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# Effects of Driving Aggressiveness to Stability of Traffic Flow with Drivers Utilizing Short-Term Memory

Rifat Sipahi   Silviu-Iulian Niculescu   Fatihcan M. Atay

**Abstract**—The main objective in this paper is to understand the stability features of traffic flow from the perspective of driver aggressiveness. We study this over a fundamental car-following traffic model, which is further enhanced by taking into account the *short-term* memory of human drivers. Physically, memory indicates that human drivers *continuously* receive stimuli from a set of continuous points distributed over the history. Specifically, it is of *practical* interest, when aggressive drivers are present, to reveal stability features of traffic flow in the parametric domain defining the physics of drivers' memory. Various traffic scenarios where vehicles are arranged in linear and ring configurations are presented and some interesting physical interpretations are discussed.

**Index Terms**—driver aggressiveness, short-term memory, distributed delay, traffic, stability

## I. INTRODUCTION AND PROBLEM STATEMENT

Traffic flow is one of the main research directions in Physics, Engineering and Mathematics since the 1930s. The main reason for this is that irregular traffic flow has direct impacts on human life (injuries, hospitalization), economy (time losses) and the environment (emission issues), [14]. There exists numerous *mathematical models* in the literature, [3], [4], [6], [9], [10], [22], [28]–[30], which describe various phenomena arising from traffic flow dynamics. Clearly, one of the most important research directions is along the lines of studying the *stability* of traffic flow, [3], [6], [7], [9], [10], [19], [20], [29], [30], since this research will help understand how to avoid traffic accidents, reduce traffic jams and lower emissions.

Systems in which humans actively participate in evolution of dynamics are richer and more complicated as they can be seen as *human-in-the-loop dynamics* [16]. Furthermore, humans exhibit *delayed* reactions as they need some time to become conscious, make decision and perform an action. From this perspective, we can state that the presence of human drivers makes traffic flow dynamics inherently *time delayed*. This is a critical observation as it is known that time delays may invite poor performance and even instability in

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dynamical systems, see [12], [17] and the references therein. In this paper, the delays are modeled as *short-term memory*, which considers that human drivers *continuously* receive stimuli from a set of continuous points distributed over the history.

Inspired by authors earlier work [26], a more realistic mathematical model of traffic flow is deployed by assuming all human drivers utilize their short-term memory. Next, the main contribution of this paper is presented over this new model: Considering presence of relatively aggressive drivers, we elaborate on the stability robustness of traffic flow in the parametric domain defining the features of short-term memory. This is practically important as it is aimed to understand risk levels (instability) in traffic when, for various reasons, some humans drive aggressively. See also [25] for a study in which the effect of number of vehicles (where vehicles are arranged around a ring) to stability is studied assuming identical driver aggressiveness.

The paper is organized as follows: Notations conclude this section. Mathematical modeling of traffic flow and delayed reactions of human drivers are summarized In Section II. In Section III, the relevance of the proposed approach and main objectives are detailed. Inspired by [26], the framework of the stability analysis is explained in Section IV. Practical scenarios are considered in Section V from stability point-of-view. Assuming all drivers utilize their memory for their control actions, we specifically study how stability is affected with respect to driver aggressiveness. Two standard configurations, ring and linear, are chosen to demonstrate the results. Section VI presents the conclusions and future research directions.

*Notation.* The notations are standard. We use  $\mathbb{R}$  for the set of real numbers, where an additional  $+$  or  $-$  sign as a subscript indicates the positive and negative real numbers, respectively. Similarly,  $\mathbb{C}_+$ , and  $\mathbb{C}_-$  denote complex numbers whose real parts are positive and negative, respectively. The imaginary axis in the complex plane is denoted by  $j\mathbb{R}$ , where  $j = \sqrt{-1}$ . We use  $s \in \mathbb{C}$  for the Laplace variable, whose values on the imaginary axis are denoted by  $s = j\omega$  where  $\omega \in \mathbb{R}$ .  $i$ th eigenvalue of a matrix  $A$  is represented by  $\lambda_i(A)$ .

## II. MATHEMATICAL MODELING

### A. Discrete delay modeling

Time delay (known to appear in a traffic flow model first time in [4]), essentially originates from the time needed for human drivers to become conscious of the changes in the

environment, make decisions, and perform control actions, see [3], [6], [9], [10], [23], [29] for time-delayed mathematical models and [11] for quantitative level of such delays. Although the above argument is straightforward, *modeling of time delays* representing the behavior of human drivers is a *challenge*. The open literature responds to this problem by suggesting *discrete delay models* [3], [6], [23], [24], [28], which may give some insight on the characteristics of traffic flow dynamics. Discrete delays indicate that an action of a human driver at time  $t$  is based on what is experienced at time  $t - \tau$ ,  $\tau \geq 0$ . Following this idea, numerous studies appear in the literature [3], [6], [19], [20], [23], [24], [28]–[30].

Discrete-delay models, on the other hand, exhibit some drawbacks, mainly due to the assumption that they are fixed (rather than representing a distribution) and that they disregard possible “memory” effects inherently present in the dynamics. From the application point-of-view, in traffic flow, one needs to incorporate memory effects arising from the physical capacities of the human drivers. Mathematically speaking, such effects can be modeled by using certain distribution functions (such as uniform and gamma distributions) which, in parallel, help represent the heterogeneity of the distribution of non-identical drivers. For these reasons, distributed nature of the time delay will yield a better representation of *reality*. Such an argument has also some potential to open new research directions in understanding human behavior [16].

Motivated by the previous arguments, short-term memory effects will be taken into account in mathematical modeling of traffic flow. In this section, a summary of authors’ preliminary work is presented in order to guide the reader, see [26] for details.

### B. Short-term memory – Distributed delay modeling

Short-term memory can be modeled by a *uniform distribution*  $f(\tau)$ , Figure 1.

$$f(\tau) = \begin{cases} 1/\delta & \text{if } h < \tau < h + \delta \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where  $\delta > 0$  and  $h \geq 0$ . Such a model is quite satisfactory for representing an average of the information available in the short-term memory as well as for introducing some heterogeneity among the drivers. See [1], [2], [5], [13] for theory and applications with different forms of distributed functions.

The uniform distribution can be defined by a combination of any of the two physical parameters, dead-time ( $h$ , before which information in the memory can be utilized for a control action), memory horizon ( $\tau_2 = h + \delta$ , representing the oldest possible information available in the memory) and memory window ( $\delta$ , indicator of the size of the memory). Note that, (1) converges to discrete delay case as  $\delta \rightarrow 0$  (where  $f(\tau)$  becomes a dirac function).

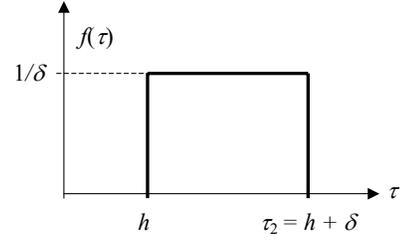


Fig. 1. Uniform distribution model for memory effects

### C. Traffic flow modeling

We consider a single-lane continuous-time microscopic car following model, in which a chain of vehicles travel at a constant velocity, the so-called quasi steady-state, without changing lanes, [4], [14], [22],

$$\dot{v}_i(t) = \kappa_i(v_{i-1}(t - \tau) - v_i(t - \tau)), \quad (2)$$

where  $v_i$  is the velocity of the  $i$ th vehicle, and  $\tau \geq 0$  is the discrete time delay. The parameter  $\kappa_i > 0$  denotes the *aggressiveness* (also known as sensitivity) of the  $i$ th driver against the velocity difference between his vehicle and the one in front. Equation (2) is also similar to consensus problems [8], [15], [18], [21], in which drivers penalize the relative velocity between two vehicles in order to achieve the agreement of  $v_i(t) \rightarrow v_{i-1}(t)$ .

Notice that the model given above carries discrete delay,  $\tau$ , only. We further improve this model by combining memory effects from (1). Hence, the generalized dynamics considered in this work becomes

$$\dot{v}_i(t) = \kappa_i \int_0^\infty f(\tau)(v_{i-1}(t - \tau) - v_i(t - \tau)) d\tau, \quad (3)$$

$i = 1, \dots, n$ , where  $n$  is the number of vehicles. We assume that the delay kernel  $f(\tau)$  is a measurable function of exponential order. For instance, the choice of a Dirac distribution for  $f(\tau)$  gives the discrete delay model (2) above.

Eq. (3) defines a traffic dynamics in which drivers perform their decisions based on what they continuously observed during a memory window, and the information in the memory is retained and used in the decision-making process. Clearly, the decisions of the drivers are limited by the capacity of this short-term memory, i.e. the size of the memory window  $\delta$ .

The system (3) can be expressed in vector form as

$$\dot{\mathbf{v}}(t) = \int_0^\infty J\mathbf{v}(t - \tau)f(\tau) d\tau \quad (4)$$

where  $\mathbf{v} = (v_1, \dots, v_n)$  and  $J \in \mathbb{R}^{n \times n}$  is a configuration matrix weighted by the driver sensitivities  $\kappa_i$ . We assume that  $J$  is diagonalizable, that is, its eigenvectors form a basis for  $\mathbb{R}^n$ .

In the *circular configuration*, we identify  $v_0 = v_n$  in (3), and the matrix  $J$  takes the form

$$J = \begin{pmatrix} -\kappa_1 & 0 & \cdots & 0 & \kappa_1 \\ \kappa_2 & -\kappa_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \kappa_n & -\kappa_n \end{pmatrix} \quad (5)$$

In the *linear configuration*, the vehicle in front (denoted with the index  $i = 1$ ) travels at a constant velocity, i.e.  $\dot{v}_1 = 0$ . Hence, the linear configuration can be derived from the circular one by setting  $\kappa_1 = 0$ , and the resulting matrix is denoted by  $J'$ .

#### D. Spatial configuration of traffic flow.

We consider two spatial configurations which are widely utilized in the literature, [19], [20], [23]: **(a)** vehicles traveling around a ring, and **(b)** vehicles arranged along a linear path. It will be interesting to obtain the stability of the traffic flow with respect to these two different configurations, and give some reasoning why one is more favorable than the other from the stability point-of-view.

### III. OBJECTIVE, PRACTICAL RELEVANCE, APPROACH

The main objective in this paper is to elaborate on how the stability of (4) is affected if aggressive drivers populate the traffic flow. In other words, it is of interest to see how stability of traffic flow defined in physical parameters of the short-term memory,  $(\delta, h)$ , vary as drivers become more aggressive. Recall that  $\kappa_i$  denotes driver aggressiveness and it will be used as one of the main parametric domain in presenting how stability is affected. We will particularly focus on understanding how memory window size of drivers,  $\delta$ , varies as  $\kappa_i$  increases for particular drivers.

The above study will be investigated both over *ring* and *linear* configuration of vehicles, as explained earlier. The scenarios considered are practically of interest, as the results will help interpret how stability is strongly/weakly robust for these two configurations. Arising results can be generalized for various dynamical systems that are also in the form of (4); see for instance consensus problems in [8], [15], [18], [21].

The approach for the stability analysis of the dynamics (4) comes from an interesting idea of rewriting the corresponding characteristic equation of (4) in a relatively different form, which decouples some parameters of interest for stability. This coupling is a representation of an interconnection scheme that enables the analytical results. We further discuss in the following section regarding how interconnection scheme idea works effectively.

### IV. STABILITY ANALYSIS

Although undesirable, the presence of time-delays in the process of decision making and performing a control-action

by human drivers is neither avoidable nor negligible. Therefore, one needs to understand the stability features of traffic dynamics (4) in presence of delays.

#### A. Characteristic equation

To perform the stability analysis, the characteristic equation (4) in Laplace domain is obtained first

$$\chi(s) = \det[sI - JF(s)] = 0, \quad (6)$$

where  $F(s)$  is the Laplace transform of the distribution function  $f(\tau)$ ,

$$F(s) = \frac{e^{-sh}(1 - e^{-s\delta})}{s\delta}. \quad (7)$$

Note that when  $\delta \rightarrow 0$ , discrete-delay model (2) with  $\tau = h$  is recovered.

One can rewrite (6) as

$$\chi(s) = \prod_{i=1}^n (s - \lambda_i(J) \frac{e^{-sh}(1 - e^{-s\delta})}{s\delta}) = 0 \quad (8)$$

where  $\lambda_i(J)$  is the  $i$ th eigenvalue of  $J$ . The roots corresponding to the  $i$ th factor of  $\chi$ , i.e., the solutions  $s$  of this equation will determine the stability/instability of traffic dynamics (4).

Notice that the configuration matrix  $J$  has a simple zero eigenvalue and all its remaining eigenvalues are complex conjugates with negative real parts, [26]. This simple zero creates an  $s = 0$  pole of the dynamics which corresponds to a rigid body motion of the entire configuration of the vehicles. Furthermore,  $s = 0$  has no influence on the stability/instability of the dynamics, thus we will assume that  $s \neq 0$  in the remaining of the paper, see details in [26].

#### B. Approach for stability analysis, Interconnection scheme

Stability analysis of the dynamics (4) is not trivial due to inherent challenges in the corresponding characteristic equation (8), thus we propose a different way to approach the problem. The need for this approach *naturally* arises *within* the following complications listed:

(a) Eq. (8) possess two independent time delay parameters,  $(\delta, h)$ , which complicate the analysis. Thus one needs to find the *regions* of stability in the  $(h, \delta)$  parameter plane.

(b) In (8), one of the time delays,  $\delta$  not only appears in an exponential term, but also in the denominator of the characteristic equation. This is an additional complication for stability analysis.

(c) A reasonable size of a traffic scenario would contain at least ten vehicles,  $n = 10$ , however, the corresponding characteristic equation (8) becomes extremely complicated for  $n > 2$ . To extract stability features from this equation is quite cumbersome and computationally inefficient. To overcome this difficulty, one may study the spectra of (8) with respect to each eigenvalue  $\lambda_i(J)$ , separately, obtaining the stability/instability features. The superposition of the spectra corresponding to individual eigenvalues will depict

the complete stability characteristics. Hence, we propose to study the spectra of the following characteristic equation for each counter value  $i = 2 \dots n$ .

$$\chi_i(s) = s - \lambda_i(J)e^{-sh} \frac{1 - e^{-s\delta}}{s\delta} = 0, \quad (9)$$

Clearly, studying (9) is much simpler, since  $\chi_i(s)$  is a  $2^{nd}$  order dynamics (if expanded by  $s$ ). Hence, the dimension of the problem is reduced from analyzing an  $n^{th}$  order dynamics to *repeatedly* ( $i = 2 \dots n$ ) analyzing one of a  $2^{nd}$  order. Clearly, such a reduction is quite beneficial when studying large scale traffic flow.

(d) Although, the idea of studying (9) reduces the dimensional difficulty of the analysis, this equation now carries complex coefficients since  $\lambda_i(J) \in \mathbb{C}_-$ , [26]. This prevents one to benefit from many stability analysis techniques available in the literature [12], [17], [27].

Since  $s \neq 0$ , one can now rearrange (9) as

$$H_i(s) \cdot \Delta(s) = 1, \quad (10)$$

$$\text{where } H_i(s) = \frac{\lambda_i(J)e^{-sh}}{s}, \quad \Delta(s) = \frac{1 - e^{-s\delta}}{s\delta} \quad (11)$$

The advantage of the form (10) lies mainly on the separation of the parameter  $\delta$  and the eigenvalues of  $J$ . This new form is an interconnection scheme which represents the characteristic equation (9) in two separate blocks. By doing so, parameters of interest ( $\delta$  and  $h$  for instance) are decoupled, and consequently an effective technique can be proposed for the stability analysis.

*Remark 1 (Continuity of roots):* The continuity of the roots of the interconnection scheme (10) holds with respect to parameters defining the short-term memory ( $\delta$  and  $h$ ), [26], therefore, the stability may be lost only when the interconnection scheme (10) attains  $s = j\omega$ .

The method for stability analysis can be summarized as follows, [26]. **(i)** Check the stability of the delay-free dynamics by setting  $h = \delta = 0$  in (8). **(ii)** Next, determine all  $s = j\omega$  roots of the interconnection scheme in the domain of delays ( $h, \delta$ ). **(iii)** Check the sensitivity of  $s = j\omega$  with respect to  $h$  and  $\delta$ , in order to determine if  $s = j\omega$  roots transit to stable  $\mathbb{C}_-$  or to unstable  $\mathbb{C}_+$ . **(iv)** Label those ( $h, \delta$ ) regions for which there exist no roots in  $\mathbb{C}_+$  as *stable*.

### C. Detection of $s = j\omega$ roots of (10)

The outline of the framework for detecting the imaginary roots of (10) is presented as follows. If  $s = j\omega$  is a root of the characteristic function  $\chi(s)$ , then the following should be satisfied,

$$H_i(j\omega) \cdot \Delta(j\omega) = 1. \quad (12)$$

Although there are several ways to analyze the interconnection scheme, we prefer the following as it also describes a practical geometric perspective. The above interconnection can be studied for its magnitude and argument in complex domain. Without loss of generality, we assume  $\omega > 0$ , and

notice that  $|e^{-jh\omega}| = 1, \forall (h, \omega) \in \mathbb{R}^2$ , thus the algorithm is formed by the following steps:

*Step 1:* Define first an intermediary variable  $z = \delta\omega/2$ . Notice that there does not exist an explicit function which maps from  $\omega$  to  $\delta$ . Hence, the additional parameter  $z$  is used to achieve this mapping. Clearly, to obtain the correspondence  $\delta \longleftrightarrow \omega$ , a sweeping on the parameter  $z$  is quite practical (see below), and such an idea resembles frequency sweeping techniques presented in [17].

*Step 2:* From the magnitude condition, one can easily find the relation

$$\delta = \frac{2z^2}{|\lambda_i(J)||\sin(z)|}, \quad (13)$$

from which  $\delta$  can be obtained by *sweeping*  $z$ . See [26] where we prove it is sufficient that  $0 \leq z \leq \pi/2$  for extracting the stability boundary of the dynamics in  $(h, \delta) \in \mathbb{R}_+ \times \mathbb{R}_+$  parameter space.

*Step 3:* By definition of  $z$  and given  $\delta, \omega$  can be solved as

$$\omega = \frac{|\lambda_i(J)||\sin(z)|}{z}. \quad (14)$$

*Step 4:* Next, by deploying the argument condition on (12),  $h$  values are obtained from

$$h = -\frac{1}{\omega} \left( \angle \left( \frac{\delta s^2}{\lambda_i(J)(1 - e^{-s\delta})} \Big|_{s=j\omega} \right) \mp 2\pi\ell \right). \quad (15)$$

Clearly, solution set  $(\omega, h, \delta)$  contains all possible imaginary roots of (10), and one has to establish the guidelines for extracting those that form the *boundary* of the stability regions in  $(h, \delta)$  domain. The sensitivity study on the imaginary root  $s = j\omega$  will enable this, see *Subsection D* below.

### D. Imaginary root transitions in $h$ domain for fixed $\delta$

Given  $\lambda_i(J)$  and  $\delta$ , the tendency of the imaginary roots  $s = j\omega$  across the imaginary axis as the corresponding  $h$  in (15) increases, can be found by the help of the following sensitivity function

$$S_i(h, \omega) = \frac{ds}{dh} = - \left( \frac{\partial \chi_i(s)}{\partial h} \right) \left( \frac{\partial \chi_i(s)}{\partial s} \right)^{-1}. \quad (16)$$

For analyzing the tendency across the imaginary axis, one needs to study the real part of  $S_i(h, j\omega)$ . If the real part is positive (or negative), it indicates the root  $s = j\omega$  crosses the imaginary axis along the direction from stable  $\mathbb{C}_-$  to unstable  $\mathbb{C}_+$  (or vice versa) of the complex plane. In other words, if  $\Re(S_i(h, j\omega)) > 0$  (or  $< 0$ ), number of roots on right half complex plane increases (or decreases) as  $h$  in (15) increases infinitesimally. Readers are directed to [26] for the details as well as some invariance properties of the sensitivity function.

*Remark 2 (Geometric approach):* The analytical framework explained above ultimately enables the *complete* stability/instability characterization of the traffic flow dynamics in

$(h, \delta)$  domain. Such a geometric approach can also be supported by an analytical framework which detects the stability boundaries *completely* and *analytically*. The characterization and its proof are suppressed here to maintain the flow of the paper, see [26] for details.

## V. ILLUSTRATIVE EXAMPLES

In the following, we present two different example scenarios for the traffic dynamics in (4). For both scenarios, linear and chain configuration of vehicles are considered. It is important to note that all results and interpretations mentioned this section are based on the particular numerical example and mathematical model considered.

First scenario takes ten drivers (number of vehicles),  $n = 10$ , and presents the effects of driver aggressiveness,  $\kappa_i$  to the memory window size,  $\delta$  of human drivers. All drivers are assumed to be identical and their aggressiveness is the range of  $\kappa_i = \kappa \in [1.5, 2.5]$ , which is realistic according to literature, [3].

The second scenario alters the first one by assuming there exists one, two or three relatively more aggressive drivers among the ten drivers. It is assumed that aggressiveness,  $\kappa_i$  of these drivers is 20% larger compared with any  $\kappa$  chosen for the remaining drivers. Assuming these aggressive drivers are present in traffic, the change in maximum allowable memory window size of drivers is studied with respect to the first scenario (where there are no aggressive drivers). Arising results are interesting as they allow us to interpret stability robustness in the two different vehicle configurations considered for the traffic flow.

In the case studies, the number of vehicles  $n$  will be kept fixed for comparison reasons. Readers interested with how  $n$  affects the stability are directed to a recent work of the authors in [25].

### A. Stability regions with respect to driver aggressiveness, $\kappa_i$

As summarized above, in this scenario we consider  $n = 10$  number of identical drivers (vehicles),  $\kappa_i = \kappa = 2.0$ , in both ring and linear configurations. Before we proceed to the main result of this subsection, the stability picture below will be helpful for the reader, Figure 2. This figure represents the stability regions (shaded) of traffic flow in the parametric domain defining the memory of the drivers given  $\kappa_i = \kappa = 2.0$ . Clearly, the linear configuration of the vehicles allows larger stability regions in this parametric domain,  $R_1 \cup R_2$ , whereas in ring configuration the stability region shrinks to  $R_2$ .

It is proven in [26] that for the traffic flow model considered, the maximum memory window size on the stability boundary,  $\max(\delta) = \bar{\delta}$ , such as in the above figure, occurs when dead-time is zero,  $h = 0$ , independent of ring or linear configuration of vehicles. Moreover, the stability boundaries shown in this figure are monotonically decreasing, [26]. Starting from this, we will use  $\bar{\delta}$  as a measure of stability, and

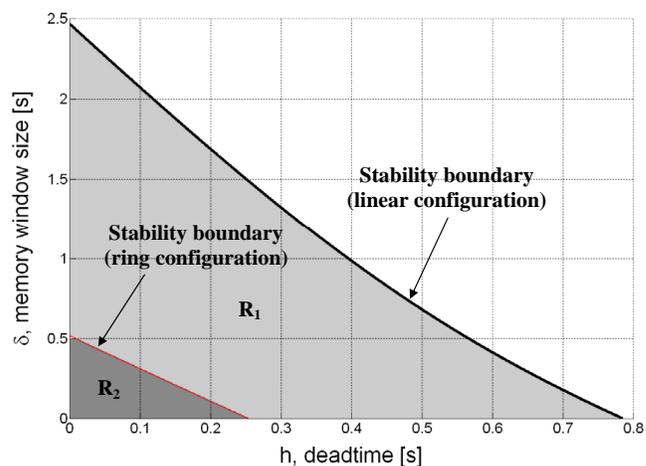


Fig. 2. Comparison of stability regions for linear and ring configurations with  $n = 10$  vehicles and driver sensitivity  $\kappa_i = \kappa = 2.0$

we will demonstrate how  $\bar{\delta}$  varies as all drivers identically become more aggressive. Arising result in  $\kappa_i = \kappa$  vs.  $\bar{\delta}$  domain is presented in Figure 3. It is interesting to see

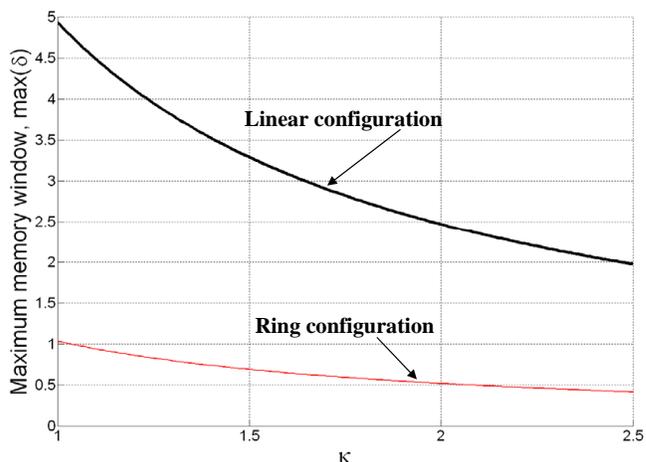


Fig. 3. Comparison of maximum memory window size,  $\max(\delta) = \bar{\delta}$ , with respect to driver aggressiveness  $\kappa_i = \kappa$ .  $n = 10$  vehicles is considered in both cases.

that linear configuration of vehicles for the selected range of  $\kappa$  allows larger memory size for the drivers without stability of the traffic flow is lost. However, as drivers become more aggressive, it can be seen in this figure that maximum allowable memory size rapidly decreases for both ring and linear configurations.

### B. Effects of Presence of Aggressive Drivers to Stability

Let us now alter the above scenario. Instead of assuming all drivers are identical, some drivers are assumed to be 20% more aggressive, i.e., their  $\kappa_i$  is 20% larger, while remaining drivers will still be assumed identical. In order to represent how maximum memory window size is affected

under presence of these relatively more aggressive drivers, arising results are compared with the previous scenario where all drivers are at same aggressiveness level, see Figure 4 and Figure 5 for ring and linear configurations, respectively.

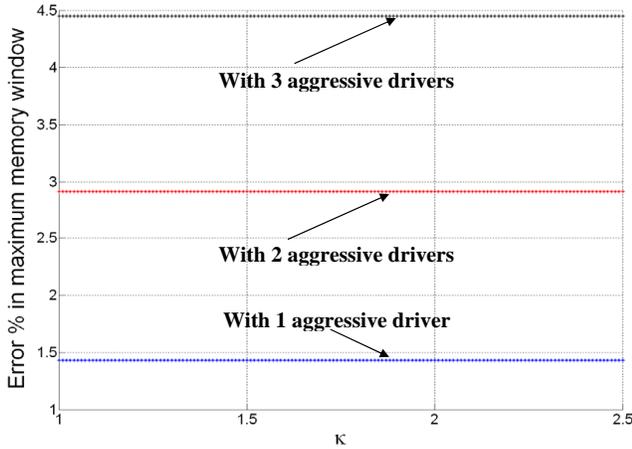


Fig. 4. Percentage of decrease (shown with positive sign) in maximum memory window size with respect to identical driver case, if 1, 2 or 3 drivers among 10 are 20% more aggressive. Ring configuration.

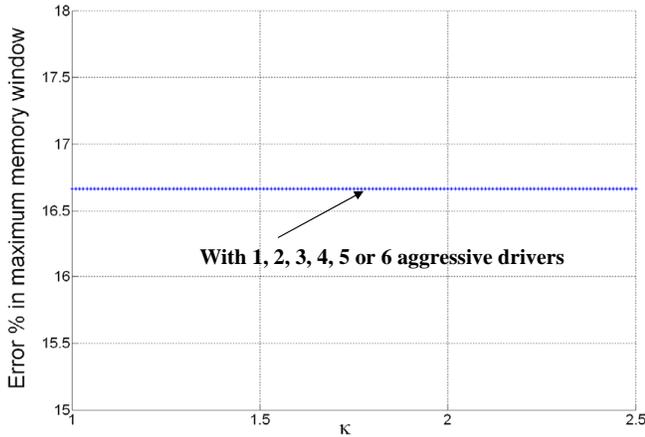


Fig. 5. Percentage of decrease (shown with positive sign) in maximum memory window size with respect to identical driver case, if 1, 2 or 3 drivers among 10 are 20% more aggressive. Linear configuration.

The results in the above figures are very interesting. In ring configuration: as the number of relatively more aggressive drivers increases, the percentage of change (decrease but its sign is kept as positive) in maximum memory window size increases. This is intuitively an expected results, as more aggressive drivers may disfavor stability measure. However, the linear configuration of vehicles is quite interesting: The decrease in memory window size is independent of the number of aggressive vehicles (decrease but shown with positive sign in figure).

Finally, we increase the number of aggressive drivers for the ring configuration, by taking four, five or six relatively

more aggressive drivers. The change of maximum memory window size (decreasing but sign is kept positive) with respect to the case where all drivers are at same level of aggressiveness is depicted in Figure 6. For this particular numerical example, when compared with Figure 4, we observe that increasing number of drivers disfavor stability measure even further.

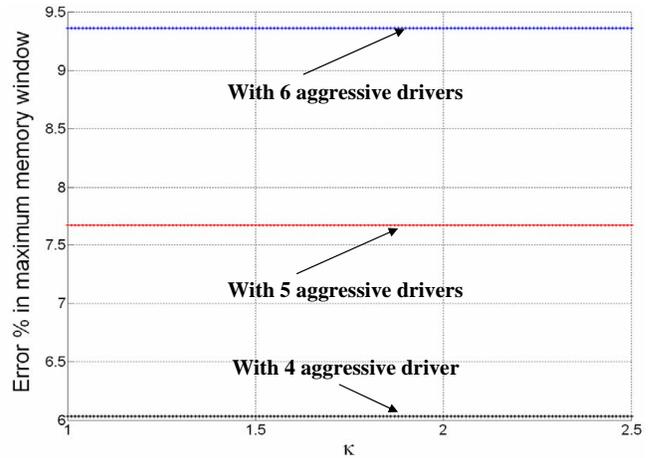


Fig. 6. Percentage of decrease (shown with positive sign) in maximum memory window size with respect to identical driver case, if 4, 5 or 6 drivers among 10 are 20% more aggressive. Ring configuration.

## VI. CONCLUSIONS

Stability of a traffic flow dynamics is studied under various scenarios in which some drivers are relatively more aggressive and all drivers utilize their memories for their control actions. In particular, short-term memory modeling is developed and effects of presence of more aggressive drivers to stability is presented in the parametric domain defining the physics of short-memory. Both linear and ring configuration of vehicles is considered and interpretations on the arising results are presented. In ring configuration, as expected, it is found that stability measure narrows down in presence of more aggressive drivers. On the other hand, we observe that stability measure does not narrow down depending on the number of aggressive drivers. To the best of our knowledge, these results are new in the context of studying driver aggressiveness to the stability of traffic flow in which drivers utilize their memory.

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