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# Efficient compressed sensing based non-sample spaced sparse channel estimation in OFDM system

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**ABSTRACT** This paper aims to develop an efficient compressed sensing (CS) based channel estimation method for non-sample spaced sparse channels in orthogonal frequency division multiplexing (OFDM) systems, which can effectively balance the channel estimation performance, spectral efficiency and computational complexity. To realize this goal, a novel delay tracking and residual norm minimization (DT-RNM) method is proposed. In this method, the idea of reference delay grids (RDG) inspired by the delay tracking (DT) method proposed in [1] is formulated, which is proven to be much more efficient than current algorithms in estimating non-sample spaced sparse channels. Both theoretical derivation and simulation results show the effectiveness of the proposed method.

**INDEX TERMS** orthogonal frequency division multiplexing (OFDM), compressed sensing (CS), channel estimation, delay tracking and residual norm minimization (DT-RNM), reference delay grids (RDG), non-sample spaced sparse channels.

## I. INTRODUCTION

As a multicarrier modulation technique, orthogonal frequency division multiplexing (OFDM) technique has the ability to provide reliable high data rate transmission in different communication scenarios [2]. In OFDM system, channel estimation is essential, because it provides an effective tool for acquiring the channel state information (CSI), which is useful for equalizing the channel distortion. However, obtaining the CSI requires the OFDM system resources, like spectral resources and power resources, which are also essential for efficient transmission. Therefore, designing effective channel estimation methods, which can effectively balance the channel estimation performance, spectral efficiency and computational complexity is very important [3], [4].

Compared with sample spaced channels, a non-sample spaced multipath channel has fractional sample spaced channel impulse response (CIR). Obviously, the non-sample spaced channel model is much more appropriate for different realistic communication environments [5].

Traditional non-sample spaced channel estimation in OFDM system is mainly based on the least squares (LS) estimator, minimum mean square error (MMSE) estimator and discrete Fourier transform (DFT) based method [6], [7]. However, both the LS and MMSE estimators have disadvantages. The performance of the LS estimator is limited by the noise effects and energy leakage of channel taps while the MMSE estimator requires the prior knowledge of the noise variance and channel statistics, moreover, it involves higher computational complexity [6]. To reduce the leakage power of CIR, [7] proposes a symmetric extension of the DFT method. In general, the traditional non-sample spaced channel estimation methods can hardly balance the channel estimation performance, spectral efficiency and computational complexity efficiently.

In recent years, as the existence of the sparse characteristics of wireless physical channel in many communication environments has been proven, more and more research is focused on the compressed sensing (CS) based non-sample

spaced sparse channel estimation [1], [8]–[13]. [8] focuses on CS based sparse channel estimation by exploring the delay-Doppler sparsity to reduce the usage of pilots and improve the spectral efficiency. [9] has mathematically described how to use the virtual representation in discrete domain to approximate the frequency selective channel and doubly selective channel by uniformly sampling the delay space and delay-Doppler space respectively at the Nyquist rate. However, for non-sample spaced channel in delay space or delay-Doppler space, using the original Nyquist rate (resolution) is not enough to reconstruct the channel with sufficient precision. In this case, high resolution sparse channel reconstruction is required [10]. [11], [12] investigate high resolution channel estimation techniques to efficiently reduce the power leakage in delay-Doppler space for doubly selective channels. For high resolution sparse channel reconstruction, the size of dictionary (measurement matrix in CS) will be dramatically increased with the oversampling factor  $R$  ( $R > 1$ ), which significantly increases the complexity of the channel reconstruction algorithm. To solve this problem, low computational complexity oriented delay estimation becomes popular [1], [13]. [1] proposes a novel adaptive delay tracking (DT) method, which achieves comparatively good channel estimation performance meanwhile significantly reduces the computational complexity by decreasing the number of coherence matching computations between the bases of the measurement matrix and the residual vector. A closed-form estimate for the tap delays is derived in [13], which can achieve effective channel estimation performance with low computational complexity. However, the delay estimation methods in [1] and [13] highly depend on the partially equispaced or equispaced pilot arrangement, which is actually not optimal in estimating non-sample spaced sparse channels. Therefore, effective non-sample spaced sparse channel estimation with optimized non-uniform pilot arrangement requires further investigation [13].

In this paper, by fully considering the spectral efficiency and exploiting the sparsity of the physical channel, we propose a novel compressed non-sample spaced sparse channel sensing scheme for OFDM system. Compared with the methods proposed in [1] and other methods, our proposed method has the following distinctive contributions.

1. Different from [1] and [13], which employ the partially equispaced or totally equispaced pilot arrangement, this paper adopts the suboptimal non-equispaced pilot arrangement proposed in [14], which is actually effective in the case of  $M < L_{cp}$  ( $M$  is the number of pilots and  $L_{cp}$  is the length of cyclic prefix). By optimizing the pilot arrangement, the proposed method has a better spectral efficiency compared with the methods proposed in [1] and [13].

2. The reference delay grid (RDG) guided residual norm minimization (RNM) method proposed in this paper is the main contribution of this work, which is new and clearly different from the current non-sample spaced sparse channel estimation methods [1], [10]. The newly proposed reference delay grid (RDG) guided residual norm minimization (RNM)

method can effectively fight against the non-uniform pilot arrangement and promote the channel delay estimation precision, therefore, the proposed method has significantly better channel estimation performance than the methods proposed in [1] and [10] throughout the whole considered  $E_b/N_0$ .

3. Although, the computational complexity of the proposed method is slightly higher than that of the method proposed in [1], it is still much lower than that of the conventional orthogonal matching pursuit (OMP) algorithm with equivalent delay resolution [10].

The rest of this paper is organized as follows. The considered OFDM system model is given in Section II. The proposed DT-RNM method is described in Section III. Section IV provides the simulation results and the conclusion is drawn in Section V.

## II. SYSTEM MODEL

### A. OFDM SYSTEM MODEL

Consider an OFDM system with  $N$  subcarriers, among which  $M$  are pilots with positions  $k_0, k_1, \dots, k_{M-1}$ . Assume that the signal is transmitted over a  $K$  sparse channel (sparse channel with  $K$  non-zero channel taps) with CIR expressed as:

$$h(t) = \sum_{l=0}^{K-1} \alpha_l \delta(t - \tau_l T_s) \quad (1)$$

where  $\alpha_l$  and  $\tau_l T_s$  ( $0 \leq \tau_l \leq L_{cp} - 1$ ) are the complex gain and delay of the  $l^{th}$  tap respectively;  $T_s$  is the sampling interval.

At the receiver, after sampling the channel frequency response (CFR) of  $h(t)$ , the CIR is given by [6]:

$$h[n] = h(nT_s) = \frac{1}{N} \sum_l \alpha_l e^{-j\frac{\pi}{N}(n+(N-1)\tau_l)} \frac{\sin(\pi\tau_l)}{\sin(\frac{\pi}{N}(\tau_l - n))} \quad (2)$$

In (2), when  $\tau_l$  is an integer, we have  $h[\tau_l] = \alpha_l$ , which means that all the energy of the  $l^{th}$  tap is mapped to the channel tap  $h[\tau_l]$  ( $n = \tau_l$ ). There is no power leakage. For sample spaced channel, its channel tap positions are exactly located in some sampling points. When  $\tau_l$  is not an integer, we have  $h(\lfloor \tau_l \rfloor T_s) \neq \alpha_l$  and  $h(\lceil \tau_l \rceil T_s) \neq \alpha_l$ , which means that not all the energy of the  $l^{th}$  tap is mapped to the channel tap  $h[n]$  ( $n = \lfloor \tau_l \rfloor$  or  $n = \lceil \tau_l \rceil$ ). There exists leakage of power to other most adjacent channel taps. For non-sample spaced channel, its channel tap positions are not exactly located in sampling points. In the presence of power leakage, the channel model is no longer a sparse channel, since most of the channel taps contain the power of channel paths.

In order to reduce the power leakage effect, high time resolution ( $T'_s = T_s/R$ ) CIR should be considered, (1) can be rewritten as:

$$h(t) = \sum_{l=0}^{K-1} \alpha_l \delta(t - R\tau_l T'_s) \quad (3)$$

The CFR of  $h(t)$  can be written as:

$$\begin{aligned} g(f) &= \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt \\ &= \sum_l \alpha_l e^{-j2\pi f R \tau_l T_s'} \end{aligned} \quad (4)$$

Taking the FFT of  $h(lT_s')$ ,  $l = 0, 1, 2, \dots, RN-1$ , we obtain the following discrete CFR:

$$g[k] = \sum_l \alpha_l e^{-j2\pi R \tau_l k / RN} \quad (5)$$

Its IFFT with size of  $RN$  is used to obtain the CIR:

$$\begin{aligned} h[n] &= \frac{1}{RN} \sum_l \alpha_l \sum_{k=0}^{RN-1} e^{j2\pi(n-R\tau_l)k/RN} \\ &= \frac{1}{RN} \sum_l \alpha_l \frac{1 - e^{j2\pi(n-R\tau_l)}}{1 - e^{j2\pi(n-R\tau_l)/(RN)}} \end{aligned} \quad (6)$$

Let  $n - R\tau_l = x$  ( $n \in [0, RN - 1]$ ), (6) becomes:

$$\frac{1}{RN} \sum_l \alpha_l \frac{1 - e^{j2\pi x}}{1 - e^{j2\pi x/(RN)}} \quad (7)$$

It is obvious that:

$$\lim_{x \rightarrow 0} \frac{1}{RN} \frac{1 - e^{j2\pi x}}{1 - e^{j2\pi x/(RN)}} = 1 \quad (8)$$

When  $R \rightarrow \infty$  and  $x \not\rightarrow 0$ , we have:

$$\lim_{R \rightarrow \infty} \lim_{x \not\rightarrow 0} \frac{1}{RN} \frac{1 - e^{j2\pi x}}{1 - e^{j2\pi x/(RN)}} = 0 \quad (9)$$

Combining the above two cases, we can obtain:

$$h[n] = \begin{cases} \alpha_l, & n = R\tau_l \\ 0, & n \neq R\tau_l \end{cases} \quad R \rightarrow \infty \quad (n \in [0, RN - 1]) \quad (10)$$

From (10), when  $R \rightarrow \infty$ ,  $h[n]$  becomes a continuous channel. Therefore, the power of tap  $\delta[n - R\tau_l]$ , with  $n = t/T_s'$  will be completely mapped into  $h[R\tau_l]$  when  $R \rightarrow \infty$ . From the above theoretical derivation, we know that the increase of  $R$  improves the channel estimation precision when estimating the non-sample spaced sparse channels. However, when  $R$  reaches a sufficiently big value (e.g.  $R > 8$ ), the channel estimation improvement will be less significant with the further increase of  $R$ . The typical values of  $R$  are  $R = 2$  or  $R = 4$  or  $R = 8$  [1], [10], [13]. Practically, the value of  $R$  should balance the computational complexity and the channel estimation performance. Therefore,  $R = 4$  or  $R = 8$  is usually recommended for the non-sample spaced sparse channel estimation in practice.

## B. COMPRESSED SENSING BASED OFDM SYSTEM

1) Compressed Sensing based OFDM System Model

Let  $\mathbf{X}_p = \text{diag}[x[k_0], x[k_1], \dots, x[k_{M-1}]] \in C^{M \times M}$  be the normalized transmitted pilot matrix. All OFDM pilot symbols are assumed to have equal transmit power. The received

pilot vector  $\mathbf{y}_p = [y[k_0], y[k_1], \dots, y[k_{M-1}]]^T \in C^{M \times 1}$  can be written as [15], [16]:

$$\mathbf{y}_p = \mathbf{X}_p \mathbf{F}_R \mathbf{h}_R + \mathbf{w}'_p \quad (11)$$

where  $\mathbf{F}_R \in C^{M \times R(L_{cp}-1)+1}$  is the dictionary matrix with element  $F_R(m, s) = e^{-j2\pi \frac{km\tau'_s}{N}}$  ( $m \in \{0, 1, \dots, M-1\}$ ,  $s \in \{0, 1, \dots, R(L_{cp}-1)\}$ ), where  $R = 1$  corresponds to the Nyquist baseband sampling factor while  $R \geq 2$  is employed as the oversampling factor,  $\mathcal{T}_R = \{0, \frac{1}{R}, \dots, L_{cp}-1\}$ ,  $\tau'_s \in \mathcal{T}_R$  and  $S = R(L_{cp}-1)+1$  is the size of the delay grid  $\mathcal{T}_R$ ;  $\mathbf{h}_R \in C^{R(L_{cp}-1)+1 \times 1}$  is the CIR with oversampling factor  $R$ ;  $\mathbf{w}'_p = [w'[k_0], w'[k_1], \dots, w'[k_{M-1}]]^T \in C^{M \times 1}$  is the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix  $\sigma_w^2 \mathbf{I}_M$ . From (11), the observed CFR can be obtained by [1], [17]:

$$\mathbf{g}_p = \mathbf{F}_R \mathbf{h}_R + \mathbf{w}_p \quad (12)$$

where  $\mathbf{g}_p = \mathbf{X}_p^{-1} \mathbf{y}_p \in C^{M \times 1}$  and  $\mathbf{w}_p = \mathbf{X}_p^{-1} \mathbf{w}'_p \in C^{M \times 1}$  are the observed CFR and the complex additive Gaussian noise vector with zero mean and covariance  $\sigma_w^2 \mathbf{I}_M$  respectively. In the following, (12) will be adopted as the CS model as in [1], [17].

2) Pilot Arrangement for Compressed Sensing based OFDM System

In pilot-aided channel estimation field, pilot arrangement is a vital task for effective channel estimation [1], [13]–[20]. In the case of  $M \geq L_{cp}$ , the equispaced or uniform pilot arrangement is optimal [18], [19]. However, this situation lacks of spectral efficiency. For better spectral efficiency,  $M < L_{cp}$  is generally considered. In this case, the optimality of the partially equispaced or equispaced pilot arrangement considered in [1], [13] is not guaranteed [20]. Therefore, in the case of  $M < L_{cp}$ , finding the suboptimal or optimal pilot arrangement for dictionary matrix  $\mathbf{F}_R$  is critical. In compressed channel sensing field, the mutual coherence between the bases of  $\mathbf{F}_R$  in (12) is an important parameter for the evaluation of the pilot arrangement methods, which can be defined as [14], [15], [17]:

$$\mu(\mathbf{F}_R) = \max_{u, v \in \mathcal{T}_R, u \neq v} \frac{|\langle \mathbf{f}_u, \mathbf{f}_v \rangle|}{\|\mathbf{f}_u\|_2 \|\mathbf{f}_v\|_2} \quad (13)$$

where  $\mathbf{f}_u = [e^{-j2\pi \frac{k_0 u}{N}}, e^{-j2\pi \frac{k_1 u}{N}}, \dots, e^{-j2\pi \frac{k_{M-1} u}{N}}]^T \in C^{M \times 1}$  ( $u \in \mathcal{T}_R$ ) is a basis (column) in  $\mathbf{F}_R$  ( $\mathbf{f}_v$  has similar definition.). According to (13), we can obtain the following coherence minimization problem regarding to the  $M$  pilots arrangement [14], [15]:

$$\arg \min_{\Lambda} \max_{u, v \in \mathcal{T}_R, u \neq v} \left\langle \sum_{m=0}^{M-1} e^{-j2\pi k_m (u-v)/N} \right\rangle \quad (14)$$

where  $\Lambda = [k_0, k_1, \dots, k_{M-1}]$  is the pilot location vector. For the optimization problem (14), there are  $\binom{N}{M}$  possible pilot arrangements, it is actually a computationally heavy task. Therefore, many suboptimal pilot arrangement criteria and

methods have been proposed in [14]–[17]. From theoretical point of view, we should optimize (14) for the non-sample spaced sparse channels. However, in the case of  $R \geq 2$ , the bases considered for the optimization in (14) will be  $R$  times as the bases considered in  $R = 1$ . Therefore, for efficiency of practical applications, we still consider  $R = 1$  and simply adopt the method proposed in [14], which firstly randomly generates a limited number of pilot arrangements, then selects the optimal one according to (14).

### III. DT-RNM FOR NON-SAMPLE SPACED SPARSE CHANNEL ESTIMATION

#### A. PROPOSED DT-RNM BASED CHANNEL ESTIMATION METHOD

This section focuses on the algorithm of OMP and its derived versions for non-sample spaced sparse channel estimation. With the CS model (12), the goal of the channel reconstruction is to find the solution of the following optimization problem [21]:

$$\hat{\mathbf{h}}_R = \arg \min_{\mathbf{h}} \|\mathbf{g}_p - \mathbf{F}_R \mathbf{h}\|_2 \quad \text{s.t.} \quad \|\mathbf{h}\|_0 \leq K_{max} \quad (15)$$

where  $K_{max}$  is the maximum sparsity (maximum possible number of non-zero channel taps) of the sparse channel. Finding the optimal solution of the above  $l_2$  norm minimization problem is a computational requiring task. Therefore, many sub-optimal solutions like OMP and its derived algorithms have been developed. The OMP algorithm provides a sub-optimal solution at the  $l^{th}$  ( $l \leq K_{max}$ ) iteration given by:

$$\hat{\mathbf{h}}'_l = \arg \min_{\mathbf{h}_l} \|\mathbf{g}_p - \hat{\mathbf{F}}'_l \mathbf{h}_l\|_2 \quad \text{s.t.} \quad \text{halting criterion} \quad (16)$$

where  $\hat{\mathbf{F}}'_l$  is the Fourier matrix with the selected bases of OMP at the  $l^{th}$  iteration. The residual vector of the  $l^{th}$  iteration is defined as  $\mathbf{r}_l = \mathbf{g}_p - \hat{\mathbf{F}}'_l \hat{\mathbf{h}}'_l$ . The halting criterion can be realized by channel sparsity or threshold, which will be discussed in the next subsection.  $\hat{\mathbf{F}}'_l = [\hat{\mathbf{F}}'_{l-1} \ \mathbf{f}_{\hat{\tau}'_l}]$ .  $\hat{\tau}'_l$  can be estimated by:

$$\hat{\tau}'_l = \arg \max_{\tau \in \mathcal{T}_R} |\langle \mathbf{f}_\tau, \mathbf{r}_{l-1} \rangle| \quad (17)$$

where  $\mathbf{r}_{l-1}$  and  $\mathbf{f}_\tau$  are the residual at the  $(l-1)^{th}$  iteration and a basis or a column of matrix  $\mathbf{F}_R$  respectively. In OMP algorithm, (17) and (16) are two essential steps known as delay tracking (DT) step and residual norm minimization (RNM) step respectively. As previously mentioned, the increase of  $R$  results in the increase of computational complexity. To solve this problem, an effective DT method is proposed in [1]. Assume that the channel sparsity  $K = 1$  and there is no intercarrier interference (ICI) and noise. Initialize the delay grids set with  $\mathcal{T}_{DT}^{(0)} = \mathcal{T}_1$  as shown in FIGURE 1 (a). Therefore, a rough estimate of delay  $\hat{\tau}_0$  can be obtained by:

$$\hat{\tau}_0 = \arg \max_{\tau \in \mathcal{T}_{DT}^{(0)}} |\langle \mathbf{f}_\tau, \mathbf{g}_p \rangle| \quad (18)$$

Based on  $\hat{\tau}_0$ , we can have the following extension of the initial delay grids set [1]:

$$\mathcal{T}_{DT}^{(1)} = \{\hat{\tau}_0 - \frac{1}{2}, \hat{\tau}_0, \hat{\tau}_0 + \frac{1}{2}\} \quad (19)$$

With the newly extended delay grids set  $\mathcal{T}_{DT}^{(1)}$ , a finer delay estimation can be obtained:

$$\hat{\tau}_0^{(1)} = \arg \max_{\tau \in \mathcal{T}_{DT}^{(1)}} |\langle \mathbf{f}_\tau, \mathbf{g}_p \rangle| \quad (20)$$

According  $\hat{\tau}_0^{(1)}$ , we can further get  $\mathcal{T}_{DT}^{(2)}$  as follows [1]:

$$\mathcal{T}_{DT}^{(2)} = \{\hat{\tau}_0^{(1)} - \frac{1}{2^2}, \hat{\tau}_0^{(1)}, \hat{\tau}_0^{(1)} + \frac{1}{2^2}\} \quad (21)$$

After  $Q$  iterations, we can get the final delay estimate  $\hat{\tau}_0^{(Q)}$  with its estimation error  $|\tau_0 - \hat{\tau}_0^{(Q)}| \leq 1/2^{Q+1}$ , which decreases with the increase of  $Q$  according to the properties of  $|\langle \mathbf{f}_\tau, \mathbf{g}_p \rangle|$  (See Appendix A) [1]. Taking  $K = 1$  and  $L_{cp} = 256$  as an example, FIGURE 1 (a) and FIGURE 1 (b) illustrate the delay grids used by the DT method proposed in [1] with  $Q = 2$  and the delay grids of the OMP method with  $R = 2^Q = 4$  respectively. The number of the calculated delay points of the OMP algorithm is 1021 while the number of the calculated delay points of the method proposed in [1] is 260. Therefore, the method proposed in [1] is actually more efficient in the delay tracking (DT) process than the classical OMP method.

With (18)-(21) and Appendix A, it is easy to know that the authors in [1] try to pursue the accurate delays of the channel taps mainly based on equation (27) in Appendix A. Based on (27) and the following two properties of  $f(\Delta\tau) = \frac{|\sin(\pi\Delta\tau)|}{|\sin(\pi\Delta\tau/M)|}$ , we know that for multipath channel (e.g.  $K \geq 2$ ), the coherence between the residual and the considered bases around each true delay is a *sinc* function or an approximate *sinc* function of  $\Delta\tau$ . Therefore, it is reasonable to pursue the accurate delay for each channel tap, which is essential for no leakage channel estimation. However, the conditions for getting (27) require  $M > L_{cp}$  and equispaced pilot arrangement. Although Algorithm 1 in [1] tries to add some additional new elements such that a new delay grid is chosen with  $2C + 1$  elements ( $C \geq 1$  is a small number) to reduce the effect of non-uniform pilot arrangement, leakage effect and interference, it still requires that the pilot arrangement is partially or globally equispaced. The same requirements also appear for the algorithms in [13]. As previously mentioned, equispaced pilot arrangement is not optimal when  $M < L_{cp}$ , especially when  $M \ll L_{cp}$  [20]. In the case of  $M < L_{cp}$ , we should optimize the pilot arrangement according to (14). After pilot arrangement optimization, the pilot can hardly be uniformly distributed, which can actually cause a serious distortion on the *sinc* function mentioned above. In this case, the true delay for the  $l^{th}$  tap may appear in a delay subset  $\mathcal{T}_{sub} = \text{sup}((\mathbf{F}_{R,\mathcal{T}}^H \mathbf{r}_{l-1})_D)$  or a comparatively small neighbourhood of a delay within  $\mathcal{T}_{sub}$  with high probability (where  $\mathcal{T} = \bigcup_{q=0}^Q (\mathcal{T}_{DT}^{(q)})$  for the DT method in [1] and  $\mathcal{T} = \mathcal{T}_R$  for OMP method et al.  $\mathcal{T}_{sub}$  contains the position

indices of the  $D$  largest components of the coherence vector  $\mathbf{F}_{R,\mathcal{T}}^H \mathbf{r}_{l-1}$ ,  $D$  is generally a small number) and only consider the delay point  $\hat{\tau}_l = \arg \max_{\tau \in \mathcal{T}} |\langle \mathbf{f}_\tau, \mathbf{r}_{l-1} \rangle|$  is not sufficient for accurate delay tracking and channel estimation with high precision. In other words, the delay points within a delay subset where the corresponding bases have high coherence with the residual vector, should be considered. With these delay points, the reference delay grid (RDG) guided RNM method is proposed in this paper to effectively fight against the non-uniform pilot arrangement and realize the near optimal delay searching of the  $l^{th}$  channel tap, which will be discussed after the DT method in this section.

According to the above discussions and taking the properties of  $|\langle \mathbf{f}_\tau, \mathbf{g}_p \rangle|$  into account, we propose the one time local delay expansion, which is given by:

$$\mathcal{T}^{(1)} = \left\{ \hat{\tau}_0 - \frac{1}{2}, \hat{\tau}_0 - \frac{1}{2} + \frac{1}{R}, \dots, \hat{\tau}_0, \dots, \hat{\tau}_0 + \frac{1}{2} - \frac{1}{R}, \hat{\tau}_0 + \frac{1}{2} \right\} \quad (22)$$

where  $\Delta\tau = -\frac{1}{2}$  and  $\Delta\tau = \frac{1}{2}$  are the two cut-off points. It is more direct, robust and convenient to consider (22) as the local delay expansion. In the following, the proposed reference delay grid (RDG) guided RNM method is described based on the local delay expansion  $\mathcal{T}^{(1)}$  to improve the performance of the channel recovery in the case of  $M < L_{cp}$ .

The algorithm in [1] uses the same RNM method as the OMP algorithm expressed by (16).  $\hat{\tau}_l'$  estimated by (17) is actually computationally efficient for RNM computation, however, it is not always optimal for RNM at the  $l^{th}$  iteration. We consider the RNM method of the OMP algorithm for simplicity,  $\hat{\mathbf{F}}_l' = [\hat{\mathbf{F}}_{l-1}' \mathbf{f}_{\hat{\tau}_l'}]$  is adopted for the RNM (16), which is only a suboptimal RNM at the  $l^{th}$  iteration. The optimal RNM at the  $l^{th}$  iteration should apply the following expression:

$$[\hat{\mathbf{h}}_{l,opt}', \hat{\tau}_{l,opt}'] = \arg \min_{\mathbf{h}_l, \tau \in \mathcal{T}_R} \|\mathbf{g}_p - \hat{\mathbf{F}}_l'' \mathbf{h}_l\|_2 \quad \text{s.t. halting criterion} \quad (23)$$

where  $\hat{\mathbf{F}}_l'' = [\hat{\mathbf{F}}_{l-1}'' \mathbf{f}_\tau]$  ( $\tau \in \mathcal{T}_R$ ), which is different from  $\hat{\mathbf{F}}_l' = [\hat{\mathbf{F}}_{l-1}' \mathbf{f}_{\hat{\tau}_l'}]$  for the OMP algorithm, in which  $\hat{\tau}_l'$  is estimated by (17).

The computational complexity of the solution to (23) is higher than both (16) and (17). By making full use of the local delay expansion in (22), we can have a subset of delay grids approaching to  $\tau_0$  (for the non-sample spaced sparse channel with sparsity  $K = 1$ , the delay grids within the shadow area are given in FIGURE 1 (c)), which is actually robust to fight against the effect of power leakage of channel taps. With the delay grids within the shadow area and the delay subset  $\mathcal{T}_{sub}$  defined previously, we can easily have the following corresponding RDG at the  $l^{th}$  tap for the proposed DT-RNM method:

$$\mathcal{T}_{RDG}(l) = \text{sup}((\mathbf{F}_{R,\mathcal{T}^{(1)}(l)}^H \mathbf{r}_{l-1})_D) \quad (24)$$

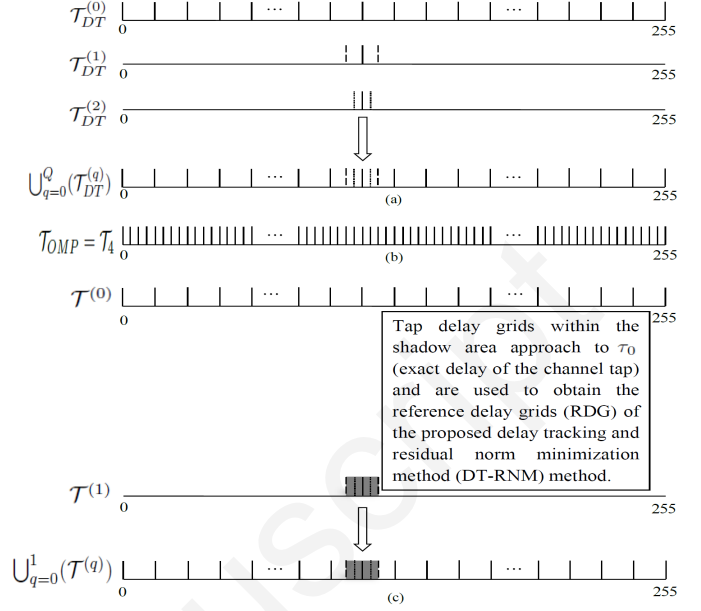


FIGURE 1: Tap delay grids of the method in [1] (a), tap delay grids of the OMP algorithm (b), tap delay grids of the proposed DT-RNM method (c).

Considering that  $D$  in  $\mathcal{T}_{sub}$  is a small number, we typically have  $D = 2, 3$  in (24). Therefore the RNM of the proposed DT-RNM method is given by:

$$[\hat{\mathbf{h}}_l, \hat{\tau}_l] = \arg \min_{\mathbf{h}_l, \tau_l \in \mathcal{T}_{RDG}(l)} \|\mathbf{g}_p - \hat{\mathbf{F}}_{RDG,l} \mathbf{h}_l\|_2 \quad (25)$$

where  $\hat{\mathbf{F}}_{RDG,l} = [\hat{\mathbf{F}}_{l-1} \mathbf{f}_{\tau_l}]$ ,  $\tau_l \in \mathcal{T}_{RDG}(l)$ . Obviously, the presence of RDG ( $D = 2, 3$ ) will increase the computational complexity mainly due to the two or three times ( $D = 2, 3$ ) of residual norm minimization computations compared with the algorithm in [1], however, we have actually significantly improved the channel estimation precision. The main steps of the proposed method (presented in Algorithm 1) will be described in the following section.

## B. MAIN CHARACTERISTICS OF THE PROPOSED DT-RNM METHOD

Compared with the algorithm in [1], the traditional OMP algorithm and other greedy pursuit algorithms, the proposed DT-RNM method has the following characteristics.

### 1) Suboptimal pilot arrangement

As previously mentioned, the case of  $M < L_{cp}$  is considered for higher spectral efficiency. In this case, the suboptimal pilot arrangement proposed in [14] is adopted, which has better spectral efficiency than the partially equispaced or equispaced pilot arrangement used in [1], [13].

### 2) RDG guided residual norm minimization

Different from the algorithm in [1] and conventional OMP method in [10], on the one hand, one time local delay expansion is proposed for delay tracking (DT), and on the other

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**Algorithm 1** Proposed DT-RNM algorithm

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**Input:**

- 1) Initial residual vector  $\mathbf{r}_{-1} = \mathbf{g}_p$ ;
- 2) Noise standard deviation  $\sigma$ ;
- 3) Oversampling factor  $R$ ;
- 4) Initial set of DT-RNM  $\mathcal{T}_{DT-RNM}(-1) = \emptyset$ ;
- 5) Initial matrix with the selected bases  $\hat{\mathbf{F}}_{-1} = \emptyset$ ;
- 6) Size of the set of reference delay grids  $D$ ;
- 7) Flag of the iteration  $flag = 0$ ;
- 8) Initial the tap number  $l = 0$ .

**Main body:**

- 1: while ( $flag == 0$ )
- 2:  $\hat{\tau}_l^{(0)} = \arg \max_{\tau \in [0, L_{cp}-1]} |\langle \mathbf{f}_\tau, \mathbf{r}_{l-1} \rangle|$ ; % Get the initial maximum coherence;
- 3:  $\mathcal{T}^{(1)}(l) = \{\hat{\tau}_l^{(0)} - \frac{1}{2}, \hat{\tau}_l^{(0)} - \frac{1}{2} + \frac{1}{R}, \dots, \hat{\tau}_l^{(0)}, \dots, \hat{\tau}_l^{(0)} + \frac{1}{2} - \frac{1}{R}, \hat{\tau}_l^{(0)} + \frac{1}{2}\}$ ; % Local delay expansion inspired by [1];
- 4:  $\mathcal{T}_{RDG}(l) = \sup((\mathbf{F}_{R, \mathcal{T}^{(1)}(l)}^H \mathbf{r}_{l-1})_D)$ ; % Obtain  $\mathcal{T}_{RDG}(l)$ ;
- 5:  $[\hat{\mathbf{h}}_l, \hat{\tau}_l] = \arg \min_{\mathbf{h}_l, \tau_l \in \mathcal{T}_{RDG}(l)} \|\mathbf{g}_p - [\hat{\mathbf{F}}_{l-1} \mathbf{f}_{\tau_l}] \mathbf{h}_l\|_2$ ;
- 6:  $\mathcal{T}_{DT-RNM}(l) = [\mathcal{T}_{DT-RNM}(l-1) \hat{\tau}_l]$ ;
- 7:  $\hat{\mathbf{F}}_l = [\hat{\mathbf{F}}_{l-1} \mathbf{f}_{\hat{\tau}_l}]$ .
- 8:  $\mathbf{r}_l = \mathbf{g}_p - \hat{\mathbf{F}}_l \hat{\mathbf{h}}_l$ ;
- 9: if  $\min(|\hat{\mathbf{h}}_l|) \leq T$  ( $T = \sqrt{2(1+a)\ln(L_{cp})}\sigma$  ( $\sigma = \frac{\sigma_w}{\sqrt{M}}$ )) % Halting criterion;
- 10:  $\hat{\mathbf{h}} = \hat{\mathbf{h}}_{l-1}$ ;
- 11:  $\mathcal{T}_{DT-RNM} = \mathcal{T}_{DT-RNM}(l-1)$ ;
- 12:  $flag = 1$ ;
- 13: end
- 14:  $l = l + 1$ ;
- 15: end

**Output:**  $\hat{\mathbf{h}}_{DT-RNM} = \hat{\mathbf{h}}(\mathcal{T}_{DT-RNM})$  % Estimated CIR with DT-RNM method.

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hand, based on the fact that the true delay for each channel tap may appear in a delay subset, the RDG guided residual norm minimization (RNM) method is proposed and analyzed to effectively fight against the non-uniform pilot arrangement and to obtain near optimal delay searching. Thanks to its efficiency in computation and effectiveness in fighting against the non-equispaced pilot arrangement, the proposed DT-RNM method has better channel estimation performance than the methods in [1], [10], much lower computational cost than the method in [10] and slightly higher computational cost than the method in [1].

### 3) Halting criterion

Unlike the method proposed in [1], which employs two thresholds. For conveniency and effectiveness in practical applications, the proposed method considers the universal threshold ( $T = \sqrt{2(1+a)\ln(L_{cp})}\sigma$  ( $\sigma = \frac{\sigma_w}{\sqrt{M}}$ )), we take  $a = 0$  in this paper [4], [18], [22] as the halting criterion.

## IV. SIMULATIONS

In the simulations, we consider a QPSK modulated OFDM system. The system has 1024 subcarriers and  $L_{cp} = 256$ . A five tap channel is considered, each channel tap has uniformly distributed delay [6]. The power of each channel tap has an exponential distribution described by  $\phi(\tau) = e^{-\frac{\tau}{\tau_{rms}}}$  with  $\tau_{rms} = \frac{L_{cp}}{4}$ . Both the delays and powers of channel taps vary from one OFDM symbol to another.

For performance evaluations, the normalized minimum mean square error (NMSE) of CFR for one OFDM symbol

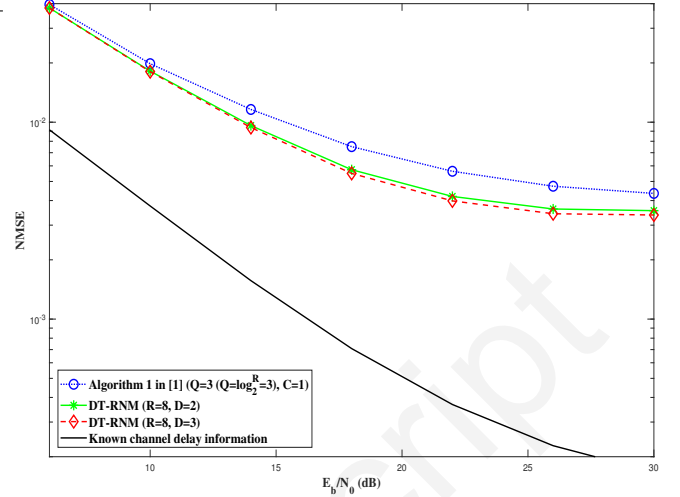


FIGURE 2: Performance of NMSE among proposed DT-RNM method with  $R = 8$ , different  $D$  ( $D = 2, 3$ ) and Algorithms 1 in [1]

defined in (26) and bit error rate (BER) are employed. Additionally, the computational complexity comparison is also presented.

$$NMSE = \frac{\sum_{k=0}^{N-1} |g[k] - \hat{g}[k]|^2}{\sum_{k=0}^{N-1} |g[k]|^2} \quad (26)$$

The compared methods are the proposed DT-RNM method, Algorithm 1 in [1], the conventional OMP method in [10] and the known channel delay information. In simulations, 96 pilots (9.375% of pilots), with positions optimized by the method proposed in [14], are employed (In order to make fair channel estimation performance comparisons, one optimized pilot arrangement is used for all the simulation algorithms mentioned above). Both Algorithm 1 in [1] and the conventional OMP method use the two thresholds  $\alpha = M\sigma_w^2$  and  $\beta = \sigma_w^2$  given in the simulation part of [1].

## A. PERFORMANCE OF THE PROPOSED DT-RNM METHOD

The performance of NMSE comparison among the proposed DT-RNM method with ( $R = 8$ ) and different  $D$  ( $D = 2, 3$ ) and Algorithms 1 with  $Q = 3$  in [1] is illustrated in FIGURE 2. Compared with Algorithms 1 in [1], the proposed DT-RNM method with different  $D$  ( $D = 2, 3$ ) achieves better NMSE performance throughout the considered  $E_b/N_0$ , especially within the  $E_b/N_0$  range (14dB-30dB), their performance gap is at least 2dB for the same NMSE performance. Additionally, with the increase of  $D$ , the performance gain of NMSE is decreased.

FIGURE 3, FIGURE 4 and FIGURE 5 show the NMSE performance comparison between the proposed DT-RNM method with  $R = 2, D = 2$  and other existing methods, the proposed DT-RNM method with  $R = 4, D = 3$  and other existing methods and the proposed DT-RNM method

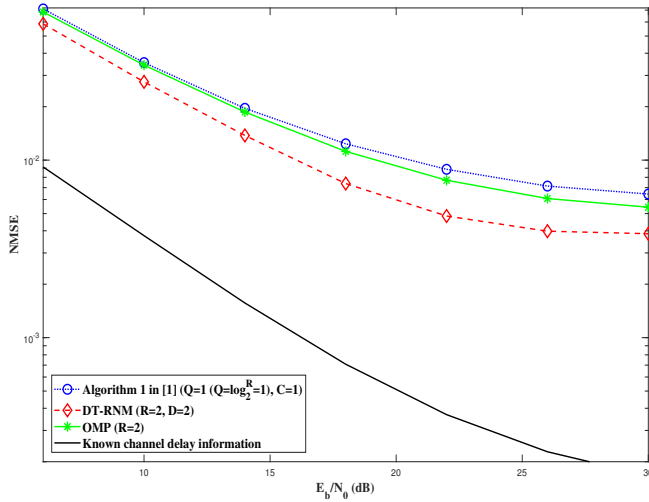


FIGURE 3: Performance of NMSE comparison between the proposed DT-RNM method with  $R = 2, D = 2$  and other existing methods

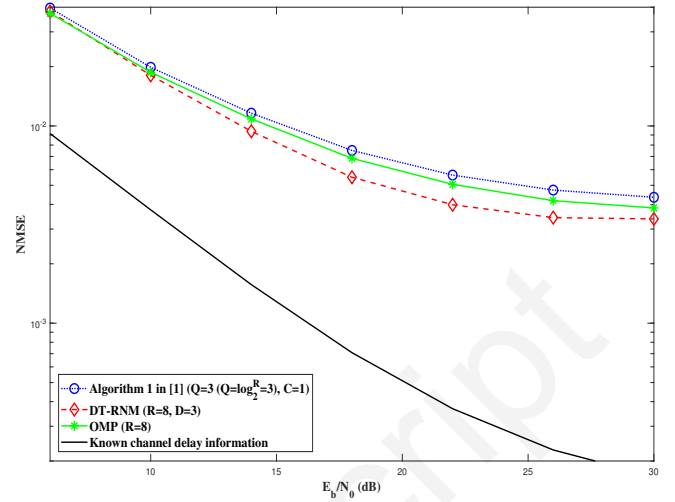


FIGURE 5: Performance of NMSE comparison between the proposed DT-RNM method with  $R = 8, D = 3$  and other existing methods

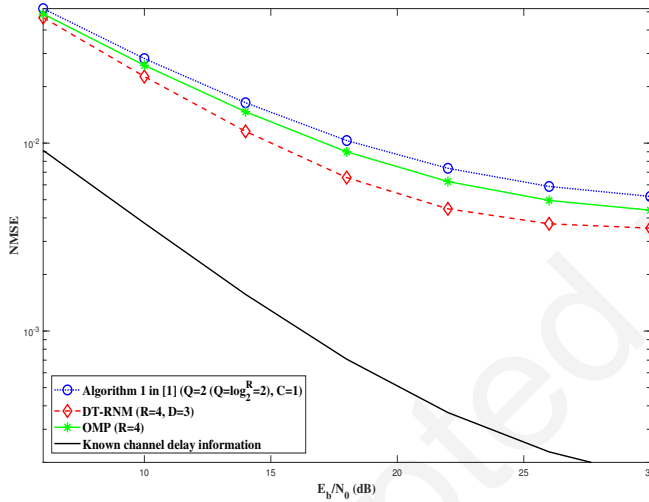


FIGURE 4: Performance of NMSE comparison between the proposed DT-RNM method with  $R = 4, D = 3$  and other existing methods

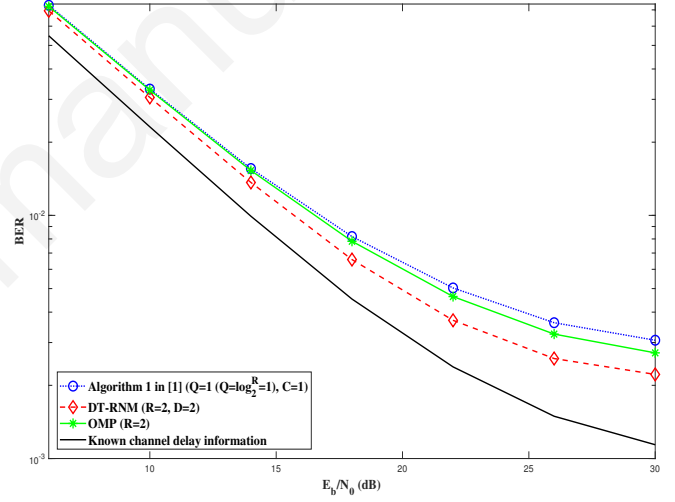


FIGURE 6: Performance of BER comparison between the proposed DT-RNM method with  $R = 2, D = 2$  and other existing methods

with  $R = 8, D = 3$  and other existing methods respectively. Generally, with the same value of  $R$  ( $R = 2, 4, 8$ ), the proposed DT-RNM method outperforms the conventional OMP channel estimation method and Algorithm 1 in [1], their performance gaps are decreased with the increase of  $R$ . Additionally, from FIGURE 3, FIGURE 4 and FIGURE 5, we know that the performance gaps of NMSE between the proposed DT-RNM method and Algorithm 1 in [1] with the corresponding  $Q$  are generally bigger compared with that between the proposed DT-RNM method and the OMP method with the same  $R$  throughout the considered  $E_b/N_0$ . The primary reason is that, theoretically, the NMSE performance of Algorithm 1 in [1] is not better than the OMP method.

The BER performance in FIGURE 6, FIGURE 7 and

FIGURE 8 has similar trends to that of NMSE performance in FIGURE 3, FIGURE 4, FIGURE 5 respectively. Although the BER performance gaps between different methods are smaller, the proposed DT-RNM method still maintains the best performance except the known channel delay information. In FIGURE 6 and FIGURE 8, the proposed DT-RNM methods with  $R = 2$  and  $R = 8$  improve the BER performance of the Algorithm 1 in [1] by no less than 2dB and 1dB respectively throughout the high  $E_b/N_0$  (20dB - 30dB).

## B. COMPUTATIONAL COMPLEXITY

The computational complexity of the conventional OMP method in [10], Algorithm 1 of [1] and the proposed DT-



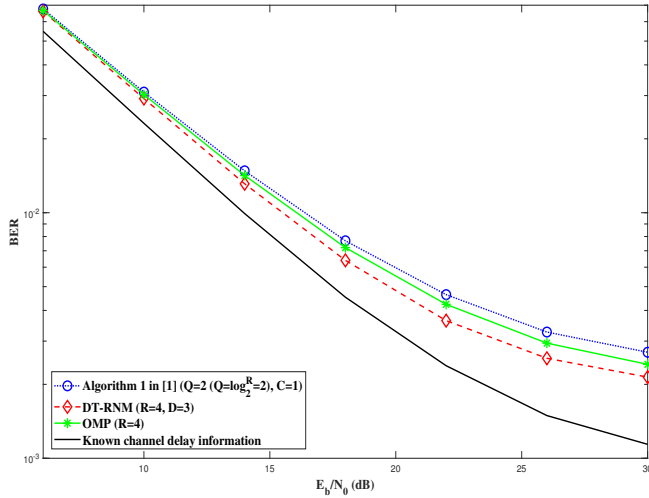


FIGURE 7: Performance of BER comparison between the proposed DT-RNM method with  $R = 4, D = 3$  and other existing methods

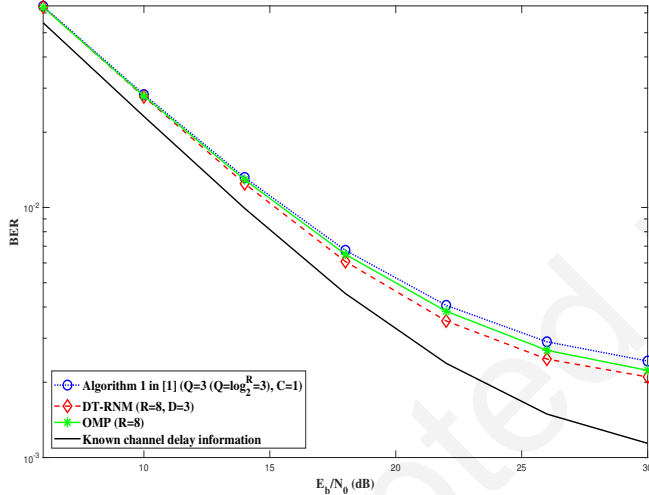


FIGURE 8: Performance of BER comparison between the proposed DT-RNM method with  $R = 8, D = 3$  and other existing methods

RNM method is given in TABLE 1.

TABLE 1: Computational complexity comparison

Algorithm	Complexity
OMP	$O(M(R(L_{cp} - 1) + 1)K)$
Alg 1 [1]	$O(M(L_{cp} + 2QC)K)$
DT-RNM	under $O(2M(L_{cp} + R)K)$

The computational complexity of the conventional OMP is  $O(M(R(L_{cp} - 1) + 1)K)$  mainly due to the coherence computation on  $R(L_{cp} - 1) + 1$  bases; the number of bases of Algorithm 1 of [1] for coherence computation is  $(L_{cp} + 2QC)$ , therefore, its complexity is  $O(M(L_{cp} + 2QC)K)$ ; the complexity of the proposed DT-RNM is composed of two parts, the first part is the complexity of Algorithm 1 in [1]

$O(M(L_{cp} + R)K)$ , the second part is the added complexity of one or two times ( $D = 2, 3$ ) of residual norm minimization computations, which includes the pseudoinverse and residual update et al, which is less than  $O(M(L_{cp} + R)K)$ , so we have the total complexity less than  $O(2M(L_{cp} + R)K)$ . Additionally, although the total complexity of the proposed DT-RNM method is a bit higher than the Algorithm 1 in [1], it is still much lower than the complexity of OMP  $O(M(R(L_{cp} - 1) + 1)K)$ .

## V. CONCLUSION

In this paper, a new DT-RNM method is proposed for the non-sample spaced sparse channel estimation. Based on the OMP algorithm and inspired by [1], the proposed method introduces the idea of reference delay grids (RDG) guided residual norm minimization (RNM), which can effectively promote the channel estimation performance by adding limited computational complexity in the case of  $M < L_{cp}$ . Simulation results and computational evaluations fully show that the proposed DT-RNM method comprehensively outperforms the conventional OMP method in [10] and the algorithm proposed in [1] and can effectively balance the channel estimation performance, spectral efficiency and computational complexity.

## APPENDIX A PROPERTIES OF $|\langle \mathbf{f}_\tau, \mathbf{g}_p \rangle|$

The mathematical expression of  $|\langle \mathbf{f}_\tau, \mathbf{g}_p \rangle|$  (channel sparsity  $K = 1$ , pilots are equispaced,  $k_n = k_0 + nd$ , where  $n = 0, 1, \dots, M - 1$ ,  $d = N/M$  and  $M > L_{cp}$ ) is given by (Appendix in [1]):

$$|\langle \mathbf{f}_\tau, \mathbf{g}_p \rangle| = |h_0 \sum_{n=0}^{M-1} e^{-j\frac{2\pi}{N}k_n\Delta\tau}| = |h_0| \frac{|\sin(\pi\Delta\tau)|}{|\sin(\pi\Delta\tau/M)|} = |h_0|f(\Delta\tau) \quad (27)$$

where  $\Delta\tau = \tau_0 - \tau$ ,  $\Delta\tau \in [-L_{cp} + 1, L_{cp} - 1]$ . According to [1],  $f(\Delta\tau) = f(-\Delta\tau)$  and  $f(\Delta\tau)$  has the following two properties:

- 1)  $f(\Delta\tau)$  is monotone increasing and monotone decreasing within the range  $[-\frac{1}{2}, 0)$  and  $(0, \frac{1}{2}]$  respectively;
- 2) For  $\Delta\tau \in [-L_{cp} + 1, -\frac{1}{2})$  and  $\Delta\tau \in (\frac{1}{2}, L_{cp} - 1]$ , we have  $f(-\Delta\tau) < f(-\frac{1}{2})$  and  $f(\Delta\tau) < f(\frac{1}{2})$  respectively.

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