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PREDICTED EFFECTS OF FLUID LOADING ON THE VIBRATION OF ELASTIC POROUS PLATES

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ABSTRACT
The effects of fluid loading on the vibration of rectangular, clamped, porous elastic plates and on their radiated sound power are considered. This requires the inclusion of an extra term in the equations for the plate vibration, corresponding to the additional external force acting on the plate. As is the case for vibrating non-porous plates, fluid-structure coupling is a very complex phenomenon since the plate modes are coupled by the fluid. The radiation impedance matrix, including direct terms and cross-coupling terms, has been defined and computed. A Gaussian quadrature scheme including twenty terms of the Legendre polynomial has been used to compute the fluid loaded plate deflection for four types of porous and elastic plate and compare the effects of the loading by water and air. The corresponding vibroacoustic indicators including mean square velocity, radiated sound power and radiation efficiency, have been calculated also. For some plate and fluid parameters, the predicted effects of fluid loading are considerable.

1. INTRODUCTION
The vibrations of fluid loaded, infinite non-porous plates have been studied extensively [1-5]. Sandman [5] has solved the governing equations of motion for the normal modes of free vibration of a fluid loaded, simply supported non-porous plate carrying a concentrated mass and has presented examples to illustrate the basic characteristics of the fluid-loaded vibration and radiation from a plate with such a discrete mass discontinuity.

Berry [6] has studied the vibration and sound radiation of fluid-loaded plates with elastic boundary conditions. Berry’s approach is based on a variational formulation of the fluid-loaded plate motion and a Rayleigh-Ritz method using polynomial displacement functions. The selection of such polynomial functions leads to a new method of solution for the radiation impedance of the plate. Theodorakopoulos and Beskos [7] have given a rigorous formulation for vibration of a simply supported rectangular plate. Leclaire [8] has developed this formulation to derive a set of equations corresponding to any type of plate (clamped, simply supported and free). Foin [9] has evaluated the acoustic pressure exerted by fluid on a baffled plate by using the Rayleigh integral.

Davies [10] and Pope and Leibowitz [11] have calculated the radiation impedance matrix for a fluid-loaded simply supported non-porous plate. The coefficients of this matrix express the cross-coupling of in vacuo modes by the fluid. The real part of the radiation impedance matrix is the radiation resistance and expresses the radiation damping of the structure. The imaginary part of the radiation impedance matrix is the radiation reactance and expresses the added mass of fluid on the structure. Berry [6] has calculated the radiation impedance matrix in terms of simple trial displacement functions. The radiation impedance matrix has been rigorously calculated by Foin [9], Sandman [4] and Nelisse [12]. Sandman and Nelisse have reduced the resulting quadruple integral into a double integral by using a specific change of variable. They integrated the double integral numerically by means of Gaussian quadrature.

The effects of the fluid loading on the vibration of a rectangular, clamped porous and elastic plate and on the radiated sound power are investigated here. The effect of fluid loading can be incorporated by inserting an extra term in the equations of the plate, corresponding to an additional external force acting on the plate. This leads to the calculation of radiation impedance matrices with non-negligible cross terms. The radiation impedance matrix for a plate has been predicted directly by simply solving the equation of the coefficients of the radiation impedance matrix [9] without interpolation, convergence and reducing the quadruple integral to a double integral with the direct terms and cross-coupling terms [13]. A Gaussian quadrature scheme including twenty terms of the Legendre polynomial has been used to compute the fluid loaded plate deflection.
2. EQUATIONS FOR THE POROUS PLATE MATION

A thin, baffled porous plate of dimension $a \times b$ and uniform thickness $h$, excited by a force density $F(x, y, t)$ is considered. The geometry of a porous plate is shown in Figure 1.

$$z > 0, \text{fluid}$$

$$z < 0, \text{vacuum}$$

![Figure1: The geometry of a baffled porous plate.](image)

The bending vibration of a rectangular porous plate of thickness $h$ is given by equations [8]:

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( W_{mn}^s \left[ D + \frac{\alpha^2 M h^3}{12} \left( I_1 I_2 + 2 I_3 I_4 + I_5 I_6 \right) - h \rho \omega^2 I_2 I_6 \right] - W_{mn} h \rho_f \omega^2 I_2 I_6 \right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} I_2 I_6,$$

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( W_{mn} \left[ \alpha M h (I_1 I_3 + I_4 I_6) + h \rho_f \omega I_2 I_6 \right] + W_{mn} h m (\omega) \omega^2 I_2 I_6 \right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Delta P_{mn} I_2 I_6. \quad (1)$$

where $D$ is the bending stiffness of the plate, $\alpha$ and $M$ are Biot’s elastic coefficients [14] that can be related to known or measurable elastic constants, $W_{mn}^s$ is the plate lateral displacement, $W_{mn}$ is the fluid solid relative displacement in the pores, $\rho$ is the bulk density of the solid-fluid mixture and $\rho_f$ is the density of the fluid. The coefficient $m(\omega)$ is a frequency dependent mass [15] which accounts for the viscous friction between the fluid and the solid. This coefficient depends on the porosity, permeability and tortuosity. The first equation corresponds to the instantaneous elastic response of the plate and the second equation describes the relative motion between the solid and the fluid, including energy losses by viscous friction [16].

The simple definite integrals $(I_1)-(I_6)$ are given by:

$$I_1 = \int_0^a X_m^{(IV)}(x) X_m(x) dx, \quad I_6 = \int_0^a X_m(x) X_m(x) dx, \quad I_3 = \int_0^a X_m^{(II)}(x) X_m(x) dx,$$

$$I_5 = \int_0^b Y_n^{(IV)}(y) Y_n(y) dy, \quad I_2 = \int_0^b Y_n(y) Y_n(y) dy, \quad I_4 = \int_0^b Y_n^{(II)}(y) Y_n(y) dy. \quad (2)$$

The exponents $(IV)$ and $(II)$ denote the fourth and second order derivative, respectively. By solving equation (1) for $m$ and $n$ modes, the coefficients $W_{mn}^s$ and $W_{mn}$ can be found. Hence the deflection of a porous plate can be calculated from;

$$w_s(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn}^s X_m(x) Y_n(y), \quad w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} X_m(x) Y_n(y). \quad (3)$$
The beam functions $X_n(x)$ and $Y_n(y)$ are given by:

$$X_n(x) = B_{m_1} \cosh(a_n x/a) + B_{m_2} \cos(a_n x/a) + B_{m_3} \sinh(a_n x/a) + B_{m_4} \sin(a_n x/a),$$

$$Y_n(y) = C_{n_1} \cosh(b_n y/b) + C_{n_2} \cos(b_n y/b) + C_{n_3} \sinh(b_n y/b) + C_{n_4} \sin(b_n y/b),$$

where $a_n$ and $b_n$ are the frequency parameters corresponding to the $m$th and $n$th roots of a characteristic equation. The constants $C_{n_1}, C_{n_2}, C_{n_3}, C_{n_4}, B_{m_1}, B_{m_2}, B_{m_3}, B_{m_4}$ [17] are determined from the boundary conditions at the edges of the plate, and allow for the imposition of any condition involving simply supported, clamped or free edges.

For a simply supported plate, the integrals have simple forms with solutions similar to Navier's algebraic solution [17] for a porous plate [7].

The coefficients $Q_{mn}$ and $\Delta P_{mn}$ correspond to the external exciting forces. For a load $F_0$ concentrated at one point $(x_0, y_0)$ of the solid surface (a shaker for example), the forces $Q_{mn}I_1I_6$ and $\Delta P_{mn}I_2I_6$ appearing in Eq.(1) are respectively equal to $F_0 X_n(x_0)Y_n(y_0)$ and 0. Detailed derivations of $Q_{mn}$ and $\Delta P_{mn}$ can be found elsewhere [18].

### 3. DEFLECTION OF FLUID LOADED POROELASTIC PLATES

By using Hamilton’s method:

$$H(w) = \int_{t_1}^{t_0} \delta(T - V + W_{\text{excitation}}) dt + \int_{t_1}^{t_0} \delta W_{\text{fluid}} dt$$

where $T$ is the kinetic energy of the plate, $V$ is the strain energy, $W_{\text{excitation}}$ is the work done by the excitation force, $W_{\text{fluid}}$ is the work done by the acoustic pressure exerted by the fluid on the plate, $w$ is the displacement of the plate, and $t_0$ and $t_1$ are arbitrary times. The formulation of porous plate motion can be written in terms of the Lagrange’s equations by using the stationary condition for the Hamiltonian $H$:

$$\frac{d}{dt} \frac{\partial T}{\partial A_{mn}} + \frac{\partial V}{\partial A_{mn}} = \frac{\partial W_{\text{excitation}}}{\partial A_{mn}} + \frac{\partial W_{\text{fluid}}}{\partial A_{mn}}.$$  

The vibrational response of the fluid-loaded porous plate can be obtained by inserting an additional external force $(-j \omega Z_{mnq} \{W_{T,pq}\})$ acting on the plate in Eq.(1). At a given angular frequency $\omega$, in the frequency domain (i.e. by Fourier transform of Eq.(6)), the equation of porous plate motion can be written in matrix form. The matrix form of the equation of fluid loaded porous plate motion is given by:

$$-\omega^2 [M_{mpq}^1 \{W_{mn}^s\} + K_{mpq}^1 \{W_{mn}^s\} + K_{mpq}^2 \{W_{mn}^s\} = (q_{mn}) - j \omega Z_{mnq} \{W_{T,pq}\}$$

$$-\omega^2 [M_{mpq}^1 \{W_{mn}^s\} + K_{mpq}^2 \{W_{mn}^s\} + K_{mpq}^1 \{W_{mn}^s\} = (p_{mn}) - j \omega Z_{mpq} \{W_{T,pq}\},$$

where $[M_{mpq}]$ is the mass matrix, $[K_{mpq}]$ is the complex stiffness matrix, $\{W_{mn}^s\}$ is the magnitude of the transverse solid lateral displacement, $\{W_{mn}\}$ is the magnitude of the transverse fluid-solid relative displacement, $\{W_{T,pq}\}$ is the total velocity in the vicinity of the plate surface, $(q_{mn})$ is the vector of excitation force, $(p_{mn})$ is the pressure difference in the fluid between two surfaces, and $[Z_{mpq}]$ is the radiation impedance matrix.
The coefficients of the mass matrix are defined by:

\[
M_{mnpq}^1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( D + \frac{\alpha^2 M h}{12} \right) (I_1 I_2 + 2I_3 I_4 + I_5 I_6),
\]

\[
M_{mnpq}^2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \alpha M h \left( I_2 I_3 + I_4 I_6 \right).
\]  

(8)

The coefficients of the complex stiffness matrixes are defined by:

\[
K_{mnpq}^1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} -h \rho \omega^2 I_2 I_6, \quad \left[ K_{mnpq}^1 \right] = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} -h \rho \omega^2 I_2 I_6,
\]

\[
K_{mnpq}^2 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} h \rho \omega^2 I_2 I_6, \quad \left[ K_{mnpq}^2 \right] = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} h m(\omega) \omega^2 I_2 I_6.
\]  

(9)

4. RADIATION IMPEDANCE MATRIX

The cross-coupling terms in the radiation impedance matrix are given by [4, 12]

\[
Z_{mnpq} = j \rho \omega \int \int S dx dy \int \int S y x G y x j Z \psi\psi \omega \rho S
\]

(10)

where \( Z_{mnpq} \) is known as the acoustic radiation impedance between the normal modes \((m, n)\) and \((p, q)\). \( \psi_{mn}(x, y) \) and \( \psi_{pq}(x', y') \) are the Eigen-functions in the case of the simply supported boundary condition given by \( \psi_{mn}(x, y) = \sin(\frac{m \pi x}{a}) \sin(\frac{n \pi y}{b}) \), and \( \psi_{pq}(x', y') = \sin(\frac{p \pi x'}{a}) \sin(\frac{q \pi y'}{b}) \), respectively. The Green function is given by

\[
G(x, y, z; x', y', 0) = \exp(-jk_0 R) / 2\pi R.
\]  

(11)

where \( R \) is the distance between vectors, \( R = \sqrt{(x-y)^2 + (x'-y')^2} \), and \( k_0 \) is the wave number. The radiation impedance matrix for a non-porous plate has been predicted directly from Eq.10, without interpolation, convergence and reducing the quadruple integral to a double integral. A Gaussian quadrature scheme with thirteen modal functions in each direction \((x, y)\) has been used. When the variables \((x, y)\) have the values \((x', y')\), for which \( G(x, y, z; x', y', 0) \) goes to infinity, care should be taken to avoid any singularity. The singularity at the origin can be avoided by taking \( G(0, 0) = 0 \).

A point force excitation is assumed at \( x = 0.08 \) m, \( y = 0.07 \) m from a corner of the plate. The assumed properties of the (non-porous) plate are given in Table 1.

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Thickness (m)</th>
<th>Density (kg/m³)</th>
<th>Young’s Modulus(Pa)</th>
<th>Loss Factor</th>
<th>Poisson Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.48</td>
<td>0.42</td>
<td>0.00322</td>
<td>2680</td>
<td>6.6x10¹⁰</td>
<td>0.005</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The results for the direct terms \( Z_{mn mn} \) are shown in Figure 2. The results for the cross coupling terms \( Z_{mn pq} \) are shown in Figure 3. The direct and cross coupling terms of the radiation impedance matrix are normalized by characteristic impedance \( \rho_0 c_0 \), where \( \rho_0 \) is the air density and \( c_0 \) is the sound speed in the air. If the values of \((m+p)\) or \((n+q)\) is odd the radiation impedance matrix is equal to zero. The values of \( m, n, p \) and \( q \) vary between \( 0 \) and \( N \). These figures show that the radiation impedance matrix exhibits a smooth variation in terms of frequency.
Figure 2: Direct terms in the real part of the radiation impedance matrix for (a) even modes and (b) odd modes.

Figure 3: First two cross coupling terms in the real part of the radiation impedance matrix.

5. VIBROACOUSTIC INDICATORS

Three vibroacoustic indicators are in common use. The mean square velocity is defined as a time-space average of the squared vibration velocity of the plate and indicates the global behavior of the plate vibration. It is given by [9]:

\[
V^2 = \frac{1}{abT} \int_0^T \int_0^\infty \int_0^\infty \left( \frac{\partial w(x, y, t)}{\partial t} \right)^2 dS dt.
\]  

where \( T = 2\pi / \omega \), and \( a, b \) are the coordinates of plates in \( x \) and \( y \) direction, respectively. \( S \) is the surface area of the plate and \( \omega \) is the angular frequency of the plate.

The radiated sound power expresses the sound energy radiated by the plate. It is obtained from the integration of the intensity over the plate surface. The radiation sound power [9] is given by

\[
W = \frac{1}{T} \int_0^T \int_0^\infty \int_0^\infty P(x, y, 0, t) \frac{\partial w(x, y, t)}{\partial t} dS dt.
\]

where \( P(x, y, 0, t) \) is the surface acoustic pressure given by [19]

\[
P(x, y, z) = -\rho_f \omega^2 \int_0^\infty \int_0^\infty w(x', y') G(x, y, z; x', y', 0) dS'.
\]

This is often called the Rayleigh integral.

The radiation efficiency expresses the ratio of the vibration energy transformed into sound power. The radiation efficiency of the plate is given [9] by;
\[ \eta = \frac{W}{\rho_0 c_0 a b V^2}. \]  

(15)

where \( \rho_0 \) is the density of air and \( c_0 \) is the sound speed in air. The radiation efficiency is calculated from the radiated sound power and the mean square velocity of the plate.

6. NUMERICAL RESULTS

The coefficients \( W_m^o \) and \( W_{mn} \) can be found by solving Eq.1 for the \( m, n \) mode. Hence the deflection of porous plates can be calculated by from Eq.3. Then the mean square velocity, the radiated sound power and radiation efficiency can be computed. A Gaussian quadrature scheme with twenty terms of the Legendre polynomial has been used to compute the fluid loaded plate deflection. The effects of the fluid loading on the plates have been studied for three different plates with the properties listed Table 2. The Coustone plate contains flint particles with a mean grain size of about 1mm and an epoxy rubber binder. The G foam and YB10 foam are fabricated from particles of plastic foam obtained from recycled car dashboards. Their grain size varies from a few tens of \( \mu m \) to 5 mm. The properties of the saturating fluid have been assumed to be either those of water or those of air.

<table>
<thead>
<tr>
<th>Table 2. Characteristics of three types of plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>G foam</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Lx (m) in x direction</td>
</tr>
<tr>
<td>Ly (m) in y direction</td>
</tr>
<tr>
<td>Thickness (m)</td>
</tr>
<tr>
<td>Density, ( \rho ) (kg/m(^3))</td>
</tr>
<tr>
<td>Young’s modulus, E(Pa)</td>
</tr>
<tr>
<td>Loss factor</td>
</tr>
<tr>
<td>The Poisson ratio, ( \nu )</td>
</tr>
<tr>
<td>Porosity, ( \phi )</td>
</tr>
<tr>
<td>Tortuosity, ( \tau_\infty )</td>
</tr>
<tr>
<td>Permeability, ( \kappa(m^2) )</td>
</tr>
</tbody>
</table>

Figure 4: Predicted deflections of the G foam plate plate in vacuo and loaded with water [13].

Figure 5: Predicted deflection of the YB10 foam plate in vacuo and loaded with either air or water [13].

The computed spectra of the plate deflections are shown in Figures 4 – 6, for G foam, YB10 foam and Coustone, respectively. The magnitudes of the predicted deflection spectra of the plate in vacuo and water-loaded deflection of the plate differ by factors of up to 1000. There is very little difference between the predicted responses of YB10 foam plate in air and in vacuo throughout frequency range. The predicted deflection spectrum of the YB10 foam plate
in water is less than the spectra predicted either *in vacuo* or in air. The Coustone plate is predicted to be relatively little affected by fluid loading. The YB10 plate (see Table 2) was manufactured at the University of Bradford from recycled car dashboards. Its properties have been used to determine the vibroacoustic indicators. A Gaussian quadrature scheme is also used to expand the mean square velocity with sixteen modal functions in each direction \((x, y)\). The plates are assumed to be excited by a point force of 1 N applied at \(x = 0.1 \ m, y = 0.09 \ m\) from a corner of the plates. The responses of the plates are calculated at the middle of each plate. Figure 7 compares the predicted frequency dependence of the mean square velocity of the YB10 foam plate in water, in air, and *in vacuo*. There is no difference between the predicted deflections of air-loaded and *in vacuo* YB10 plates. But air loading is predicted to decrease the mean square velocity. The effects of fluid loading may be explained by different patterns of vibration. Although fluid (air) loading does not change the shape of curve, it decreases the mean square velocity by about 15 dB. With water loading the shape of the velocity curve is changed, and also the velocity is decreased by about 20 dB. The differences between the predicted responses in water and in air are very small. So it can be said that fluid loading decreases the mean square velocity of poroelastic materials.

The frequency dependence of the radiated sound power of the YB10 foam plate predicted in air and in water is shown in Figure 8. The radiated sound power predicted *in vacuo* is zero since the sound speed and density in vacuum is zero. Air loading decreases the radiated sound

![Figure 6: Predicted deflections of the Coustone plate *in vacuo* and loaded with water [13].](image)

![Figure 7: Predicted mean square velocity of the YB10 plate *in vacuo* and loaded with air or water [13].](image)

![Figure 8: Predicted Radiated Sound Power from YB10 plate loaded by air and water [13].](image)

![Figure 9: Predicted radiation efficiency of YB10 plate loaded with air and water [13].](image)
power by about 30 dB in comparison with water loading response. The predicted radiation efficiencies of the YB10 plate loaded with air and water are compared in Figure 9. The predicted radiation efficiency of the plate in air is higher than that predicted in water. With water loading the radiation efficiency of the porous plate is decreased by between 10 dB and 20 dB. The predicted radiation efficiency of the plate in-vacuo response is zero because the radiated sound power in-vacuo is zero.

7. CONCLUSION
The fluid-loaded vibration of a clamped porous elastic plate has been predicted by the solving the governing equations for flexural vibration. A suitable trial function has been used to calculate the radiation impedance matrix. Calculations have been presented to illustrate the effects of fluid-loading on the vibration and radiation from different porous plates. The deflections of fluid-loaded porous plates have been calculated and compared to the in-vacuo deflection of porous plates for three sets of plate properties and assuming either water or air saturation. The predicted frequency responses of the fluid-loaded plate vibration and radiation have been investigated. Generally speaking, with air and water loading, the predicted radiation efficiency, mean square velocity, and radiated sound power of porous plates are less than the predicted in vacuo values.

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