New detection thresholds and stop rules for CUSUM online detection

Nassim Sahki, Anne Gégout-Petit, Sophie Wantz-Mézières

INSTITUT ÉLIE CARTAN DE LORRAINE

INRIA BIGS TEAM

ENBIS-19 in Budapest
2 - 4 September 2019

New Detection Thresholds and Stop Rules for CUSUM Online Detection

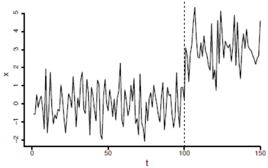
Nassim Sahki, Anne Gégout-Petit, Sophie Wantz-Mézières

03 September 2019

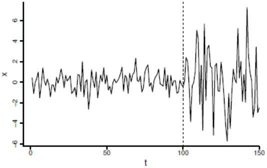
Outline

1. Preamble
2. Online change-point detection
3. New detection threshold
4. Simulation results
5. Perspectives

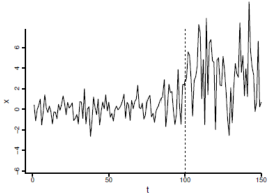
Change-point



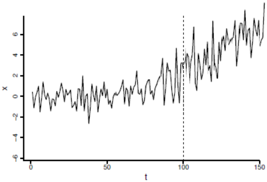
(a) Changepoint in the mean



(b) Changepoint in the variance



(c) Changepoint in the mean and variance

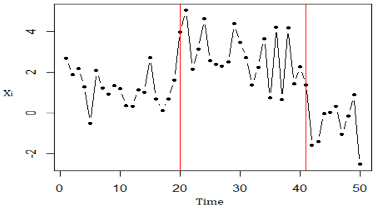


(d) Slope changepoint

Context of analysis

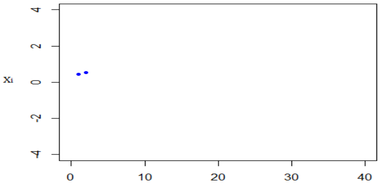
Offline context :

- All data are received and processed in one go ;
- The primary aim is accurate detection of changes ;
- Inference about all change-points simultaneously.



Online context :

- Data arrives either as single data-points or in batches ;
- Data must be processed quickly before new data arrives ;
- The aim is the quickest detection of a change after it has occurred ;
- Inference about most recent change only.



Online change-point detection

Hypothesis test

Let $\{X_i\}_{i=1,\dots,n}$ a series of observations **sequentially observed.**

X_n is the last observed point in the dataset.

Statistically, the problem of change-point detection is to sequentially test for each new observation x_n, the hypotheses :

$$\begin{cases} H_{0,n} : v > n & X_i \sim f_0(\cdot) \quad \forall\, i = 1, \dots, n \\ H_{1,n} : \exists\, v \leq n, & X_i \sim f_0(\cdot) \quad \forall\, i = 1, \dots, (v-1) \\ & X_i \sim f_1(\cdot) \quad \forall\, i = v, \dots, n \end{cases} \tag{1}$$

Where ("distribution pre-change") $f_0 \neq f_1$ ("distribution post-change")

The "instantaneous" Log Likelihood Ratio (LLR) is defined by :

$$L_i = \log(\Lambda_i) = \log\left(\frac{f_1(x_i)}{f_0(x_i)}\right), \quad i \geq 1$$

Hypothesis test

Let $\{X_i\}_{i=1,\dots,n}$ a series of observations **sequentially observed.**

X_n is the last observed point in the dataset.

Statistically, the problem of change-point detection is to sequentially test for each new observation x_n, the hypotheses :

$$\begin{cases} H_{0,n} : v > n & X_i \sim f_0(\cdot) \quad \forall\, i = 1, \dots, n \\ H_{1,n} : \exists\, v \leq n, & X_i \sim f_0(\cdot) \quad \forall\, i = 1, \dots, (v-1) \\ & X_i \sim f_1(\cdot) \quad \forall\, i = v, \dots, n \end{cases} \tag{1}$$

Where ("distribution pre-change") $f_0 \neq f_1$ ("distribution post-change")

The "instantaneous" Log Likelihood Ratio (LLR) is defined by :

$$L_i = \log(\Lambda_i) = \log\left(\frac{f_1(x_i)}{f_0(x_i)}\right), \quad i \geq 1$$

Recursive detection statistics

- **The Cumulative Sum "CUSUM" statistics is written recursively [Page(1954)] :**

$$W_n = \max\{0,\ W_{n-1} + L_n\}, \qquad n \geq 1, \quad W_0 = 0 \tag{2}$$

When the two distributions f_0 and f_1 are unknown ;

$\Rightarrow$ [Tartakovsky, A. G. and all (2006)] suggests replacing the log likelihood ratio L_n through a score function $S_n = S_n(X_1, .., X_n)$.

Recursive detection statistics

- **The Cumulative Sum "CUSUM" statistics is written recursively [Page(1954)] :**

$$W_n = \max\{0,\ W_{n-1} + L_n\}, \qquad n \geq 1, \quad W_0 = 0 \tag{2}$$

When the two distributions f_0 and f_1 are unknown ;

$\Rightarrow$ [Tartakovsky, A. G. and all (2006)] suggests replacing the log likelihood ratio L_n through a score function $S_n = S_n(X_1, .., X_n)$.

Score function

The score S_n is defined for a mean and variance change-point detection by :

$$S_n(\delta, q) = C_1 \cdot Y_n + C_2 \cdot Y_n^2 - C_3 \tag{3}$$

$Y_n = (X_n - \mu_0)/\sigma_0$: the centered and standardized data under H_0.

$$C_1 = \delta \cdot q^2, \;\; C_2 = \frac{1-q^2}{2}, \;\; C_3 = \frac{\delta^2 . q^2}{2} - log(q)$$

$\delta = (\mu_1 - \mu_0)/\sigma_0$ $\qquad$ $q = \sigma_0/\sigma_1$

- δ : minimum level of change in the mean that is required to be detected.

  No changepoint detection on the mean :

  $\mu_1 = \mu_0 \Rightarrow \delta = 0$, therefore $C_1 = 0$.

- q : minimum level of change in the variance that is required to be detected.

  No changepoint detection on the variance :

  $\sigma_1^2 = \sigma_0^2 \Rightarrow q = 1$, therefore $C_2 = 0$.

Statistics model

Note

The score function can only be used to the knowledge of the parameters **mean and variance of pre-change data** μ_0, σ_0^2.

- Use a portion of observed data on the normal state without change-point

  $\Rightarrow$ **Estimate** μ_0 and σ_0^2.

- Depending on **the objective (mean and/ or variance) and level of change that we want to detect** : $\delta = (\mu_1 - \mu_0)/\sigma_0$, $q = \sigma_0/\sigma_1$

  $\Rightarrow$ **Fixed** μ_1 and σ_1^2.

Statistics model

Note

The score function can only be used to the knowledge of the parameters **mean and variance of pre-change data** μ_0, σ_0^2.

- Use a portion of observed data on the normal state without change-point

  $\Rightarrow$ **Estimate** μ_0 and σ_0^2.

- Depending on **the objective (mean and/ or variance) and level of change that we want to detect** : $\delta = (\mu_1 - \mu_0)/\sigma_0$, $q = \sigma_0/\sigma_1$

  $\Rightarrow$ **Fixed** μ_1 and σ_1^2.

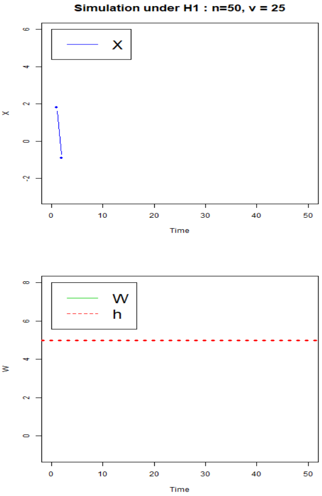
Stopping rule

- The statistics is calculated recursively :

  $$W_n = \max\{0, W_{n-1} + S_n\}, \quad n \geq 1, \quad W_0 = 0$$

- Online detection is based on a **Stopping Rule** :

  $$T_h = \min\{n \geq 1 : W_n \geq h \}, \quad h \geq 0 \text{ : threshold.}$$



When W exceeds the threshold h :

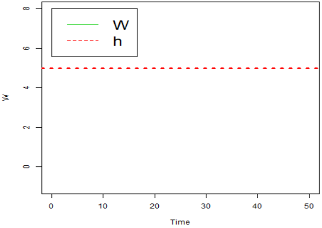
$\Rightarrow$ **The procedure triggers an alarm (Stopping Time) to signal that a change-point has occurred.**

Stopping rule

Denote T a stopping time, such as :

$T = \min\{i \geq 1 : W_i \geq h\}$

- $* \; T \geq v$: detection with a delay $(T - v)$.
- $* \; T < v$: false alarm.
- $* \; T = +\infty$: non detection.



Detection parameters

Let $\mathbb{P}_0[.]$, $\mathbb{E}_0[.]$: respectively the probability and the expectation **before** the change-point v.
Let $\mathbb{P}_1[.]$, $\mathbb{E}_1[.]$: respectively the probability and the expectation **after** the change-point v.

Parameters evaluated under $\mathbb{P}_0$.

Mean Time Between False Alarm (MTBFA)

$$MTBFA = \mathbb{E}_0[T] \tag{4}$$

Instantaneous False Alarm Rate (IFAR)

$$\alpha = \frac{1}{\mathbb{E}_0[T]} \tag{5}$$

Parameter evaluated under $\mathbb{P}_1$.

Average Detection Delay (ADD)

$$ADD = \mathbb{E}_1[T] \tag{6}$$

Detection threshold

The conventional detection threshold used in the literature is based on the Wald's inequality [Egea-Roca et al (2017)].

$\Rightarrow$ This threshold is constant. It is given after fixing α "the tolerated $IFAR$", by :

$$h_\alpha \leq -\ln(\alpha) \tag{7}$$

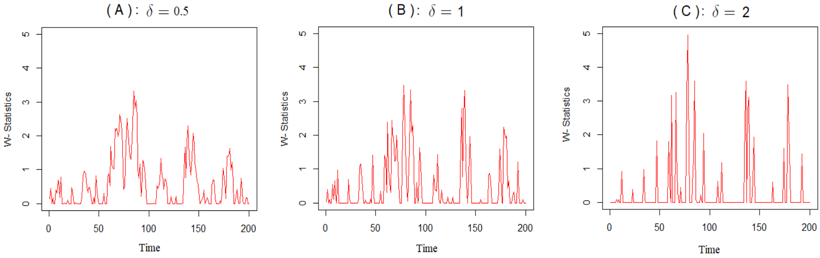
***Daniel Egea-Roca**, Gonzalo Seco-Granados, and Jose A Lopez-Salcedo. Comprehensive overview of quickest detection theory and its application to GNSS threat detection. Gyroscopy and Navigation, 8(1) : 1-14, 2017.*

***Abraham Wald.** Sequential tests of statistical hypotheses. The annals of mathematical statistics, 16(2) : 117-186, 1945.*

1. New detection thresholds constructed by an empirical method ;
2. New stopping rules by modifying the classical rule.

CUSUM statistics under pre-change regime

- Simulate a series X_n of $n = 200$ observations of Gaussian distribution ($\mu_0 = 0$ et $\sigma_0^2 = 1$) ;
- Compute W- statistics according to different levels of δ, ($q = 1$).

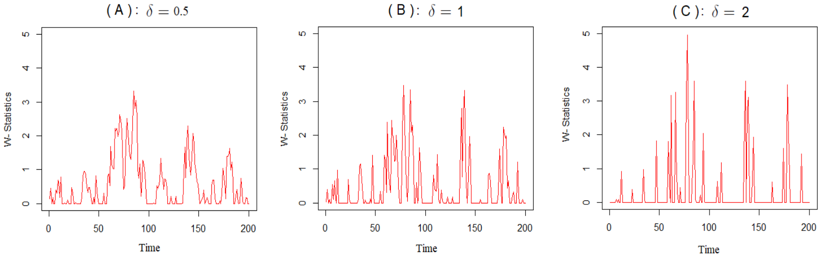


The behavior (variability) of the W-statistics depends on the level of δ ;

$\Rightarrow$ Build thresholds according to δ.

CUSUM statistics under pre-change regime

- Simulate a series X_n of $n = 200$ observations of Gaussian distribution ($\mu_0 = 0$ et $\sigma_0^2 = 1$) ;
- Compute W- statistics according to different levels of δ, ($q = 1$).



The behavior (variability) of the W-statistics depends on the level of δ ;

$\Rightarrow$ Build thresholds according to δ.

Empirical constant threshold

Empirical method : perform simulations of the statistics under $\mathbb{P}_0$ and build the detection threshold according to empirical quantile of law of the statistics under pre-change regime.

Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,.,n;\ j=1,.,B}$.

Empirical constant threshold

Empirical method : perform simulations of the statistics under $\mathbb{P}_0$ and build the detection threshold according to empirical quantile of law of the statistics under pre-change regime.

Construction steps :

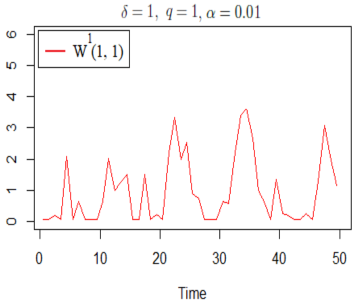
1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,.,n;\ j=1,.,B}$.

Empirical constant threshold

Empirical method : perform simulations of the statistics under $\mathbb{P}_0$ and build the detection threshold according to empirical quantile of law of the statistics under pre-change regime.

Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,.,n;\ j=1,.,B}$.
2. Choice of the objective of detection (δ, q) and compute $w_i^j(\delta, q)$;
3. Choice of instantaneous false alarm rate α tolerated ;



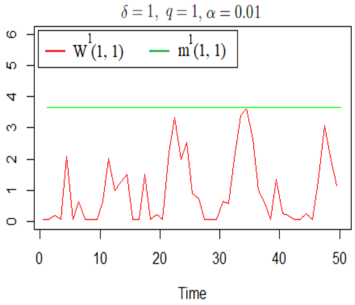
Empirical constant threshold

Empirical method : perform simulations of the statistics under $\mathbb{P}_0$ and build the detection threshold according to empirical quantile of law of the statistics under pre-change regime.

Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,.,n;\ j=1,.,B}$.
2. Choice of the objective of detection (δ, q) and compute $w_i^j(\delta, q)$;
3. Choice of instantaneous false alarm rate α tolerated ;
4. For each series $\{x_i^j\}_{1\leq i\leq n}$, compute the **maximum of statistics** :

$$\mathrm{m}^j(\delta, q) = \max_{1\leq i\leq n} w_i(\delta, q).$$



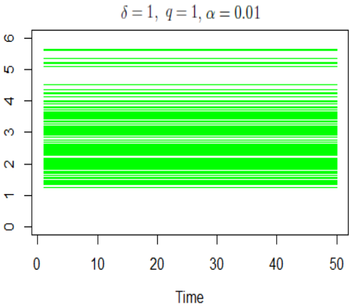
Empirical constant threshold

Empirical method : perform simulations of the statistics under $\mathbb{P}_0$ and build the detection threshold according to empirical quantile of law of the statistics under pre-change regime.

Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,..,n;\ j=1,..,B}$.
2. Choice of the objective of detection (δ, q) and compute $w_i^j(\delta, q)$;
3. Choice of instantaneous false alarm rate α tolerated ;
4. For each series $\{x_i^j\}_{1\leq i\leq n}$, compute the **maximum of statistics** :

$$\mathrm{m}^j(\delta, q) = \max_{1\leq i\leq n} w_i(\delta, q).$$



Empirical constant threshold

Empirical method : perform simulations of the statistics under $\mathbb{P}_0$ and build the detection threshold according to empirical quantile of law of the statistics under pre-change regime.

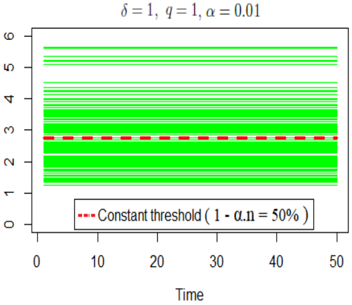
Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,..,n;\ j=1,..,B}$.
2. Choice of the objective of detection (δ, q) and compute $w_i^j(\delta, q)$;
3. Choice of instantaneous false alarm rate α tolerated ;
4. For each series $\{x_i^j\}_{1\leq i\leq n}$, compute the **maximum of statistics** :

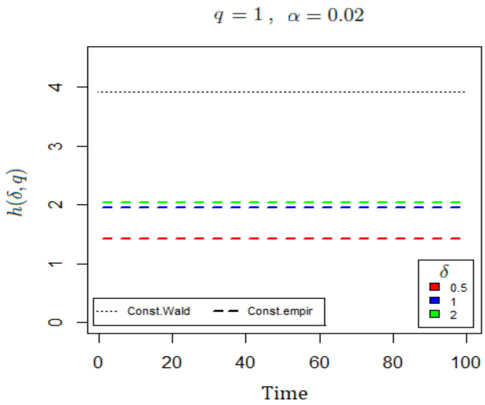
$$\mathrm{m}^j(\delta, q) = \max_{1\leq i\leq n} w_i(\delta, q).$$

5. The constant threshold would be the empirical quantile of order (1-αn) :

$$h(\delta, q) = \mathbf{q}_{(1-n\cdot\alpha)}\left[\left(m^j(\delta, q)\right)_{1\leq j\leq B}\right]$$



Empirical constant threshold



Empirical instantaneous threshold

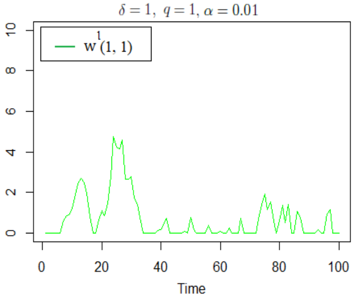
Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,..,n;\ j=1,..,B}$.

Empirical instantaneous threshold

Construction steps :

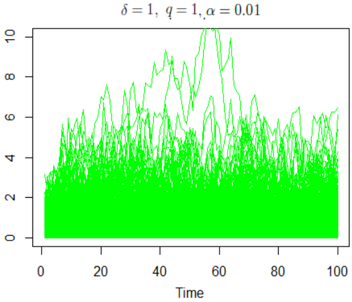
1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,.,n;\ j=1,.,B}$.
2. Choice of the objective of detection $(\delta,\, q)$ and compute $w_i^j\,(\delta,\, q)$;
3. Choice of instantaneous false alarm rate α tolerated ;



Empirical instantaneous threshold

Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,.,n;\ j=1,.,B}$.
2. Choice of the objective of detection (δ, q) and compute $w_i^j(\delta, q)$;
3. Choice of instantaneous false alarm rate α tolerated ;

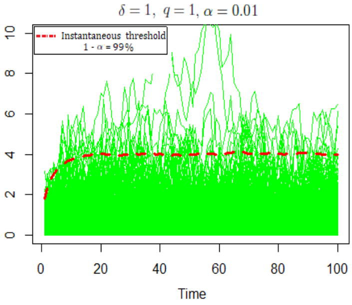


Empirical instantaneous threshold

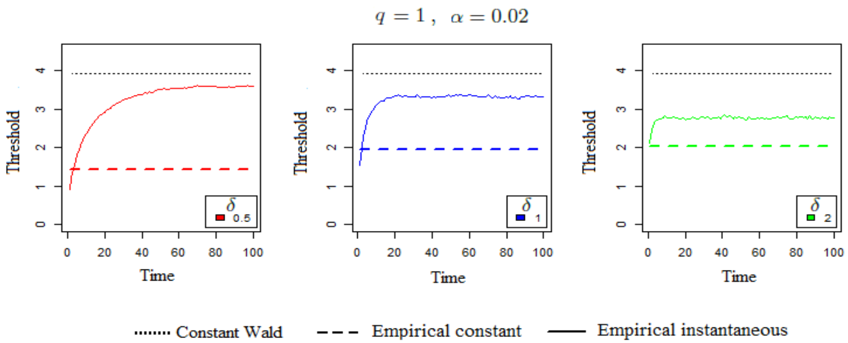
Construction steps :

1. Under $\mathbb{P}_0$: simulate B series of n observations $\{X_i^j\}_{i=1,.,n;\ j=1,.,B}$.
2. Choice of the objective of detection (δ, q) and compute $w_i^j(\delta, q)$;
3. Choice of instantaneous false alarm rate α tolerated ;
4. The instantaneous threshold would be the empirical quantile of order (1-α) :

$$h_t(\delta, q) = \mathbf{q}_{(1-\alpha)} \left[\left(w_t^j(\delta, q) \right)_{1 \leq j \leq B} \right],$$
$$t = 1, ., n$$



Empirical instantaneous threshold

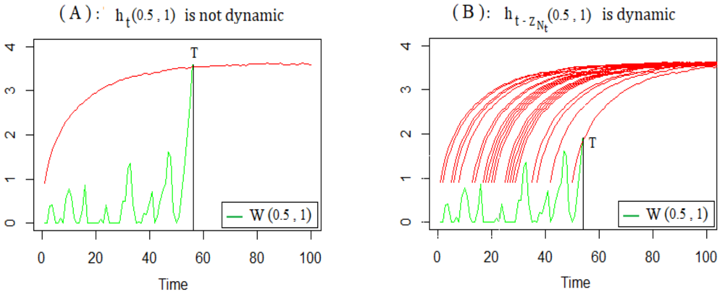


Dynamic instantaneous threshold

$\Rightarrow$ Propose a **dynamic instantaneous threshold (data-driven)** : $h_{t-Z_{N_t}}(\delta, q)$

- Use the built instantaneous threshold and adapt it to the behavior of the statistic ;
- Moving the threshold whenever statistics returns to its initial value (zero).

Where $N_t = \sum_{i=1}^{t} \mathbf{1}_{\{W_i=0\}}$, and $Z_{N_t} = \inf\{i \geq Z_{N_t - 1}; W_i = 0\}$ (renewal process)

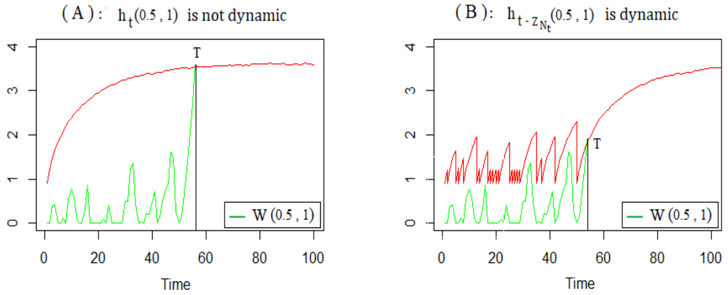


Dynamic instantaneous threshold

$\Rightarrow$ Propose a **dynamic instantaneous threshold (data-driven)** : $h_{t-Z_{N_t}}(\delta, q)$

- Use the built instantaneous threshold and adapt it to the behavior of the statistic ;
- Moving the threshold whenever statistics returns to its initial value (zero).

Where $N_t = \sum_{i=1}^{t} \mathbf{1}_{\{W_i=0\}}$, and $Z_{N_t} = \inf\{i \geq Z_{N_t-1}; W_i = 0\}$ (renewal process)

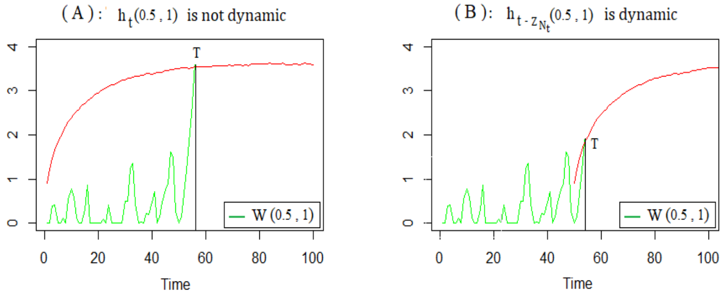


Dynamic instantaneous threshold

$\Rightarrow$ Propose a **dynamic instantaneous threshold (data-driven)** : $h_{t-Z_{N_t}}(\delta, q)$

- Use the built instantaneous threshold and adapt it to the behavior of the statistic ;
- Moving the threshold whenever the statistical returns to its initial value (zero).

Where $N_t = \sum_{i=1}^{t} \mathbf{1}_{\{W_i=0\}}$, and $Z_{N_t} = \inf\{i \geq Z_{N_t-1}; W_i = 0\}$ (renewal process)



Simulation results

Simulation

- **Objective**

  - Evaluate the different detection thresholds.

- **Data simulation**

  - Choice of real pre-change regime $\mathbb{P}_0^R$: μ_0^R, σ_0^R (supposed known) ;

  - Choice of real post-change regime $\mathbb{P}_1^R$: μ_1^R, σ_1^R (and fixed δ^R, q^R).

- **Objective of detection**

  - Choice the type and level of the expected change (δ, q).

- **Estimation of $MTBFA$, α and ADD**

  - Knowing that we simulated series limited to n = 100 observations each, we used an empirical estimate taking into account the censoring (survival analysis).

Simulation

- **Objective**

  - Evaluate the different detection thresholds.

- **Data simulation**

  - Choice of real pre-change regime $\mathbb{P}_0^R$: μ_0^R, σ_0^R (supposed known) ;

  - Choice of real post-change regime $\mathbb{P}_1^R$: μ_1^R, σ_1^R (and fixed δ^R, q^R).

- **Objective of detection**

  - Choice the type and level of the expected change (δ, q).

- **Estimation of $MTBFA$, α and ADD**

  - Knowing that we simulated series limited to n = 100 observations each, we used an empirical estimate taking into account the censoring (survival analysis).

Simulation

- **Objective**

  - Evaluate the different detection thresholds.

- **Data simulation**

  - Choice of real pre-change regime $\mathbb{P}_0^R$: μ_0^R, σ_0^R (supposed known) ;

  - Choice of real post-change regime $\mathbb{P}_1^R$: μ_1^R, σ_1^R (and fixed δ^R, q^R).

- **Objective of detection**

  - Choice the type and level of the expected change (δ, q).

- **Estimation of $MTBFA$, α and ADD**

  - Knowing that we simulated series limited to n = 100 observations each, we used an empirical estimate taking into account the censoring (survival analysis).

Simulation

- **Objective**

  - Evaluate the different detection thresholds.

- **Data simulation**

  - Choice of real pre-change regime $\mathbb{P}_0^R$: μ_0^R, σ_0^R (supposed known) ;

  - Choice of real post-change regime $\mathbb{P}_1^R$: μ_1^R, σ_1^R (and fixed δ^R, q^R).

- **Objective of detection**

  - Choice the type and level of the expected change (δ, q).

- **Estimation of $MTBFA$, α and ADD**

  - Knowing that we simulated series limited to n = 100 observations each, we used an empirical estimate taking into account the censoring (survival analysis).

Results under $\mathbb{P}_0$: $MTBFA$ and $IFAR$

TABLE – $B = 100000$, $n = 100$, $\alpha = 0.02$

Threshold		δ	$\widehat{MTBFA}$	$\widehat{\alpha}$	Nbr. FA
Wald	3.91	0.5	779	0.001	12156
		1	318	0.003	27125
		2	239	0.004	34283

- The more the objective δ is large, the more we have false alarms.

Results under $\mathbb{P}_0$: *MTBFA* and *IFAR*

TABLE – $B = 100000, n = 100, \alpha = 0.02$

Threshold		δ	$\widehat{MTBFA}$	$\widehat{\alpha}$	Nbr. FA
Wald	3.91	0.5	779	0.001	12156
		1	318	0.003	27125
		2	239	0.004	34283
Const. Empir	1.42	0.5	35	0.028	95139
	1.94	1	36	0.028	94385
	2.04	2	37	0.027	93740

- Similar $\widehat{\alpha}$ whatever is δ ; $\widehat{\alpha}$ slightly exceeds the tolerated α.

Results under $\mathbb{P}_0$: *MTBFA* and *IFAR*

TABLE – $B = 100000, n = 100, \alpha = 0.02$

Threshold		δ	$\widehat{MTBFA}$	$\widehat{\alpha}$	Nbr. FA
Wald	3.91	0.5	779	0.001	12156
		1	318	0.003	27125
		2	239	0.004	34283
Const. Empir	1.42	0.5	35	0.028	95139
	1.94	1	36	0.028	94385
	2.04	2	37	0.027	93740
Inst. Empir	$h_t(0.5)$	0.5	291	0.003	27953
	$h_t(1)$	1	147	0.007	48564
	$h_t(2)$	2	73	0.014	74391

- The same behavior of Wald's threshold results, with higher levels of FA but always respecting the tolerated α.

Results under $\mathbb{P}_0$: *MTBFA* and *IFAR*

TABLE – $B = 100000, n = 100, \alpha = 0.02$

Threshold		δ	$\widehat{MTBFA}$	$\widehat{\alpha}$	Nbr FA
Wald	3.91	0.5	779	0.001	12156
		1	318	0.003	27125
		2	239	0.004	34283
Const. Empir	1.42	0.5	35	0.028	95139
	1.94	1	36	0.028	94385
	2.04	2	37	0.027	93740
Inst. Empir	$h_t(0.5)$	0.5	291	0.003	27953
	$h_t(1)$	1	147	0.007	48564
	$h_t(2)$	2	73	0.014	74391
Inst. Empir Dynam	$h_t(0.5)$	0.5	75	0.013	73466
	$h_t(1)$	1	65	0.015	78544
	$h_t(2)$	2	58	0.017	81940

- More homogeneous results ;
  $\widehat{\alpha}$ is close to the tolerated one but never exceeds it.

Results under $\mathbb{P}_1$: ADD

TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold		δ	$\delta^R = 1$			$\delta^R = 2$		
			$\widehat{ADD}$	$\widehat{Mdn}$	No-detect	$\widehat{ADD}$	$\widehat{Mdn}$	No-detect
Wald	3.91	0.5	9.45	9	0	4.31	4	0
		1	7.44	6	2	2.94	3	0
		2	9.37	7	212	2.58	2	0

- Detection is not so fast when $\delta^R > \delta$;
  The change-point is quickly detected when δ^R is large, whatever is δ.

Results under $\mathbb{P}_1$: ADD

TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold		δ	$\delta^R = 1$		
			$\widehat{ADD}$	$\widehat{Mdn}$	No-detect
Wald	3.91	0.5	9.45	9	0
		1	7.44	6	2
		2	9.37	7	212
Const. Empir	1.42	0.5	3	3	0
	1.94	1	3.62	3	0
	2.04	2	4.54	3	0

- $\widehat{ADD}$ is considerably better ;
  Change-point is quickly detected as long as $\delta^R \geqslant \delta$.

Results under $\mathbb{P}_1$: *ADD*

TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold		δ	$\delta^R = 1$ $\widehat{ADD}$	$\widehat{Mdn}$	No-detect
Wald	3.91	0.5	9.45	9	0
		1	7.44	6	2
		2	9.37	7	212
Const. Empir	1.42	0.5	3	3	0
	1.94	1	3.62	3	0
	2.04	2	4.54	3	0
Inst. Empir	$h_t(0.5)$	0.5	8.47	8	0
	$h_t(1)$	1	6.22	5	0
	$h_t(2)$	2	6.23	5	11

- Comparable to Wald's but with faster detection.

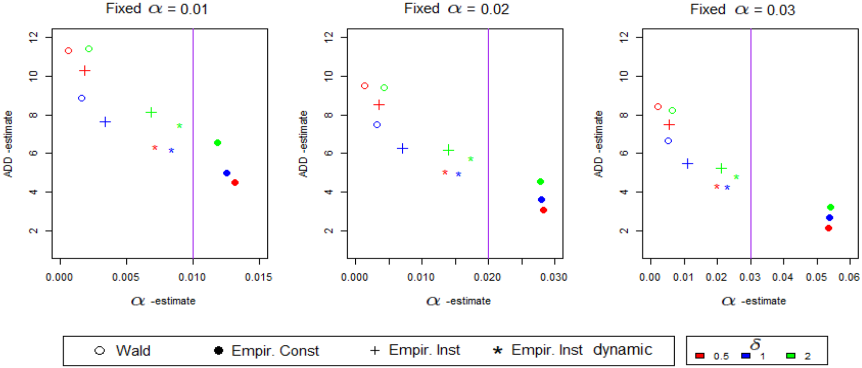
Results under $\mathbb{P}_1$: ADD

TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold		δ	$\delta^R = 1$: $\widehat{ADD}$	$\widehat{Mdn}$	No-detect
Wald	3.91	0.5	9.45	9	0
		1	7.44	6	2
		2	9.37	7	212
Const. Empir	1.42	0.5	3	3	0
	1.94	1	3.62	3	0
	2.04	2	4.54	3	0
Inst. Empir	$h_t(0.5)$	0.5	8.47	8	0
	$h_t(1)$	1	6.22	5	0
	$h_t(2)$	2	6.23	5	11
Inst. Empir Dynam	$h_t(0.5)$	1	5.1	4	0
	$h_t(1)$	2	5	4	0
	$h_t(2)$	3	5.8	4	3

- It detects more quickly than the fixed threshold and that of Wald.

Summary



Fixed α = 0.01
Fixed α = 0.02
Fixed α = 0.03
ADD -estimate
α -estimate
Wald
Empir. Const
Empir. Inst
Empir. Inst dynamic
δ
0.5
1
2

Perspectives

- Theoretical study on the behavior of detection statistics (understand results given by the thresholds) ;
- The case where the parameters of the pre-change regime are unknown : estimation methods ;
- Use the detection methods in the multivariate case ;
- Thesis framework : prediction of a dreaded event during online monitoring of lung transplant patients.

Thank you !

Annex

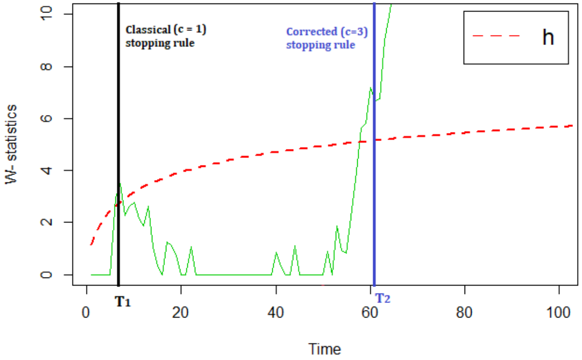
New stopping rule

Classical stopping rule

signals the existence of a changepoint when the detection statistic exceeds the instantaneous threshold.

Corrected stopping rule

signals the existence of a change-point when the detection statistic exceeds the instantaneous threshold during a time $c \geq 1$.



Results : corrected stop rule

TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold		δ	Sous $\mathbb{P}_0$: $\widehat{\alpha}$			Sous $\mathbb{P}_1$: $\widehat{ADD}$		
			Stop rule "c" 1	2	3	1	2	3
Wald	3.91	0.5	0.001	0.0009	0.0007	9.45	10.74	11.9
		1	0.003	0.002	0.001	7.44	9.03	10.4
		2	0.004	0.001	0.0004	9.37	13.1	15.8
Const. empir	1.42	0.5	0.028	0.018	0.014	3	4.34	5.5
	1.94	1	0.028	0.014	0.008	3.62	5.2	6.5
	2.04	2	0.027	0.007	0.002	4.54	7.7	10.5
Inst. empir.	h_t(0.5)	0.5	0.003	0.002	0.001	8.47	9.76	10.9
	h_t(1)	1	0.007	0.003	0.002	6.22	7.82	9.2
	h_t(2)	2	0.014	0.004	0.001	6.23	9.73	12.6
Inst. Empir Dynam	h_t(0.5)	0.5	0.013	0.007	0.004	5.1	6.6	7.84
	h_t(1)	1	0.015	0.006	0.003	5	6.81	8.3
	h_t(2)	2	0.017	0.004	0.001	5.8	9.4	12.36