted to the issue of the present subsection, see Mitri [61], [93], [94] in subsection 3.2 devoted to laser sheets, Devi and De [273] in subsection 3.6 devoted to pulses and laser pulses, Zhang and Yang [232], Ambrosio and Ferreira [206] Li *et al.* [103], Zemanek *et al.* [328], Simpson *et al.* [104], Li *et al.* [211], Mitri [106], [107], Chen *et al.* [213] in section 4 devoted to mechanical effects of light, Rondo-Ojeda and Soto-Eguibar [219] in section 6 devoted to optical theorem, Khlebtsov and Khlebtsov [453] in section 9 devoted to miscellaneous issues, and Bui *et al.* [352], Polimeno *et al.* [355] in section 10 devoted to reviews and tutorials.

## 8.5 Other numerical techniques.

For a general introduction to the issue, the reader might refer to Cui and Han [454] for a review. In this subsection, we are mainly concerned with Surface Integral Equation Methods (SIEM) in which the unknowns are defined only on the surface(s) of the particle(s) leading to surface integral equations and Finite Element Methods (FEM) in which the surface of the scatterer is discretized, e.g. with triangular patches, or the whole volume of the scatterer is discretized, e.g. with tetrahedral elements. These numerical methods can in principle handle arbitrary shaped particles and do not require the expansion of the incident beam in terms of vector wave functions, although such expansions may indeed be used. Let us note that there is a common point between SIEM and EBCM, namely the use of the Schelkunoff optical/equivalence theorem which establishes a relationship of equivalence between scattered fields and electromagnetic surface currents.

For SIEM, let us begin with Cui *et al.* [221] who studied the scattering of zeroth-order Bessel beam by arbitrary shaped composite with core-shell structure. Validations were carried out by comparing the obtained results with the

already investigated cases of concentric and eccentric spheres. On-axis normally incident beam scattered by a composite spheroid with an eccentrically located spheroidal inclusion, off-axis normally incident beam scattered by a composite hexagonal prism with a spherical air bubble at the center, and off-axis obliquely incident beam scattered by a rough spherical particle with a spheroidal inclusion at the center were considered. Yang *et al.* [113] used SIEM to carry out a computational study of radiation torque on arbitrary shaped homogeneous particles, specifically triaxial ellipsoids, prolate particles, prolate and oblate spheroids, and bi-concave cell-like particles.

This is to be completed by Yang *et al.* [225] who computed radiation pressure forces exerted on arbitrary shaped homogeneous particles by high-order Bessel vortex beams using SIEM. The arbitrarily shaped particles were modeled by using surface triangular patches and the SIEMs were discretized by following the procedure of the method of moments (MoM). The MLFMA (multilevel fast multipole algorithm) was utilized to speed up matrix-vector multiplication in the iterative solution step of the final matrix equation system. See also Yang *et al.* [224] for a similar study in which the SIEM code was validated against GLMT comparisons for spherical particles. Zhang [114] used SIEM to study the shaped beam scattering by a spheroidal object. For oblique incidence of a Gaussian beam and of a zeroth-order Bessel beam, numerical results of the normalized differential cross section were presented. In the case of spheroids, comparisons with those obtained by using a GLMT for spheroids were carried out, leading to an excellent agreement. See also Gao and Zhang [115], Wang *et al.* [117] using SIEM as well in a similar study but in the case of the scattering by chiral spheroids and Chen *et al.* [455] dealing similarly with a shaped beam scattering by an anisotropic particle. Wang *et al.* [116] presented a SIEM procedure concerning the scattering of an arbitrary shaped beam by a dielectric particle, applied to the calculation of normalized differential scattering cross sections in the cases of a spheroid and of a circular cylinder of finite length illuminated by a Gaussian beam, a zeroth-order Bessel beam and a Hertzian electric dipole radiation. The Gaussian beam was described by using the Davis-Barton first-order approximate expressions. SIEM was compared with GLMT for spheroids, in which the expansion coefficients of the Gaussian beam were evaluated by using an extrinsic method with a localized approximation. Although there was in general a good agreement between the results, differences became obvious for very focused beams when the beam waist radius approaches its theoretical largest bound  $\lambda$ . Yu *et al.* [150] dealt with the electromagnetic scattering by multiple particles under the illumination of (non-diffracting) high-order Bessel beams, using SIEM. Numerical results were presented in terms of differential scattering cross sections in the far-field region. The following configurations have been studied: interaction with two homogeneous dielectric spherical particles (with validations of the results by using the GLMT for assemblies of spheres and aggregates [16]), four dielectric cylinders arranged in three different ways, and a 5x5x5 array of homogeneous dielectric spherical particles. Effects of the beam order and of its half-cone angle were investigated.

SIEM is used as well by Cui *et al.* [118] who dealt with the scattering

of polarized non-diffracting Lommel beams with different polarizations by non-spherical homogeneous particles (homogeneous spheroids, cylinders, cones, and hexagonal prisms). Results were provided in the form of differential scattering cross section and validated by using GLMT computations in the case of prolate spheroids. Liu *et al.* [456] dealt with the transmission of a shaped beam through a rotationally uniaxial anisotropic sphere using SIEM. Numerical results were given for the normalized internal and near-surface field intensity distributions for incidence of a Gaussian beam, zeroth-order Bessel beam and Hertzian electric dipole radiation. When the relations among the expansion coefficients of the incident beam, scattered and internal fields, were obtained, the approach used is equivalent to the exact analytical solution based on the GLMT. Yu *et al.* [234] dealt with the scattering of a Laguerre-Gauss beam by complicated shaped biological cells such as fusiform cells, using SIEM. To verify the correctness of the theoretical treatment and of the computation codes, numerical results in the case of homogeneous dielectric spherical particles were calculated using the GLMT leading to very good coincidences.

After SIEM, we now turn to FEM, starting with Mangini *et al.* [457] who studied the interaction of an electromagnetic field with a composite core-shell structure, made of a sphere eccentrically located inside a host sphere, and looked for an effective-medium analysis in order to deduce the properties of the scatterer as a homogenized sphere. A quasi-static analysis allowed to obtain an analytic expression for the polarizability and for the effective permittivity of the scatterer. Quasi-static results were validated through comparisons with a FEM based on a commercial software. Cui *et al.* [222] dealt with a numerical investigation of scattering of an arbitrarily incident Bessel beam by fractal soot aggregates, using a hybrid vector finite element-boundary integral-domain decomposition method. Sundaram and Wen [223] examined the light scattering from nonspherical symmetric pure dielectric structures, by using a finite element full-wave analysis. The goal of this study was to find the geometry of dielectric scatterers that can provide nanoscale forward scattering with high focusing intensity that is comparable with standard spherical metallic scatterers. Four cases were considered, namely scattering of radially polarized Bessel beams by (i) dielectric nanospheres, (ii) snowman scatterers, (iii) teardrop scatterers and (iv) combination of a teardrop scatterer and a far-field microlens.

Pan *et al.* [345] dealt with the prediction of metallic nano-optical trapping forces by finite-element-boundary integral method. Reid and Johnson [346] used a boundary-element-method for efficient computation of power, force, and torque in scattering calculations. The approach computed the quantities of interest directly from surface currents with no need ever to compute the scattered electromagnetic fields.

Patel *et al.* [300] dealt with the generation of highly confined photonic nanojets using crescent-shape refractive index profile (CSR) in microspheres (Crescent shapes may be defined as multilayered spheres which are nonconcentric with a common point of attach on the external surface). The results showed that photonic nanojets with FWHM down to  $\sim \lambda/4.5$  can be achieved in free space with multilayer CSR lossless dielectric microspheres of  $3 \mu\text{m}$  diameter.

In the presence of commonly used substrates with higher refractive index this would enable writing features as small as  $\sim \lambda/6$  or even  $\sim \lambda/6.7$ . Theoretical simulations were carried out by FEM.

Next, Yu *et al.* [458] used a Volume Integral Equation Method (VIEM) to propose a numerical scheme to deal with complex metallic nanostructures in the context of thermoplasmonics which leads to enhanced heat generation due to the localized surface plasmon resonances. The use of VIEM is accompanied by the steady-state heat diffusion equation. The authors concluded that their simulations showed that VIEM is more efficient than the approach based on SIEM under comparable accuracy.

Related to volume integral equations, Mishchenko and Yurkin [459] dealt with impressed sources and fields in the volume-integral-equation formulation of electromagnetic scattering by a finite object. They provided a tutorial discussing how to account for the actual existence of impressed source currents rather than impressed incident fields. Specifically, they outlined a self-consistent theoretical formalism describing electromagnetic scattering by an arbitrary finite object in the presence of arbitrarily distributed impressed currents, some of which can be far removed from the object and some can reside in its vicinity, including inside the object. Here, the term "impressed" means that primary source currents are unaffected by (independent of) the total electromagnetic field existing in the presence of a scattering object. This is the point of view taken when speaking of an "incident" field which is not disturbed by the forthcoming interaction with a scatterer (although the paper deals with impressed currents rather than with impressed fields). The study is carried out in the volume integral equation formulation of electromagnetic scattering to make the resulting formalism applicable to a wide range of scattering-object morphologies.

For FDTD, we have Nayak *et al.* [299] who carried out a theoretical investigation on the tuning of a photonic jet through varying the salt concentration in a propagating medium made from a NaCl aqueous solution. It was found from simulations that the behavior of photonic jet varies with the different concentration of aqueous solution. Wu *et al.* [119] dealt with the generation of Bessel beam sources in FDTD. The incident Bessel beam was described as a superposition of plane waves whose wavevectors covered a conical surface. This decomposition of Bessel beam was then approximated by a finite collection of plane waves which were injected in FDTD simulations. The correctness and accuracy of the present method were verified by comparing the reconstructed field in FDTD with the original intended field. Far-field scattered diagrams of a dielectric sphere and a spheroid particle illuminated by a zeroth-order or a higher-order Bessel beam were calculated. The results were compared with those calculated using the GLMT and SIEM. In a similar study, Chen *et al.* [120] investigated the scattering of complex particles illuminated by Bessel beams using FDTD. As an example, differential scattering cross sections for the scattering of a spheroid illuminated by a zeroth-order Bessel beam were calculated and FDTD results were compared with those of a GLMT. In the same series, we have Wu *et al.* [226] who used the ASD and 3D FDTD to generate Bessel beam sources. The correctness and accuracy of the method were verified by comparing

the reconstructed field in the FDTD framework with the original field. Other validations came from the comparisons of scattering diagrams calculated by using either FDTD and GLMT in the case of a dielectric sphere illuminated by a Bessel beam.

Finally, Salgado-Remacha *et al.* [460] studied the diffraction of a focused Gaussian beam by a microrefractive cylinder in order to carry out the positioning of the beam inside the cylinder, with the aim of achieving laser micromachining of optic fibers for optical sensor manufacturing, specifically producing a novel technique for ablation of optical fibers. They relied on a numerical analysis using a Fourier transform propagator performed in Matlab, simulating the propagation of an electromagnetic field under a scalar approach so that the field is obtained at any point of a propagation map. Numerical results were experimentally validated.

For other papers devoted to the issue of the present subsection, see Zhu *et al.* [131] in subsection 2.3 devoted to cylinders and cylindrical coordinates, Kozlova *et al.* [135] in subsection 3.6 devoted to pulses and laser pulses, Tapp *et al.* [305], Pan *et al.* [306], Yang *et al.* [100], [209], Brzobohaty *et al.* [112], [325], Xu *et al.* [105], Ge *et al.* [337] in section 4 devoted to mechanical effects of light, Krasavin *et al.* [370] in section 6 devoted to the optical theorem, Zhou *et al.* [413] in subsection 7.4 devoted to imaging and microscopy, Yang *et al.* [438] in subsection 8.2 devoted to ray optics, Khlebtsov and Khlebtsov [453] in section 9 devoted to miscellaneous issues, Han *et al.* [121], Polimeno *et al.* [355] in section 10 devoted to reviews and tutorials.

## 9 Miscellaneous issues.

Bressel and Reich [461] dealt with the theoretical and experimental study of the diffuse transmission of light through highly concentrated absorbing and scattering materials relying on monte-Carlo simulations. Hanske *et al.* [428] dealt with the creation of assemblies of plasmonic nanoparticles that are both strongly coupling and at the same time homogeneous in their optical properties on macroscopic areas. They demonstrated that, utilizing purely bottom-up, template-assisted approach, highly ordered, linear arrangements of closely spaced plasmonic nanoparticles could be fabricated with unprecedented uniformity over centimeter-squared areas, then allowing the emergence of strongly coupled plasmonic modes on macroscopic areas via template-assisted colloidal self-assembly. Comparisons between experimentally determined extinction spectra of single particles, dimer, and tetramer chain ensembles were compared to simulation data modeled for ideal chains via GMM. Gagnon *et al.* [151] investigated lasing thresholds in a representative photonic molecule (PM) composed of two coupled active cylinders of slightly different radii. They used a 2D-GLMT to compute the eigenstates of the PM. This kind of theory can also be used for the computation of the scattering of arbitrary beams by a complex arrangement of dielectric cylinders. The approach used in this paper is not limited to

diatomic PMs. For instance, it could readily be applied to the computation of modes of random lasers for an arbitrary number of active scattering centers.

Ramm and Andriychuk [462] studied the electromagnetic wave scattering on a set of small particles and dealt with the creation of materials with desired refraction coefficient and magnetic permeability, obtained by embedding many small particles into a given material. The opportunity of this study was taken to overview author's results related to electromagnetic wave scattering by many small particles, embedded in a homogeneous medium. Thin (nano) perfectly conducting scattering wires have been considered, and the results were given under the form of explicit analytical formulas for the refraction coefficient and magnetic permeability of the resulting media. Khlebtsov and Khlebtsov [453] dealt with Raman molecules which were trapped in a nanometer-sized interior gap between the metallic core and the shell of a multilayered particle, and considered hollow gaps as well. Extinction spectra of such structures were measured and simulations were carried out using a multilayered Mie solution, a dipole approximation, and also a FDTD-solver. This paper is to be completed with Khlebtsov and Khlebtsov [463] who furthermore derived explicit analytical expressions for surface and volume averaged intensities of local field within a multilayered sphere or of near field around the sphere surface. The expansion coefficients of the analytical solutions were calculated through efficient recurrent schemes.

Kalume *et al.* [347] dealt with the detection and characterization of the presence of chemical agent aerosols in various complex atmospheric environments in the context of a defense mission, relying on Raman spectroscopy. The authors reported on a single-particle Raman spectrometer system that can measure strong spontaneous, stimulated and resonance Raman spectral peaks from a single laser-trapped chemical aerosol particle. Using this system, time-resolved Raman spectra and elastic scattered intensities were recorded to monitor the chemical properties and the variation of the trapped particle. In addition to the spontaneous Raman peaks reported in bulk liquid, the microdroplet Raman spectrum exhibited additional features, namely stimulated Raman scattering (SRS) and MDRs/WGMs leading to noticeable WGM resonance frequencies of Raman peaks. In parallel to the chemical composition characterization, the authors were able to determine the particle size of the trapped microdroplet by profiling the intensity of the elastic scattered light as a function of time. Buccini *et al.* [464] dealt with the issue of inversion of light scattering data, for inverse problems taking the form of a Fredholm integral equation, with the goal to accurately and fastly recover sample particle-size distributions over the largest possible range of particle radii. Luo *et al.* [363] proposed and demonstrated a manipulative dual-microsphere lasing based on nanosilica waveguide integrated in a simplified hollow-core microstructured optical fiber, including a discussion of in-cavity WGMS distributions. Steiner *et al.* [465] investigated the formation of chains of few plasmonic nanoparticles – known as plasmonic oligomers – by strain-induced fragmentation of linear particle assemblies. Experimental observations were quantitatively explained by using GMM.

Bi *et al.* [435] discussed the depolarization capabilities of nonspherical par-

ticles in a super-ellipsoidal shape space allowing one to describe salient characteristics of most atmospheric particles such as aspect ratio, roundness and concavity, using both EBCM and invariant imbedding T-matrix method. To be precise enough, the equation of a superellipsoid relies on the usual three semi-axes of ellipsoids, together with two roundness parameters to specify the morphology variation of the particle. Kiani *et al.* [466] dealt with an analytical method for analysis of electromagnetic plane wave scattering from inhomogeneous spherical structures using duality principles. In contrast with the usual approach which would be a "multilayer model", the inhomogeneous radial structure is embedded in an 1D integral radiation in the radial direction. A duality principle then was used to relate this 1D structure to the simpler study of a dual structure which is an inhomogeneous planar layer obliquely illuminated. It is worth mentioning that the proposed duality operates between each harmonic of spherical wave (in which the incident plane wave is expanded) and each angle of the incident plane wave in the dual planar structure. The proposed method then substitutes a complex problem in spherical geometry that has a large computational load, with a simple, well-posed problem, in planar geometry having less time cost. Shiloah [152] dealt with an algorithmic issue, namely with canonical scattering coefficients upward recursion algorithm for Bessel or Riccati-Bessel functions, for multilayered sphere or long cylinder with large size parameters. Canonical recommendations prescribed using both upward and downward recursions for different types of functions, making the computational algorithm rather cumbersome. The authors justified the procedure using the upward recursion only, whose results are as stable as the canonical recommendations. It was shown that by modifying basic relations, one can use only the upward recursion without losing the stability of the process. The necessary modification involved the ratios of the different type Riccati-Bessel (or Bessel for the cylinder case) functions. By excluding the downward recursions, the authors found that the calculation algorithm was more compact. Lamprianidis and Miroshnichenko [467] dealt with the excitation of nonradiating magnetic anapole states with azimuthally polarized vector beams. The authors noticed that, in quantum mechanics, it was postulated that electrons live at certain orbits, which do not radiate. They then consider classical analogs which may be viewed in confined classical trajectories of charged particles that do not produce far-field radiation, the key ingredient for such occurrences being destructive interferences in the far-field. It then happens that nonradiating sources have attracted a great deal of attention in various nanoscale structures. Recently, it was suggested that silicon nanodisks can support a nonradiating excitation known as anapole. In this context, the authors discussed in detail the possibility of magnetic anapole excitation under various configurations and suggested how they could be tested experimentally. The theoretical analysis relied on a multipolar decomposition of focused beams and on EBCM T-matrix method for far-field scattering calculations. Téllez-Quinones *et al.* [468] discussed the retarded potential of a non-homogeneous wave equation and the Green function in the language of distributions. The authors expected at least to motivate the reader on the importance of considering sequences of approximating functions (which are inroads

to distributions) in the framework of electromagnetism. By the way, the concept of distributions has been invaluable for the development of GLMTs for cylinders (with circular or elliptical cross sections), see [469], [470] for examples and [471] for an easy introduction. Cheng *et al.* [472] dealt with the average intensity and spreading of a radially polarized multi-Gaussian beam in anisotropic turbulence. Yu and Sun [473] dealt with the visibility reduction due to the spray created by water film on pavement and caused by high-speed vehicles, relying on Mie scattering and Monte-Carlo simulations. Alonso *et al.* [22] expressed the GLMT in the case of a wide class of focused electromagnetic fields, known as "complex focus fields", allowing one to obtain closed-form expressions for their multipolar expansions and to compute the field scattered by a spherical particle and the resulting forces and torques exerted on it, for any relative position between the field's focus and the particle. The authors claim that the fields considered are elements of complete bases, and therefore can be superposed to describe arbitrary incident fields. However, from a private communication, the authors "did not claim to have obtained analytical expressions for the shape coefficients of any electromagnetic beam". They mentioned that "for some beams, particularly those mimicking experimental scenarios, numerical calculations are unavoidable". The author of the present review did not examine in all details the content of this paper by Alonso *et al.* (a task which would be in practice impossible considering the comprehensiveness of the present review), but believes that the contents of the quoted paper is very rich. It would deserve a more precise and accurate examination which is postponed to a future work. See also Gutiérrez-Cuevas *et al.* [348] on the same issue.

I end this section with a paper by Domínguez-Juárez *et al.* [474] who studied the influence of solvent polarity on light-induced thermal cycles in plasmonic nanofluids, and in particular dealt with an oscillatory behavior caused by light grazing a nanofluid meniscus. This paper, which is quoting GLMT, has however little relevance with GLMT. The author of the present review confess that he did not resist to the pleasure of quoting it because it dealt with a phenomenon elsewhere named "thermal lensing" (a particular case of local heating below an interface) to which he devoted a significant amount of time, e.g. [475], [476].

## 10 Reviews and tutorials.

Gogoi *et al.* [477] described Webscat, an online web application to simulate and display electromagnetic wave scattering from small particles, having spherical, spheroidal and cylindrical shapes. Simpson [349] reviewed experimental and theoretical work on the relationship between particle geometry and composition (particularly inhomogeneous and anisotropic particles) and optical trapping, focusing on the physical behavior of trapping setups and applications. A comprehensive range of associated metrological techniques was discussed as well. Cui and Han [454] provided a very comprehensive review devoted to the



numerical investigation on the scattering of Gaussian beam by complex particles with arbitrary shape and structure, including arbitrarily shaped conducting particles, dielectric particles, composite particles, as well as random discrete particles and fractal soot aggregates, when one has to resort to numerical methods. The numerical techniques used are SIEM for conducting particles, homogeneous dielectric particles, composite particles with conducting or dielectric inclusions, and FEM (under a hybrid form) to efficiently characterize the scattering by multiple randomly distributed particles, and soot aggregates. GLMT is used to provide reference data when usable.

Santos *et al.* [364] extensively provided a description of the origin of localized surface plasmon resonance through the works developed by Mie, Maxwell and Maxwell-Garnett and a description of many examples of applications of plasmonic nanoparticles on biosensors and solar cells, detailing the contribution of these plasmonic nanoparticles on the performance of these devices. Stangner *et al.* [350] published a book chapter which has the flavour of a review devoted primarily to the biophysical investigation of receptor-ligand interactions on a single molecule level, focusing on optical tweezers-assisted dynamic force spectroscopy. Let us precise that dynamic force spectroscopy is the investigation of a force-induced dissociation of a single bond and provide information about kinetic parameters as well as parameters that characterize the energy landscape of the binding from the distribution of rupture forces. By using this technique it was possible to investigate receptor-ligand interactions on a single-molecule level, and to extract specific binding parameters. Haussman [478] reviewed recent advances in observation and theory concerning the rainbow in nature, specifically rainbows in natural rainfall including the effects of natural non-spherical raindrops, as well as natural broad polydisperse raindrop distributions into the classical rainbow theory of Lorenz-Mie and Debye scattering. The description of the classical history of rainbow theories up to Lorenz-Mie and Debye scattering was followed by theoretical consideration towards natural non-spherical drop shapes and broad drop-size distributions. Barbosa *et al.* [380] provided an introduction to light scattering extinction spectrometry (LES) as a diagnostic for dust particle characterization in dusty plasmas, under the form of a review. LES allows the direct in situ measurement of the particle size distribution and absolute concentration of dust clouds levitating in plasmas. Using a relatively simple and compact experimental set-up, the dust cloud parameters can be recovered with a good accuracy making minimal assumptions on their physical properties. Special emphasis was given to the inversion procedure of light extinction spectra. Two exemplifying measurements were discussed. Calculation of extinction matrix was reviewed as well as required theoretical approaches, including GLMT and EBCM.

Sazhin [479] provided a review on the modelling of fuel droplet heating and evaporation, dealing with recent results and unsolved problems. GLMTs were mentioned in connection with the issue of radiative heating of droplets, making a connection with one of the original motivation of GLMT, namely the development of optical particle characterization techniques for multiphase flows issues, such as in spray combustion or plasma spraying. In a similar network

of motivation, Fuqiang *et al.* [480] provided a comprehensive review of radiative heat transfer in particle solar thermochemical reactor. Tu *et al.* [381] dealt with a review of experimental techniques for measuring micro- and nano-particle-laden gas flows, in particular optical particle characterization, including phase-Doppler anemometry with a discussion of the use of GLMTs and of particle trajectory effects, particle image velocimetry, scattering intensity measurements, laser-induced fluorescence techniques for temperature and composition measurements, among others. Conteduca *et al.* [351] reviewed photonic and plasmonic resonant technologies for biomolecule trapping, manipulation, and sensing at the nanoscale, with a special emphasis on hybrid photonic/plasmonic nanodevices allowing a very strong light-matter interaction. The state-of-the-art of competing technologies, e.g. electronic, magnetic, acoustic and carbon nanotube-based nanotweezers, and a description of their applications were also included. More specifically, the following issues were considered: non-optical trapping techniques, optical trapping techniques (ray optics regime, Rayleigh regime, LMT and GLMT regimes), photonic and plasmonic nanotweezers configurations (optical waveguides and nanowires, optical dielectric resonators, plasmonic resonators, hybrid photonic/plasmonic cavities). They concluded that optically based techniques have demonstrated high efficiency and very stable trapping conditions over a long trapping time, as well as low power consumption, making it possible to manipulate nanoparticles down to single molecules. The performance reached by the state-of-the-art optical nanotweezers highlighted the advantage of optically based techniques compared to other trapping approaches, making integrated optical nanotweezers a suitable tool for several applications in medicine, biology, and chemistry. Luk'yanchuk *et al.* [301] reviewed some of the phenomena and possible applications arising from the interaction of light with particles having a refractive index less than 2, encompassing a historical background, the basic properties of photonic nanojets, and other issues such as concerning caustic structures, shape effects, microscopic dielectric particles, with an extended focus on the issue of super-resolution imaging and the possibility of achieving optical resolutions on the level of  $\lambda/10$  or even better. Zhang and Xuab [481] published a book chapter to review the issue of nanophotonics, i.e. of light scattering by small metallic particles. Han *et al.* [121] reviewed the issue of light wave propagation and scattering through particles illuminated by Gaussian beams, dealing with spheres, spheroids, and circular cylindrical particles in the framework of GLMTs, SIEM and FEM. Sukhov and Dogariu [353] acknowledged that laser tweezers are the most recognized application of optically induced mechanical action, and that their operation is usually described in terms of conservative forces originating from intensity gradients. They afterward noticed that the fundamental optical action on matter is however non-conservative and therefore reviewed different manifestations of non-conservative optical forces, discussing their dependence on the specific spatial properties of optical fields that generate them. From the point of view of terminology, "non-conservative", in the context of radiation fields, means that part of the energy is dissipated (absorbed or scattered) to produce mechanical action. Different approaches for describing optical forces (and torques) under plane waves or

structured field illuminations were considered.

Bui *et al.* [352] very comprehensively reviewed the theory and practice of simulation of optical tweezers. The paper is easy-reading, avoiding complicated equations to get the gist of the involved phenomena to be considered, and allows the reader to optimize his/her strategy to enter the field. In general, trapping simulation requires to deal not only with optical forces (and torques) but also with forces which are exerted on the particle in the trap due to its interaction with the surrounding environment, mainly Brownian motion and viscous drag, but also weight, buoyancy, and others (such as adhesion forces, electrostatic forces, magnetic forces). Computing times, even limited to force maps, may be prohibitive, ranging from less than 1 minute for the simplest cases to 30 years in the most complicated case dealing with mapping force for a non-spherical particle lacking rotational symmetry in a trap lacking rotational symmetry, or if the trapping beam varies in time, or even more when several particles are simultaneously present in the trap. This was a motivation for the development of a skeletonization process discussed in [105]. Optical forces and torques were discussed using a T-matrix formalism (such as with GLMTs or EBCM, among others), finite element and finite difference methods, and approximate methods, notably Rayleigh scattering and ray optics. Qiu and Shen [248] provided a review on the calculation of BSCs for a Gaussian beam using localized approximations, quadratures and ASD. In particular, the formulations of the BSCs of the TEM<sub>00</sub> Gaussian beam in the quadrature and the ASD methods were simplified into one-dimensional integrations (the finite series technique is however not considered). Gong *et al.* [354] dealt with the issue of optical trapping and manipulation of single particles in air, specifically with principles, technical details, and applications. They reviewed recently demonstrated optical configuration for trapping and manipulating single airborne particles. Based on different trapping principles, they tentatively categorized them into radiation-pressure traps, photophoretic traps, and universal optical traps. The last ones can trap a variety of different types of particles, including transparent or absorbing, spherically or irregularly shaped, and liquid or solid particles. Furthermore, recent updates on optical methods for characterizing and monitoring single particles in air were discussed, such as light scattering, Raman spectroscopy, and cavity ringdown spectroscopy. Polimeno *et al.* [355] discussed optical tweezers and their applications. From a theoretical point of view, the paper discussed ray optics, dipole approximation and electromagnetic theory and modeling to deal with various kinds of particles using analytical or semi-analytical, e.g. EBCM, as well as numerical methods. A whole section was devoted to the experimental practice of optical tweezers with a comprehensive focus on various calibration methods for spheres, while applications were compiled in another section, dealing with mechanical properties of red blood cells, microbubbles, chiral optical forces, nanotechnology applications, optical binding, optical force positioning, aggregation, spectroscopy, optical Casimir forces, stochastic thermodynamics, and active matter.

For other reviews already quoted in previous sections, see Gouesbet *et al.* [12] in the Introduction (section 1) to discuss the electromagnetic scattering

of arbitrary shaped beams by arbitrary shaped particles, Han *et al.* [80] for a review on intrinsic and extrinsic methods in subsection 2.2 on spheroids and spheroidal coordinates, Gouesbet *et al.* [85] to discuss and compare different versions of GLMTs for spheroids again in subsection 2.2 on spheroids and spheroidal coordinates, Bouchal [179] and Recami *et al.* [181] reviewing non-diffracting beams in subsection 3.3 on non-diffracting beams, Gouesbet *et al.* [235] reviewing localized approximations in subsection 3.5 on localized approximations and associated warnings, Thomas *et al.* [283], [284] for analogies and differences between optical and acoustical scatterings in subsection 3.7 devoted to acoustical beams, Gouesbet [81] for GLMTs and mechanical effects of laser-light in section 4 on mechanical effects of light, Foreman *et al.* [361] reviewing whispering-gallery modes in section 5 on internal fields, Gouesbet [377] for a review on optical particle sizing and velocimetry before the advent of GLMTs and Gouesbet and Gréhan [378] for a recent review on OPC at the beginning of section 7 on OPC, Lock and Laven [280] reviewing Debye series in subsection 8.3 on Debye series, and Cui and Han [454] reviewing numerical techniques at the beginning of section 8 devoted to different to alternative light scattering approaches.

## 11 Conclusion.

Pursuing what can now be viewed as a tradition, the present paper presents a review concerning laser-light scattering for structured beams both from a fundamental theoretical point of view (GLMTs, EBCM, ray optics, Rayleigh scattering, numerical techniques) and from the point of view of applications, i.e. encompassing the theory and the practice of light scattering, however restricted to the period 2014-2018. A particular attention is paid as well to the issues of mechanical effects of light, internal fields and resonances, optical theorem while applications were mainly devoted to the field of optical particle characterization which has been the original motivation for the development of GLMTs. I hope that this review can be useful for the confirmed scatterist as well as for the newcomer which, expectedly, will find in this review a general overview of a rich landscape explored by using a bit less than 500 references.

## References

- [1] G. Gouesbet, B. Maheu, and G. Gréhan. Light scattering from a sphere arbitrarily located in a Gaussian beam, using a Bromwich formulation. *Journal of the Optical Society of America A*, 5,9:1427–1443, 1988.

- [2] G. Gouesbet, G. Gréhan, and B. Maheu. *Combustion measurements, edited by N. Chigier*, chapter : Generalized Lorenz-Mie theory and applications to optical sizing, pages 339–384. Hemisphere Publishing Corporation, New-York, USA, 1991.
- [3] G. Gouesbet and G. Gréhan. *Generalized Lorenz-Mie theories, second edition*. Springer International Publishing AG, 2017.
- [4] G. Gouesbet and G. Gréhan. Sur la généralisation de la théorie de Lorenz-Mie. *Journal of Optics*, 13,2:97–103, 1982.
- [5] P.C. Waterman. Symmetry, unitarity, and geometry in electromagnetic scattering. *Physical Review D*, 3, 4:825–839, 1971.
- [6] M.I. Mishchenko, L.D. Travis, and A.A. Lacis. *Scattering, absorption, and emission of light by small particles*. Cambridge University Press, Cambridge, 2002.
- [7] M.I. Mishchenko, J.W. Hovenier, and L.D. Travis, editors. *Light scattering by nonspherical particles. Theory, measurements and applications*. Academic Press, San Diego, 2000.
- [8] M.I. Mishchenko. *Electromagnetic scattering by particles and particle groups, an introduction*. Cambridge University Press, Cambridge, UK, 2014.
- [9] A. Doicu, T. Wriedt, and Y.A. Eremin. *Light scattering by systems of particles*. Springer, Berlin, 2006.
- [10] T.A. Nieminen, V.L.Y. Loke, A.B. Stilgoe, N.R. Heckenberg, and H. Rubinsztein-Dunlop. T-matrix method for modelling optical tweezers. *Journal of Modern Optics*, 58, 5-6:528–544, 2011.
- [11] G. Gouesbet and J.A. Lock. List of problems for future research in generalized Lorenz-Mie theories and related topics, review and prospectus; commemorative invited paper, for the 50th anniversary of "Applied Optics". *Applied Optics*, 52, 5:897–916, 2013.
- [12] G. Gouesbet and J.A. Lock. On the electromagnetic scattering of arbitrary shaped beams by arbitrary shaped particles: A review. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:31–49, 2015.
- [13] G. Gouesbet and J.A. Lock. On the description of electromagnetic arbitrary shaped beams: The relationship between beam shape coefficients and plane wave spectra. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:18–30, 2015.
- [14] G. Gouesbet. T-matrix formulation and generalized Lorenz-Mie theories in spherical coordinates. *Optics Communications*, 283, 4:517–521, 2010.

- [15] N. T. Zakharova, G. Videen, and N.G. Khlebtsov. Comprehensive T-matrix reference database: A 2009-2011 update. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 113:1844–1852, 2012.
- [16] G. Gouesbet and G. Gréhan. Generalized Lorenz-Mie theory for assemblies of spheres and aggregates. *Journal of Optics A: Pure and Applied Optics*, 1, 6:706–712, 1999.
- [17] G. Gouesbet and G. Gréhan. Generalized Lorenz-Mie theory for a sphere with an eccentrically located spherical inclusion. *Journal of Modern Optics*, 47,5:821–837, 2000.
- [18] T.A. Nieminen, H. Rubinsztein-Dunlop, and N.R. Heckenberg. Calculation of the T-matrix: general considerations and applications of the point-matching method. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 79:1019–1029, 2003.
- [19] R.H.T. Bates and D.J.N. Wall. Null field approach to scalar diffraction. I. General method. *Philosophical Transactions of the Royal Society of London, Series A*, 287, 1339:45–78, 1977.
- [20] R.H. Hackman and R. Lim. Development and application of the spheroidal coordinate based T matrix solution to elastic wave scattering. *Radio Science*, 29, 4:1035–1049, 1994.
- [21] A. Doicu and T. Wriedt. Formulations of the extended boundary condition method for incident Gaussian beams using multiple-multipole expansions. *Journal of Modern Optics*, 44, 4:785–801, 1997.
- [22] M. Alonso, R. Gutiérrez-Cuevas, and N. Moore. Lorenz-Mie theory scattering of focused light via complex focus fields: an analytic treatment. *Physical Review A*, 97, 5:Paper 053848, 2018.
- [23] A. Berlemont, A. Picart, and G. Gouesbet. Prediction of turbulence fields and particles dispersion using the code DISCO-2,. *Computational techniques for fluid flow - fifth volume in the series on recent advances in numerical methods in fluids*, Ed: C. Taylor, J.A. Johnson, W.R. Smith, Pineridge Press Ltd:281–313, 1986.
- [24] G. Gouesbet, C. Letellier, K.F. Ren, and G. Gréhan. Discussion of two quadrature methods of evaluating beam shape coefficients in generalized Lorenz-Mie theory. *Applied Optics*, 35,9:1537–1542, 1996.
- [25] G. Gouesbet, G. Gréhan, and B. Maheu. Localized interpretation to compute all the coefficients  $g_n^m$  in the generalized Lorenz-Mie theory, using finite seriesgm in the generalized Lorenz-Mie theory. *Journal of the Optical Society of America A*, 7,6:998–1007, 1990.
- [26] K.F. Ren, G. Gouesbet, and G. Gréhan. Integral localized approximation in generalized Lorenz-Mie theory. *Applied Optics*, 37,19:4218–4225, 1998.

- [27] G. Gouesbet, G. Gréhan, and B. Maheu. Expressions to compute the coefficients  $g_n^m$  in the generalized Lorenz-Mie theory, using finite series. *Journal of Optics (Paris)*, 19,1:35–48, 1988.
- [28] G. Gouesbet. Generalized Lorenz-Mie theory and applications. *Particle and Particle Systems Characterization*, 11, 1:22–34, 1994.
- [29] G. Gouesbet and G. Gréhan. Generalized Lorenz-Mie theories, from past to future. *Atomization and Sprays*, 10,3-5:277–333, 2000.
- [30] G. Gouesbet. Generalized Lorenz-Mie theories, the third decade: A perspective. *Journal of Quantitative Spectroscopy and Radiative Transfer, Invited Review Paper*, 110:1223–1238, 2009.
- [31] J.A. Lock and G. Gouesbet. Generalized Lorenz-Mie theory and applications. *Journal of Quantitative Spectroscopy and Radiative Transfer, Invited Review Paper*, 110:800–807, 2009.
- [32] G. Gouesbet. Latest achievements in generalized Lorenz-Mie theories: A commented reference database. *Annalen der Physik*, 526, 11-12:461–489, 2014.
- [33] A.D. Kiselev and D.O. Plutenko. Mie scattering of Laguerre-Gaussian beams: Photonic nanojets and near-field optical vortices. *Physical Review A*, 89, 4:Article Number: 043803, 2014.
- [34] G. Gouesbet, J.J. Wang, and Y.P. Han. Transformations of spherical beam shape coefficients in generalized Lorenz-Mie theories through rotations of coordinate system. IV. Plane waves. *Optics Communications*, 283, 17:3244–3254, 2010.
- [35] F. Onofri, G. Gréhan, and G. Gouesbet. Electromagnetic scattering from a multilayered sphere located in an arbitrary beam. *Applied Optics*, 34,30:7113–7124, 1995.
- [36] F. Onofri, D. Blondel, G. Gréhan, and G. Gouesbet. On the optical diagnosis and sizing of spherical coated and multilayered particles with phase-Doppler anemometry. *Particle and Particle Systems Characterization*, 13, 2:104–111, 1996.
- [37] Z.S. Wu, L.X. Guo, K.F. Ren, G. Gouesbet, and G. Gréhan. Improved algorithms for electromagnetic scattering of plane waves and shaped beams by multilayered spheres. *Applied Optics*, 36,21:5188–5198, 1997.
- [38] J.J. Wang, G. Gouesbet, Y.P. Han, and G. Gréhan. Study of scattering from a sphere with an eccentrically located spherical inclusion by generalized Lorenz-Mie theory: Internal and external field distributions. *Journal of the Optical Society of America A*, 28, 1:24–39, 2011.

- [39] J.J. Wang, G. Gouesbet, G. Gréhan, and S. Saengkaew. Morphology-dependent resonances in an eccentrically layered sphere illuminated by a tightly focused off-axis Gaussian beam. *Journal of the Optical Society of America A*, 28, 9:1849–1859, 2011.
- [40] P. Briard, J.J. Wang, and Y.P. Han. Shaped beam scattering by an aggregate of particles using generalized Lorenz-Mie theory. *Optics Communications*, 365:186–193, 2016.
- [41] J.J. Wang, L. Han, Y.P. Han, G. Gouesbet, X.C. Wu, and Y.C. Wu. Shaped beam scattering from a single lymphocyte cell by generalized Lorenz-Mie theory. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 133:72–80, 2014.
- [42] B. Zhang and X. Li. Electromagnetic scattering by a partially charged multilayered sphere. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 148:228–232, 2014.
- [43] M. Selmke. Mie and potential scattering by a refractive  $1/r$  inhomogeneity : electromagnetic scattering by an infinite Coulomb-like scatterer. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:103–113, 2015.
- [44] H. Tang and J.Z. Lin. Scattering of randomly distributed spherical particles illuminated with a Gaussian beam. *Laser physics letters*, 12, 4:Article Number: 046001, 2015.
- [45] J.A. Lock. Scattering of the evanescent components in the angular spectrum of a tightly focused electromagnetic beam by a spherical particle. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:95–102, 2015.
- [46] Q-C Shang, Z-S Wu, T. Qu, Z-J Li, and L. Bai. Scattering from a multilayered chiral sphere using an iterative method. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 173:72–82, 2016.
- [47] X. Zambrana-Puyalto and N. Bonod. Tailoring the chirality of light emission with spherical Si-based antennas. *Nanoscale*, 8:10441–10452, 2016.
- [48] R.P.M. Höller, M. Dulle, S. Thomä, M. Mayer, A.M. Steiner, S. Förster, A. Fery, C. Kuttner, and M. Chanana. Protein-assisted assembly of modular 3D plasmonic raspberry-like core/satellite nanoclusters: correlation of structure and optical properties. *ACS Nano*, 10:5740–5750, 2016.
- [49] F. Frezza and F. Mangini. Electromagnetic scattering of an inhomogeneous elliptically polarized plane wave by a multilayered sphere. *Journal of Electromagnetic Waves and Applications*, 30, 4:492–504, 2016.
- [50] C. Ding, X. Han, and R. Li. Scattering of a cylindrical vector beam by a sphere. *Optik*, 127:2748–2754, 2016.



- [51] J. Bai, Z. Wu, C. Ge, Q. Shang, and Z. Li. Electromagnetic scattering for multilayered spheres induced by laser sheet beam. *Antennas, propagation and EM theory*, 11th International Symposium on Antennas, propagation and EM theory:287–290, 2016.
- [52] X. Jia. Calculation of auxiliary functions related to Riccati-Bessel functions in Mie scattering. *Journal of Modern Optics*, 63, 21:2348–2355, 2016.
- [53] G. Gréhan and G. Gouesbet. Mie theory calculations: new progress, with emphasis on particle sizing. *Applied Optics*, 18,20:3489–3493, 1979.
- [54] R.J. Wang, B. Deng, Y.L. Qin, H.Q. Wang, and X. Li. Study of scattering characteristics for metallic spheres at terahertz frequencies. *Journal of Electromagnetic Waves and Applications*, 28, 14:1786–1797, 2014.
- [55] K. Ladutenko, U. Pal, and d O. Pena-Rodriguez A. Rivera a. Mie calculation of electromagnetic near-field for a multilayered sphere. *Computer Physics Communications*, 214:225–230, 2017.
- [56] J.A. Lock. Electromagnetic scattering of a plane wave by a radially inhomogeneous sphere in the short wavelength limit. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 202:126–135, 2017.
- [57] J.J. Wang, T. Wriedt, Y.P. Han, L. Mädler, and Y.C. Jian. Internal field distribution of a radially inhomogeneous droplet illuminated by an arbitrary shaped beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 210:19–34, 2018.
- [58] G. Gouesbet and J. Lock. Comments on localized and integral localized approximations in spherical coordinates. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 179:132–136, 2016.
- [59] J.R. Frisvad. Phase function of a spherical particle when scattering an inhomogeneous electromagnetic plane wave. *Journal of the Optical Society of America A*, 35, 4:669–680, 2018.
- [60] X. Zambrana-Puyalto, X. Vidal, P. Wozniak, P. Banzer, and G. Molina-Terriza. Tailoring multipolar Mie scattering with helicity and angular momentum. *ACS Photonics*, 5,7:2936–2944, 2018.
- [61] F.G. Mitri. Pulling and spinning reversal of a subwavelength absorptive sphere in adjustable vector Airy light-sheets. *Applied Physics Letters*, 110:Paper 181112 (5 pp), 2017.
- [62] Z.Y. Chen, Y.P. Han, Z.W. Cui, and X.W. Shi. Scattering of a zero-order Bessel beam by a concentric sphere. *Journal of Optics*, 16, 5:Article number: 055701, 2014.
- [63] Z.Y. Chen, Y.P. Han, Z.W. Cui, and X.W. Shi. Scattering analysis of Bessel beam by a multilayered sphere. *Optics Communications*, 340:5–10, 2015.

- [64] T.C. Preston and J.P. Reid. Angular scattering of light by a homogeneous spherical particle in a zeroth-order Bessel beam and its relationship to plane wave scattering. *Journal of the Optical Society of America A*, 32, 6:1053–1062, 2015.
- [65] Z.J. Li, Z.S. Wu, T. Qu, H.Y. Li, L. Bai, and L. Gong. Light scattering of a non-diffracting zero-order Bessel beam by uniaxial anisotropic bisphere. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:56–65, 2015.
- [66] Y. Wen, Q. Xi, R. Li, S. Qin, and C. Ding. Scattering of a vector Bessel-Gaussian beam by a sphere. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 204:165–178, 2018.
- [67] J.J. Wang, Y.P. Han, J.Y. Chang, and Z.Y. Chen. Light scattering of a Bessel beam by a nucleated biological cell: An eccentric sphere model. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 206:22–30, 2018.
- [68] T. Qu, Z. Wu, Q. Shang, J. Wu, and L. Bai. Interactions of high-order Bessel vortex beam with a multilayered chiral sphere: Scattering and orbital angular momentum spectrum analysis. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 217:363–372, 2018.
- [69] S. Gong, R. Li, Y. Liu, and J. Zhang. Scattering of a vector Bessel vortex beam by a charged sphere. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 215:13–24, 2018.
- [70] T. Qu, Z.-S. Wu, Q.-C. Shang, Z.-J. Li, L. Bai, and L. Gong. Analysis of the radiation force of a Laguerre Gaussian vortex beam exerted on a uniaxial anisotropic sphere. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:103–113, 2015.
- [71] T. Qu, Z-S. Wu, Q-C Shang, and Z-J Li. Light scattering of a Laguerre-Gaussian vortex beam by a chiral sphere. *Journal of the Optical Society of America A*, 33, 4:475–482, 2016.
- [72] A. Al-Jarro, C.G. Biris, and N.C. Panolu. Resonant mixing of optical orbital and spin angular momentum by using chiral silicon nanosphere clusters. *Optics Express*, 24, 7:6945–6958, 2016.
- [73] Z.J. Li, Z.S. Wu, T. Qu, H.Y. Li, L. Bai, and L. Gong. Radiation torque exerted on a uniaxial anisotropic sphere: Effects of various parameters. *Optics and Laser Technology*, 64:269–277, 2014.
- [74] H.C. Yu and W.L. She. Radiation force exerted on a sphere by focused Laguerre-Gaussian beams. *Journal of the Optical Society of America A*, 32, 1:130–142, 2015.

- [75] R. Li, C. Ding, and F.G. Mitri. Optical spin torque induced by vector Bessel (vortex) beams with selective polarizations on a light-absorptive sphere of arbitrary size. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 196:53–68, 2017.
- [76] N. Leartprapun, R. R. Iyer, and S.G. Adie. Depth-resolved measurement of optical radiation pressure forces with optical coherence tomography. *Optics Express*, 26, 3:2410–2426, 2018.
- [77] N. Leartprapun, R.R. Iyer, G.R. Untracht, J.A. Mulligan, and S.G. Adie. Photonic force optical coherence elastography for three-dimensional mechanical microscopy. *Nature Communications*, 9:Article number: 2079, 2018.
- [78] R.X. Li, L.X. Guo, and C.Y. Ding. Debye series analysis of optical forces induced by an axicon-generated Bessel beam. *Journal of Modern Optics*, 62, 6:493–502, 2015.
- [79] L. Guo, C.-Ge Shi, Q. Huang, M. Cheng, and J. Li. Polarization remote sensing of atmospheric coated-spherical aerosol based on optical vortex and parallel acceleration. *Proc SPIE 10792, High-Performance Computing in Geoscience and Remote Sensing VIII*, 1079201:doi: 10.1117/12.2325383, 9 October 2018.
- [80] L. Han, G. Gouesbet, and Y.P. Han. Intrinsic method for the evaluation of beam shape coefficients in spheroidal coordinates for oblique illumination. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 224:312–318, 2019.
- [81] G. Gouesbet. Generalized Lorenz-Mie theories and mechanical effects of laser light, on the occasion of Arthur Ashkin’s receipt of the 2018 Nobel prize in physics for his pioneering work in optical levitation and manipulation: A review. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 225:258–277, 2019.
- [82] G. Gouesbet and J.J. Wang. On the structures of some light scattering theories depending on whether or not the Bromwich formulation may be used, e.g. spherical versus spheroidal coordinates. *Optics Communications*, 285, 21-22:4200–4206, 2012.
- [83] G. Gouesbet, J.J. Wang, and Y.P. Han. Solution to the intrinsic method for the evaluation of beam shape coefficients in spheroidal coordinates. *Optics Communications*, 294:29–35, 2013.
- [84] L. Han, G. Gouesbet, Y.P. Han, G. Gréhan, and J.J. Wang. Intrinsic method for the evaluation of beam shape coefficients in spheroidal coordinates for on-axis standard configuration. *Optics Communications*, 310:125–137, 2014.

- [85] G. Gouesbet, F. Xu, and Y.P. Han. Expanded description of electromagnetic arbitrary shaped beam in spheroidal coordinates for use in light scattering theories: A review. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 112:2249–2267, 2011.
- [86] W.J. Zhao, Y.P. Han, and L. Han. Scattering of an oblique incident focused Gaussian beam by a spheroidal particle. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 145:88–94, 2014.
- [87] X.M. Sun and H.H. Wang. Scattering of on-axis polarized Gaussian light beam by spheroidal water coating aerosol particle. *Chinese Optics Letters*, 12, 1:Article Number : 012901, 2014.
- [88] X.M. Sun, S. Xiao, L.X. Ma, and B.C. Su. Scattering of polarized Gaussian light by a spheroidal particle. *Optics Communications*, 336:24–29, 2015.
- [89] H. Tang and W. Zheng. Modeling of scattering intensity of spheroid particles with a Gaussian beam. *IOP Conf. Series: Earth and Environmental Science*, 61:paper 012133, 2017.
- [90] W. Zheng and H. Tang. Modeling of scattering cross section for mineral aerosol. *Journal of Nanotechnology*, Article ID6513634:7 pages, 2018.
- [91] H. Zhang, Y. Yuan, M. Li, and Z. Huang. Gaussian beam scattering by a particle above a plane surface. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 217:22–28, 2018.
- [92] Y. Yuan, H. Zhang, B. Wu, and Z. Huang. Gaussian beam scattering from a sphere on or near a plane surface. *Conference 2018, Cross Strait Quad-Regional Radio Science and Wireless Technology*, DOI: 10.1109/CSQRWC.2018.8455536:3 pages, 2018.
- [93] F.G. Mitri. Negative optical radiation force and spin torques on sub-wavelength prolate and oblate spheroids in fractional Bessel-Gauss pinchers light-sheets. *Journal of the Optical Society of America A*, 34, 7:1246–1254, 2017.
- [94] F.G. Mitri. Adjustable vector Airy light-sheet single optical tweezers: negative radiation forces on a subwavelength spheroid and spin torque reversal. *The European Physical Journal D*, 72: 21:<https://doi.org/10.1140/epjd/e2017-80632-5>, 2018.
- [95] Z.Y. Chen, Y.P. Han, Z.W. Cui, and X.W. Shi. Scattering of Bessel beam by a conducting spheroidal particle with dielectric coating. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 148:197–202, 2014.
- [96] L. Han, Y.P. Han, Z.W. Cui, and J.J. Wang. Expansion of a zero-order Bessel beam in spheroidal coordinates by generalized Lorenz-Mie theory. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 147:279–287, 2014.

- [97] L. Han, Y.P. Han, J.J. Wang, and Z.W. Cui. Internal and near-surface electromagnetic fields for a dielectric spheroid illuminated by a zero-order Bessel beam. *Journal of the Optical Society of America A*, 31, 9:1946–1955, 2014.
- [98] L. Han. Scattering of a high-order Bessel beam by a spheroidal particle. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 211:129–137, 2018.
- [99] Z. Gong, W. Li, F.G. Mitri, Y. Chai, and Y. Zhao. Arbitrary scattering of an acoustic Bessel beam by a rigid spheroid with large aspect ratio. *Journal of Sound and Vibration*, 383:233–247, 2016.
- [100] M.L. Yang, K.F. Ren, Y.Q. Wu, and X.Q. Sheng. Computation of stress on the surface of a soft homogeneous arbitrarily shaped particle. *Physical Review E*, 89, 4:Article Number: 043310, 2014.
- [101] A.V. Arzola, P. Jakl, L. Chvatal, and P. Zemanek. Rotation, oscillation and hydrodynamic synchronization of optically trapped oblate spheroidal microparticles. *Optics Express*, 22, 13:16207–16221, 2014.
- [102] J-W Liaw, Y-S Chen, and M-K Kuo. Spinning gold nanoparticles driven by circularly polarized light. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 175:46–53, 2016.
- [103] M. Li, S. Yan, B. Yao, Y. Liang, G. Han, and P. Zhang. Optical trapping force and torque on spheroidal Rayleigh particles with arbitrary spatial orientations. *Journal of the Optical Society of America A*, 33, 7:1341–1347, 2016.
- [104] S.H. Simpson, L. Chvatal, and P. Zemanek. Synchronization of colloidal rotors through angular optical binding. *Physical Review A*, 93:Paper 023842 (12 pp), 2016.
- [105] K. Xu, X.-M. Pan, R. Li, and X.-Q. Sheng. Fast and accurate algorithm for repeated optical trapping simulations on arbitrarily shaped particles based on boundary element method. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:76–84, 2017.
- [106] F.G. Mitri. Optical Bessel beam illumination of a subwavelength prolate gold (Au) spheroid coated by a layer of plasmonic material: radiation force, spin and orbital torques. *Journal of Physics Communications*, 1, 1:Paper 015 001 (13 pp), 2017.
- [107] F. G. Mitri. Reverse orbiting and spinning of a Rayleigh dielectric spheroid in a  $J_0$  Bessel optical beam. *Journal of the Optical Society of America B*, 34, 10:2169–2178, 2017.

- [108] K.W. Seo, H.J. Byeon, and S.J. Lee. Measuring the light scattering and orientation of a spheroidal particle using in-line holography. *Optics Letters*, 39, 13:3915–3918, 2014.
- [109] Z. Yi, C. Jun, Y. Hui-Nan, L. Tan, and S. Ming-Xu. Influence of spheroidal particle shape on particle size characterization by multi-wavelength light extinction method. *Results in Physics*, 10:22–27, 2018.
- [110] C. B. Chang, W.-X. Huang, and K. H. Lee. Optical separation of ellipsoidal particles in a uniform flow. *Physics of Fluids*, 26:Paper 062001, 20 pages, 2014.
- [111] W. Lin, L. Bi, D. Liu, and K. Zhang. Use of Debye’s series to determine the optimal edge-effect terms for computing the extinction efficiencies of spheroids. *Optics Express*, 25, 17:20298–20312, 2017.
- [112] O. Brzobohaty, A.V. Arzola, M. Siler, L. Chvatal, P. Jakl, S. Simpson, and P. Zemanek. Complex rotational dynamics of multiple spheroidal particles in a circularly polarized, dual beam trap. *Optics Express*, 23, 6:7273–7287, 2015.
- [113] M.L. Yang, K.F. Ren, T. Petkov, B. Pouligny, J.C. Loudet, and X.Q. Sheng. Computational study of radiation torque on arbitrary shaped particles with MLFMA. *Optics Express*, 23, 18:23365–23379, 2015.
- [114] H. Zhang. Shaped beam scattering by a spheroidal object. *Optics Communications*, 381:135–139, 2016.
- [115] X. Gao and H. Zhang. Scattering of shaped beam by a chiral spheroid. *Optik*, 129:43–49, 2017.
- [116] W. Wang, H. Zhang, and Y. Sun. Semi-analytical solution to arbitrarily shaped beam scattering. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:114–118, 2017.
- [117] W. Wang, Y. Sun, and H. Zhang. Scattering of arbitrarily shaped beam by a chiral object. *Optics Communications*, 385:54–58, 2017.
- [118] Z. Cui, P. S. Yuanfei Hui, W. Zhai, and Y.P. Han. Scattering of polarized non-diffracting Lommel beams by nonspherical homogeneous particles. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 219:238–247, 2018.
- [119] Z. Wu, Y.P. Han, J.J. Wang, and Z. Cui. Generation of Bessel sources in FDTD. *Optics Express*, 26, 22:28727–28737, 2018.
- [120] A. Chen, J.J. Wang, Y.P. Han, Z. Cui, and M. Yu. Implementation of nondiffracting Bessel beam sources in FDTD for scattering by complex particles. *Optics Express*, 26, 20:26766–26775, 2018.

- [121] Y.P. Han, Z.W. Cui, and J.J. Wang. *Light wave propagation and scattering through particles*, chapter 5, pages 81–105. Intech, 2017.
- [122] M.J. Wang, H.Y. Zhang, G.S. Liu, Y.L. Li, and Q.F. Dong. Reflection and transmission of Gaussian beam by a uniaxial anisotropic slab. *Optics Express*, 22, 3:3705–3711, 2014.
- [123] Z. Chen, H. Zhang, X. Wu, and Z. Huang. Transmission of a Gaussian beam through a gyrotropic cylinder. *Journal of the Optical Society of America A*, 31, 9:1931–1935, 2014.
- [124] B. Yan, C.H. Liu, H.Y. Zhang, and Y. Shi. Transmission of Gaussian beam through a coated chiral cylinder. *Optics Communications*, 338:261–264, 2015.
- [125] Z. Huang, F. Xu, B. Wang, M. Li, and H. Zhang. Propagation of Gaussian beam through a uniaxial anisotropic slab. *Optics Communications*, 380:336–341, 2016.
- [126] H. Li, F.Honary, J.J. Wang, J. Liu, Z.S. Wu, and L. Bai. Intensity, phase, and polarization of a vector Bessel vortex beam through multilayered isotropic media. *Applied Optics*, 57, 9:1967–1976, 2018.
- [127] H. Li, J. Liu, L. Bai, and Z. Wu. Deformations of circularly polarized Bessel vortex beam reflected and transmitted by a uniaxial anisotropic slab. *Applied Optics*, 57, 25:7353–7362, 2018.
- [128] H. Li, F. Honary, Z.S. Wu, and L. Bai. Reflection and transmission of Laguerre-Gaussian beams in a dielectric slab. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:35–43, 2017.
- [129] N. Iqbal, P.K. Choudhury, and P.S. Menon. Scattering of waves by chiral medium-coated dielectric elliptical/circular cylinders with PEC sheath helix loading. *Journal of Electromagnetic Waves and Applications*, 30, 11:1504–1518, 2016.
- [130] F.G. Mitri. Scattering cross-section of a cylindrical conducting particle illuminated by electromagnetic plane wave near a conducting quarter-space. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 215:77–83, 2018.
- [131] D. Zhu, B. Wu, H. Zhang, and Z. Huang. Transmission of arbitrary electromagnetic beam through uniaxial anisotropic cylinder. *Conference 2018, Cross Strait Quad-Regional Radio Science and Wireless Technology*, DOI: 10.1109/CSQRWC.2018.84554953:3 pages, 2018.
- [132] F.G. Mitri. Nonparaxial scalar Airy light-sheets and their higher-order spatial derivatives. *Applied Physics Letters*, 110:Paper 091104 (4 pp), 2017.

- [133] F.G. Mitri. Cylindrical particle manipulation and negative spinning using a nonparaxial Hermite-Gaussian light-sheet beam. *Journal of Optics*, 18:Article 105402 (11pp), 2016.
- [134] F.G. Mitri. Radiation force and torque of light-sheets. *Journal of Optics*, 19, 6:Paper 065403 (13 pp), 2017.
- [135] E.S. Kozlova, V.V. Kotlyar, and S.A. Degtyarev. Modeling the resonance focusing of a picosecond laser pulse using a dielectric microcylinder. *Journal of the Optical Society of America B*, 32, 11:2352–2357, 2015.
- [136] F.G. Mitri. Interaction of an acoustical 2D-beam with an elastic cylinder with arbitrary location in a non-viscous fluid. *Ultrasonics*, 62:244–252, 2015.
- [137] F.G. Mitri. Extended optical theorem for scalar monochromatic acoustical beams of arbitrary wavefront in cylindrical coordinates. *Ultrasonics*, 67:129–135, 2016.
- [138] X. Zhang, Z. Song, D. Chen, G. Zhang, and H. Cao. Finite series expansion of a Gaussian beam for the acoustic radiation force calculation of cylindrical particles in water. *Journal of Acoustical Society of America*, 137, 4:1826–1833, 2015.
- [139] X. Zhang, Q. Yun, G. Zhang, and X. Sun. Computation of the acoustic radiation force on a rigid cylinder in off-axis Gaussian beam using the translational addition theorem. *Acta Acustica united with Acustica*, 102, 2:334–340, 2016.
- [140] Z. Gong, W. Li, Y. Chai, Y. Zhao, and F.G. Mitri. T-matrix method for acoustical Bessel beam scattering from a rigid finite cylinder with spheroidal endcaps. *Ocean Engineering*, 129:507–519, 2017.
- [141] F.G. Mitri. Pushing, pulling and electromagnetic radiation force cloaking by a pair of conducting cylindrical particles. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 206:142–150, 2018.
- [142] F.G. Mitri. Active electromagnetic invisibility cloaking and radiation force cancellation. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 207:48–53, 2018.
- [143] D. Gagnon and L.J. Dubé. Lorenz-Mie theory for 2D scattering and resonance calculations. *Journal of Optics*, 17, 10:Article Number: 103501, 2015.
- [144] F.G. Mitri. Generalization of the optical theorem for monochromatic electromagnetic beams of arbitrary wavefront in cylindrical coordinates. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 166:81–92, 2015.



- [145] G. Swirniak and J. Mroczka. Approximate solution for optical measurements of the diameter and refractive index of a small and transparent fiber. *Journal of the Optical Society of America A*, 33, 4:667–676, 2016.
- [146] G. Swirniak and J. Mroczka. Numerical analysis of primary rainbows from a homogeneous cylinder and an optical fiber for incident low-coherent light. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:176–188, 2017.
- [147] G. Swirniak and G. Glomb. A tunable fiber-optic LED illumination system for non-invasive measurements of the characteristics of a transparent fiber. *Modeling aspects in optical metrology VI, Proceedings of the SPIE*, edited by B. Bodermann, K. Frenner and R.M. Silver, 10330:Paper 1033019, 2017.
- [148] Q. Duan, R. Zhong, X. Han, and K.F. Ren. Influence of spatial curvature of a liquid jet on the rainbow positions: ray tracing and experimental study. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:156–163, 2017.
- [149] F. Laurendeau, O. Léon, F. Chedevergne, J.-M. Senoner, and G. Casalis. Particle image velocimetry experiment analysis using large-eddy simulation: Application to plasma actuators. *AIAA Journal*, 55, 11:3767–3780, 2017.
- [150] M. Yu, Y.P. Han, Z. Cui, and A. Chen. Electromagnetic scattering by multiple dielectric particles under the illumination of unpolarized high-order vortex beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:107–113, 2017.
- [151] D. Gagnon, J. Dumont, J.-L. Déziel, and L.J. Dubé. Ab initio investigation of lasing thresholds in photonic molecules. *Journal of the Optical Society of America B*, 31, 8:1867–1873, 2014.
- [152] N. Shiloah. Canonical scattering coefficients upward recursion algorithm for multilayered sphere or long cylinder with large size parameters. *AIP Advances*, 8:Paper 075227, 2018.
- [153] L.A. Ambrosio. Extracting metamaterial properties of negative-index and plasmonic scatterers from the Mie coefficients. *Journal of Microwaves, Optoelectronics and Electromagnetic Applications*, 15, 2:146–156, 2016.
- [154] L.A. Ambrosio. Symmetry relations in the generalized Lorenz-Mie theory for lossless negative refractive index media. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 180:147–153, 2016.
- [155] K.F. Ren, G. Gréhan, and G. Gouesbet. Symmetry relations in generalized Lorenz-Mie theory. *Journal of the Optical Society of America A*, 11,6:1812–1817, 1994.

- [156] N. Wang, S. Wang, and J. Ng. Electromagnetic stress tensor for amorphous metamaterial medium. *Physical Review A*, 97:Paper 033839, 2018.
- [157] D.P. Chrissoulidis and E. Richalot. Wave-amplitude synthesis applied to Gaussian beam scattering by an off-axis sphere. *Journal of the Optical Society of America A*, 34, 4:558–567, 2017.
- [158] H. Yu, H. Sun, J. Shen, and C. Tropea. Measurement of refractive index and size of a spherical drop from Gaussian beam scattering in the primary rainbow region. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 207:83–88, 2018.
- [159] Y. Wu, M. Brunel, X. Wu, J. Wang, J. Chen, D. Lebrun, S. Coëtmelec, and G. Gréhan. Tensor ABCD law for misaligned inline particle holography of inclusions in a host droplet. *Applied Optics*, 56, 5:1526–1535, 2017.
- [160] M. Selmke and F. Cichos. Energy-redistribution signatures in transmission microscopy of Rayleigh and Mie particles. *Journal of the Optical Society of America A*, 31, 11:2370–2384, 2014.
- [161] D.W. Mackowski and M.I. Mishchenko. Direct simulation of multiple scattering by discrete random media illuminated by Gaussian beams. *Physical Review A*, 83:Article number: 013804, 9 pages, 2011.
- [162] K.F. Ren, G. Gréhan, and G. Gouesbet. Laser sheet scattering by spherical particles. *Particle and Particle Systems Characterization*, 10, 3:146–151, 1993.
- [163] G. Gréhan, K.F. Ren, G. Gouesbet, A. Naqwi, and F. Durst. Evaluation of a particle sizing technique based on laser sheets. *Particle and Particle Systems Characterization*, 11, 1:101–106, 1994.
- [164] K.F. Ren, G. Gréhan, and G. Gouesbet. Evaluation of laser sheet beam shape coefficients in generalized Lorenz-Mie theory by use of a localized approximation. *Journal of the Optical Society of America A*, 11,7:2072–2079, 1994.
- [165] K.F. Ren, G. Gréhan, and G. Gouesbet. Electromagnetic field expression of a laser sheet and the order of approximation. *Journal of Optics (Paris)*, 25,4:165–176, 1994.
- [166] F.G. Mitri. Nonparaxial fractional Bessel and Bessel-Gauss auto-focusing light-sheet pincers and their higher-order spatial derivatives. *Journal of Optics*, 19:Paper 055602 (6 pp), 2017.
- [167] F.G. Mitri. Self-bending scalar and vector bottle sheets. *Journal of the Optical Society of America A*, 34, 7:1194–1200, 2017.

- [168] Y.-L. Li, J. Li, M.-J. Wang, and Q.-F. Dong. A solution of scattered field of particle in electromagnetic beam based on beam series expansion. *IEEE Transactions on Antennas and Propagation*, 62, 12:6375–6381, 2014.
- [169] J. Li and X. Feng. Characteristics of scattering for anisotropic particles in photoelectric electromagnetic beam. *Progress in Electromagnetics Research M*, 66:41–52, 2018.
- [170] J. Bai, Z.S. Wu, C.-X. Ge, Z.-L. Li, T. Qu, and Q.-C. Shang. Analysis of lateral binding force exerted on a bi-sphere induced by an elliptic Gaussian beam. *Journal of the Optical Society of America A*, 35, 2:336–345, 2018.
- [171] Y. Wu, C. Crua, H. Li, S. Saengkaew, L. Mädler, X. Wu, and G. Gréhan. Simultaneous measurement of monocomponent droplet temperature/refractive index, size and evaporation rate with phase rainbow refractometry. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 214:146–157, 2018.
- [172] Y. Wu, X. Wu, L. Yao, M. Brunel, S. Coëtmelec, R. Li, D. Lebrun, H. Zhou, G. Gréhan, and K. Cen. Characterizations of transparent particle holography in near-field using Debye series. *Applied Optics*, 55, 3:A60–A70, 2016.
- [173] Y. Wu, M. Brunel, R. Li, L. Lan, W. Ao, J. Chen, X. Wu, and G. Gréhan. Simultaneous amplitude and phase contrast imaging of burning fuel particle and flame with digital inline holography: Model and verification. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 199:26–35, 2017.
- [174] R. Meng, B. Ge, Q. Lu, and X. Yu. Determining the refractive index of particles using glare-point imaging technique. *Review of Scientific Instruments*, 89:Article 043107, 6 pages, 2018.
- [175] S.J. Satinover, J.D. Dove, and M.A. Borden. Single-particle optical sizing of microbubbles. *Ultrasound in Medicine and Biology*, 40, 1:138–147, 2014.
- [176] J. Durnin, Jr. J.J. Miceli, and J.H. Eberly. Diffraction-free beams. *Physical Review Letters*, 58, 15:1499–1501, 1987.
- [177] J. Durnin. Exact solutions for nondiffracting beams. I. The scalar theory. *Journal of the Optical Society of America A*, 4, 4:651–654, 1987.
- [178] S.R. Mishra. A vector wave analysis of a Bessel beam. *Optics Communications*, 85:159–161, 1991.
- [179] Z. Bouchal. Nondiffracting optical beams: physical properties, experiments, and applications. *Czechoslovak Journal of Physics*, 53,7:537–578, 2003.

- [180] H.E. Hernandez-Figueroa, M. Zamboni-Rached, and E. Recami, editors. *Localized waves*. John Wiley and Sons, Inc, 2008.
- [181] E. Recami, M. Zamboni-Rached, and H.E. Hernandez-Figueroa. *Non-diffracting waves*, chapter Non-diffracting waves: An introduction, pages 1–67. Wiley-VCH Verlag GmbH, 2014.
- [182] H.E. Hernandez-Figueroa, E. Recami, and M. Zamboni-Rached, editors. *Nondiffracting waves*. Wiley-VCH Verlag GmbH, 2014.
- [183] G. Gouesbet and J.A. Lock. Consequences of the angular spectrum decomposition of a focused beam including slower than c beam propagation. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 178:142–151, 2016.
- [184] F.G. Mitri, R.X. Li, L.X. Guo, and C.Y. Ding. Resonance scattering of a dielectric sphere illuminated by electromagnetic Bessel non-diffracting (vortex) beams with arbitrary incidence and selective polarization. *Annals of Physics*, 361:120–147, 2015.
- [185] A. Chafiq and A. Belafhal. Radiation pressure cross section exerted on homogeneous dielectric spherical particle by zeroth order Mathieu beams. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 179:170–176, 2016.
- [186] A. Chafiq and A. Belafhal. Scattering of Lommel beams by homogeneous spherical particle in generalized Lorenz-Mie theory. *Optical and Quantum Electronics*, 2/2018, 2018.
- [187] J.J. Wang, T. Wriedt, J.A. Lock, and L. Mädler. General description of circularly symmetric Bessel beams of arbitrary order. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 184:218–232, 2016.
- [188] J.J. Wang, T. Wriedt, J.A. Lock, and Y.C. Jiao. General description of transverse mode Bessel beams and construction of basis Bessel fields. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:8–17, 2017.
- [189] J.J. Wang, T. Wriedt, L.Mädler, Y.P. Han, and P. Hartmann. Multipole expansion of circularly Bessel beams of arbitrary order for scattering calculations. *Optics Communications*, 387:102–109, 2017.
- [190] P. Briard, Y.P. Han, Z. Chen, X. Cai, and J.J. Wang. Scattering of aggregated particles illuminated by a zeroth-order Bessel beam. *Optics Communications*, 391:42–47, 2017.
- [191] J. Chen, J. Ng, P. Wang, and Z. Lin. Analytical partial wave expansion of vector Bessel beam and its application to optical binding. *Optics Letters*, 35, 10:1674–1676, 2010.

- [192] J. Chen, J. Ng, P. Wang, and Z. Lin. Analytical partial wave expansion of vector Bessel beam and its application to optical binding: erratum. *Optics Letters*, 36, 7:1243, 2011.
- [193] L.A. Ambrosio. Circularly symmetric frozen waves: Vector approach for light scattering calculations. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 204:112–119, 2018.
- [194] V.S. de Angelis and L.A. Ambrosio. Microstructured light fields for optical trapping: zero order continuous vector frozen waves in the Rayleigh regime. *Conference paper*, <https://www.sbfoton.org.br/>, *SBFoton conference, Campinas, Brazil, 22-24 October 2018, technical session OS-L-TUE-03 (Lasers 2)*, 2018.
- [195] L.A. Ambrosio and M. Zamboni-Rached. Analytical approach of ordinary frozen waves for optical trapping and micromanipulation. *Applied Optics*, 54, 10:2584–2593, 2015.
- [196] L.A. Ambrosio, M. Zamboni-Rached, and G. Gouesbet. Discrete vector frozen waves in generalized Lorenz-Mie theory: linear, azimuthal and radial polarization. *Applied Optics*, 57, 12:3293–3300, 2018.
- [197] L.A. Ambrosio. Analytical descriptions of finite-energy Bessel beams in the generalized Lorenz-Mie theory. *Conference paper*, <https://www.sbfoton.org.br/>, *SBFoton conference, Campinas, Brazil, 22-24 October 2018, technical session OS-L-TUE-03 (Lasers 2)*, 2018.
- [198] F.G. Mitri and G.T. Silva. Generalization of the extended optical theorem for scalar arbitrary-shape acoustical beams in spherical coordinates. *Physical Review E*, 90:Paper 053204, 2014.
- [199] J.P. Leao-Neto, J. H. Lopes, and G.T. Silva. Scattering of a longitudinal Bessel beam by a sphere embedded in an isotropic elastic fluid. *J. Acoust. Soc. Am.*, 142, 5:2881–2889, 2017.
- [200] C. Jiang, X. Liu, J. Liu, Y. Mao, and P.L. Marston. Acoustic radiation force on a sphere in a progressive and standing zero-order quasi-Bessel-Gauss beam. *Ultrasonics*, 76:1–9, 2017.
- [201] F.G. Mitri. Acoustics of finite asymmetric exotic beams: Examples of Airy and fractional Bessel beams. *J. Appl. Phys.*, 122:Article 224903, 2017.
- [202] D.B. Ruffner and D.G. Grier. Universal, strong and long-ranged trapping by optical conveyors. *Optics Express*, 22, 22:26834–26843, 2014.
- [203] S.K. Song, N. Wang, W.L. Lu, and Z.F. Lin. Optical force on a large sphere illuminated by Bessel beams: comparisons between ray optics method and generalized Lorenz-Mie theory. *Journal of the Optical Society of America A*, 31, 10:2192–2197, 2014.

- [204] T.C. Preston, B.J. Mason, J.P. Reid, D. Luckhaus, and R. Signorell. Size-dependent position of a single aerosol droplet in a Bessel beam trap. *Journal of Optics*, 16, 2:Article Number: 025702, 2014.
- [205] N. Wang, W.L. Lu, J. Ng, and Z.F. Lin. Optimized optical "tractor beam" for core-shell nanoparticles. *Optics Letters*, 39, 8:2399–2402, 2014.
- [206] L.A. Ambrosio and M.D. Ferreira. Time-average forces over Rayleigh particles by superposition of equal-frequency arbitrary-order Bessel beams. *Journal of the Optical Society of America B*, 32, 5:B67–B74, 2015.
- [207] L.A. Ambrosio and M. Zamboni-Rached. Optical forces experienced by arbitrary-sized spherical scatterers from superpositions of equal-frequency Bessel beams. *Journal of the Optical Society of America B*, 32, 5:B37–B46, 2015.
- [208] F.G. Mitri, R.X. Li, R.P. Yang, L.X. Guo, and C.Y. Ding. Optical pulling force on a magneto-dielectric Rayleigh sphere in Bessel tractor polarized beams. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 184:360–381, 2016.
- [209] R. Yang, R. Li, S. Qin, C. Ding, and F.G. Mitri. Direction reversal of the optical spin torque on a Rayleigh absorptive sphere in vector Bessel polarized beams. *Journal of Optics*, 19:Paper 025602, 17pp, 2016.
- [210] F.G. Mitri, R.X. Li, L.X. Guo, and C.Y. Ding. Optical tractor Bessel polarized beams. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 187:97–115, 2017.
- [211] R. Li, R. Yang, C. Ding, and F.G. Mitri. Optical torque on a magneto-dielectric-Rayleigh absorptive sphere by a vector Bessel (vortex) beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 191:96–115, 2017.
- [212] W. Lu, H. Chen, S. Liu, and Z. Lin. Rigorous full-wave calculation of optical forces on dielectric and metallic microparticles immersed in a vector Airy beam. *Optics Express*, 25, 19:23238–23253, 2017.
- [213] H. Chen, W. Lu, X. Yu, C. Xue, S. Liu, and Z. Lin. Optical torque on small chiral particles in generic optical fields. *Optics Express*, 25, 26:32867–32878, 2017.
- [214] C. McKell and K.D. Bonin. Optical corral using a standing-wave Bessel beam. *Journal of the optical Society of America B*, 35, 8:1910–1920, 2018.
- [215] J. Bai, Z.S. Wu, C.X. Ge, Q.C. Shang, Z.J. Li, and L. Gong. Analysis of lateral binding force exerted on multilayered spheres induced by high-order Bessel beams with arbitrary polarization angles. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 221:183–193, 2018.

- [216] J. Bai, Z.S. Wu, C.X.Ge, Z.J. Li, T. Qu, and Q.C. Shang. Analytical description of lateral binding force exerted on bi-sphere induced by high-order Bessel beams. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 214:71–81, 2018.
- [217] R. Li, P. Li, J. Zhang, C. Ding, and Z. Cui. Optical Bessel tractor polarized beams on a charged sphere of arbitrary size. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 219:186–198, 2018.
- [218] H.J. Chen, S.Y. Liu, J. Zi, and Z.F. Lin. Fano resonance-induced negative optical scattering force on plasmonic nanoparticles. *ACS Nano*, 9, 2:1926–1935, 2015.
- [219] I. Rondon-Ojeda and F. Soto-Eguibar. Generalized optical theorem for propagation invariant beams. *Optik*, 137:17–24, 2017.
- [220] S. Qin, R. Li, R. Yang, and C. Ding. Debye series analysis of internal and near-surface fields for a homogeneous sphere illuminated by an axicon-generated vector Bessel beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:26–34, 2017.
- [221] Z.W. Cui, Y.P. Han, Z.Y. Chen, and L. Han. Scattering of Bessel beam by arbitrarily shaped composite particles with core-shell structure. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 144:108–116, 2014.
- [222] Z.W. Cui, Y.P. Han, and M.P. Yu. Numerical investigation on scattering of an arbitrarily incident Bessel beam by fractal soot aggregates. *Chinese Physics Letters*, 31, 6:Article Number: 064206, 2014.
- [223] V.M. Sundaram and S.B. Wen. Nanoscale high-intensity light focusing with pure dielectric nonspherical scatterer. *Optics Letters*, 39, 3:582–585, 2014.
- [224] M. Yang, K.F. Ren, M. Gou, and X. Sheng. Computation of radiation pressure force on arbitrarily shaped homogeneous particles by multilevel fast multipole algorithm. *Optics Letters*, 38, 11:1784–1786, 2013.
- [225] M. Yang, Y. Wu, K.F. Ren, and X. Sheng. Computation of radiation pressure force exerted on arbitrary shaped homogeneous particles by high-order Bessel vortex beams using MLFMA. *Optics Express*, 24, 24:27979–27992, 2016.
- [226] Z. Wu, J. Wang, Y. Han, L. Zhu, Z. Cui, and W. Zhao. Computation of Bessel beams in the FDTD. *12th International Symposium on Antennas, Propagation and EM theory*, doi: 10.1109/ISAPE.2018.8634306, 3-6 December 2018.
- [227] G. Gouesbet and J.A. Lock. A darkness theorem for the beam shape coefficients and its relationship to higher-order non vortex Bessel beams. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 201:229–235, 2017.

- [228] G. Gouesbet. Poynting theorem in terms of beam shape coefficients and applications to axisymmetric, dark and non-dark, vortex and non-vortex beams. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 201:184–196, 2017.
- [229] L.A. Ambrosio and G. Gouesbet. On the validity of the use of a localized approximation for helical beams. II. Numerical aspects. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 215:41–50, 2018.
- [230] F.G. Mitri. Superposition of nonparaxial vectorial complex-source spherically focused beams: Axial Poynting singularity and reverse propagation. *Physical Review A*, 94:Paper 023801 (10 pp), 2016.
- [231] F.G. Mitri. Vector spherical quasi-Gaussian vortex beams. *Physical Review E*, 89, 2:Article number: 023205, 2014.
- [232] D.J. Zhang and Y.J. Yang. Radiation forces on Rayleigh particles using a focused anomalous vortex beam under paraxial approximation. *Optics Communications*, 336:202–206, 2015.
- [233] L. Guo, Q. Huang, M. Cheng, J. Li, and X. Yan. Remote sensing for aerosol particles in marine atmosphere using scattering of optical vortex. *Proceedings of SPIE, High-performance in geoscience and remote sensing VII*, edited by Bormin Huang, Sebastian Lopez, Zhensen Wu, Jose M. Nascimento, Jun Li, Valeriy V. Strotov, Vol 10430:Article 104300M, 8 pages, 2017.
- [234] M. Yu, Y. Han, Z. Cui, and H. Sun. Scattering of a Laguerre-Gaussian beam by complicated shaped biological cells. *Journal of the Optical Society of America A*, 35, 9:1504–1510, 2018.
- [235] G. Gouesbet, J.A. Lock, and G. Gréhan. Generalized Lorenz-Mie theories and description of electromagnetic arbitrary shaped beams: localized approximations and localized beam models, a review. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 112:1–27, 2011.
- [236] J.J. Wang and G. Gouesbet. Note on the use of localized beam models for light scattering theories in spherical coordinates. *Applied Optics*, 51, 17:3832–3836, 2012.
- [237] G. Gouesbet. Second modified localized approximation for use in generalized Lorenz-Mie theories and other theories revisited. *Journal of the Optical Society of America A*, 30, 4:560–564, 2013.
- [238] X. Jia, J. Shen, and H. Yu. Calculation of generalized Lorenz-Mie theory based on the localized beam models. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:44–54, 2017.



- [239] J.A. Lock and G. Gouesbet. Rigorous justification of the localized approximation to the beam shape coefficients in generalized Lorenz-Mie. I. On-axis beams. *Journal of the Optical Society of America A*, 11,9:2503–2515, 1994.
- [240] G. Gouesbet, J.A. Lock, and G. Gréhan. Partial wave representations of laser beams for use in light scattering calculations. *Applied Optics*, 34,12:2133–2143, 1995.
- [241] G. Gouesbet. Higher-order descriptions of Gaussian beams. *Journal of Optics (Paris)*, 27,1:35–50, 1996.
- [242] K.F. Ren, G. Gréhan, and G. Gouesbet. Prediction of reverse radiation pressure by generalized Lorenz-Mie theory. *Applied Optics*, 35,15:2702–2710, 1996.
- [243] J.A. Lock. Beam shape coefficients of the most general focused Gaussian laser beam for light scattering applications. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 126:16–24, 2013.
- [244] G. Gouesbet, G. Gréhan, and K.F. Ren. Rigorous justification of the cylindrical localized approximation to speed up computations in the generalized Lorenz-Mie theory for cylinders. *Journal of the Optical Society of America A*, 15,2:511–523, 1998.
- [245] J. Shen, X. Jia, and H. Yu. Compact formulation of the beam shape coefficients for elliptical Gaussian beam based on localized approximation. *Journal of the Optical Society of America A*, 33, 11:2256–2263, 2016.
- [246] W. Wang and J. Shen. Beam shape coefficients calculation for an elliptical Gaussian beam with 1-dimensional quadrature and localized approximation methods. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 212:139–148, 2018.
- [247] J. Shen, X. Liu, W. Wang, and H. Yu. Calculation of light scattering of an elliptical Gaussian beam by a spherical particle. *Journal of the Optical Society of America A*, 35, 8:1288–1298, 2018.
- [248] J. Qiu and J. Shen. Beam shape coefficient calculation for a Gaussian beam: localized approximation, quadrature and angular spectrum decomposition methods. *Applied Optics*, 57, 2:302–313, 2018.
- [249] G. Gouesbet and J.A. Lock. Rigorous justification of the localized approximation to the beam shape coefficients in generalized Lorenz-Mie theory. II. Off-axis beams. *Journal of the Optical Society of America A*, 11,9:2516–2525, 1994.
- [250] G. Gouesbet. Validity of the localized approximation for arbitrary shaped beams in generalized Lorenz-Mie theory for spheres. *Journal of the Optical Society of America A*, 16,7:1641–1650, 1999.

- [251] G. Gouesbet. On the validity of localized approximations for Bessel beams: all N-Bessel beams are identically equal to zero. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 176:82–86, 2016.
- [252] L.A. Ambrosio and H.E. Hernandez-Figueroa. Integral localized approximation description of ordinary Bessel beams and application to optical trapping forces. *Biomedical Optics Express*, 2, 7:1893–1906, 2011.
- [253] L.A. Ambrosio and H.E. Hernandez-Figueroa. Integral localized approximation description of  $\nu$ -th order Bessel beams in the Generalized Lorenz-Mie theory and applications to optical trapping. In *PIERS Proceedings, Marrakesh, Morocco, March 20-23, pp 294-298*, 2011.
- [254] G. Gouesbet, J.A. Lock, L.A. Ambrosio, and J.J. Wang. On the validity of localized approximation for an on-axis zeroth-order Bessel beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:18–25, 2017.
- [255] L.A. Ambrosio, J.J. Wang, and G. Gouesbet. On the validity of the integral localized approximation for Bessel beams and associated radiation pressure forces. *Applied Optics*, 56, 19:5377–5387, 2017.
- [256] L.A. Ambrosio, L.F.M. Votto, G. Gouesbet, and J.J. Wang. Assessing the validity of the localized approximation for discrete superposition of Bessel beams. *Journal of the Optical Society of America B*, 35, 11:2690–2698, 2018.
- [257] A. Chafiq, L.A. Ambrosio, G. Gouesbet, and A. Belafhal. On the validity of the integral localized approximation for on-axis zeroth-order Mathieu beams. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 204:27–34, 2018.
- [258] G. Gouesbet and L.A. Ambrosio. On the validity of the use of a localized approximation for helical beams. I. Formal aspects. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 208:12–18, 2018.
- [259] L.A. Ambrosio and G. Gouesbet. On localized approximations for Laguerre-Gauss beams focused by a lens. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 218:100–114, 2018.
- [260] G. Gouesbet and G. Gréhan. Generic formulation of a generalized Lorenz-Mie theory for a particle illuminated by laser pulses. *Particle and Particle Systems Characterization*, 17,5-6:213–224, 2000.
- [261] L. Méès, G. Gréhan, and G. Gouesbet. Time-resolved scattering diagrams for a sphere illuminated by plane wave and focused short pulses. *Optics Communications*, 194,1-3:59–65, 2001.
- [262] L. Méès, G. Gouesbet, and G. Gréhan. Scattering of laser pulses (plane wave and focused Gaussian beams) by spheres. *Applied Optics*, 40,15:2546–2550, 2001.

- [263] L. Méès, G. Gouesbet, and G. Gréhan. Interaction between femtosecond pulses and a spherical microcavity: Internal fields. *Optics Communications*, 199, 1-4:33–38, 2001.
- [264] L. Méès, J.P. Wolf, G. Gouesbet, and G. Gréhan. Two-photon absorption and fluorescence in a spherical micro-cavity illuminated by using two laser pulses: Numerical simulations. *Optics Communications*, 208, 4-6:371–375, 2002.
- [265] L. Méès, G. Gouesbet, and G. Gréhan. Numerical predictions of microcavity internal fields created by femtosecond pulses, with emphasis on whispering gallery modes. *Journal of Optics A: Pure and Applied Optics*, 4:S150–S153, 2002.
- [266] Y.P. Han, L. Méès, K.F. Ren, G. Gréhan, Z.S. Wu, and G. Gouesbet. Far scattered field from a spheroid under a femtosecond pulsed illumination in a generalized Lorenz-Mie theory framework. *Optics Communications*, 231, 1-6:71–77, 2004.
- [267] L. Méès, G. Gouesbet, and G. Gréhan. Transient internal and scattered fields from a multi-layered sphere illuminated by a pulsed laser. *Optics Communications*, 282, 21:4189–4193, 2009.
- [268] H.Y. Li, Z.S. Wu, L. Bai, and Z.J. Li. Scattering and propagation of terahertz pulses in random soot aggregate systems. *Chinese Physics B*, 23, 5:Article Number: 054201, 2014.
- [269] H.Y. Li, Z.S. Wu, and L. Bai. Scattering and propagation of UV pulses in soot aerosols. *Optik*, 125, 16:4229–4232, 2014.
- [270] N. du Preez-Wilkinson, A.B. Stilgoe, T. Alzaidi, H. Rubinsztein-Dunlop, and T.A. Nieminen. Forces due to pulsed beams in optical tweezers: linear effects. *Optics Express*, 23, 6:7190–7208, 2015.
- [271] A.M. Shakhov, A.A. Astafiev, D.O. Plutenko, O.M. Sarkisov, A.I. Shushin, and V.A. Nadochenko. Femtosecond optical trap-assisted nanopatterning through microspheres by a single Ti-sapphire oscillator. *Journal of physical chemistry*, 119, 22:12562–12571, 2015.
- [272] A. Devi and A.K. De. Generalized Lorenz-Mie theory of optical Kerr effect in femtosecond laser trapping of dielectric nanoparticles. *The International Conference on Fiber Optics and Photonics, OSA2016*, page 3 pages, 2016.
- [273] A. Devi and A.K. De. Theoretical estimation of nonlinear optical force on dielectric spherical particles of arbitrary size under femtosecond pulsed excitation. *Physical Review A*, 96:Article 023856, 9 pages, 2017.

- [274] C. Zensen, N. Villadsen, F. Winterer, and S.R. Kelding. Pushing nanoparticles with light - A femtonewton resolved measurement of optical scattering forces. *APL Photonics*, 1, 026102:1–8, 2016.
- [275] J. A. Lock and P. Laven. Co-polarized and cross-polarized scattering of an off-axis focused Gaussian beam by a spherical particle. 1. Exact GLMT formalism. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 221:260–272, 2018.
- [276] J. A. Lock and P. Laven. Co-polarized and cross-polarized scattering of an off-axis focused Gaussian beam by a spherical particle. 2. Sum over azimuthal modes. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 221:273–285, 2018.
- [277] P. Laven and J. A. Lock. Co-polarized and cross-polarized scattering of an off-axis focused Gaussian beam by a spherical particle. 3. Diffraction, the Debye series. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 221:286–299, 2018.
- [278] I. Minin and O. Minin. *Diffraction optics and nanophotonics*, chapter 4: Photonic jets formation by nonspherical axially and spatially asymmetric 3D dielectric particles, pages 31–54. Springer-Briefs in Physics, 2016.
- [279] G. Swirniak. Optical modeling of light scattering for refractive-index detection of liquids in a microcapillary with low-coherence rainbow diffractometry. *Proceedings of SPIE, Optical Modeling and Performance Predictions IX*, edited by Mark A. Kahan, Marie B. Levine-West, doi: 10.1117/12.2273009, Vol. 10374:Article 103740K, 9 pages, 2017.
- [280] J.A. Lock and P. Laven. *Light scattering reviews, Vol 11*, chapter :The Debye series and its use in time-domain scattering, pages 219–297. Springer-Verlag, Berlin, 2016.
- [281] G. Gouesbet and J.A. Lock. Quantum arbitrary shaped beams revisited. *Optics Communications*, 273:296–305, 2007.
- [282] G. Gouesbet. On the optical theorem and non-plane-wave scattering in quantum mechanics. *Journal of Mathematical Physics*, 50:Article number : 112302, 5 pages, 2009.
- [283] J.-L. Thomas, R. Marchiano, and D. Baresch. Acoustical and optical radiation pressures and the development of single beam acoustical tweezers. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:55–65, 2017.
- [284] J.-L. Thomas, D. Baresch, and R. Marchiano. Single-beam acoustical tweezers. *Complex light and optical forces XI, Proceedings of SPIE*, 10120:Paper 101200Y (9 pp), 2017.

- [285] G. Gouesbet. Asymptotic quantum inelastic generalized Lorenz-Mie theory. *Optics Communications*, 278:215–220, 2007.
- [286] G.T. Silva, A.L. Baggio, J.H. Lopes, and F.G. Mitri. Computing the acoustic radiation force exerted on a sphere using the translational addition theorem. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency control*, 62, 3:576–583, 2015.
- [287] Y.-Y. Wang, J. Yao, D.-J. Wu, and X.-J. Liu. Modulation of acoustic radiation forces on three-layered nucleate cells in a focused Gaussian beam. *EPL*, 124:Article 24004, 6 pages, 2018.
- [288] L. Han, Y.P. Han, G. Gouesbet, J.J. Wang, and G. Gréhan. Photonic jet generated by spheroidal particle with Gaussian-beam illumination. *Journal of the Optical Society of America B*, 31, 7:1476–1483, 2014.
- [289] L. Han, Y.P. Han, J.J. Wang, G. Gouesbet, and G. Gréhan. Controllable and enhanced photonic jet generated by fiber combined with spheroid. *Optics Letters*, 39, 6:1585–1588, 2014.
- [290] A.A.A.R. Neves. Photonic nanojets in optical tweezers. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:122–132, 2015.
- [291] X.R. Mao, Y. Yang, H.T. Dai, D. Luo, B.L. Yao, and S.H. Yan. Tunable photonic nanojet formed by generalized Luneberg lens. *Optics Express*, 23, 20:26426–26433, 2015.
- [292] C.-Y. Liu and F.-C. Lin. Geometric effect on photonic nanojet generated by dielectric microcylinders with non-cylindrical cross-sections. *Optics Communications*, 380:287–296, 2016.
- [293] S.A. Degtyarev, A.P. Porfirev, and S.N. Khonina. Photonic nanohelix generated by a binary spiral axicon. *Applied Optics*, 55, 12:B44–B48, 2016.
- [294] A. Darafsheh and D. Bollinger. Systematic study of the characteristics of the photonic nanojets formed by dielectric microcylinders. *Optics Communications*, 402:270–275, 2017.
- [295] H. Xing, W. Zhou, and Y. Wu. Side lobes controlled photonic nanojet with a horizontal graded-index microcylinder. *Optics Letters*, 43, 17:4292–4295, 2018.
- [296] D.A. Kozlov, E.S. Kozlova, and V.V. Kotlyar. Analysis of dielectric circular cylinder light spot narrowing by whispering gallery modes and influence of material absorption. *Proc. SPIE 10774, Optical Technologies in Telecommunications*, doi: 10.1117/12.23117848:7 pages, 2017, 107740U (6 june 2018).

- [297] F.G. Mitri. Ultrasonic superlensing jets and acoustic-forks sheets. *Physics Letters A*, 381, 19:1648–1654, 2017.
- [298] M.P.L. Sentis, F.R.A. Onofri, and F. Lamadie. Photonic jet reconstruction for particle refractive index measurement by digital in-line holography. *Optics Express*, 25, 2:867–873, 2017.
- [299] C. Nayak, S. Mukharjee, and A. Saha. Theoretical investigation of tuning the photonic jet through varying the salt concentration in propagating medium using FDTD method. *Optik*, 127:7258–7261, 2016.
- [300] H.S. Patel, P.K. Kushwaha, and M.K. Swami. Generation of highly confined photonic nanojet using crescent-shape refractive index profile in microsphere. *Optics Communication*, 415:140–145, 2018.
- [301] B.S. Luk’Yanchuk, R. Paniagua-Dominguez, I. Minin, O. Minin, and Z. Wang. Refractive index less than two: photonic nanojets yesterday, today and tomorrow [invited]. *Optical Materials Express*, 7, 6:1820–1847, 2017.
- [302] N. Wang, X. Li, J. Chen, Z. Lin, and J. Ng. Gradient and scattering forces of anti-reflection-coated spheres in an aplanatic beam. *Scientific Reports*, Article number: 17423, 2018.
- [303] H. Zheng, X. Yu, W. Lu, J. Ng, and Z. Lin. Decomposition of optical force into gradient and scattering parts. *Computer Physics Communications*, <https://doi.org/10.1016/j.cpc.2018.11.002> pages, 2018.
- [304] D.B. Phillips, M.J. Padgett, S. Hanna, Y.L.D Ho, D.M. Carberry, M.J. Miles, and S.H. Simpson. Shape-induced force fields in optical trapping. *Nature photonics*, 8, 5:400–405, 2014.
- [305] D. Tapp, J.M. Taylor, A.S. Lubansky, C.D. Bain, and B. Chakrabarti. Theoretical analysis for the optical deformation of emulsion droplets. *Optics Express*, 22, 4:4523–4538, 2014.
- [306] X.M. Pan, M.J. Gou, and X.Q. Sheng. Prediction of radiation pressure force exerted on moving particles by the two-level skeletonization. *Optics Express*, 22, 8:10032–10045, 2014.
- [307] Z.P. Li, S.P. Zhang, L.M. Tong, P.J. Wang, B. Dong, and H.X. Xu. Ultrasensitive size-selection of plasmonic nanoparticles by Fano interference optical force. *ACS Nano*, 8, 1:701–708, 2014.
- [308] A. Gloppe, P. Verlot, E. Dupont-Ferrier, A. Siria, P. Poncharal, G. Bachelier, P. Vincent, and O. Arcizet. Bidimensional nano-optomechanics and topological backaction in a non-conservative radiation force field. *Nature nanotechnology*, 9:920–926, 2014.

- [309] J. Burgin, S. Si, M.H. Delville, and J.P. Delville. Enhancing optofluidic actuation of micro-objects by tagging with plasmonic particles. *Optics Express*, 22, 9:10139–10150, 2014.
- [310] Y.Y. Cao, L.X. Chen, W.Q. Ding, F.K. Sun, and T.T. Zhu. Optical collection of multiple spheres in single tightly focused beams. *Optics Communications*, 311:332–337, 2014.
- [311] N. Wang, Z. Lin, and J. Ng. Laser induced surface stress on water droplets. *Optics Express*, 22, 20:23770–23775, 2014.
- [312] M. Siler and P. Zemanek. Optical trapping in secondary maxima of focused laser beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:114–121, 2015.
- [313] R.S. Dutra, N.B. Viana, P.A.M. Neto, and H.M. Nussenzveig. Absolute calibration of forces in optical tweezers. *Physical Review A*, 90, 1:Article Number: 013825, 2014.
- [314] A. Ashkin. *Optical trapping and manipulations of neutral particles using lasers: A reprint volume with commentaries*. World Scientific, Singapore, 2006.
- [315] L.W. Davis. Theory of electromagnetic beams. *Physical Review*, 19, 3:1177–1179, 1979.
- [316] G. Gouesbet, B. Maheu, and G. Gréhan. Scattering of a Gaussian beam by a sphere using a Bromwich formulation: case of an arbitrary location. *Journal of Particle and Particle Characterization*, 5, 1:1–8, 1988.
- [317] B. Maheu, G. Gouesbet, and G. Gréhan. A concise presentation of the generalized Lorenz-Mie theory for arbitrary location of the scatterer in an arbitrary incident profile. *Journal of Optics (Paris)*, 19,2:59–67, 1988.
- [318] J.A. Lock. Calculation of the radiation trap force for laser tweezers by use of generalized Lorenz-Mie theory. I. Localized model description of an on-axis tightly focused laser beam with spherical aberration. *Applied Optics*, 43, 12:2532–2544, 2004.
- [319] J.A. Lock. Calculation of the radiation trapping force for laser tweezers by use of generalized Lorenz-Mie theory. II. On-axis trapping force. *Applied Optics*, 43,12:2545–2554, 2004.
- [320] J.A. Lock, S.Y. Wrbanek, and K.E. Weiland. Scattering of a tightly focused beam by an optically trapped particle. *Applied Optics*, 45,15:3634–3645, 2006.
- [321] R.B. Bai and Z.G. Suo. Optomechanics of soft materials. *Journal of applied mechanics. Transactions of the ASME. Special issue.*, 82, 7:Article number: 071011, 2015.

- [322] V.V. Datsyuk and O.R. Pavlyniuk. Maxwell stress on a small dielectric sphere in a dielectric. *Physical Review A*, 91, 2:Article Number: 023826, 2015.
- [323] G. Ranjit, D.P. Atherton, J.H. Stutz, M. Cunningham, and A.A. Geraci. Attonewton force detection using microspheres in a dual-beam optical trap in high vacuum. *Physical Review A*, 91, 5:Article Number: 051805, 2015.
- [324] L. Chvatal, O. Brzobohaty, and P. Zemanek. Binding of a pair of Au nanoparticles in a wide Gaussian standing wave. *Optical Review*, 22, 1:157–161, 2015.
- [325] O. Brzobohaty, M. Siler, J. Trojek, L. Chvatal, V. Karasek, and P. Zemanek. Non-spherical gold nanoparticles trapped in optical tweezers: shape matters. *Optics Express*, 23, 7:8179–8189, 2015.
- [326] D. De Coster, H. Ottevaere, M. Vervaeke, J. Van Erps, M. Callewaert, P. Wuytens, S.H. Simpson, S. Hanna, W. De Maische, and H. Thienpont. Mass-manufacturable polymer microfluidic device for dual fiber optical trapping. *Optics Express*, 23, 24:30991–31009, 2015.
- [327] W. Lee, H. Kim, and C.H. Oh. Study on particle size dependence of axial trapping efficiency. *Applied Optics*, 54, 4:901–907, 2015.
- [328] P. Zemanek, M. Siler, O. Brzobohaty, P. Jakl, and R. Filip. Noise-to-signal transition of a Brownian particle in the cubic potential: II. optical trapping geometry. *Journal of Optics*, 18, Paper 065402:10pp, 2016.
- [329] Z. Li, H. Hu, Z. Fu, Q. Zhu, and Y. Shen. Theoretical research and comparison of forces in optical tweezers based on ray optics method and T-matrix method. *Proc SPIE*, 10155:10 pages, 2016.
- [330] A. Giroto, N. Danné, A. Würger, T. Bickel, K.F. Ren, J.C. Loudet, and B. Pouligny. Motion of optically heated spheres at the water-air interface. *Langmuir*, 32, 11:2687–2697, 2016.
- [331] Z. Zhou, Y. Zhang, and L. Zhu. Theoretical and experimental studies on optical trapping using radially polarized beams. *Optical and Quantum Electronics*, 48: 404, 2016.
- [332] D.W. Pilat, B. Pouligny, A. Best, T.A. Nick, R. Berger, and H.J. Butt. Surface forces between colloidal particles at high hydrostatic pressure. *Physical Review E*, 93:Paper 022608 (8 pp), 2016.
- [333] H. Chen, C. Liang, S. Liu, and Z. Lin. Chirality sorting using two-wave-interference-induced lateral optical force. *Physical Review A*, 93:Paper 053833 (5 pp), 2016.
- [334] X. Yu, Q. Ye, H. Chen, S. Liu, and Z. Lin. Simple algorithm for partial wave expansion of plasmonic and evanescent fields. *Optics Express*, 25, 4:4201–4215, 2017.



- [335] J. Herranen, J. Markkanen, and K. Muinonen. Dynamics of small particles in electromagnetic radiation fields: a numerical solution. *Radio Science*, 25, 8:1016–1029, 2017.
- [336] J. Gargiulo, I.L. Violi, S. Cerrota, L. Chvatal, E. Cortés, E.M. Perassi, F. Diaz, P. Zemanek, and D. Stefani. Accuracy and mechanistic details of optical printing of single Au and Ag nanoparticles. *ACS Nano*, 11:9678–9688, 2017.
- [337] C.-X. Ge, Z.-S. Wu, J. Bai, and L. Gong. Effect of nanoscale roughness on optical trapping properties of surface plasmon polaritons exerted on nanoparticle. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 219:339–349, 2018.
- [338] A. Rafferty and T.C. Preston. Measuring the size and complex refractive index of an aqueous aerosol particle using electromagnetic heating and cavity-enhanced Raman scattering. *Phys. Chem. Chem. Phys.*, 20:17038–17047, 2018.
- [339] T.A. Nieminen, N. du Preez-Wilkinson, A.B. Stilgoe, V.L.Y. Loke, A.A.M. Bui, and H. Rubinsztein-Dunlop. Optical tweezers: Theory and modelling. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 146:59–80, 2014.
- [340] C. Ciminelli, D. Conteduca, F. Dell’Olio, G. Brunetti, T. F. Krauss, and M.N. Armenise. Photonic, plasmonic and hybrid nanotweezers for single particle nanoparticle trapping and manipulation. *Icton 2017*, 2017.
- [341] A. Bain, A. Rafferty, and T.C. Preston. Determining the size and refractive index of single aerosol particles using angular light scattering and Mie resonances. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 221:61–70, 2018.
- [342] J.-C. Loudet, B.M. Mihiretie, and B. Pouligny. Optically driven oscillations of ellipsoidal particles. Part II: Ray-optics calculations. *The European Physical Journal E*, 37, 125:15 pages, 2014.
- [343] M. Haghshenas-Jaryani, B. Black, S. Ghaffari, J. Drake, A. Bowling, and S. Mohanty. Dynamics of microscopic objects in optical tweezers: experimental determination of underdamped regime and numerical simulation using multiscale analysis. *Nonlinear Dynamics*, 76, 2:1013–1030, 2014.
- [344] U. Delabre, K. Feld, E. Crespo, G. Whyte, C. Sykes, U. Seifert, and J. Guck. Deformation of phospholipid vesicles in an optical stretcher. *Soft Matter*, 11, 30:6075–6088, 2015.
- [345] X.M. Pan, K.J. Xu, M.L. Yang, and X.Q. Sheng. Prediction of metallic nano-optical trapping forces by finite element-boundary integral method. *Optics Express*, 23, 5:6130–6144, 2015.

- [346] M.T.H. Reid and S.G. Johnson. Efficient computation of power, force, and torque in BEM scattering calculations. *IEEE Transactions on antennas and propagation*, 63, 8:3588–3598, 2015.
- [347] A. Kalume, L.A. Beresnev, J. Santarpia, and Y.-L. Pan. Detection and characterization of chemical aerosol using laser-trapping single-particle Raman spectroscopy. *Applied Optics*, 56, 23:6577–6582, 2017.
- [348] R. Gutiérrez-Cuevas, N. J. Moore, and M. A. Alonso. Lorenz-Mie scattering in terms of complex focus fields. *Frontiers in Optics/ Laser Science*, JW3A.34.pdf:2 pages, 2018.
- [349] S.H. Simpson. Inhomogeneous and anisotropic particles in optical traps: Physical behaviour and applications. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 146:81–99, 2014.
- [350] T. Stangner, C. Wagner, C.F. Gutsche, K. Stangner, D. Singer, S. Angeliotti-Uberti, and F. Kremer. *Encyclopedia of Biocolloid and Biointerface Science, Volume 2*, chapter 74. Probing receptor-ligand interactions on a single molecule level using optical tweezers, pages 915–932. 2016.
- [351] D. Conteduca, F. Dell’Olio, T.F. Krauss, and C. Ciminelli. Photonic and plasmonic nanotweezing of nano- and microscale properties. *Applied Spectroscopy*, 71, 3:367–390, 2017.
- [352] A.A.M. Bui, A.B. Stilgoe, I.C.D. Lenton, L.J. Gibson, A.V. Kashchuk, S. Zhang, H. Rubinsztein-Dunlop, and T.A. Nieminen. Theory and practice of simulation of optical tweezers. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:66–75, 2017.
- [353] S. Sukhov and A. Dogariu. Non-conservative optical forces: a key issues review. *Rep. Prog. Phys.*, 80, 11:Paper 112001, 2017.
- [354] Z. Gong, Y.L. Pan, G. Videen, and C. Wang. Optical trapping and manipulation of single particles in air: Principles, technical details and applications. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 214:94–119, 2018.
- [355] P. Polimeno, A. Magazzu, M.A. Iati, F. Patti, R. Saija, C.D. Esposti Boschi, M.G. Donato, P.G. Gucciardi, P.H. Jones, G. Volpe, and O.M. Marago. Optical tweezers and their applications. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 218:131–150, 2018.
- [356] G. Gouesbet, S. Meunier-Guttin-Cluzel, and G. Gréhan. Periodic orbits in Hamiltonian chaos of the annular billiard. *Physical Review E*, 65, 1, Art 016212:1–18, 2001.

- [357] G. Gouesbet, S. Meunier-Guttin-Cluzel, and G. Gréhan. Morphology-dependent resonances and/or whispering gallery modes for a two-dimensional dielectric cavity with an eccentrically located spherical inclusion, a Hamiltonian point of view with Hamiltonian (optical) chaos. *Optics Communications*, 201, 4-6:223–242, 2002.
- [358] D. Buker. An analysis on the ability of colloidal silver to act as a biosensor for the steroid testosterone via host-guest interactions. Technical report, Alleghany College DSpace Repository, Projects by Department of Interdivisional Program, Academic Year 2016-2017.
- [359] T.X. Hoang, S.N. Nagelberg, M. Kolle, and G. Barbastathis. Fano resonances from coupled whispering-gallery modes in photonic molecules. *Optics Express*, 25, 12:13125–13144, 2017.
- [360] R.N.S. Suryadharma, M. Fruhnert, I. Fernandez-Corbaton, and C. Rockstuhl. Studying plasmonic resonance modes of hierarchical self-assembled meta-atoms based on their transfer matrix. *Physical Review B*, 96:Paper 045406 (14 pp), 2017.
- [361] M.R. Foreman, J.D. Swaim, and F. Vollmer. Whispering gallery mode sensors. *Advances in optics and photonics*, 7, 2:168–240, 2015.
- [362] M.S. Murib, W.S. Yeap, D. Martens, X. Liu, P. Bienstman, M. Fahlman, M.J. Schoning, L. Michiels, K. Haenen, and A. Serpenguzel et al. Photonic studies on polymer-coated sapphire-spheres: A model system for biological ligands. *Sensors and actuators A - Physical*, 222:212–219, 2015.
- [363] M. Luo, Y. Liu, Z. Wang, Z. Li, W.-Y. Zhou, J. Guo, W. Huang, and X. Liu. Manipulative dual-microsphere lasing based on nanosilica waveguide integrated in a simplified hollow-core microstructured optical fiber. *Journal of lightwave technology*, 35, 11:2183–2189, 2017.
- [364] J.F.L. Santos, M.J.L. Santos, A. Thesing, F. Tavares, J.Griep, and R.F. Rodrigues. Localized surface plasmon resonance applied to biosensor and solar cells (in Portuguese). *Quim. Nova*, 39, 9:1098–1111, 2016.
- [365] J.A. Lock, J.T. Hodges, and G. Gouesbet. Failure of the optical theorem for Gaussian-beam scattering by a spherical particle. *Journal of the Optical Society of America A*, 12,12:2708–2715, 1995.
- [366] G. Gouesbet, C. Letellier, G. Gréhan, and J.T. Hodges. Generalized optical theorem for on-axis Gaussian beams. *Optics Communications*, 125:137–157, 1996.
- [367] L.A. Ambrosio, M. Zamboni-Rached, and G. Gouesbet. Zeroth-order continuous frozen waves for light scattering: exact multipole expansion in the generalized Lorenz-Mie theory. *Journal of the Optical Society of America B*, 36, 1:81–89, 2019.

- [368] Yu A. Eremin. Generalization of the optical theorem for a multipole based on integral transform. *Differential equations*, 53, 9:1121–1126, 2017.
- [369] Y.A. Eremin and A.G. Sveshnikov. Generalized optical theorem to a multipole source excitation in the scattering theory. *Russian journal of mathematical physics*, 24, 2:207–215, 2017.
- [370] A.V. Krasavin, P. Segovia, R. Dubrovka, N. Olivier, G.A. Wurtz, P. Ginzburg, and A.V. Zayats. Generalization of the optical theorem: experimental proof for radially polarized beams. *Light : Science and Applications*, 7:Article number: 36, 2018.
- [371] G. Gouesbet. A scientific story of generalized Lorenz-Mie theories with epistemological remarks. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 126:7–15, 2013.
- [372] G. Gouesbet and G. Gréhan, editors. *Proceedings of the International Symposium: Optical Particle Sizing, Theory and Practice, held in Rouen, 12-15 may, 1987*. Plenum Publishing Corporation, 1988.
- [373] G. Gouesbet and M. Brunel. Lasers and Interactions with particles, 2012 : Optical particle characterization follow-up. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 126:1–6, 2013.
- [374] G. Gouesbet and F.R.A. Onofri. Preface: Laser-light and Interactions with Particles (LIP), 2014. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 162:1–7, 2015.
- [375] G. Gouesbet and F. Onofri. Laser-light and Interactions with particles (LIP), 2016. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:1–7, 2017.
- [376] G. gouesbet, P. Yang, and F. Onofri. Afterword. Laser-light and Interactions with Particles (LIP), 2018. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 225:45–49, 2019.
- [377] G. Gouesbet. Optical sizing, with emphasis on simultaneous measurements of velocities and sizes of particles embedded in flows. In *Measurement techniques in heat and mass transfer*, edited by R.I. Soloukhin and N.H. Afgan, pp 27-50, Springer-Verlag, 1985.
- [378] G. Gouesbet and G. Gréhan. Laser-based optical measurement techniques of discrete particles: A review. *International Journal of Multiphase Flow*, 72:288–297, 2015.
- [379] R.L. Xu. Light scattering: A review of particle characterization applications. *Particuology*, 18:11–21, 2015.

- [380] S. Barbosa, F.R.A. Onofri, L. Couëdel, M. Wozniak, C. Montet, C. Pelcé, C. Arnas, L. Boufendi, E. Kovacevic, J. Berndt, and C. Grisolia. An introduction to light extinction spectrometry as a diagnostic for dust particle characterization in dusty plasmas. *J. Plasma Phys.*, 82:Paper 615820403, 2016.
- [381] C. Tu, Z. Yin, J. Lin, and F. Bao. A review of experimental techniques for measuring micro- to nano-particle-laden gas flows. *Applied Sciences*, 7, 120:21 pp, 2017.
- [382] G. Gréhan, G. Gouesbet, A. Naqwi, and F. Durst. Particle trajectory effects in phase-Doppler systems: computations and experiments. *Particle and Particle Systems Characterization*, 10, 6:332–338, 1993.
- [383] G. Gréhan, G. Gouesbet, A. Naqwi, and F. Durst. Trajectory ambiguities in phase-Doppler systems: study of a near-forward and a near-backward geometry. *Particle and Particle Systems Characterization*, 11, 2:133–144, 1994.
- [384] F. Onofri, T. Girasole, G. Gréhan, G. Gouesbet, G. Brenn, J. Domnick, T.H. Xu, and C. Tropea. Phase-Doppler anemometry with the dual burst technique for measurement of refractive index and absorption coefficient simultaneously with size and velocity. *Particle and Particle Systems Characterization*, 13, 2:112–124, 1996.
- [385] H. Mignon, G. Gréhan, G. Gouesbet, T.H. Xu, and C. Tropea. Measurement of cylindrical particles with phase-Doppler anemometry. *Applied Optics*, 35,25:5180–5190, 1996.
- [386] N. Yokoi and Y. Aizu. Measurements of size and absorption coefficient of a single moving particle by using dual burst Doppler signal. *Measurement*, 59:51–62, 2015.
- [387] L. Xie, X.Q. Dou, and J. Zhou. Sizing charged particles by phase Doppler anemometry. *Applied Optics*, 55, 12:3279–3286, 2016.
- [388] A. Verdier, J. Marrero Santiago, A. Vandel, S. Saengkaew, G. Cabot, G. Gréhan, and B. Renou. Experimental study of local flame structures and fuel droplet properties of a spray jet flame. *Proceedings of the Combustion Institute*, 36, 2:2595–2602, 2017.
- [389] X.C. Wu, Z.L. Xue, H.F. Zhao, L.C. Yao, L.H. Chen, C.H. Zheng, and X. Gao. Measurement of slurry droplets by digital holographic microscopy : Fundamental research. *Fuel*, 158:697–704, 2015.
- [390] G. Swirniak and G. Glomb. Step-index optical fiber sizing with a rainbow technique. *Proc. SPIE 10678, Optical Micro- and Nanotechnology VII*, doi: 10.1117/12.2311842:10 pages, 1067815 (24 mai 2018).

- [391] C.D. Rosebrock, S. Shirinzadeh, M. Soeken, N. Riefler, T. Wriedt, R. Drechsler, and L. Mädler. Time-resolved detection of diffusion limited temperature gradients inside single isolated burning droplets using Rainbow Refractometry. *Combustion and Flame*, 168:255–269, 2016.
- [392] X. Han, K.F. Ren, Z. Wu, F. Corbin, G. Gouesbet, and G. Gréhan. Characterization of initial disturbances in liquid jet by rainbow sizing. *Applied Optics*, 37,36:8498–8503, 1998.
- [393] J. Promvongsa, P. Vallikul, B. Fungtammasan, A. Garo, G. Gréhan, and S. Saengkaew. Multicomponent fuel droplet evaporation using 1D global rainbow technique. *Proceedings of the Combustion Institute*, 36, 2:2401–2408, 2017.
- [394] H. Li, C.D. Rosebrock, T. Wriedt, and L. Mädler. The effect of initial diameter on rainbow positions and temperature distributions of burning single-component n-Alkane droplets. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 195:164–175, 2017.
- [395] M. Ouattara, F. Lamadie, M.P.L. Sentis, and F.R.A. Onofri. Droplet sizing and mixture fraction measurement in liquid-liquid- flows with rainbow-angle diffractometry. *Applied Optics*, 56, 29:8109–8120, 2017.
- [396] M. Luo, Y. Wu, and O. J. Haidn. Temperature and size measurements of cryogenic propellant spray droplets with global rainbow refractometry. *2nd Joint Conference of Aerospace Propulsion*, August 23-27, Dalian, China, 2017.
- [397] C. Li, X.-C. Wu, J.-Z. Cao, L.-H. Chen, G. Gréhan, and K.-F. Cen. Application of rainbow refractometry for measurement of droplets with solid inclusions. *Optics and Laser Technology*, 98:354–362, 2018.
- [398] S. Coetmellec, W. Wichitwong, G. Gréhan, D. Lebrun, M. Brunel, and A.J.E.M. Janssen. Digital in-line holography assessment for general phase and opaque particle. *Journal of the European Optical Society - Rapid Publications*, 9:Article Number: 14021, 2014.
- [399] S. Coetmellec, D. Pejchang, D. Allano, G. Gréhan, D. Lebrun, M. Brunel, and A.J.E.M. Janssen. Digital in-line holography in a droplet with cavitation air bubbles. *J. Europ. Opt. Soc. Rap. Public.*, 9:Paper 14056 (10 pp), 2014.
- [400] T. Latychevskaia and H.W. Fink. Holographic time-resolved particle tracking by means of three-dimensional volumetric deconvolution. *Optics Express*, 22, 17:20994–21003, 2014.
- [401] J.L. Marie, N. Grosjean, L. Méès, M. Seifi, C. Fournier, B. Barbier, and M. Lance. Lagrangian measurements of the fast evaporation of falling diethyl ether droplets using in-line digital holography and a high-speed camera. *Experiments in Fluids*, 55, 4:Article Number: 1708, 2014.

- [402] H. Ling and J. Katz. Separating twin images and locating the center of a microparticle in dense suspensions using correlations among reconstructed fields of two parallel holograms. *Applied Optics*, 53, 27:G1–G11, 2014.
- [403] J. Hattay, S. Belaid, D. Lebrun, and W. Naanaa. Digital in-line particle holography: twin-image suppression using sparse blind source separation. *Signal image and video processing*, 9, 8:1767–1774, 2015.
- [404] J. Hattay, S. Belaid, T. Aguilí, and D. Lebrun. A new wavelet-based reconstruction algorithm for twin image removal in digital in-line holography. *Optics and lasers in Engineering*, 82:159–172, 2016.
- [405] L.C. Yao, X.C. Wu, Y.C. Wu, J. Yang, X. Gao, L.H. Chen, G. Gréhan, and K.F. Cen. Characterization of atomization and breakup of acoustically levitated drops with digital holography. *Applied Optics*, 54, 1:A23–A31, 2015.
- [406] L.C. Yao, X.C. Wu, J. Yang, Y.C. Wu, X. Gao, L.H. Chen, G. Gréhan, and K.F. Cen. Simultaneous 3D temperature and velocity field measurements of micro-flow with laser-induced fluorescence and micro-digital holographic particle tracking velocimetry: Numerical study. *Chinese Optics Letters*, 13, 7:Article Number : 072801, 2015.
- [407] C. Wang, F.C. Cheong, D.B. Ruffner, X. Zhong, M.D. Ward, and D.G. Grier. Holographic characterization of colloidal fractal aggregates. *Soft Matter*, 42:8774–8780, 2016.
- [408] F. Jolivet, F. Momey, L. Denis, L. Méès, N. Faure, N. Grosjean, F. Pinston, J.-L. Marié, and C. Fournier. Regularized reconstruction of absorbing and phase objects from a single in-line hologram, application to fluid mechanics and micro-biology. *Optics Express*, 26, 7:8923–8934, 2018.
- [409] N. De Cock, M. Massinon, D. Nuyttens, and D. Dekeyser. Measurements of reference ISO nozzles by high-speed imaging. *Crop protection*, 89:105–115, 2016.
- [410] M.J. Klaiber, Z. Wang, and S. Simon. *Process spray*, chapter 8: A real-time process analysis system for the simultaneous acquisition of spray characteristics, pages 265–305. Springer International Publishing Switzerland, 2016.
- [411] L.L. Lulbadda Waduge, S. Zigan, L.E. Stone, A. Belaidi, and P. Garcia-Trinanes. Predicting concentrations of fine particles in enclosed vessels using a camera based system and CFD simulations. *Process safety and environmental protection*, 105:262–273, 2017.
- [412] S. Berisha, T. van Dijk, R. Bhagarva, P. Scott Carney, and D. Mayerich. BIM-Sim : Interactive simulation of broadband imaging using Mie theory. *Frontiers in Physics, Technology report*, 5:Paper 5 (15 pp), 2017.

- [413] Y. Zhou, Y. Tang, Q. Deng, L. Zhao, and S. Hu. Contrast enhancement of microsphere-assisted super-resolution imaging in dark-field microscopy. *Applied Physics Express*, 10:Article 082501, 4 pages, 2017.
- [414] A. Hollerbach, D. Logsdon, K. Iyer, A. Li, J.A. Schaber, and R.G. Cooks. Sizing sub-diffraction limit electrosprayed droplets by structured illumination microscopy. *Analyst*, 143:232–240, 2018.
- [415] S. Kang, W.-T. Lai, S.-C. Chen, and D.Y.H. Pui. Measurement of dispersity of ISO A2 fine dusts by shadowgraph method. *Powder Technology*, 323:186–194, 2018.
- [416] M. Gragston, Y. Wu, C.D. Smith, and Z. Zhang. Experimental characterization of ethanol and jet-A liquid spray flames. *AIAA Aerospace Sciences Meeting*, 2018.
- [417] L. Xu, T. Wei, J. Zhou, and Z. Cao. Modified Landweber algorithm for robust particle sizing by using Fraunhofer diffraction. *Applied Optics*, 53, 27:6185–6193, 2014.
- [418] S. Bosch and J. Sancho-Parramon. Procedures for the measurement of the extinction cross section of one particle using a Gaussian beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 180:77–83, 2016.
- [419] F. El Bassri, C. Lefort, E. Capitaine, C. Louot, D. Pagnoux, V. Couderc, and P. Leproux. Nanosecond coherent anti-Stokes Raman scattering for particle size characterization. *Proceedings volume 9703, Optical Biopsy XIV*, Paper 970314 (7pp), 2016.
- [420] S. Rosenkranz, W. Schäfer, C. Tropea, and A.M. Zoubir. Modeling photon transport in turbid media for measuring colloidal concentration in drops using the time-shift technique. *Applied Optics*, 55, 34:9703–9711, 2016.
- [421] Q. Huang, M. Cheng, L. Guo, J. Li, X. Yan, and S. Liu. Scattering of aerosol particles by a Hermite-Gaussian beam in marine atmosphere. *Applied Optics*, 56, 19:5329–5335, 2017.
- [422] Y. Wang and J.E. Muth. An optical-fiber-based airborne particle sensor. *Sensors*, 17, 2110:10 pp, 2017.
- [423] G.K. Oo d’Amore, M. Caniato, A. Travan, G. Turco, L. Marsich, A. Ferluga, and C. Schmid. Innovative thermal and acoustic insulation foam from recycled waste glass powder. *Journal of cleaner production*, 165:1306–1315, 2017.
- [424] H. Xie, L. Xu, H. Niu, and Z. Cao. Particle sizing from Fraunhofer diffraction pattern using a digital micro-mirror device and a single photodiode. *Powder Technology*, 332:351–358, 2018.



- [425] L. Songfang and S. Jianqi. Particle size analysis based on backward-scattered light (in Chinese). *Optical Instruments*, 40, 4:1–8, 2018.
- [426] K.F. Ren, G. Gréhan, and G. Gouesbet. Radiation pressure forces exerted on a particle arbitrarily located in a Gaussian beam by using the generalized Lorenz-Mie theory, and associated resonance effects. *Optics Communications*, 108, 4-6:343–354, 1994.
- [427] L. de Rond, F.A.W. Coumans, R. Nieuwland, T.G. van Leeuwen, and E. van der Pol. Deriving extracellular vesicle size from scatter intensities measured by flow cytometry. *Current Protocols in Cytometry e43*, doi: 10.1002/cpcy.43:14 pages, 2018.
- [428] C. Hanske, M. Tebbe, C. Kuttner, V. Bieber, V. V. Tsukruk, M. Chanana, T.A.F. König, and A. Fery. Strongly coupled plasmonic modes on macroscopic areas via template-assisted colloidal self-assembly. *Nano Letters*, October 27:6863–6871, 2014.
- [429] Z. Chen, H. Zhang, Z. Huang, and X. Wu. Scattering of on-axis Gaussian beam by a uniaxial anisotropic object. *Journal of the Optical Society of America A*, 31, 11:2545–2550, 2014.
- [430] J.J. Wang, A.T. Chen, Y.P. Han, and P. Briard. Light scattering from an optically anisotropic particle illuminated by an arbitrary shaped beam. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 167:135–144, 2015.
- [431] M. Zheng, H.Y. Zhang, Y.F. Sun, and Z.G. Wang. Scattering of on-axis Gaussian beam by an arbitrarily shaped chiral object. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 151:192–197, 2015.
- [432] M. Zheng, H.Y. Zhang, and Y.F. Sun. On-axis Gaussian beam scattering by an object with a chiral inclusion. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 151:269–274, 2015.
- [433] Z. Chen, H. Zhang, X. Wu, and Z. Huang. Gaussian beam scattering by a gyrotropic anisotropic object. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 180:1–6, 2016.
- [434] M.I. Mischenko and J.M. Dlugach. Scattering of Gaussian beams by disordered particulate media. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 183:85–89, 2016.
- [435] L. Bi, W. Lin, D. Liu, and K. Zhang. Assessing the depolarization capabilities of nonspherical particles in a super-ellipsoidal shape space. *Optics express*, 26, 2:1726–1742, 2018.
- [436] T. Wu, R.Y. Wang, and X.D. Zhang. Plasmon-induced strong interaction between chiral molecules and orbital angular momentum of light. *Scientific Reports*, 5:Article Number : 18003, 2015.

- [437] A. Ungut, G. Gréhan, and G. Gouesbet. Comparisons between geometrical optics and Lorenz-Mie theory for transparent particles in forward directions. *Applied Optics*, 20,17:2911–2918, 1981.
- [438] M.L. Yang, Y.Q. Wu, X.Q. Sheng, and K.F. Ren. Comparison of scattering diagrams of large non-spherical particles calculated by VCRM and MLFMA. *Journal of Quantitative Spectroscopy and Radiative Transfer, special issue*, 162:143–153, 2015.
- [439] H.T. Yu, J.Q. Shen, and C. Tropea. Application of vector ray tracing to the computation of Möbius shifts for the primary and secondary rainbows. *Applied Optics*, 54, 31:9093–9101, 2015.
- [440] P.G. Stegmann, C. Tropea, E. Järvinen, and M. Schnaiter. Comparison of measured and computed phase functions of individual tropospheric ice crystals. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 178:379–389, 2016.
- [441] A. Haussmann. Polarization-resolved simulations of multiple-order rainbows using realistic raindrop shapes. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 175:76–89, 2016.
- [442] A. Haussmann. Light scattering from sessile water drops and raindrop-shaped glass beads as a validation tool for rainbow simulations. *Applied Optics*, 56, 19:G136–G144, 2017.
- [443] J.A. Lock and G.P. Können. Rainbows by elliptically deformed drops. I. Möbius shift for higher-order rainbows. *Applied Optics*, 56, 19:G88–G97, 2017.
- [444] B. Sun, P. Yang, G.K. Kattawar, and X. Zhang. Physical-geometric optics method for large size faceted particles. *Optics Express*, 25, 20:24044–24060, 2017.
- [445] G. Gouesbet. Debye series formulation for generalized Lorenz-Mie theory with the Bromwich method. *Particle and Particle Systems Characterization*, 20,6:382–386, 2003.
- [446] L. Bi, P. Yang, G.W. Kattawar, and M.I. Mishchenko. Optical tunneling by arbitrary macroscopic three-dimensional objects. *Physical Review A*, 92, 1:Article Number: 013814, 2015.
- [447] S.Y. Li, S. Qin, D.H. Li, and Q.H. Wang. Using a laser source to measure the refractive index of glass beads and Debye theory analysis. *Applied Optics*, 54, 33:9688–9694, 2015.
- [448] C. Zhou. Coherent backscattering enhancement in single scattering. *Optics Express*, 26, 10:A508–A519, 2018.

- [449] L. Bi, F. Xu, and G. Gouesbet. Depolarization of nearly spherical particles: The Debye series. *Physical Review A*, 98, 5:Paper 053809, 2018.
- [450] M. Li, S. Yan, B. Yao, M. Lei, Y. Yang, J. Min, and D. Dan. Intrinsic optical torque of cylindrical vector beams on Rayleigh absorptive spherical particles. *Journal of the Optical Society of America A*, 31, 8:1710–1715, 2014.
- [451] K. Skorupski. Using the DDA (Discrete Dipole Approximation) method in determining the extinction cross section of black carbon. *Metrology and measurement systems*, 22, 1:153–164, 2015.
- [452] J. Li and X.-Y. Feng. Modeling of electric multipole moment in target scattering with system identification. *Indian J. Phys.*, doi.org/10.1007/s12648-018-1247-8:9 pages, 2018.
- [453] B. N. Khlebtsov and N.G. Khlebtsov. Surface morphology of a gold core controls the formation of hollow or bridged nanogaps in plasmonic nanomatryoshkas and their SERS responses. *The Journal of Physical Chemistry*, 120:15385–15394, 2016.
- [454] Z.W. Cui and Y.P. Han. A review of the numerical investigation on the scattering of Gaussian beam by complex particles. *Physics Reports - Review Section of Physics Letters*, 538, 2:39–75, 2014.
- [455] Z. Chen, H. Zhang, Z. Huang, and X. Wu. Shaped beam scattering by an anisotropic particle. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 189:238–242, 2017.
- [456] D. Liu, T. Liao, and H. Zhang. Transmission of shaped beam through a rotationally uniaxial anisotropic sphere. *Optik*, 154:731–737, 2018.
- [457] F. Mangini, N. Tedeschi, F. Frezza, and A. Sihvola. Electromagnetic interaction with two eccentric spheres. *Journal of the Optical Society of America A*, 31, 4:783–789, 2014.
- [458] D.-M. Yu, Y.-N. Liu, F.-L. Tian, X.-M. Pan, and X.-Q. Sheng. Accurate thermoplasmonic simulation of metallic nanoparticles. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 187:150–160, 2017.
- [459] M.I. Mishchenko and M. A. Yurkin. Impressed sources and fields in the volume-integral-equation formulation of electromagnetic scattering by a finite object: A tutorial. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 214:158–167, 2018.
- [460] F.J. Salgado-Remacha, J.A. Sanchez-Martin, and E. Bernabeu. Positioning a focused Gaussian beam inside a refractive cylinder. *Optics and Lasers in Engineering*, 55:53–58, 2014.

- [461] L. Bressel and O. Reich. Theoretical and experimental study of the diffuse transmission of light through highly concentrated absorbing and scattering materials. Part I: Monte-Carlo simulations. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 146:190–198, 2014.
- [462] A. Ramm and M. Andriychuk. EM wave scattering on a set of small particles and creation of materials with desired refraction coefficient and magnetic permeability. *2016 International conference on "Mathematical methods in electromagnetic theory"*, pages 410–413, 2016.
- [463] N.G. Khlebtsov and B.N. Khlebtsov. Optimal design of gold nanomatryoshkas with embedded Raman reporters. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 190:89–102, 2017.
- [464] A. Buccini, M. Donatelli, and F. Ferri. Weakly constrained Lucy-Richardson with applications to inversion of light scattering data. *J. Sci. Comput.*, <https://doi-org/10.1007/s10915-017-0461-4>, 2017.
- [465] A. M. Steiner, M. Mayer, M. Seuss, S. Nikolov, K.D. Harris, A. Alexeev, C. Kuttner, T.A.F. König, and A. Fery. Macroscopic strain-induced transition from quasi-infinite gold nanoparticles chains to defined plasmonic oligomers. *ACS Nano*, 11:8871–8880, 2017.
- [466] M. Kiani, A. Abdolali, and M. Safari. Analytical method for analysis of electromagnetic scattering from inhomogeneous spherical using duality principle. *Physical Review E*, 97:Article 032137, 9 pages, 2018.
- [467] A.G. Lamprianidis and A.E. Miroshnichenko. Excitation of nonradiating magnetic anapole states with azimuthally polarized vector beams. *Beilstein Journal of Nanotechnology*, 9:1478–1490, 2018.
- [468] A. Téllez-Quinones, J.C. Valdivievo-Navarro, A. Salazar-Garibay, and A.A. Lopez-Caloca. The retarded potential of a non-homogeneous wave equation: introductory analysis through the Green function. *Revista Mexicana de Física E*, 64:26–38, 2018.
- [469] G. Gouesbet. Interaction between Gaussian beams and infinite cylinders, by using the theory of distributions. *Journal of Optics (Paris)*, 26,5:225–239, 1995.
- [470] E. Lenglart and G. Gouesbet. The separability "theorem" in terms of distributions with discussion of electromagnetic scattering theory. *Journal of Mathematical Physics*, 37,9:4705–4710, 1996.
- [471] G. Gouesbet. Theory of distributions and its application to beam parametrization in light scattering. *Particle and Particle Systems Characterization*, 16:147–159, 1999.

- [472] M. Cheng, L. Guo, J. Li, X. Yan, K. Dong, and Y. You. Average intensity and spreading of a radially polarized multi-Gaussian Schell-model beam in anisotropic turbulence. *Journal of Quantitative Spectroscopy and Radiative Transfer, LIP2018 special issue*, 218:12–20, 2018.
- [473] B. Yu and Y. Sun. Simulation of impact of water-film spray on visibility. *J. Transp. Eng., Part B: Pavements*, 144, 4:Paper 04018054, 2018.
- [474] J.L. Dominguez-Juarez, S. Vallone, A. Lempel, M. Moocarme, J. Oh, H.D. Gafney, and L.T. Vuong. Influence of solvent polarity on light-induced thermal cycles in plasmonic nanofluids. *Optica*, 2, 5:447–453, 2015.
- [475] G. Gouesbet, C. Rozé, and S. Meunier-Guttin-Cluzel. Instabilities by local heating below an interface, a review. *Journal of Nonequilibrium Thermodynamics*, 25:337–379, 2000.
- [476] G. Gouesbet, S. Meunier-Guttin-Cluzel, and O. Ménard, editors. *Chaos and its reconstruction*. Novascience Publishers, New York, 2003.
- [477] A. Gogoi, P. Rajkhowa, G.P. Saikia, G.A. Ahmed, and A. Choudhury. WEBSCAT: A web application for the analysis of electromagnetic scattering from small particles. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 146:270–279, 2014.
- [478] A. Haussmann. Rainbows in nature: recent advances in observation and theory. *European Journal of Physics*, 37:Paper 063001, 30pp, 2016.
- [479] S.S. Sazhin. Modelling of fuel droplet heating and evaporation: Recent results and unsolved problems. *Fuel*, 196:69–101, 2017.
- [480] W. Fuqiang, M. Lanxin, C. Ziming, T. Jianyu, H. Xing, and L. Linhua. Radiative heat transfer in solar thermochemical particle reactor: A comprehensive review. *Renewable and sustainable energy reviews*, 73:935–949, 2017.
- [481] S. Zhang and H. Xuab. *Nanophotonics Manipulating light with plasmons*, chapter 2 : Light scattering by small metallic particles : Mie theory and more. 2017.