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A double distribution lattice Boltzmann scheme for **unsteady Conjugate Heat Transfer : the DD-CHT LB method.**

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Abstract

In this paper, we propose an innovative formulation of the "well-known" thermal lattice Boltzmann method for solving transient conduction heat transfer problems in heterogeneous media : **the DD-CHT LB method for Double Distribution - Conjugate Heat Transfer**. The main idea of this new formulation is to bring out a second distribution function that represent the effect of thermal inertia and its variation in a heterogeneous media. By adopting this original formulation we achieved several goals, mainly conservation of local properties of the studied media and ensuring the stability of the proposed formulation by avoiding any alteration of the "well-known" TLBM collision step. Validation of the formulation is fulfilled through three different benchmark conduction heat transfer problems in heterogeneous media. All results are compared with those obtained by the Control Volume Method (CVM) and excellent agreements were found.

1. Introduction

Conjugate Heat Transfer (CHT) takes place when heat is transferred between two materials with different transport properties. The major difficulty when solving CHT is the necessity of satisfying two fundamental conditions: the temperature continuity and the normal heat flux continuity.

Generally when adopting conventional Computational Fluid Dynamic (CFD) solvers in solving CHT problems, extrapolations and iterative schemes have to be applied. As a result extra computational cost and complex codes are to be expected.

In earlier researches trying to solve CHT with the LB method, most of the proposed solutions are based on solving the convection-diffusion equation in heterogeneous media based

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on temperature as a dependent variable ([Chen and Han, 2000](#); [Meng et al., 2008](#); [Hu et al., 2015](#)). In these works precautions must be taken regarding tracking and correcting for the presence of interfaces. By doing so, the overall solution procedure become more complex especially in case of curved interfaces, moving or evolving interfaces over time.

Wang et al. ([Wang et al., 2017](#)) extended a previous work of Le et al. ([Le et al., 2015](#)) in order to include the interface discontinuity. By presenting their model they proposed a solution for treatment of discontinuity at interfaces. Their solution is opting for the Counter-extrapolation method which provides extrapolations of the computed transport scalar along the local normal direction of the conjugate interface and combining that with bilinear interpolations. Patel et al. ([Patel et al., 2016](#)) presented a spectral element discontinuous Galerkin thermal LB model for fluid-solid conjugate heat transfer problems. The authors employed a double distribution thermal LB model for solving flow field and CHT problems on unstructured non-uniform grids. The results presented are mostly at the steady state. Overall comparisons are made with finite difference and continuous Spectral elements solutions. Recently Pareschi et al. ([Pareschi et al., 2016](#)) proposed a CHT model based on the two populations entropic LB method. In their study, the authors chose the entropic multi-relaxation LB method and validated their formulation by solving a pure conduction transfer problem, a CHT problem with a backward facing step flow and finally tested their model with a three dimensional turbulent flow around a heated mounted cube.

Some other researchers opted for an enthalpy based LB formulation to recover the convection-diffusion equation under its enthalpic form. Lately Huang et al. ([Huang and Wu, 2016](#)) proposed a total enthalpy based MRT LB model associated with an adaptive mesh refinement near the solid-liquid phase change interface where thermodynamic properties vary significantly. Several melting problems in a square cavity with different values of Prandtl number and Rayleigh number were simulated. More recently Liu et al. ([Liu et al., 2017](#)) simulated solid-liquid phase change heat transfer problem in metal foam. Their formulation comes under the form of an enthalpic MRT LB model used with the local thermal non-equilibrium (LTNE) condition. The authors implicitly tracked the moving liquid-solid interface by mean of the liquid fraction expression.

Another attempt to solve CHT problems with the LB method was performed by Hamila et al. ([Hamila et al., 2017](#)). In their work, the authors proposed a general formulation of the convection-diffusion equation in multicomponent media at its enthalpic form. The main idea of the their work is to introduce an extra source term to the enthalpic form of the con-

vection diffusion equation. The same idea was also tested in the three dimensional case (Nouri et al., 2017).

In this paper we propose a completely different approach from what we mentioned above. The basic idea of the proposed formulation is a double distribution approach of thermal transfer, in other words the concept is an internal reformulation of the lattice Boltzmann equation. Without any further calculations of term source or specific interface tracking method, the proposed model aim to solve CHT problems in heterogeneous media with local properties conservation and stability of the model proposed.

2. Mathematical formulation

2.1. Thermal Lattice Boltzmann formulation

The Thermal Lattice Boltzmann formulation for heat transfer problems based on BGK collision operator takes the following form:

$$f_k(r + e_k \delta t, t + \delta t) = f_k(r, t) - \frac{\delta t}{\tau} [f_k(r, t) - f_k^{(eq)}(r, t)] \quad (1)$$

where f_k is the distribution function, denoting the number of particles at lattice node r and time t , τ is the relaxation time, $f_k^{(eq)}$ is the equilibrium distribution function, and δt is the time step. In two dimensional problems, when using the $D2Q9$ lattice scheme τ and $f_i^{(eq)}$ are as below :

$$\tau = \frac{2\alpha}{|e_k|^2} + \frac{\delta t}{2} \quad (2)$$

$$f_k^{(eq)} = w_k T(r, t) \left[1 + \frac{e_k \cdot U}{c_s^2} \right] \quad (3)$$

where $\alpha = \frac{k}{\rho C_p}$ is the thermal diffusivity, k and ρC_p are thermal conductivity and heat capacitance, respectively. The nine velocities $\vec{e}_k$ and their corresponding weights w_k in the $D2Q9$ lattice scheme are the following:

$$e_0 = (0, 0) \cdot U \quad e_{1,3} = (\pm 1, 0) \cdot U \quad e_{2,4} = (0, \pm 1) \cdot U; \quad e_{5,6,7,8} = (\pm 1, \pm 1) \cdot U \quad (4)$$

$$w_0 = \frac{4}{9}, \quad w_{1,2,3,4} = \frac{1}{9} \quad w_{5,6,7,8} = \frac{1}{36} \quad (5)$$

once the values of population f_k are known, temperature T is calculated as follows:

$$T(\vec{r}, t) = \sum_k f_k(\vec{r}, t) \quad (6)$$

Equation 1 based on Chapman-Enskog expansion and BGK collision model recover the following macroscopic equation

$$\frac{\partial T}{\partial t} = \nabla \cdot (\alpha \nabla T) \quad (7)$$

When analyzing the macroscopic energy equation recovered by thermal lattice Boltzmann formulation, we notice that the method can only provide correct results in homogenous media or in heterogenous media with identical heat capacity in either sides of the existing interfaces (Hamila et al., 2017; Karani and Huber, 2015). Indeed, as the distribution function is used to compute both the flux between points of the lattice and the temperature via equation (6), temperature and enthalpy are equal in this approach, which implicitly imposes $\rho C_p = 1$.

2.2. The double distribution lattice Boltzmann equation for unsteady heat conduction in heterogeneous media

In this paper we propose a more generalized formulation of the thermal LBM. The formulation allows to distinguish temperature and enthalpy when solving heat transfer problems. To do so, a second distribution function is added. This function is very simple to compute as it contains only one component. Through these changes we will be able to correctly simulate thermal transfer in heterogeneous media without further considerations of heat flux and temperature continuities at the interface.

Writing the energy equation in the enthalpy form is as following:

$$\frac{\partial h}{\partial t} = \nabla \cdot (k \nabla T) \quad (8)$$

with $h = \rho C_p T$, and k represent the conductivity.

By operating a slight change on Eq. 8, we aim to express this equation with temperature as a dependent variable. So by adding in both sides of equation $\frac{\partial T}{\partial t}$ the equation takes the

following form:

$$\underbrace{\frac{\partial(\rho C_p T - T)}{\partial t}}_{II} + \underbrace{\frac{\partial(T)}{\partial t}}_I = \nabla \cdot (k \nabla T) \quad (9)$$

Eq. 9 can be seen as the sum of two parts. Part I resolves the energy equation for a reference medium in which the volumetric heat capacity is equal to unity $\rho C_p = 1$ and Part II account for the actual thermal inertia.

In order to distinguish temperature and enthalpy, we introduce a second distribution g which contains a unique component g_0 with a zero velocity. g is defined to obtain the enthalpy :

$$\begin{aligned} T &= \sum f_i \\ h &= \rho C_p T = g_0 + \sum f_i \end{aligned} \quad (10)$$

These expressions tell us that a simple relation relies g_0 and $\sum f_i$:

$$g_0 = (\rho C_p - 1) \sum f_i \quad (11)$$

At each time-iteration, from t to $t + 1$, solving Part I using Eq. 1 accounts for the right heat flux, driven by the temperature field, but estimates a wrong temperature change as this equation assumes the heat capacity to be equal to unit $\rho C_p = 1$. Let's name the new temperature fields obtained after the streaming-collision steps $T^{*(t+1)} = \sum f_i^{*(t+1)}$. At this stage, g didn't change as the single component of this distribution is at rest and g is not involved in the collision step. For this reason, the set $g_0^{(t)}$ and $f_i^{*(t+1)}$ do not respect equation 13. A correction is therefore needed at constant enthalpy :

$$h^{(t+1)} = g_0^{(t)} + \sum_i f_i^{*(t+1)} = g_0^{(t+1)} + \sum_i f_i^{(t+1)} \quad (12)$$

Together with the definition of the enthalpy :

$$g_0^{(t+1)} = (\rho C_p - 1) \sum_i f_i^{(t+1)} \quad (13)$$

This leads to the following expression :

$$T^{(t+1)} = \beta \times T^{*(t+1)} \quad (14)$$

Where :

$$\beta = \frac{1 + g_0^{(t)} / T^{*(t+1)}}{\rho C_p} \quad (15)$$

This correction is applied to each component of distribution f (Fig. 1):

$$f_i^{(t+1)} = \beta \times f_i^{*(t+1)} \quad (16)$$

The value of g_0 at $t + 1$, $g_0^{(t+1)}$ is then updated using equation 12. The algorithm to be implemented is detailed hereafter :

Algorithm 1 The DD-CHT LBM algorithm

Initialize Lattice Boltzmann distribution $f_i^{(t=0)}$
Initialize thermal inertia $g_0^{(t=0)} \leftarrow (\rho C_p - 1) \times \sum_i f_i^{(t=0)}$
loop
 Streaming step¹⁾
 Collision step¹⁾
 Correct temperature distribution $f_i^{(t+1)} \leftarrow \beta \times f_i^{*(t+1)}$
 Update inertia distribution $g_0^{(t+1)} \leftarrow g_0^{(t)} + T^* \times (1 - \beta)$
end loop

¹⁾Boundary nodes need the same treatment as for the classical LBM.

3. Numerical simulations

3.1. Transient heat conduction in a two-layer medium with inclined interface

To demonstrate the ability of the proposed formulation to solve conjugate conduction heat transfer problems in heterogeneous media, a transient heat transfer problem in a two-layer medium with an inclined interface is proposed first. Initially the medium is uniformly at the initial temperature T_{int} . At ($t > 0$), a Dirichlet condition at the hot temperature T_0 is applied to all sides (Fig.2a). Simulation is carried out on a (250×250) mesh. The same problem was solved using the control volume method for validation purposes. Different sets of thermo-physical properties were tested for the two sub-domains 1 and 2 to obtain contrasted values

of the thermal diffusivity either due to the heat capacity or to the thermal conductivity. The values used are : $\rho C p_1 = 20$, $\rho C p_2 = 1$, $k_1 = 1$, $k_2 = 6$ for the first simulation (Fig.2b) and $\rho C p_1 = \rho C p_2 = 2$, $k_1 = 1$, $k_2 = 4$ for the second simulation (Fig.2c). The results obtained with the new LBM formulation and the CVM solutions are depicted as isotherms after 30 seconds. We also plotted in (Fig. 2d) and (Fig.2e) the dimensionless temperature profile along the diagonal for a set of selected times for case 1 and case 2, respectively. Excellent agreements were found for the two case studies.

3.2. Unsteady conduction heat transfer problem inside a square medium with a circular inclusion

The problem investigated in this part of the paper is a 2D unsteady conduction heat transfer problem inside a square medium with circular inclusion. The two mediums have different values of thermophysical properties. Initially the temperature of the entire medium is set to T_{ini} . At ($t > 0$), the left-hand side of the square medium is heated to T_0 while the other sides are kept insulated. In this test case we choose to perform two different simulations, in other word in the first simulation we imposed similar thermal conductivity values of the sub-domains and imposed non unitary ratio between the heat capacity's values, while in the second simulation we chose to impose non unitary ration between the heat capacity's values and between the thermal conductivity's values (Fig.3a) . Both simulations were carried out on a (150×150) mesh.

(Fig.3b -3c) present the results as computed with the new LBM method for the first simulation case. In this figure, the isotherm lines are compared with the same solution computed with a the control volumes method(CVM). We can see that excellent agreement are found both in transient period (750 s) and at steady-state.

In the second configuration, the conduction heat transfer problem is solved in a square medium with a circular inclusion, the values of thermophysical properties are $k_1/k_2 = 20$ and $\rho C p_2/\rho C p_1 = 20$. We intentionally selected the same ratio of properties to obtain the same thermal diffusivity even though the medium is heterogeneous. The first validation for this set of thermophysical properties is plotted as isotherms at $t=100s$ (Fig. 3d).

4. Conclusion

In this paper, a rigorous reformulation of the "well-known" thermal lattice Boltzmann method is detailed. The proposed approach aim to correctly simulate thermal transfer in heterogeneous media which obviously mean dealing with both different conductivities and heat capacities on both sides of the interface. Mainly the basic idea of the paper involve a second distribution function, with a single component, that allows distinguishing temperature and enthalpy in the TLBM algorithm. The new formulation offer an innovative method for treatment of unsteady conduction heat transfer problems in heterogeneous media. The method is presented as a novel LBM algorithm which is easy to implement and works without need to track the interface of the heterogeneities. After showing the ability of the presented formulation to simulate conduction heat transfer in homogeneous media, the method has been tested and validated with three unsteady conduction heat transfer problems in heterogeneous media.

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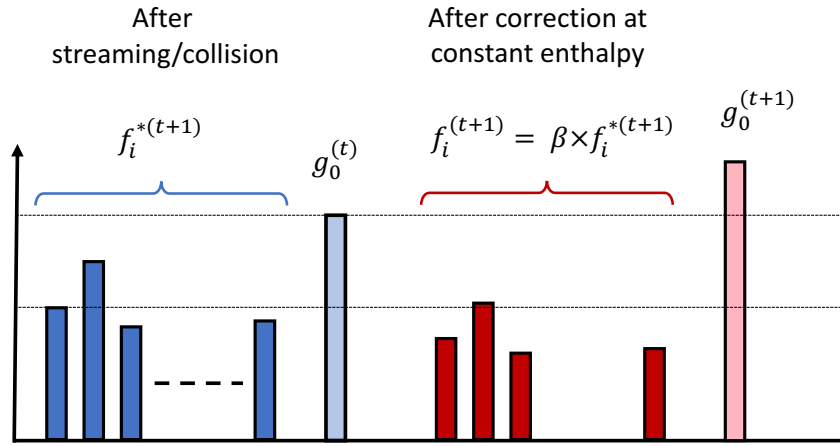


Figure 1: Modified distribution functions with the DD-CHT LBM algorithm.

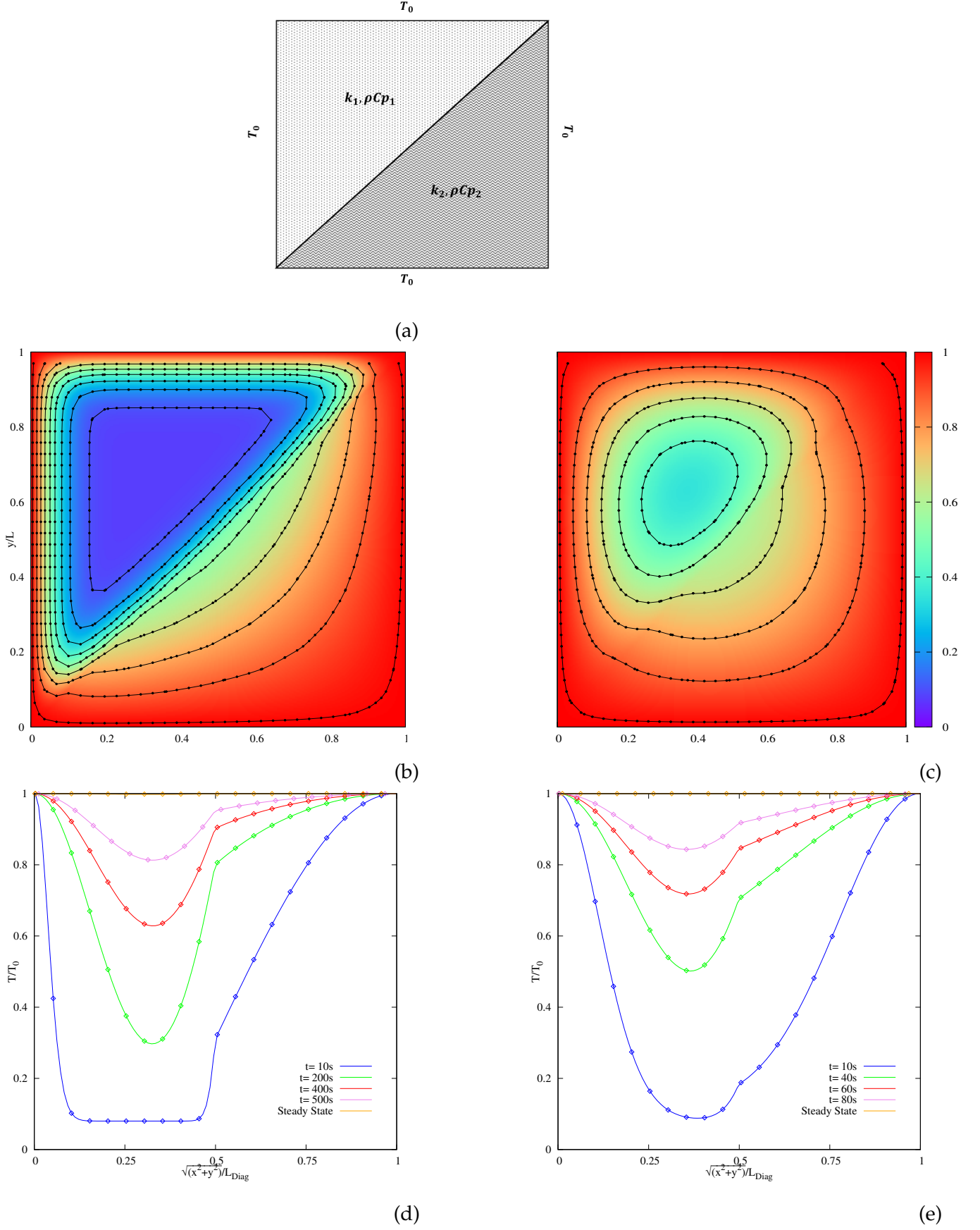


Figure 2: (a):Heterogeneous configuration; Isotherms computed at 30 seconds for two contrasted sub-dmain (b): $\rho C p_1 = 20, \rho C p_2 = 1, k_1 = 1, k_2 = 6$ and (c): $\rho C p_1 = \rho C p_2 = 2, k_1 = 1, k_2 = 4$; Dimensionless temperature profile along the diagonal at selected elapsed times for (d):case 1, (e):case 2; (symbols) for CVM and (solid line) for the DD-CHT of LBM.

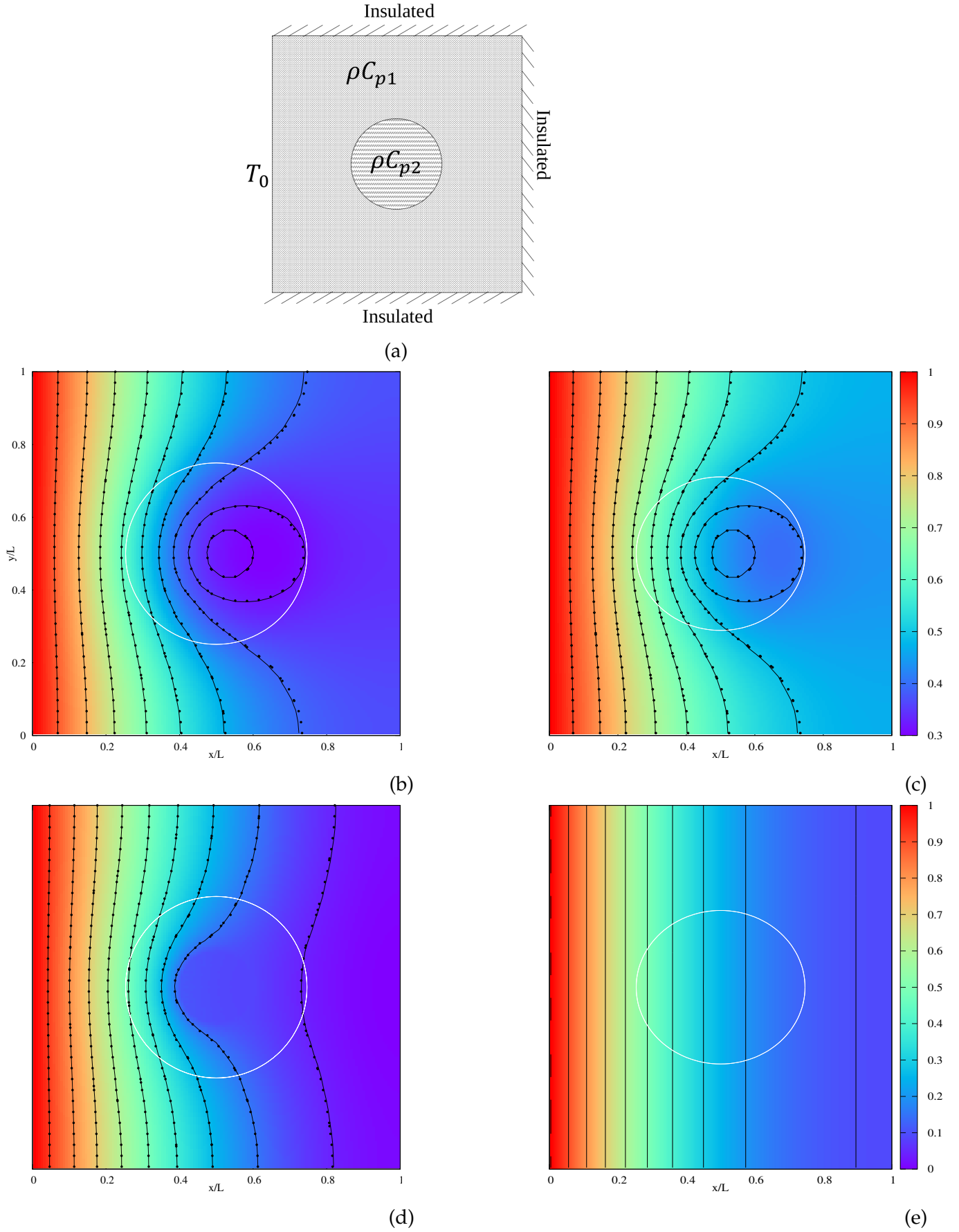


Figure 3: (a):heterogeneous media; Comparison of isotherms for (Configuration-1) $k_1/k_2 = 1$, $\rho C_{p2}/\rho C_{p1} = 250$ (b):Isotherm at $t=750s$, (c):Isotherm at steady state; for (Configuration-2): $k_1/k_2 = 20$, $\rho C_{p2}/\rho C_{p1} = 20$ at $t=100s$ (d):Isotherm computed with DD-CHT-CVM (e):Isotherm computed with LBM-CVM - for Fig.3 (b),(c),(d):(symbols) stands for CVM and (solid line) for the DD-CHT solution,and for Fig.3(e):(solid line)stands for conventional LBM