

# Non-classical regression laws for droplet vaporization

**Roger Prud'homme\*, Stéphane Préau**

Laboratoire de Modélisation en Mécanique – UPMC/CNRS

Case 162 - 4 place Jussieu 75252 Paris cédex 05

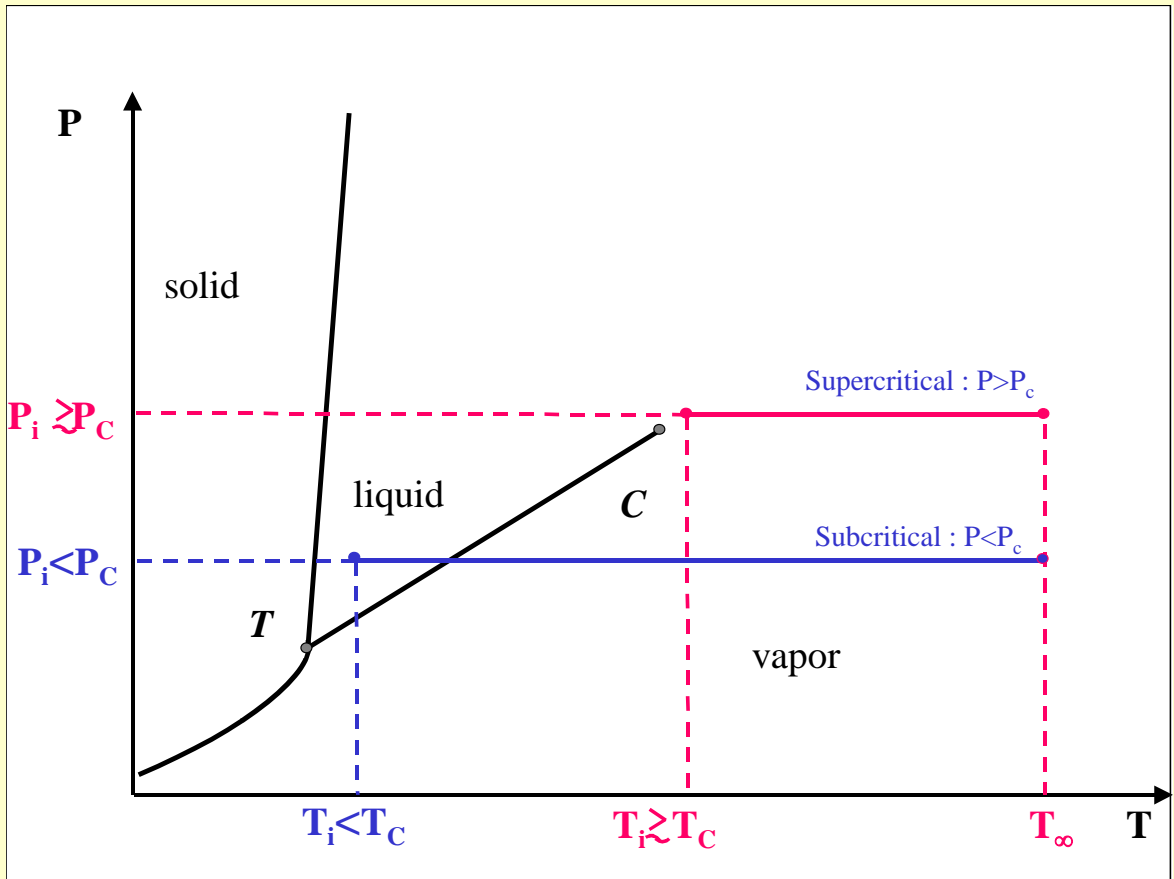
\*Correspondence author: Fax: (33)1 44 27 52 59 Email: [prudhom@ccr.jussieu.fr](mailto:prudhom@ccr.jussieu.fr)

# INTRODUCTION

Droplet vaporization or combustion is usually studied assuming quasi-steady hypotheses for the vapor and a constant and uniform temperature for the liquid. A “ $D^2$  law” is then obtained for the diameter evolution.

This is not the case for near-critical droplets in the general case, and also for sub-critical droplets in transient heat-up regime. We present here these two situations: near-critical vaporization in a pure fluid, and pure liquid vaporization in a gaseous mixture.

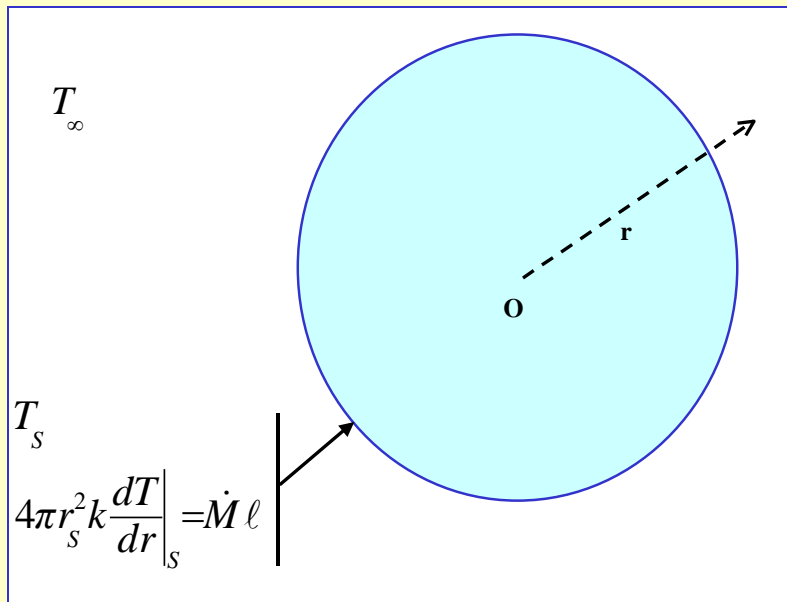
# VAPORIZATION OF A “DROPLET” UNDER SUPERCRITICAL PRESSURE



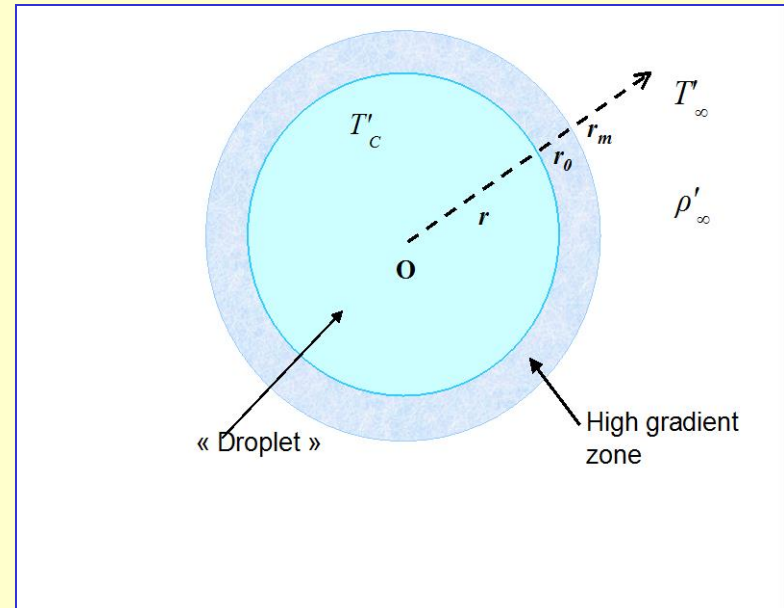
$C_{pr} \rightarrow \infty$   
 $\lambda \rightarrow \infty$   
 $T_{sat} = T_c$   
 $L = 0$   
 $\Delta\rho = 0$

## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE

Subcritical :  $P < P_c$

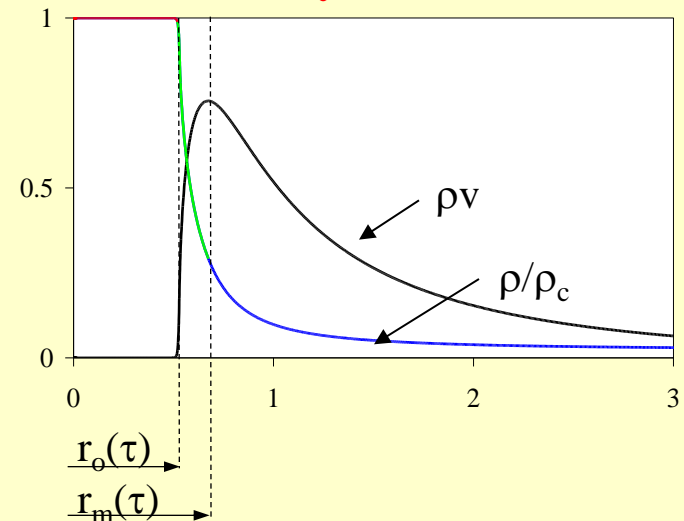
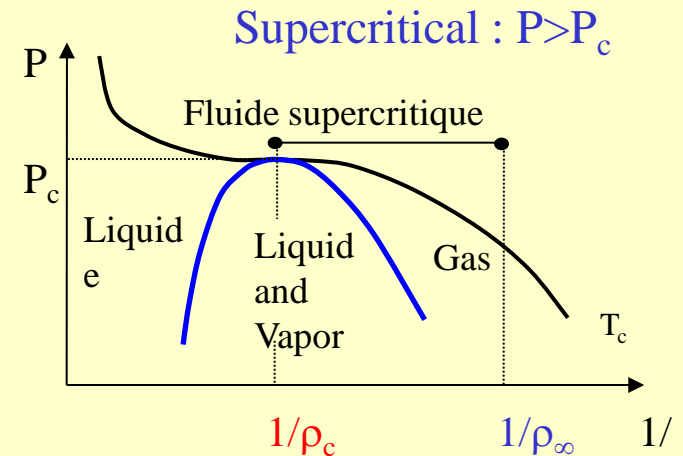
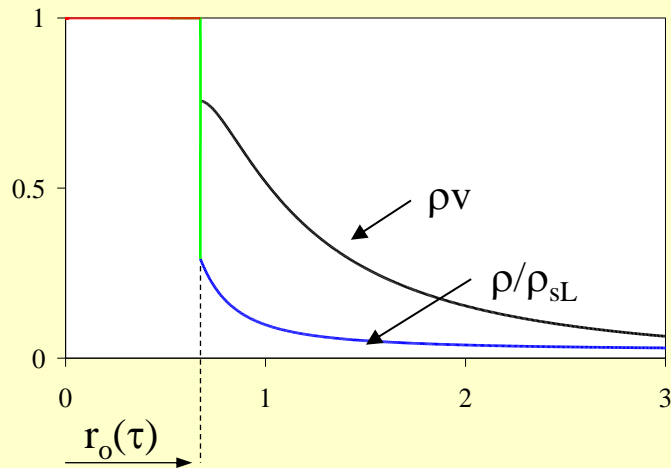
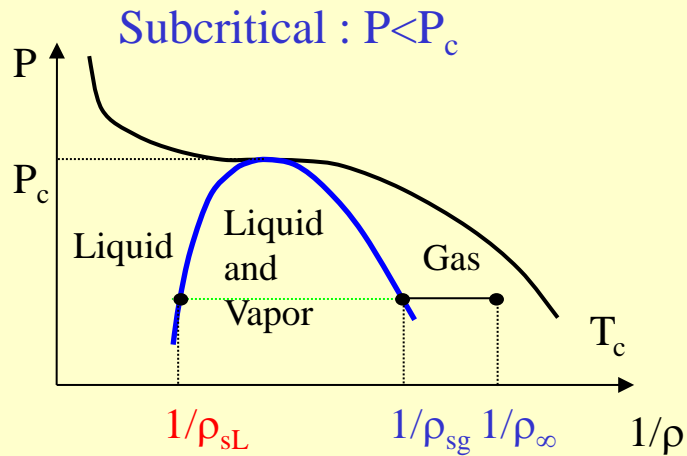


Supercritical :  $P > P_c$



## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE

### Subcritical and supercritical cases



## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE

### Subcritical and supercritical cases

Hypotheses :

★ Configuration: isobaric

$$★ \gamma_{\infty} = \frac{t_{diff}(\rho_{\infty})}{t_{vie}} \ll 1$$

#### Subcritical case

★ Interface

⇒ Mass and energy fluxes

★ Gaseous zone

#### Supercritical case

★ Thick interface

Resolution

Hypothesis : steady in the considered frame

$$X = \frac{r - r_0(\tau)}{r_m(\tau) - r_0(\tau)}$$

⇒ Flux for  $r = r_m(\tau)$

★ External zone

Resolution of the equations

Quasi-steady hypothesis

## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE

### Results

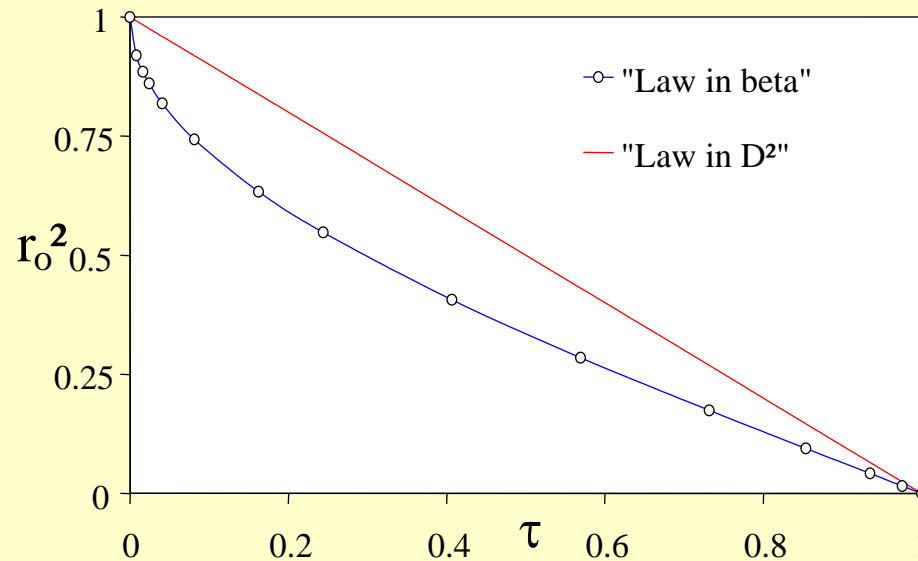
Subcritical case

★  $r_o^2 = 1 - \tau$

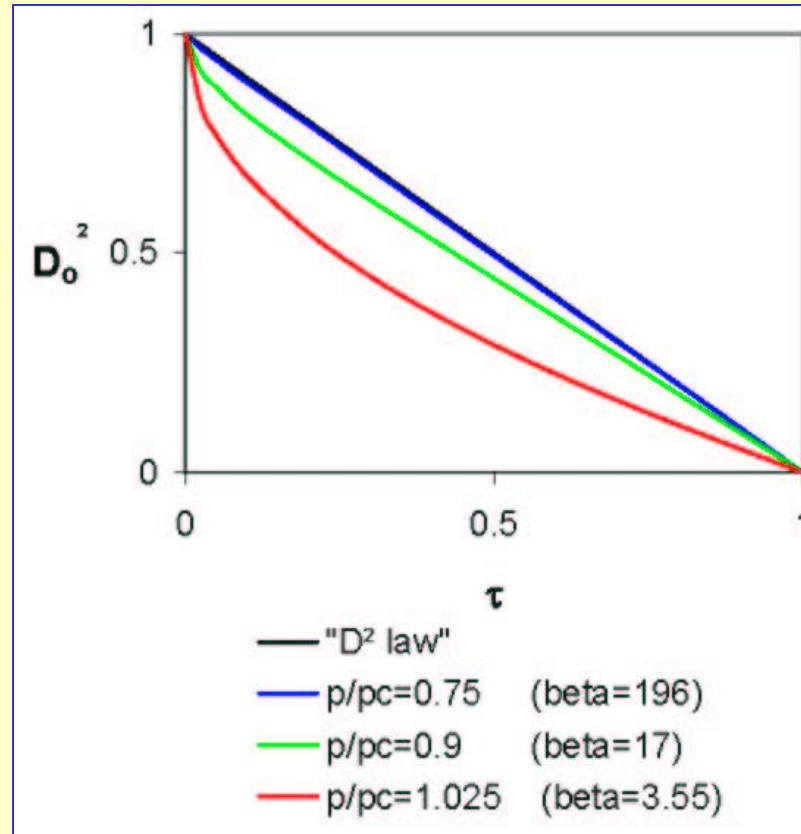
Supercritical case

★  $r_o^\beta - \beta r_o = (1 - \beta)\sqrt{1 - \tau}$

★  $r_m^2 = 1 - \tau$



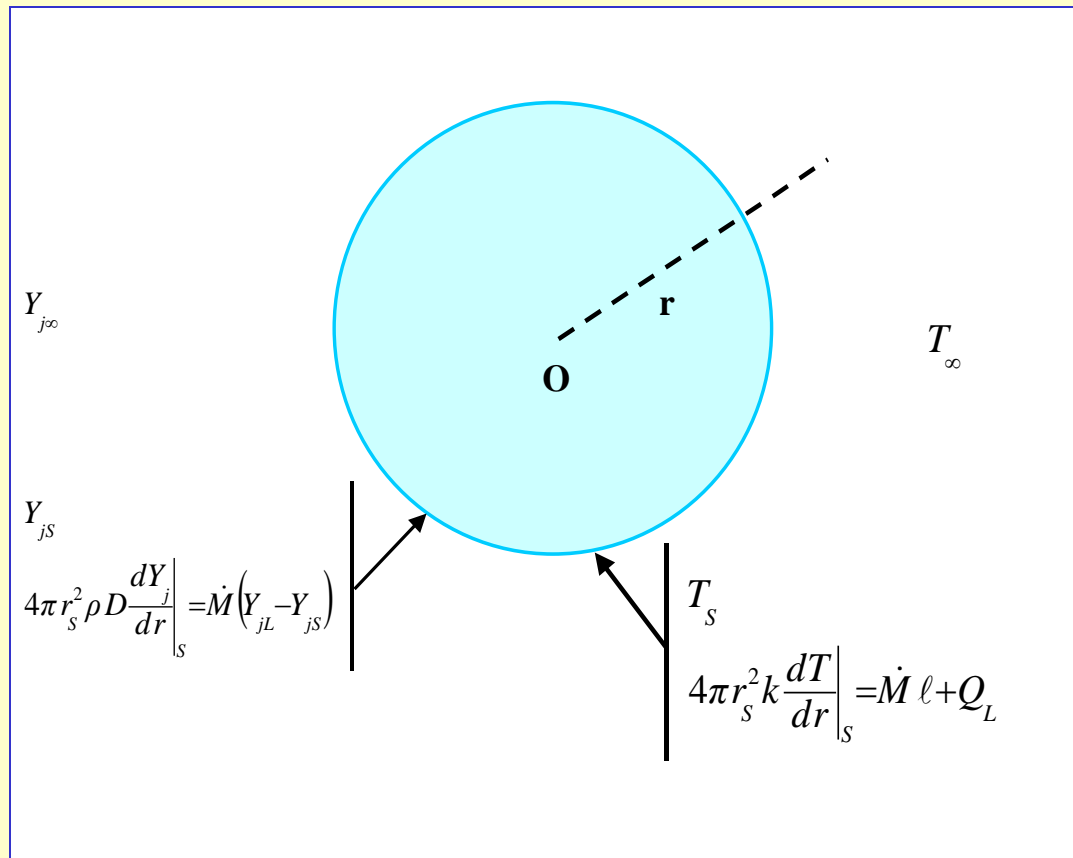
## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE



Square of the droplet diameter as a function of the reduced time  $\tau$   
 for different pressures ( $p_c$  is the critical pressure).



# VAPORIZATION WITH HEAT-UP OF A LIQUID DROPLET IN A GASEOUS MIXTURE



## VAPORIZATION WITH HEAT-UP OF A LIQUID DROPLET

$$\begin{cases} \dot{M} = 4\pi r^2 \dot{m}, & \dot{m} = \rho u \\ \dot{M} \frac{dY_j}{dr} - 4\pi \rho D r \frac{d^2(rY_j)}{dr^2} = 0 \\ \dot{M} c_p \frac{dT}{dr} - 4\pi k r \frac{d^2(rT)}{dr^2} = 0 \end{cases}$$

### Boundary conditions

- at infinity  $r = \infty$  :  $Y_j = Y_{j\infty}, T = T_\infty$

- at the droplet surface  $r_s$  :

$$\begin{cases} Y_{js}, & 4\pi \rho D r_s^2 \frac{dY_j}{dr} \Big|_s = -\dot{M} (Y_{jL} - Y_{js}) \\ T_s, & 4\pi \frac{k}{c_p} r_s^2 \frac{dT}{dr} \Big|_s = \dot{M} \frac{\ell}{c_p} \end{cases}$$

$$\begin{aligned} \dot{M} &= 4\pi \frac{k}{c_p} r_s \ln(1 + B_T) \\ &= 4\pi \rho D r_s \ln(1 + B_M) \end{aligned}$$

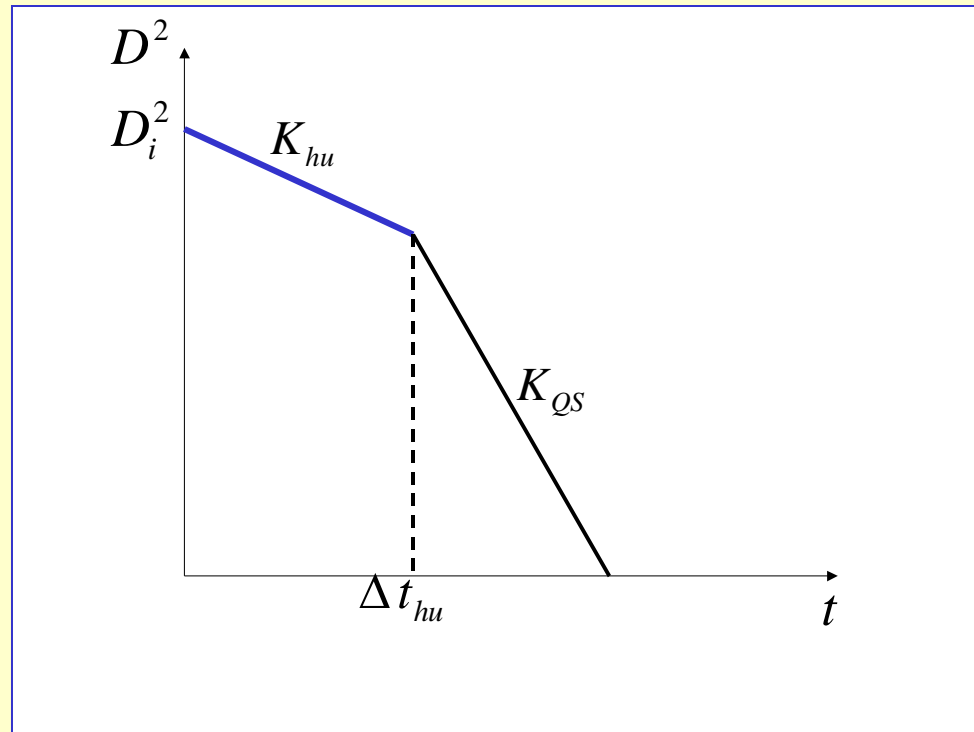
$$\begin{cases} \frac{dT_s}{dt} = \frac{12k\ell}{\rho_L c_L c_p D^2} \left( \frac{B'_T}{B_T} - 1 \right) \ln(1 + B_T) \\ D \frac{dD}{dt} = \frac{4k}{\rho_L c_p} \ln(1 + B_T) \end{cases}$$

$$B_M = \frac{Y_{FS} - Y_{F\infty}}{1 - Y_{FS}}$$

$$B_T = c_p (T_\infty - T_s) / (\ell + Q_L / \dot{M}) \quad B'_T = c_p (T_\infty - T_s) / \ell$$

## VAPORIZATION WITH HEAT-UP OF A LIQUID DROPLET

### Approximation of Chin et Lefebvre (1985)



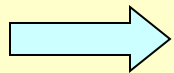
## VAPORIZATION WITH HEAT-UP OF A LIQUID DROPLET

### A new approximation

$$D^2 \frac{dT_s}{dt} = F(T_s), \quad \frac{dD^2}{dt} = G(T_s)$$

$$T_s = T_{QS} + (T_i - T_{QS}) e^{-t/\tau_T}$$

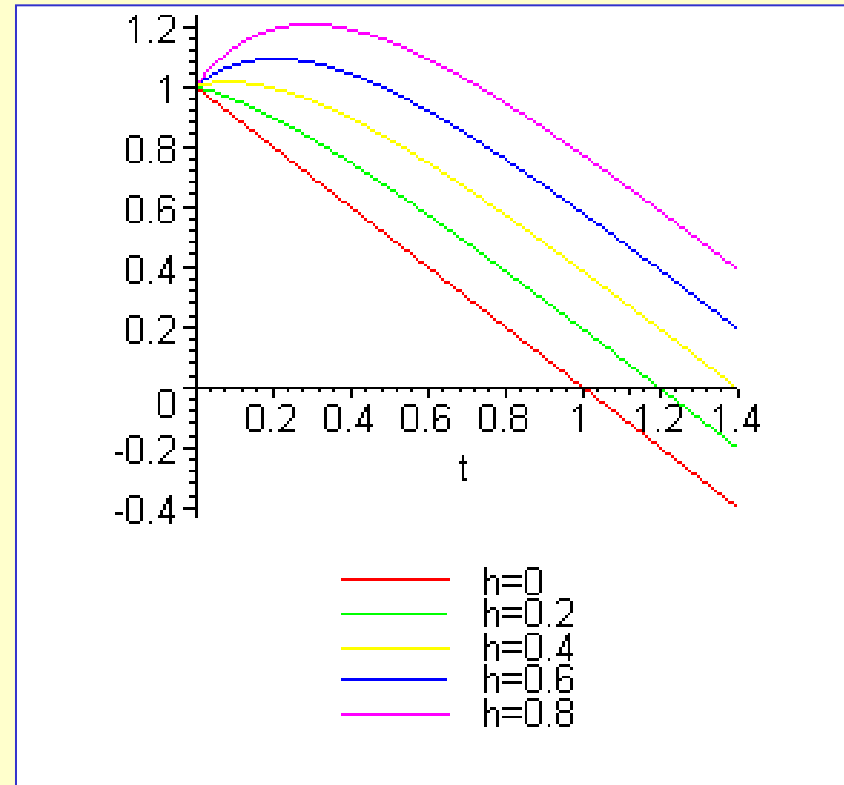
$$G(T_s) = -K_{QS} + h(T_s - T_{QS})$$



$$F(T_s) \approx \frac{T_{QS} - T_s}{\tau_T} \left( D_i^2 - \frac{h}{\tau_T} (T_s - T_i) + \tau_T K_{QS} \ln \left( \frac{T_{QS} - T_s}{T_{QS} - T_i} \right) \right)$$

$$D^2 \approx D_i^2 - \frac{h}{\tau_T} (T_{QS} - T_i) t + \frac{h}{\tau_T} (T_{QS} - T_i) e^{-t/\tau_T}$$

## VAPORIZATION WITH HEAT-UP OF A LIQUID DROPLET



$(D/D_i)^2$  as a function of reduced time  $\tau = K_{QS} t / D_i$ , for  $\tau_T = 0.3$ , for a subcritical droplet.

## CONCLUSIONS AND PERSPECTIVES

**Two cases** of droplet regression have been examined:

- **A near critical droplet** in its pure vapor has been studied in section 1, which gives well a  $D^2$  law but not for the droplet diameter  $D_0$  as usually defined, but for a particular diameter  $D_m$ . On the contrary a non linear law was found for  $D_0$  as a function of time. An approximate law was given, depending on a coefficient  $\beta$ .

- **A subcritical vaporizing droplet** in a hotter gaseous mixture. A simplifying assumption was that the droplet temperature keeps its uniformity. This corresponds to an infinite conductivity for the liquid. Chin and Lefebvre who consider two successive  $D^2$  law, one for the heat up regime and the other for the following QS regime. In this section, we proposed an alternative method with simple hypotheses. A continuous profile is obtained.

- The  $D^2$  law is not valid too in **other cases of heat-up regime**, when the droplet is not assumed to have a uniform temperature. Law & Sirignano studied the case of a *purely conducting droplet* numerically and using series expansions. To simplify the continuous conduction problem, it is possible to consider a *two-layer model* or a *n-layer model*. For not purely conductive droplets, Abramson & Sirignano proposed to assimilate the heat transfer inside the droplet to a purely conductive problem, and doing that they introduce an *effective conduction coefficient*.

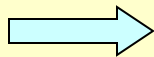
## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE

### Resolution in the supercritical case

$$\begin{cases} \frac{\gamma_\infty}{\varepsilon} \dot{r}_0 (r_m - r_0) = J(\theta_0) - J(\theta_m) \\ r_m^2 = 1 - \tau, \quad r_0(0) = 1 \end{cases} \quad \theta = \frac{T'}{T'_\infty}, \quad \varepsilon = \frac{\rho'_\infty}{\rho'_{in}}, \quad J(\theta) = \int_\theta^1 \frac{\lambda}{\int_{\theta_{in}}^{\theta'} C_p d\theta''} d\theta', \quad \gamma_\infty = \frac{t'_{dif \infty}}{t'_{life}}$$

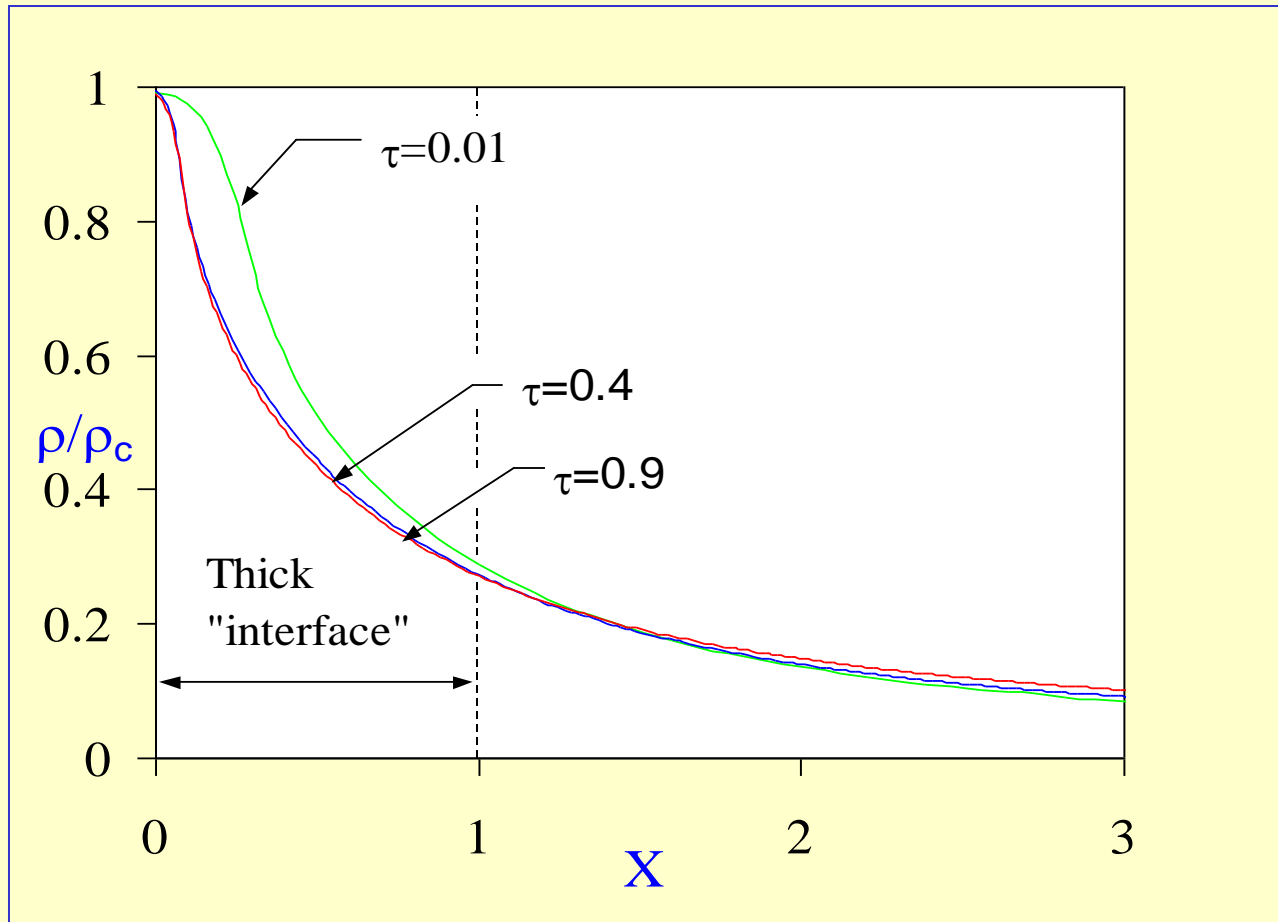
Approximate resolution

$$r_m / r_0 = 1 + u, \quad u \ll 1 \quad \Rightarrow \quad dr_m / dr_0 = \beta u \quad \beta = \gamma_\infty / 2\varepsilon [J(\theta_0) - J(\theta_m)]$$



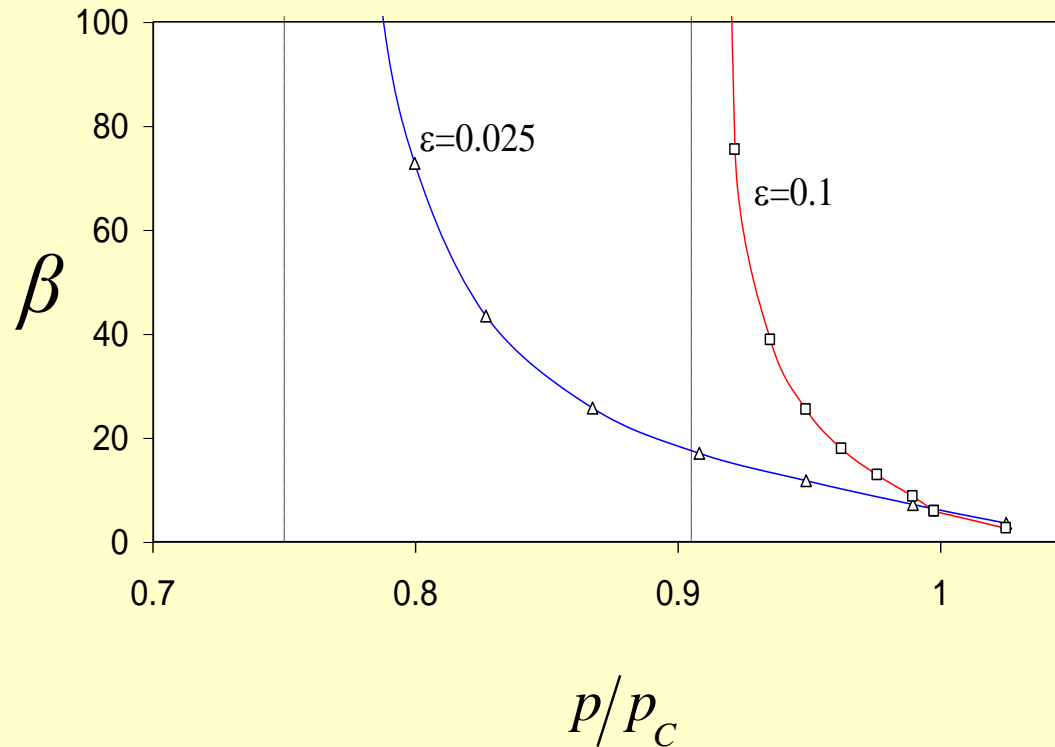
$$r_0^\beta - \beta r_0 = (1 - \beta) \sqrt{1 - \tau}$$

## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE





## VAPORIZATION OF A "DROPLET" UNDER SUPERCRITICAL PRESSURE



$r_0 \approx \sqrt{1-\tau}$  for small  $p$  ( $\beta \rightarrow \infty$ ) : the  $d^2$  law is recovered

INTRODUCTION

# The quasi-steady sub-critical case

$$\begin{cases}
 \dot{M} = 4\pi r^2 \dot{m}, & \dot{m} = \rho u \\
 \dot{M} \frac{dY_j}{dr} - 4\pi \rho D r \frac{d^2(rY_j)}{dr^2} = 0 \\
 \dot{M} c_p \frac{dT}{dr} - 4\pi k r \frac{d^2(rT)}{dr^2} = 0
 \end{cases}$$

## Boundary conditions

- at infinity  $r = \infty$  :  $Y_j = Y_{j\infty}, T = T_\infty$
- at the droplet surface  $r_s$  :  $Y_{jS}, \quad 4\pi \rho D r_s^2 \left. \frac{dY_j}{dr} \right|_S = -\dot{M} (Y_{jL} - Y_{jS})$
- $T_s, \quad 4\pi \frac{k}{c_p} r_s^2 \left. \frac{dT}{dr} \right|_S = \dot{M} \frac{\ell}{c_p}$

$$\begin{aligned}
 \dot{M} &= 4\pi \frac{k}{c_p} r_s \ln(1 + B_T) & B_T &= \frac{c_p (T_\infty - T_s)}{\ell} \\
 &= 4\pi \rho D r_s \ln(1 + B_M) & B_M &= \frac{Y_{FS} - Y_{F\infty}}{1 - Y_{FS}}
 \end{aligned}$$

$$r_s \frac{dr_s}{dt} = \frac{k}{\rho_L c_p} \ln(1 + B_T)$$

