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INSTITUT ÉLIE CARTAN DE LORRAINE

INRIA BIGS TEAM

ENBIS-19 in Budapest

2 - 4 September 2019

New Detection Thresholds and Stop Rules for CUSUM Online Detection

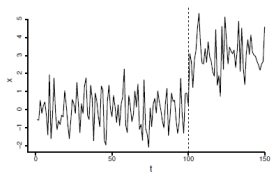
Nassim Sahki, Anne Gégout-Petit, Sophie Wantz-Mézières

03 September 2019

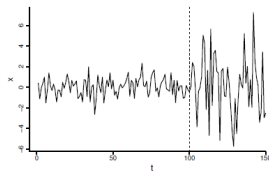
Outline

- 1 Preamble
- 2 Online change-point detection
- 3 New detection threshold
- 4 Simulation results
- 5 Perspectives

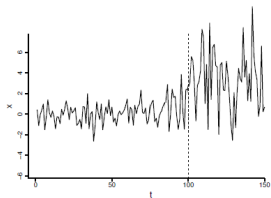
Change-point



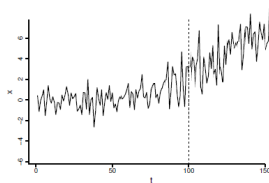
(a) Changepoint in the mean



(b) Changepoint in the variance



(c) Changepoint in the mean and variance

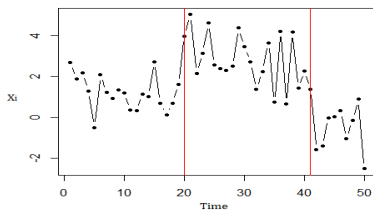


(d) Slope changepoint

Context of analysis

Offline context :

- All data are received and processed in one go ;
- The primary aim is accurate detection of changes ;
- Inference about all change-points simultaneously.

**Online context :**

- Data arrives either as single data-points or in batches ;
- Data must be processed quickly before new data arrives ;
- The aim is the quickest detection of a change after it has occurred ;
- Inference about most recent change only.

Online change-point detection

Hypothesis test

Let $\{X_i\}_{i=1,\dots,n}$ a series of observations **sequentially observed**.

X_n is the last observed point in the dataset.

Statistically, the problem of change-point detection is to sequentially test for each new observation x_n , the hypotheses :

$$\left\{ \begin{array}{lll} H_{0,n} : v > n & X_i \sim f_0(\cdot) & \forall i = 1, \dots, n \\ H_{1,n} : \exists v \leq n, & \begin{array}{l} X_i \sim f_0(\cdot) \\ X_i \sim f_1(\cdot) \end{array} & \begin{array}{l} \forall i = 1, \dots, (v-1) \\ \forall i = v, \dots, n \end{array} \end{array} \right. \quad (1)$$

Where ("distribution pre-change") $f_0 \neq f_1$ ("distribution post-change")

The "instantaneous" Log Likelihood Ratio (LLR) is defined by :

$$L_i = \log(\Lambda_i) = \log\left(\frac{f_1(x_i)}{f_0(x_i)}\right), \quad i \geq 1$$

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Recursive detection statistics

- The Cumulative Sum "CUSUM" statistics is written recursively [Page(1954)] :

$$W_n = \max\{0, W_{n-1} + L_n\}, \quad n \geq 1, \quad W_0 = 0 \quad (2)$$

When the two distributions f_0 and f_1 are unknown ;

⇒ [Tartakovsky, A. G. and all (2006)] suggests replacing the log likelihood ratio L_n through a score function $S_n = S_n(X_1, \dots, X_n)$.

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Score function

The score S_n is defined for a mean and variance change-point detection by :

$$S_n(\delta, q) = C_1 \cdot Y_n + C_2 \cdot Y_n^2 - C_3 \quad (3)$$

$Y_n = (X_n - \mu_0)/\sigma_0$: the centered and standardized data under H_0 .

$$C_1 = \delta \cdot q^2, \quad C_2 = \frac{1-q^2}{2}, \quad C_3 = \frac{\delta^2 \cdot q^2}{2} - \log(q)$$

$$\delta = (\mu_1 - \mu_0)/\sigma_0 \quad q = \sigma_0/\sigma_1$$

- δ : minimum level of change in the mean that is required to be detected.

No changepoint detection on the mean :

$$\mu_1 = \mu_0 \Rightarrow \delta = 0, \text{ therefore } C_1 = 0.$$

- q : minimum level of change in the variance that is required to be detected.

No changepoint detection on the variance :

$$\sigma_1^2 = \sigma_0^2 \Rightarrow q = 1, \text{ therefore } C_2 = 0.$$

Statistics model

Note

The score function can only be used to the knowledge of the parameters **mean and variance of pre-change data** μ_0, σ_0^2 .

- Use a portion of observed data on the normal state without change-point
⇒ **Estimate** μ_0 and σ_0^2 .
- Depending on the **objective (mean and/ or variance)** and level of change that we want to **detect** : $\delta = (\mu_1 - \mu_0)/\sigma_0, q = \sigma_0/\sigma_1$
⇒ **Fixed** μ_1 and σ_1^2 .

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Stopping rule

- The statistics is calculated recursively :

$$W_n = \max\{0, W_{n-1} + S_n\}, \quad n \geq 1, \quad W_0 = 0$$

- Online detection is based on a **Stopping Rule** :

$$T_h = \min\{n \geq 1 : W_n \geq h\}, \quad h \geq 0 : \text{threshold.}$$

When W exceeds the threshold h :

⇒ **The procedure triggers an alarm (Stopping Time) to signal that a change-point has occurred.**

Stopping rule

Denote T a stopping time, such as :

$$T = \min\{i \geq 1 : W_i \geq h\}$$

- * $T \geq v$: detection with a delay ($T - v$).
- * $T < v$: false alarm.
- * $T = +\infty$: non detection.

Detection parameters

Let $\mathbb{P}_0[\cdot], \mathbb{E}_0[\cdot]$: respectively the probability and the expectation **before** the change-point v .

Let $\mathbb{P}_1[\cdot], \mathbb{E}_1[\cdot]$: respectively the probability and the expectation **after** the change-point v .

Parameters evaluated under \mathbb{P}_0 .

Mean Time Between False Alarm (MTBFA)

$$MTBFA = \mathbb{E}_0 [T] \quad (4)$$

Instantaneous False Alarm Rate (IFAR)

$$\alpha = \frac{1}{\mathbb{E}_0 [T]} \quad (5)$$

Parameter evaluated under \mathbb{P}_1 .

Average Detection Delay (ADD)

$$ADD = \mathbb{E}_1 [T] \quad (6)$$

Detection threshold

The conventional detection threshold used in the literature is based on the Wald's inequality [Egea-Roca et al (2017)].

⇒ This threshold is constant. It is given after fixing α "the tolerated *IFAR*", by :

$$h_{\alpha} \leq -\ln(\alpha) \quad (7)$$

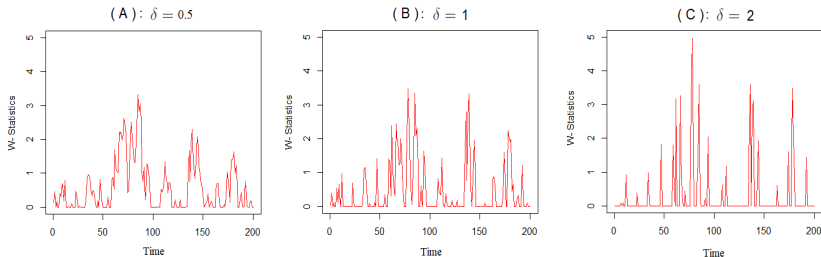
Daniel Egea-Roca, Gonzalo Seco-Granados, and Jose A Lopez-Salcedo. Comprehensive overview of quickest detection theory and its application to GNSS threat detection. Gyroscopy and Navigation, 8(1) : 1-14, 2017.

Abraham Wald. Sequential tests of statistical hypotheses. The annals of mathematical statistics, 16(2) : 117-186, 1945.

- 1 New detection thresholds constructed by an empirical method ;
- 2 New stopping rules by modifying the classical rule.

CUSUM statistics under pre-change regime

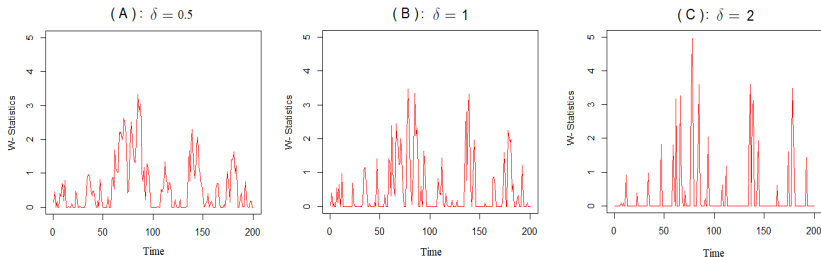
- Simulate a series X_n of $n = 200$ observations of Gaussian distribution ($\mu_0 = 0$ et $\sigma_0^2 = 1$);
- Compute W- statistics according to different levels of δ , ($q = 1$).



The behavior (variability) of the W-statistics depends on the level of δ ;
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Empirical constant threshold

Empirical method : perform simulations of the statistics under \mathbb{P}_0 and build the detection threshold according to empirical quantile of law of the statistics under pre-change regime.

Construction steps :

- 1 Under \mathbb{P}_0 : simulate B series of n observations

$$\{X_i^j\}_{i=1,\dots,n; j=1,\dots,B}$$

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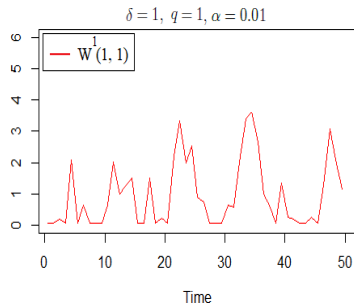
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 $\{X_i^j\}_{i=1, \dots, n; j=1, \dots, B}$
- 2 Choice of the objective of detection (δ, q) and compute $w_i^j(\delta, q)$;
- 3 Choice of instantaneous false alarm rate α tolerated;



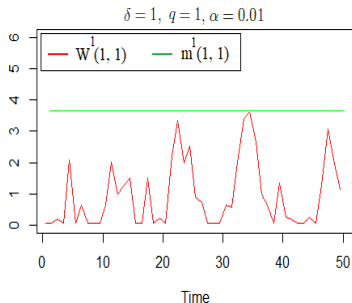
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$$m^j(\delta, q) = \max_{1 \leq i \leq n} w_i(\delta, q).$$



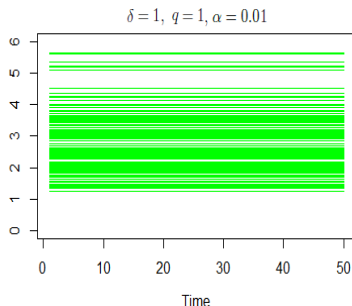
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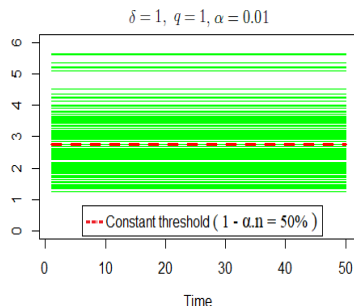
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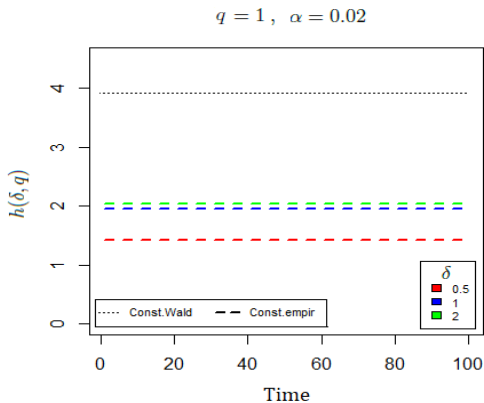
$$m^j(\delta, q) = \max_{1 \leq i \leq n} w_i(\delta, q).$$

- The constant threshold would be the empirical quantile of order $(1-\alpha)n$:

$$h(\delta, q) = \mathbf{q}_{(1-n \cdot \alpha)} \left[\left(m^j(\delta, q) \right)_{1 \leq j \leq B} \right]$$



Empirical constant threshold



Empirical instantaneous threshold

Construction steps :

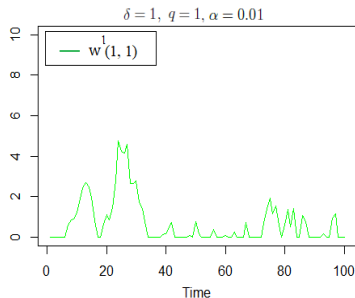
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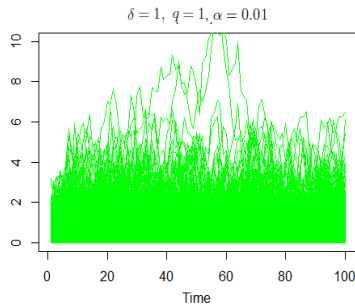
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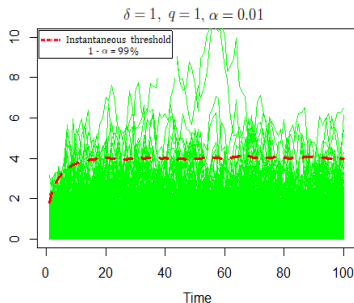
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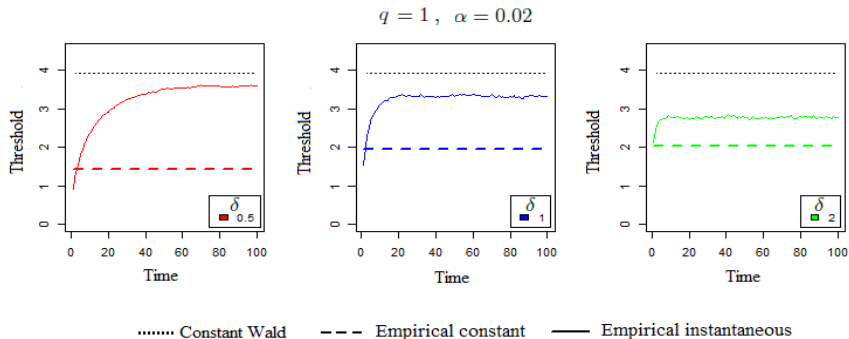
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- 2 Choice of the objective of detection (δ, q) and compute $w_t^j(\delta, q)$;
- 3 Choice of instantaneous false alarm rate α tolerated;
- 4 The instantaneous threshold would be the empirical quantile of order $(1-\alpha)$:

$$h_t(\delta, q) = \mathbf{q}_{(1-\alpha)} \left[\left(w_t^j(\delta, q) \right)_{1 \leq j \leq B} \right],$$

$$t = 1, \dots, n$$



Empirical instantaneous threshold

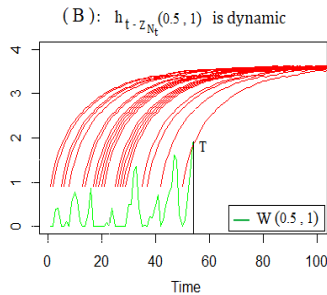
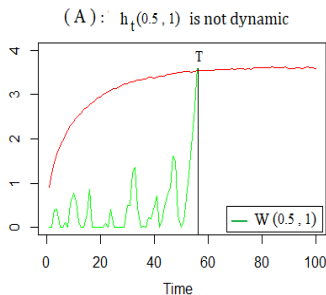


Dynamic instantaneous threshold

⇒ Propose a **dynamic instantaneous threshold (data-driven)** : $h_{t-Z_{N_t}}(\delta, q)$

- Use the built instantaneous threshold and adapt it to the behavior of the statistic;
- Moving the threshold whenever statistics returns to its initial value (zero).

Where $N_t = \sum_{i=1}^t \mathbf{1}_{\{W_i=0\}}$, and $Z_{N_t} = \inf\{i \geq Z_{N_t-1}; W_i = 0\}$ (renewal process)

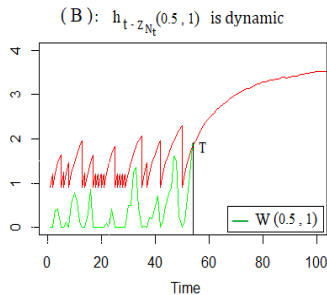
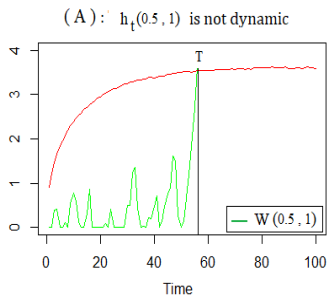


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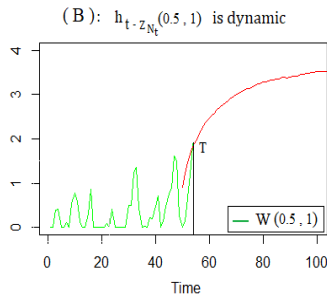
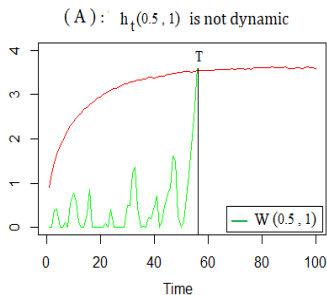


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Simulation results

Simulation

- **Objective**

- Evaluate the different detection thresholds.

- **Data simulation**

- Choice of real pre-change regime $\mathbb{P}_0^R : \mu_0^R, \sigma_0^R$ (supposed known);
- Choice of real post-change regime $\mathbb{P}_1^R : \mu_1^R, \sigma_1^R$ (and fixed δ^R, q^R).

- **Objective of detection**

- Choice the type and level of the expected change (δ, q) .

- **Estimation of *MTBFA*, α and *ADD***

- Knowing that we simulated series limited to $n = 100$ observations each, we used an empirical estimate taking into account the censoring (survival analysis).

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Results under \mathbb{P}_0 : *MTBFA* and *IFAR*TABLE – $B = 100000$, $n = 100$, $\alpha = 0.02$

Threshold	δ	\widehat{MTBFA}	$\widehat{\alpha}$	Nbr. FA	
Wald	3.91	0.5	779	0.001	12156
		1	318	0.003	27125
		2	239	0.004	34283

- The more the objective δ is large, the more we have false alarms.

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Const. Empir	1.42	0.5	35	0.028	95139
		1	36	0.028	94385
		2	37	0.027	93740

- Similar $\hat{\alpha}$ whatever is δ ;
 $\hat{\alpha}$ slightly exceeds the tolerated α .

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Inst. Empir	$h_t(0.5)$	0.5	291	0.003	27953
	$h_t(1)$	1	147	0.007	48564
	$h_t(2)$	2	73	0.014	74391

- The same behavior of Wald's threshold results, with higher levels of FA but always respecting the tolerated α .

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Inst. Empir	$h_t(0.5)$	0.5	291	0.003	27953
	$h_t(1)$	1	147	0.007	48564
	$h_t(2)$	2	73	0.014	74391
Inst. Empir Dynam	$h_t(0.5)$	0.5	75	0.013	73466
	$h_t(1)$	1	65	0.015	78544
	$h_t(2)$	2	58	0.017	81940

- More homogeneous results ;
 $\hat{\alpha}$ is close to the tolerated one but never exceeds it.

Results under $\mathbb{P}_1 : ADD$ TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold	δ	$\delta^R = 1$			$\delta^R = 2$			
		\widehat{ADD}	\widehat{Mdn}	No-detect	\widehat{ADD}	\widehat{Mdn}	No-detect	
Wald	3.91	0.5	9.45	9	0	4.31	4	0
		1	7.44	6	2	2.94	3	0
		2	9.37	7	212	2.58	2	0

- Detection is not so fast when $\delta^R > \delta$;
The change-point is quickly detected when δ^R is large, whatever is δ .

Results under $\mathbb{P}_1 : ADD$ TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold		δ	$\delta^R = 1$			
			\widehat{ADD}	\widehat{Mdn}	No-detect	
Wald	3.91	0.5	9.45	9	0	
		1	7.44	6	2	
		2	9.37	7	212	
Const. Empir		1.42	0.5	3	3	0
		1.94	1	3.62	3	0
		2.04	2	4.54	3	0

- \widehat{ADD} is considerably better ;
Change-point is quickly detected as long as $\delta^R \geq \delta$.

Results under $\mathbb{P}_1 : ADD$ TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold		δ	$\delta^R = 1$		
			\widehat{ADD}	\widehat{Mdn}	No-detect
Wald	3.91	0.5	9.45	9	0
		1	7.44	6	2
		2	9.37	7	212
Const. Empir	1.42	0.5	3	3	0
		1	3.62	3	0
		2	4.54	3	0
Inst. Empir	$h_t(0.5)$	0.5	8.47	8	0
		1	6.22	5	0
		2	6.23	5	11

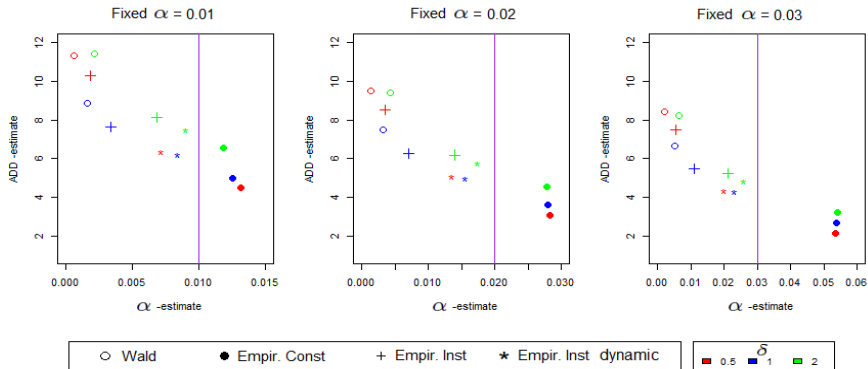
- Comparable to Wald's but with faster detection.

Results under $\mathbb{P}_1 : ADD$ TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

		$\delta^R = 1$			
Threshold	δ	\widehat{ADD}	\widehat{Mdn}	No-detect	
Wald	3.91	0.5	9.45	9	0
		1	7.44	6	2
		2	9.37	7	212
Const. Empir	1.42	0.5	3	3	0
	1.94	1	3.62	3	0
	2.04	2	4.54	3	0
Inst. Empir	$h_t(0.5)$	0.5	8.47	8	0
	$h_t(1)$	1	6.22	5	0
	$h_t(2)$	2	6.23	5	11
Inst. Empir Dynam	$h_t(0.5)$	1	5.1	4	0
	$h_t(1)$	2	5	4	0
	$h_t(2)$	3	5.8	4	3

- It detects more quickly than the fixed threshold and that of Wald.

Summary



Perspectives

- Theoretical study on the behavior of detection statistics (understand results given by the thresholds);
- The case where the parameters of the pre-change regime are unknown : estimation methods ;
- Use the detection methods in the multivariate case ;
- Thesis framework : prediction of a dreaded event during online monitoring of lung transplant patients.

Thank you !

Annex

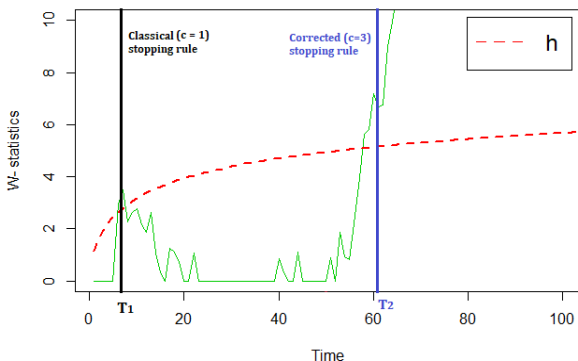
New stopping rule

Classical stopping rule

signals the existence of a change-point when the detection statistic exceeds the instantaneous threshold.

Corrected stopping rule

signals the existence of a change-point when the detection statistic exceeds the instantaneous threshold during a time $c \geq 1$.



Results : corrected stop rule

TABLE – $B = 100000, n = 100, v = 50, \alpha = 0.02$

Threshold	δ	Sous $\mathbb{P}_0 : \hat{\alpha}$			Sous $\mathbb{P}_1 : \widehat{ADD}$			
		1	2	3	1	2	3	
Wald	3.91	0.5	0.001	0.0009	0.0007	9.45	10.74	11.9
		1	0.003	0.002	0.001	7.44	9.03	10.4
		2	0.004	0.001	0.0004	9.37	13.1	15.8
Const. empir	1.42	0.5	0.028	0.018	0.014	3	4.34	5.5
	1.94	1	0.028	0.014	0.008	3.62	5.2	6.5
	2.04	2	0.027	0.007	0.002	4.54	7.7	10.5
Inst. empir.	$h_t(0.5)$	0.5	0.003	0.002	0.001	8.47	9.76	10.9
	$h_t(1)$	1	0.007	0.003	0.002	6.22	7.82	9.2
	$h_t(2)$	2	0.014	0.004	0.001	6.23	9.73	12.6
Inst. Empir Dynam	$h_t(0.5)$	0.5	0.013	0.007	0.004	5.1	6.6	7.84
	$h_t(1)$	1	0.015	0.006	0.003	5	6.81	8.3
	$h_t(2)$	2	0.017	0.004	0.001	5.8	9.4	12.36