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ON ROBUST STABILITY VIA COMPLETE LYAPUNOV KRASOVSKII FUNCTIONAL

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Abstract: Stability of linear systems with norm-bounded uncertainties and uncertain time-varying delays is considered. The delays are supposed to be bounded and fast-varying (without any constraints on the delay derivative). Sufficient stability conditions are derived via complete Lyapunov-Krasovskii functional (LKF). A new LKF construction, which was recently introduced for systems with uncertain delays, is extended to the case of norm-bounded uncertainties: to a nominal LKF, which is appropriate to the system with the nominal value of the coefficients and of the delays, terms are added that correspond to the perturbed system and that vanish when the uncertainties approach 0. Numerical examples illustrate the efficiency of the method.

Keywords: uncertain delay, time-varying delay, norm-bounded uncertainties, Lyapunov-Krasovskii functional

1. INTRODUCTION

The stability and control of time-delay systems is a subject of recurring interest, and a lot of research has been devoted to the field in the last decade using both frequency- and time-domain methods. Most of the results devoted to the the robust stability of systems with norm-bounded uncertainties and uncertain delays consider as assumption the *stability* of the *system free of delays*, and next, in the time-domain, use appropriate Lyapunov-Razumikhin functions or Lyapunov-Krasovskii functionals combined with Linear Matrix Inequalities (LMIs) to derive some bounds on the delay values μ_i (finite), such that the uncertain system will be stable for all delays intervals of the form $[0, \mu_i]$. Without any loss of generality, we can define such a case as stability characterization of *uncertain 'small' delays* (see e.g. Li & de Souza, 1997, Moon *et al.*, 2001, Fridman & Shaked, 2002).

Systems with uncertain '*non-small*' delays, where the *nominal delay values are non-zero* and constant appears in different applications such as high-speed networks, biological systems (Kolmanovskii & Myshkis, 1999). Such systems may be not stable for the zero values of the delays and thus their stability can not be analyzed via *simple* LKFs (Gu, Kharitonov & Chen, 2003). The analysis in such case becomes largely more complicated than in the 'small' delay case.

Only a few works have been devoted to the stability analysis of such systems. Thus, for example, the Lyapunov-based methods have been developed in the case of known constant delays and norm-bounded uncertainties (Kharitonov & Zhabko, 2003), (Mondie *et al.*, 2005) or in the case of uncertain '*non-small*' delays and known coefficients (Kharitonov & Niculescu, 2003). Robust stability of uncertain systems with '*non-small*' de-

lay has been analyzed also in the frequency domain (Gu *et al.*, 2003; Kao & Lincoln, 2004). However, only Lyapunov-based methods can be applied to the problems, where the knowledge of the initial function is important (see e.g. Tarbouriech & Gomes da Silva, (2000) for application of LKF to the estimate on the domain of attraction of the nonlinear system, modeled as a linear uncertain system).

Recently, a new construction of LKF for stability analysis of systems with uncertain delays was suggested (Fridman, 2004): to a nominal LKF, which is appropriate to the nominal system (with nominal delays and coefficients), the terms are added which correspond to the perturbed system and which vanish when the uncertainties approach 0. In (Fridman, 2004) the simple descriptor nominal LKF was considered. In (Fridman, 2006) such LKF construction was extended to the complete nominal LKF. Unlike the existing complete LKFs (see e.g. Repin, 1965; Datko, 1971; Huang, 1989; Kharitonov & Zhabko, 2003; Kharitonov & Niculescu, 2003), the derivative of the complete nominal LKF of (Fridman, 2006) along the trajectories of the nominal system depends on the state and the *state derivative* which allows a less conservative treatment of the *delay perturbation*.

To the best of our knowledge, the stability of the systems with both, norm-bounded uncertainties and uncertain non-small delays, has not been studied yet via complete LKF. Note that the discretized Lyapunov functional method (Gu *et al.*, 2003), which gives the sufficient conditions only, can not always be applicable. In the present paper we provide stability analysis of such uncertain systems via complete LKF. We extend the construction of LKF started in (Fridman, 2006) to the case of norm-bounded uncertainty. In the case of known constant delays, the derivative of the complete nominal LKF along the trajectories of the nominal system depends only on the state. The results for this case are simpler and less restrictive than the existing ones. The numerical examples illustrate the efficiency of the method.

Notation: Throughout the paper the superscript ‘ T ’ stands for matrix transposition, \mathcal{R}^n denotes the n dimensional Euclidean space with vector norm $|\cdot|$, $\mathcal{R}^{n \times m}$ is the set of all $n \times m$ real matrices, and the notation $P > 0$, for $P \in \mathcal{R}^{n \times n}$ means that P is symmetric and positive definite. The symmetric elements of the symmetric matrix will be denoted by $*$.

2. PROBLEM FORMULATION

We consider the following linear system with uncertain coefficients and uncertain time-varying delays $\tau_i(t)$ ($i=1,2$):

$$\begin{aligned} \dot{x}(t) = & (A_0 + H\Delta E_0)x(t) \\ & + \sum_{i=1}^2 (A_i + H\Delta E_i)x(t - \tau_i(t)), \end{aligned} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the system state, $A_0, A_1, A_2, H, E_0, E_1$ and E_2 are constant matrices of appropriate dimensions and $\Delta(t)$ is a time-varying uncertain matrix that satisfies

$$\Delta(t)^T \Delta(t) \leq I. \quad (2)$$

The uncertain delays $\tau_i(t)$ are supposed to have the following form:

$$\tau_i(t) = h_i + \eta_i(t), \quad i = 1, 2, \quad h_1 > 0, h_2 = 0, \quad (3)$$

where h_i are nominal constant values and $\eta_i(t)$ are time-varying perturbations satisfying the following inequalities: $|\eta_1(t)| \leq \mu_1 \leq h_1, 0 \leq \eta_2(t) \leq \mu_2$ with the known upper bounds μ_1 and μ_2 .

We derive stability sufficient conditions via Lyapunov-Krasovskii technique. As suggested in (Fridman, 2004) we consider the following form of LKF:

$$V = V_n + V_a, \quad (4)$$

where V_n is a nominal LKF which corresponds to the nominal system

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^2 A_i x(t - h_i), \quad (5)$$

and V_a consists of additional terms and depends on $\mu_i, H, E_i (i = 1, 2)$ and $V_a \rightarrow 0$ for $\mu_1 + \mu_2 \rightarrow 0, H \rightarrow 0, E_1 \rightarrow 0, E_2 \rightarrow 0$. The latter will guarantee that if the conditions for the stability of the nominal system are feasible, then the stability conditions for the perturbed system will be feasible for small enough delay perturbations and norm-bounded uncertainties.

For the nominal system in the present paper we choose the complete LKF. The results are easily generalized to the case of a finite number of small uncertain delays.

3. MAIN RESULTS

We assume that $h_2 = 0$ and that the nominal system (5) is asymptotically stable. Then (Fridman, 2006) there exists the nominal ‘complete’ LKF $V_n(x_t)$ such that $V_n(x_t) > \epsilon |x(t)|^2, \epsilon > 0$ and along the trajectories of the nominal system (5)

$$\dot{V}_n = -x^T(t)W_0x(t) - \dot{x}^T(t)W_1\dot{x}(t). \quad (6)$$

It has the following form:

$$\begin{aligned}
V_n(\phi) &= \phi^T(0)U(0)\phi(0) \\
&+ 2\phi^T(0) \int_0^0 U(h_1 + \theta)A_1\phi(\theta)d\theta \\
&+ \int_{-h_1}^0 \int_{-h_1}^0 \phi^T(\theta_2)A_1^T U(\theta_2 - \theta_1)A_1\phi(\theta_1)d\theta_1 d\theta_2 \\
&+ \bar{V}_n,
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
U(\theta) &= U_0(\theta) + U_1(\theta), \quad \theta \in R, \\
U_0(\theta) &= \int_0^\infty K^T(t)W_0K(t + \theta)dt, \quad \theta \in R, \\
U_1(\theta) &= \int_0^\infty \dot{K}^T(t)W_1\dot{K}(t + \theta)dt, \quad \theta \in R,
\end{aligned} \tag{8}$$

$$\begin{aligned}
\bar{V}_n &= \int_{-h_1}^0 \phi^T(\theta_2)A_1^T W_1\{A_1\phi(\theta_2) \\
&+ 2[A_0e^{A_0(\theta_2+h_1)}\phi(0) \\
&+ \int_{-h_1}^{\theta_2} A_0e^{A_0(\theta_2-\theta_1)} A_1\phi(\theta_1)d\theta_1]\}d\theta_2.
\end{aligned}$$

Here $K(t)$ is a fundamental matrix associated with the nominal system (5), i.e. $K(t)$ is an $n \times n$ -matrix function satisfying

$$\dot{K}(t) = (A_0 + A_2)K(t) + A_1K(t - h_1), \quad t \geq 0,$$

with the initial condition $K(0) = I$ and $K(t) = 0$ for $t < 0$.

Similarly to (Kharitonov & Niculescu, 2003), (Fridman, 2006) we represent the perturbed system in the form:

$$\begin{aligned}
\dot{x}(t) &= (A_0 + A_2 + H\Delta(E_0 + E_2))x(t) \\
&+ (A_1 + H\Delta E_1)x(t - h_1) - (A_1 + H\Delta E_1) \times \\
&\times \int_{t-h_1-\eta_1}^{t-h_1} \dot{x}(s)ds - (A_2 + H\Delta E_2) \int_{t-\eta_2}^t \dot{x}(s)ds.
\end{aligned} \tag{9}$$

Differentiating V_n along the trajectories of (9), we find

$$\begin{aligned}
\dot{V}_n(x_t) &= -x^T(t)W_0x(t) - \dot{x}^T(t)W_1\dot{x}(t) \\
&+ \sum_{i=0}^4 \delta_i(t),
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
\delta_0(t) &= -2 \int_{t-h_1-\eta_1}^{t-h_1} \dot{x}^T(s)A_1^T[U(0)x(t) \\
&+ \int_{-h_1}^0 Q^T(h_1 + \theta)A_1x(t + \theta)d\theta]ds,
\end{aligned}$$

$$Q(h_1 + \theta) = U(h_1 + \theta) + W_1A_0e^{A_0(h_1+\theta)}, \tag{11}$$

$$\begin{aligned}
\delta_1(t) &= -2 \int_{t-\eta_2(t)}^t \dot{x}^T(s)A_2^T[U(0)x(t) \\
&+ \int_{-h_1}^0 Q^T(h_1 + \theta)A_1x(t + \theta)d\theta]ds,
\end{aligned}$$

$$\begin{aligned}
\delta_2(t) &= 2[x^T(t)(E_0^T + E_2^T) \\
&+ x^T(t - h_1)E_1^T]\Delta^T H^T[U(0)x(t) \\
&+ \int_{-h_1}^0 Q^T(h_1 + \theta)A_1x(t + \theta)d\theta]ds,
\end{aligned}$$

$$\begin{aligned}
\delta_3(t) &= -2 \int_{t-h_1-\eta_1(t)}^{t-h_1} \dot{x}^T(s)E_1^T\Delta^T H^T[U(0)x(t) \\
&+ \int_{-h_1}^0 Q^T(h_1 + \theta)A_1x(t + \theta)d\theta]ds,
\end{aligned}$$

$$\begin{aligned}
\delta_4(t) &= -2 \int_{t-\eta_2(t)}^t \dot{x}^T(s)E_2^T\Delta^T H^T[U(0)x(t) \\
&+ \int_{-h_1}^0 Q^T(h_1 + \theta)A_1x(t + \theta)d\theta]ds.
\end{aligned}$$

By applying standard bounding, for $n \times n$ -matrices $R_1 > 0, R_{11} > 0$ and scalars $r_2 > 0, r_{12} > 0, \rho_{ij} > 0, i = 1, \dots, 4, j = 1, 2$ the following is obtained:

$$\begin{aligned}
\delta_0(t) &\leq | \int_{t-h_1-\eta_1(t)}^{t-h_1} \dot{x}^T(s)A_1^T(R_1^{-1} + r_2^{-1}I)A_1\dot{x}(s)ds | \\
&+ | \int_{t-h_1-\eta_1(t)}^{t-h_1} x^T(t)U(0)R_1U(0)x(t)dt | \\
&+ r_2 | \int_{t-h_1-\eta_1(t)}^{t-h_1} \int_{-h_1}^0 x^T(t + \theta)A_1^T Q(h_1 + \theta) \\
&\times Q^T(h_1 + \theta)A_1x(t + \theta)d\theta ds | \\
&\leq \int_{t-h_1+\mu_1}^{t-h_1} \dot{x}^T(s)A_1^T(R_1^{-1} + r_2^{-1}I)A_1\dot{x}(s)ds \\
&+ \mu_1 x^T(t)U(0)R_1U(0)x(t) \\
&+ \mu_1 r_2 \int_{-h_1}^0 x^T(t + \theta)A_1^T Q(h_1 + \theta) \\
&\times Q^T(h_1 + \theta)A_1x(t + \theta)d\theta,
\end{aligned} \tag{12}$$

$$\begin{aligned}
& \delta_1(t) \\
& \leq \int_{t-\mu_2}^t \dot{x}^T(s) A_2^T (R_{11}^{-1} + r_{12}^{-1} I) A_2 \dot{x}(s) ds \\
& + \mu_2 x^T(t) U(0) R_{11} U(0) x(t) \\
& + \mu_2 r_{12} \int_{-h_1}^0 x^T(t+\theta) A_1^T Q(h_1+\theta) \\
& \times Q^T(h_1+\theta) A_1 x(t+\theta) d\theta,
\end{aligned} \tag{13}$$

$$\begin{aligned}
& \delta_2(t) \\
& \leq (\rho_{11} + \rho_{12}) x^T(t) U^T(0) H H^T U(0) x(t) \\
& + (\rho_{21} + \rho_{22}) \int_{-h_1}^0 x^T(t+\theta) A_1^T Q(h_1+\theta) H \\
& \times H^T Q^T(h_1+\theta) A_1 x(t+\theta) d\theta \\
& + (\rho_{11}^{-1} + \rho_{21}^{-1}) x^T(t) (E_0 + E_2)^T \\
& \times (E_0 + E_2) x(t) \\
& + (\rho_{12}^{-1} + \rho_{22}^{-1}) x^T(t-h_1) E_1^T E_1 x(t-h_1),
\end{aligned} \tag{14}$$

$$\begin{aligned}
& \delta_3(t) \leq \rho_{31} \mu_1 x^T(t) U^T(0) H H^T U(0) x(t) \\
& + (\rho_{31}^{-1} + \rho_{32}^{-1}) \int_{t-h_1-\mu_1}^{t-h_1+\mu_1} \dot{x}^T(s) E_1^T E_1 \dot{x}(s) \\
& + \rho_{32} \mu_1 \int_{-h_1}^0 x^T(t+\theta) A_1^T Q(h_1+\theta) H \\
& \times H^T Q^T(h_1+\theta) A_1 x(t+\theta) d\theta,
\end{aligned} \tag{15}$$

$$\begin{aligned}
& \delta_4(t) \leq \rho_{41} \mu_2 x^T(t) U^T(0) H H^T U(0) x(t) \\
& + (\rho_{41}^{-1} + \rho_{42}^{-1}) \int_{t-\mu_2}^t \dot{x}^T(s) E_2^T E_2 \dot{x}(s) ds \\
& + \rho_{42} \mu_2 \int_{-h_1}^0 x^T(t+\theta) A_1^T Q(h_1+\theta) H \\
& \times H^T Q^T(h_1+\theta) A_1 x(t+\theta) d\theta.
\end{aligned} \tag{16}$$

We choose

$$\begin{aligned}
V(x_t) &= V_n(x_t) + \sum_{i=1}^3 V_{a_i}(x_t), \\
V_{a1}(x_t) &= (\rho_{12}^{-1} + \rho_{22}^{-1}) \int_{t-h_1}^t x^T(s) E_1^T E_1 x(s) ds, \\
V_{a2}(x_t) &= \int_{-h_1}^{\mu_1} \int_{t-h_1}^t \dot{x}^T(s) [A_1^T (R_1^{-1} + r_2^{-1} I) A_1 \\
& + E_1^T (\rho_{31}^{-1} + \rho_{32}^{-1}) E_1] \dot{x}(s) ds d\theta, \\
& + \int_0^t \int_{-h_1}^t \dot{x}^T(s) [A_2^T (R_{11}^{-1} + r_{12}^{-1} I) A_2 \\
& + E_2^T (\rho_{41}^{-1} + \rho_{42}^{-1}) E_2] \dot{x}(s) ds d\theta, \\
V_{a3}(x_t) &= \int_{-h_1}^0 \int_{t+\theta}^t x^T(s) A_1^T Q(h_1+\theta) [(\mu_1 r_2 \\
& + \mu_2 r_{12}) I + H H^T (\rho_{21} + \rho_{22} + \mu_1 \rho_{32} \\
& + \mu_2 \rho_{42})] Q^T(h_1+\theta) A_1 x(s) ds d\theta.
\end{aligned}$$

where V_n is defined by (7), we obtain that $\dot{V} < 0$ if the following inequalities are satisfied:

$$\begin{bmatrix} \Phi & E_0^T + E_2^T & E_0^T + E_2^T & E_1^T & E_1^T \\ * & -\rho_{11} I & 0 & 0 & 0 \\ * & * & -\rho_{21} I & 0 & 0 \\ * & * & * & -\rho_{12} I & 0 \\ * & * & * & * & -\rho_{22} I \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} \Psi & 2\mu_1 E_1^T & 2\mu_1 E_1^T & \mu_2 E_2^T & \mu_2 E_2^T \\ * & 0 & 0 & 0 & 0 \\ * & -2\mu_1 \rho_{31} I & 0 & 0 & 0 \\ * & * & -2\mu_1 \rho_{32} I & 0 & 0 \\ * & * & * & -\rho_{41} I & 0 \\ * & * & * & * & -\rho_{42} I \end{bmatrix} < 0, \tag{18}$$

where

$$\begin{aligned}
\Psi &= \begin{bmatrix} -W_1 & 2\mu_1 A_1^T & 2\mu_1 A_1^T & \mu_2 A_2^T & \mu_2 A_2^T \\ * & -2\mu_1 R_1 & 0 & 0 & 0 \\ * & * & -2\mu_1 r_2 I & 0 & 0 \\ * & * & * & -\mu_2 R_{11} & 0 \\ * & * & * & * & -\mu_2 r_{12} I \end{bmatrix}, \\
\Phi &= -W_0 + [\rho_{11} + \rho_{12} + \mu_1 \rho_{31} \\
& + \mu_2 \rho_{41}] U^T(0) H H^T U(0) \\
& + U^T(0) (\mu_1 R_1 + \mu_2 R_{11}) U(0) \\
& + [\mu_1 r_2 + \mu_2 r_{12}] A_1^T Q A_1 \\
& + [\rho_{21} + \rho_{22} + \mu_1 \rho_{32} + \mu_2 \rho_{42}] A_1^T Q_H A_1, \\
\mathcal{Q} &= \int_{-h_1}^0 Q(h_1+\theta) Q^T(h_1+\theta) d\theta, \\
\mathcal{Q}_H &= \int_{-h_1}^0 Q(h_1+\theta) H H^T Q^T(h_1+\theta) d\theta.
\end{aligned} \tag{19}$$

We have proved the following

Theorem 1. Assume that $h_2 = 0$ and that the nominal system (5) is asymptotically stable. Let $U(\theta), Q(\theta), \theta \in [0, h]$, \mathcal{Q} and \mathcal{Q}_H be defined by (8), (11), (19). Then (1) is asymptotically stable for all piecewise-continuous delays $|\eta_1(t)| \leq \mu_1 \leq h_1, 0 \leq \eta_2(t) \leq \mu_2$ if there exist $n \times n$ matrices W_0, W_1, R_1, R_{11} and scalars $r_2, r_{12}, \rho_{ij}, i = 1, \dots, 4, j = 1, 2$ that satisfy (17), (18).

In the case when delays are known ($\eta_1 = 0, \eta_2 = 0$), we choose $W_1 = 0$. Here $\delta_0 = \delta_1 = \delta_3 = 0$ and we obtain the following:

Corollary 2. Assume that $h_2 = 0, \eta_1 = \eta_2 = 0$ and that the nominal system (5) is asymptotically stable. Let

$$\begin{aligned}
U(\theta) &= U_0(\theta) \\
&= \int_0^\infty K^T(t) W_0 K(t+\theta) dt, \theta \in \mathbb{R},
\end{aligned} \tag{20}$$

\mathcal{Q} and \mathcal{Q}_H be given by (19), where $Q(\theta) = U_0(\theta), \theta \in [0, h]$. Then (1) is asymptotically stable if there exist a $n \times n$ -matrix W_0 and scalars $\rho_{ij}, i = 1, 2, j = 1, 2$ that satisfy (17), where

$$\begin{aligned}
\Phi &= -W_0 + [\rho_{11} + \rho_{12}] U_0^T(0) H H^T U_0(0) \\
& + [\rho_{21} + \rho_{22}] A_1^T Q_H A_1.
\end{aligned}$$

In the case when the delays are uncertain, but system coefficients are known ($\Delta = 0$ or $H = 0$, $E_i = 0$, $i = 0, 1, 2$), Theorem 1 implies the result of (Fridman, 2006).

Example 1: (Kharitonov & Niculescu, 2003) Consider (1) with

$$A_0 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}, H = I, (21)$$

$$E_i = e_i I, i = 0, 1, e_i \in \mathbb{R}, \tau_1 \equiv 1,$$

where $A_2 = E_2 = 0$. It was found in (Mondie *et al.* 2005) that the system is robustly stable for $e_0 = 0.016$ and $e_1 = 0.02$. The latter bounds on the perturbations of the system matrices were less conservative than those given in (Kharitonov & Zhabko, 2003). The improvement was achieved due to the cross term which was inserted into the time derivative of the complete LKF. By applying a simpler nominal complete LKF of (7), (6), where $W_0 = I$ and $W_1 = 0$ and Corollary 2, we obtain a larger stability margins $e_0 = 0.04$ and $e_1 = 0.05$. The result is less conservative because of the proper construction of LKF in the form of (4), where (similar to simple LKFs) the terms of V_a compensate the perturbations (and not complete LKF itself).

Example 2: Consider (1) with

$$A_1 = \begin{bmatrix} 0 & 0 \\ -0.4 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}, H = I, (22)$$

$$A_0 = E_0 = 0, E_i = e_i I, i = 1, 2, e_i \in \mathbb{R}.$$

which was analyzed in (Kharitonov & Niculescu, 2003) for $\tau_2 = 0$. The nominal non-delayed system (i.e. (22) with $\tau_1 = \tau_2 = 0$ and $H = 0$) is not asymptotically stable and thus simple nominal LKFs are not applicable. For the case of constant delay $\tau_1 = 4 + \eta_1$ and $e_1 = e_2 = 0$ the following stability interval was found by the frequency domain analysis (Kharitonov & Niculescu, 2003): $\tau_2 = 0$, $-0.6209 < \eta_1(t) < 0.7963$.

For $h_1 = 4, h_2 = 0, \Delta = 0$ the following stability interval was found in (Fridman, 2006) by using complete LKF of (7), (6), where $W_0 = W_1 = I$: $0 \leq \eta_2(t) \leq 0.002, |\eta_1(t)| \leq 0.002$.

By Theorem 1 for $e_1 = e_2 = 10^{-4}$ we verify that the system is asymptotically stable for $\eta_2 = 0, |\eta_1(t)| \leq 0.01$ and for $0 \leq \eta_2(t) \leq 0.001, |\eta_1(t)| \leq 0.001$.

4. CONCLUSIONS

Stability of linear retarded type system with uncertain time-varying delays and norm-bounded uncertainties is analyzed via complete LKF. A new Lyapunov functional construction, which

was recently introduced for systems with uncertain delays, is extended to the case of norm-bounded uncertainties. This leads to effective method for robust stability of linear uncertain systems with time-varying delays in the cases, where other Lyapunov-based methods are not applicable. Moreover, the new results improve and simplify the existing ones.

References

- Datko, R. (1971). An algorithm for computing Lyapunov functionals for some differential-difference equations. In L. Weiss (Ed.), *Ordinary differential equations*, Academic Press, 387-398.
- Fridman, E. (2001). New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems. *Systems & Control Letters*, 43, 309-319.
- Fridman, E. (2004). Stability of linear functional differential equations: A new Lyapunov technique. In *Proceedings of MTNS 2004, Leuven*.
- Fridman, E. (2006). Stability of systems with uncertain non-small delay: a new "complete" Lyapunov-Krasovskii". *IEEE Trans. on Automat. Contr.*, 51, No.5, 2006.
- K. Gu, V. Kharitonov and J. Chen (2003), *Stability of time-delay systems*. Birkhauser: Boston.
- Hale, J. & Lunel, S. (1993). *Introduction to functional differential equations* (New York: Springer-Verlag).
- Huang, W. (1989). Generalization of Lyapunov's theorem in a linear delay system. *J. of Math. Analysis and Appl.*, 142, 83-94.
- Kao CY, Lincoln B. (2004). Simple stability criteria for systems with time-varying delays. *Automatica* 40, 1429-1434.
- Kharitonov, V. & Niculescu, S. (2003). On the stability of linear systems with uncertain delay. *IEEE Trans. on Automat. Contr.*, 48, 127-132.
- Kharitonov, V. & Zhabko, A. (2003). Lyapunov-Krasovskii approach to the robust stability analysis of time-delay systems. *Automatica*, 39, 15-20.
- Kolmanovskii, V. & Myshkis, A. (1999). *Applied Theory of functional differential equations*, Kluwer.
- Li, X. & C. de Souza (1997). Criteria for robust stability and stabilization of uncertain linear systems with state delay. *Automatica*, 33, 1657-1662.
- Mondie, S., Kharitonov, V. & Omar Santos (2005). Complete Lyapunov-Krasovskii functional with a given cross term in the time derivative, In: *Proc. of*

- 44th Conf. on Decision and Control*,
Sevilla, Spain, 2005, 5060-5064.
- Niculescu, S.I. (2001). *Delay effects on stability: A Robust Control Approach*, Lecture Notes in Control and Information Sciences, 269, Springer-Verlag, London, 2001.
- Repin, Y. (1965). Quadratic Lyapunov functionals for systems with delay. *Prikladnaya Matematika Mehanika*, 29, 564-566 (in Russian).
- Tarbouriech S. & Gomes da Silva, J. (2000). Synthesis of Controllers for Continuous-Time Delay Systems with Saturating Controls via LMI's. *IEEE Trans. Automat. Contr.*, 45, No. 1, 105-111.