Conditional reliability-based importance measures

Phuc Do Van, Christophe Bérenguer

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Phuc Do[1]*and Christophe Bérenguer[2]

[1]Université de Lorraine, CRAN, UMR 7039, Campus Sciences, BP 70239, Vandoeuvre-les-Nancy, 54506, France

[2]Univ. Grenoble Alpes, CNRS, Grenoble-INP, GIPSA-lab, F-38000 Grenoble, France

Abstract

Importance measures have been widely used as important decision-aiding indicators in various purposes such as reliability studies, risk analyses and maintenance optimization. In this work, we propose a novel time-dependent importance measure for systems composed of multiple non repairable components. The proposed importance measure of a component/group of components is defined as its ability to improve the system reliability during a mission given the current conditions (states or degradation levels) of its components. To take into account economic aspects (e.g., maintenance costs, economic dependence between components and the cost benefit thanks to maintenance operations), an extension of the proposed importance measure is then investigated. Thanks to these proposed importance measures, the component/group of components can be "optimally" selected for preventive maintenance regarding to the reliability criteria and/or the financial issues. A numerical example is introduced to illustrate the use and the advantages of the proposed importance measures.

Keywords: Importance measure, conditional reliability, maintenance decision-making, economic dependence, multi-component system.

1 Introduction

Importance measures providing information about the importance of a component or a group of components on the system performance (reliability/availability, productivity, safety, or any performance metrics of interest) can help to identify design weakness

*Corresponding author: phuc.do@univ-lorraine.fr

or operation bottlenecks and to suggest optimal modifications for system upgrades and maintenances [20]. In the literature, a large number of importance measures have been developed and successfully applied for various purposes, see [14] for an overview about recent advances on importance measures. In risk analyses, importance measures are used in risk-informed decision-making, see for instance [5, 6, 12, 21, 23]. In reliability engineering, importance measures are used to prioritize components in a system for reliability improvement [1, 2, 4, 8, 11, 19]. Recently, importance measures have been applied for maintenance optimization and spare parts management [3, 10, 16, 24, 26]. More specifically, Birnbaum structural importance measure is used to build a decision indicator for maintenance optimization of multi-component systems with complex structure, see [16, 24]. Differential importance measure is proposed to use in inventory management in [3]. More recently, the link between component importance and preventive maintenance policy has been discussed in [26].

In the framework of condition-based maintenance optimization, the current condition of components (e.g., failure state, working state, deterioration level) is an important issue and needs to be taken into account in decision-making. However, very few existing importance measures allow incorporating the actual condition of components over time. Moreover, in practice, positive economic dependence, which implies that joint maintenance of several components is cheaper than performing maintenance on components separately, often exists and should be integrated in maintenance decision-making in the framework of maintenance optimization. To the best of our knowledge, no existing importance measure allows taking into account this kind of interaction between components. To face this issue, in this paper, a novel time-dependent importance measure based on the conditional reliability evaluation of the system, namely RIM measure, is proposed. At a given time and given the real condition of the components of a system, the proposed importance measure can be used to rank the components or groups of components according to their ability to improve the system reliability for a given mission. The proposed RIM measure is then extended to take into account the maintenance cost and the economic dependence between components. Indeed, the extended RIM measure is defined as the ratio of the cost benefit given by the maintenance of a component/group of components to its total maintenance cost. This indicator can help to identify the most cost-effective components/group of components for preventive maintenance before a given mission.

This paper is organized as follows. Section 2 is devoted to the description of general assumptions and different reliability metrics. Different kinds of information on components at given time are also discussed and integrated in the evaluation of reliability metrics. Section 3 focuses on the definition of the proposed time-dependent importance measure, namely RIM . The influence of information level on the RIM measure and RIM -based importance ranking is also investigated. An extension of RIM measure is developed in section 4. Maintenance cost structures and economic dependence between components are also formulated and discussed. To illustrate the uses of RIM measure and its extension, a numerical of a 5-component system is introduced in Section 5. In addition some numerical results are herein discussed. Finally, the last section presents the conclusions drawn from this work.

Notation

c_0^i, c_0^G	set-up cost of component i and of group G respectively
C_p^i, C_p^G	preventive maintenance cost of component i and of group G respectively
c_d	downtime cost rate of the system
d^i, d^G	duration of a replacement for component i of group G respectively
$s^i(t)$	state of component i at time t
$(X_t^i)_{t\geq 0}$	stochastic degradation process component i over time t
x_t^i	degradation level of component i at time t
Z_i	failure threshold of component i
$\mathfrak{F}_t^i, \mathfrak{F}_t^S$	available information on component i and on the system respectively at t
$R^i(t), R(t)$	reliability of component i and of the system respectively
$R^i(t+u\vert\mathfrak{F}_t^i)$	conditional reliability of component i
$R(t+u\vert\mathfrak{F}_t)$	conditional reliability of the system
$f_{\tau_i}(u)$	probability density function for the failure time of component i
$f_{X_t^i}(u)$	probability density function for the deterioration level of component i at time t

2 System modeling & reliability metrics

2.1 System description and assumptions

We consider a coherent system composed of n non-identical components which are interconnected according to a complex structure in terms of reliability block diagram (RBD). Each component is subject to an underlying aging/deterioration process which can cause random failures. It is also assumed that only kind maintenance action, that brings a maintained component to as-good-as new, is possible for each component.

Let $\mathbf{s(t)} = (s_1(t), s_2(t), ..., s_n(t))$ be the state vector with $s_i(t)$ $(i = 1, ..., n)$ is binary variable representing the state of the component i.

$$s_i(t) = \begin{cases} 1 & \text{if component } i \text{ is functioning at time } t \\ 0 & \text{if component } i \text{ is in a failed state at time } t \end{cases}$$

The structure function of the system, denoted $\phi(\mathbf{s(t)})$, can be defined as follows

$$\phi(\mathbf{s(t)}) = \phi(s_1, s_2, \ldots, s_n) = \begin{cases} 1 & \text{if the system is functioning at time } t \\ 0 & \text{if the system is in a failed state at time } t \end{cases}$$

It is important to note that $s_i(t)$ $(i = 1, ..., n)$ are independent since all components are assumed to be stochastically independent. The system structure function can be expressed as a function of its minimal cut sets or its minimal path sets, [20]. For example, as a function of the minimal cut sets, the structure function writes :

$$\phi(\mathbf{s(t)}) = \prod_{j=1}^{n_K} [1 - \prod_{i \in K_j} (1 - s_i(t))], \tag{1}$$

where K_j is j-th minimal cut set, and n_K is the total number of minimal cut sets of the system.

2.2 Reliability metrics

To evaluate the reliability performance of a system, different reliability metrics (either instantaneous, average or asymptotic) such as reliability/availability, MTTF (mean time to failure), conditional reliability, have been introduced and successfully applied in different applications [20]. The conditional reliability that allows taking into account the real/current information on the components/system is now recognized as an important indicator for decision-making in risk analysis, maintenance and production scheduling optimization, see for instance [17, 24].

2.2.1 Reliability

The reliability of a system is defined as the probability that the system operates correctly until time t. Mathematically, for non repairable components the system reliability, denoted $R(t)$, is expressed as:

$$R(t) = \mathbb{E}[\phi(\mathbf{s(t)})], \tag{2}$$

where $E[.]$ indicates the mathematical expectation. As an example, we consider a 5-non repairable components system whose RBD is shown in Figure 1.

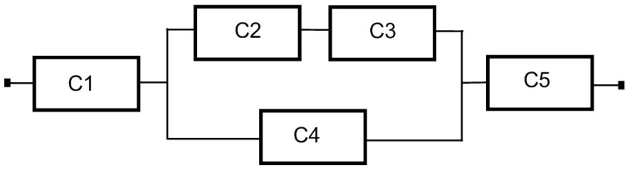

Figure 1: An example of 5-component system.

The considered system has four minimal cut sets: $K_1 = \{C1\}, K_2 = \{C2, C4\}, K_3 = \{C3, C4\}$ and $K_4 = \{C5\}$. According to Eq. (1), the structure function can be written as:

$$\begin{aligned}\phi(\mathbf{s(t)}) &= s_1(t) \cdot [s_2(t) + s_4(t) - s_2(t) \cdots_4 (t)] \cdot [s_3(t) + s_4(t) - s_3(t) \cdots_4 (t)] \cdot s_5(t). \\ &= s_1(t) \cdot [s_4(t) + s_2(t) \cdot s_3(t) - s_2(t) \cdot s_3(t) \cdot s_4(t)] \cdot s_5(t).\end{aligned}$$

By using Eq. (2), and under the assumption of independent components, the reliability of the system is expressed by:

$$R(t) = R^1(t) \cdot [R^4(t) + R^2(t) \cdot R^3(t) - R^2(t) \cdot R^3(t) \cdot R^4(t)] \cdot R^5(t). \tag{3}$$

where $R^i(t) = E[s_i(t)]$ $(i = 1, ..., 5)$ is the reliability of component i.

Depending on the level of information available on the state of a component, two different approaches can be followed to assess its reliability: degradation based model or lifetime failure model [20, 17]. Two cases are thus considered for the evaluation of $R^i(t)$:

1. if component i gradually deteriorates and its deterioration evolution is assumed to be described by a stochastic process X_t^i, the reliability of component i can be expressed as:

   $$R^i(t) = \mathbb{P}(X_t^i < Z_i) = \int_0^{Z_i} f_{X_t^i}(u)du, \tag{4}$$

   with Z_i is the failure threshold above which component i is considered as failed when $X_t^i \geq Z_i$; $f_{X_t^i}(u)$ is the probability density function of the deterioration level of component i at time t. Gamma stochastic processes have been widely used in the literature for modelling the degradation process of components [22] ; in this work, we use them as an example among several other possible ones. A detailed description is given in appendix A.

2. if the failure behavior of component i is described by a lifetime failure model due to a technical and/or economical issue (e.g., degradation inspection on component i is not allowed or impossible; its inspection cost is too expensive or component i is very reliable and its inspection are then unnecessary), the reliability of component i can be then evaluated by

$$R^i(t) = (\tau_i > t) = 1 - \int_0^t f_{\tau_i}(u)\,du, \tag{5}$$

where τ_i is the time to failure of component i and follows a probability law with probability density function (pdf) $f_{\tau_i}(u)$. The Weibull law has been widely used in the literature for modeling the failure behavior of components [20] and we use it as an example among several other possible ones. A detailed description is given in appendix B.

2.2.2 Conditional reliability

Let $\mathfrak{F}_t^S$ be the available information associated to the system state at time t: $\mathfrak{F}_t^S = 1$ if the system is working at time t, $\mathfrak{F}_t^S = 0$ for otherwise.

Assume now the system is functioning at time t ($\mathfrak{F}_t^S = 1$), the conditional reliability of the system within the interval horizon $(t, t+u)$ (with $u > 0$) is then expressed as:

$$R(t+u|\mathfrak{F}_t^S = 1, \mathfrak{F}_t^1, \mathfrak{F}_t^2, ..., \mathfrak{F}_t^n) = \prod_{j=1}^{n_c}[1 - \prod_{i \in C_j}(1 - R^i(t+u|\mathfrak{F}_t^i))], \tag{6}$$

where $R^i(t+u|\mathfrak{F}_t^i)$ is the conditional reliability of component i given the available information on the component i at time t, $\mathfrak{F}_t^i$. Four levels are herein specified for the available information $\mathfrak{F}_t^i$. More precisely, $\mathfrak{F}_t^i$ can take the following values:

$$\mathfrak{F}_t^i = \begin{cases} 0 & \text{if component } i \text{ is in a failed state at time } t; \\ 1 & \text{if component } i \text{ is working but its deterioration level is not measured at time } t; \\ 2 & \text{if component } i \text{ is working and its deterioration level is measured at time } t; \\ 3 & \text{if component } i \text{ is replaced by a new one at time } t. \end{cases}$$

According the information level given at time t on a component, the conditional reliability can thus be specified as follows:

- If component i is not working at time t, i.e., $\mathfrak{F}_t^i = 0$, its predicted reliability is then $R^i(t+u|\mathfrak{F}_t^i = 0) = 0$;

- If component i is working but its deterioration level is not measured, i.e., $\mathfrak{F}_t^i = 1$, its predicted reliability is calculated as:

$$R^i(t+u|\mathfrak{F}_t^i = 1) = \frac{R^i(t+u)}{R^i(t)}. \tag{7}$$

  It is important to note that if the failure behaviour of component i follows an exponential distribution, then $R^i(t+u|\mathfrak{F}_t^i = 1) = R^i(u)$. This means that component i is considered as a new one if it is observed that it has survived at time t, consequently, there is no need for preventive maintenance if this is known.

- If component i is working and its deterioration level is measured at time t ($\mathfrak{F}_t^i = 2$), the predicted reliability of component i is then calculated as:

$$R^i(t+u|\mathfrak{F}_t^i = 2) = R^i(t+u|X_t^i = x_t^i) = \mathbb{P}(X_{t+u}^i < Z_i|X_t^i = x_t^i) = \int_{x_t^i}^{Z_i} f_{X_{t+u}^i}(v)dv, \tag{8}$$

  with $X_t^i = x_t^i$ is the deterioration level of component i at time t. An illustration for Eq. (8) is shown in Figure 2.

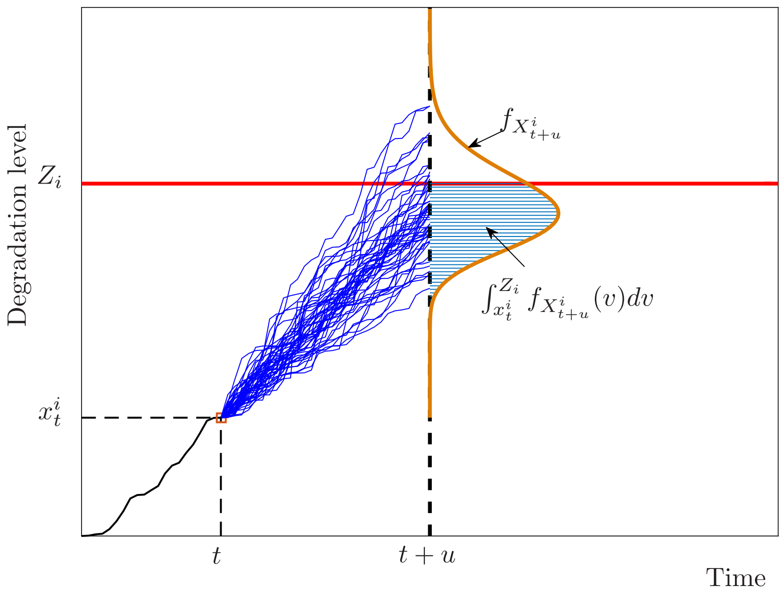

Figure 2: Illustration for the predicted reliability assessment given by Eq. (8).

- If component i is replaced by a new one at time t ($\mathfrak{F}_t^i = 3$), its predicted reliability is then evaluated as: $R^i(t+u|\mathfrak{F}_t^i = 3) = R^i(u)$.

An illustration of the predicted reliability of component i regarding to different levels of information available at time $t = 20$ is shown in Figure 3. It should be noticed that in this illustration, gamma process with shape and scale parameter $\gamma_i = 1, \eta_i = 2$ (see Appendix A) is used to simulate the degradation process of component i. The failure threshold is set by $Z_i = 100$.

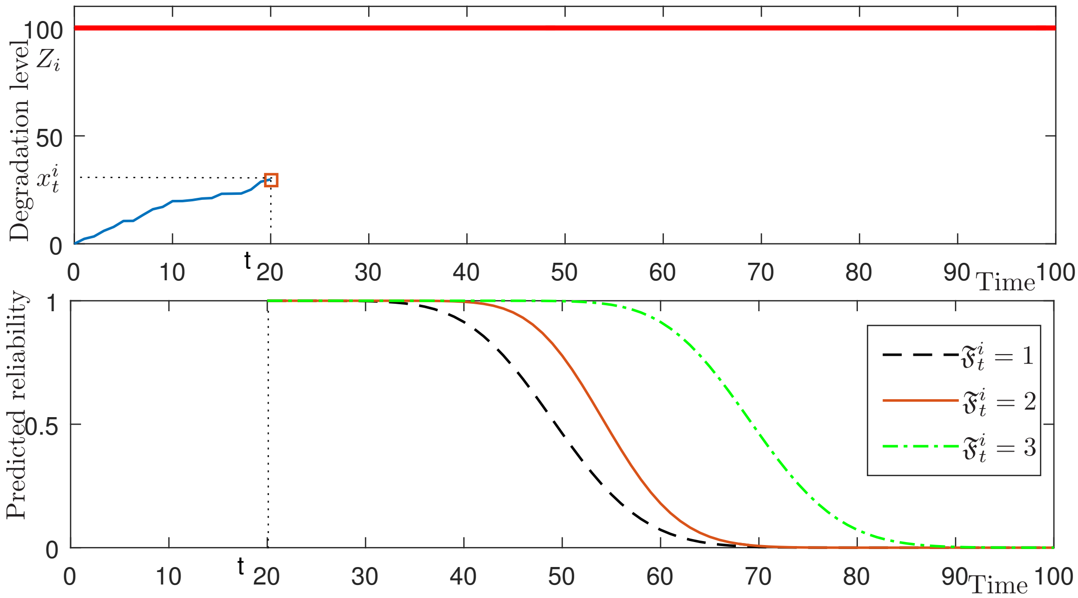

Figure 3: Illustration of the predicted reliability of component i under a given information level $\mathfrak{F}_t^i$

The numerical results in Figure 3 show that the information level ($\mathfrak{F}_t^i$) is crucial for the reliability assessment of a component.

3 Conditional reliability-based importance measures

3.1 Definition

Conditional reliability-based importance measure, namely RIM, for component i is defined as follows:

$$RIM^i(u,t) = R(t+u|\mathfrak{F}_t^1, \mathfrak{F}_t^2, ..., \{\mathfrak{F}_t^i = 3\}, ..., \mathfrak{F}_t^n) - R(t+u|\mathfrak{F}_t^1, \mathfrak{F}_t^2, ..., \mathfrak{F}_t^n). \tag{9}$$

By definition, this importance measure provides the potential improvement in the system reliability within the interval horizon $(t, t+u)$ when component i is replaced at time t

given the current condition of all components of the system.

According to the definition, several properties can be established for the proposed RIM measure :

- For a coherent system $R(t+u|\mathfrak{F}_t^1, \mathfrak{F}_t^2, ..., \{\mathfrak{F}_t^i = 3\}, ..., \mathfrak{F}_t^n) \geq R(t+u|\mathfrak{F}_t^1, \mathfrak{F}_t^2, ..., \mathfrak{F}_t^n)$, consequently $RIM^i(u,t)$ is not negative;
- $RIM^i(u,t)$ is bounded, i.e., $0 \leq RIM^i(u,t) \leq 1$, more precisely:
  - $RIM^i(u,0) = 0$ since component i is new at time $t = 0$;
  - $RIM^i(0,t) = 0$ if the system is still working at time t;
  - $RIM^i(u,t) = 0$ if component i is still working at time t and the failure rate of component i is time-independent, e.g., it is described by an exponential distribution;
  - $RIM^i(0,t) = 1$ if component i is critical and it is not working at time t, i.e., the system fails before time t. The replacement of component i brings the system to an operational state.
- $RIM^i(u,t)$ may depend not only on t and u but also on the condition (state or degradation level) of all components at time t.

It is important to note that if $u = 0$, the proposed RIM measure becomes a variant of a classical importance measure called improvement potential factor [20]. Indeed, the improvement potential factor for component i is defined as:

$$IP^i(t) = R(t|\{\mathfrak{F}_t^i = 3\}) - R(t). \tag{10}$$

It is clear that the improvement potential factor allows considering the current condition of only component i while that of the other components are ignored. In addition, unlike RIM measure, the improvement potential factor does not allow considering a given mission $(t, t+u)$.

3.2 Influence of information level on RIM measure

To study the influence of information level on RIM measure and RIM -based importance ranking, we consider a series structure composed of two components $C1$ and $C2$. It is assumed that the degradation process of each component is described by a gamma process (see Appendix A) with shape and scale parameter (γ_i, η_i) with $i = 1, 2$ shown in Figure 4. The failure threshold of C1 and C2 is $Z_1 = Z_2 = 100$.

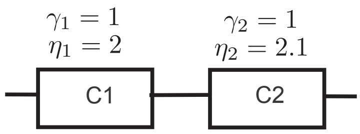

Figure 4: An example of 2-component system.

Assume that both components are still functioning at time $t = 20$. We consider herein two cases according the available information level on each component:

Case 1 Both components are working at time t but their deterioration levels are unknown, i.e., $\mathfrak{F}_t^1 = \mathfrak{F}_t^2 = 1$. RIM measure of each component is evaluated by using Eq.(9) and Eq.(7) and the obtained results for the interval $(t, t+u)$ with $u = 30$ are shown in Table 1. According to RIM 's value, component C2 is more important than C1. This is quite natural since without any information about the current level on the degradation process of both components, we can only take into account the fact that the degradation speed of C2 is higher than that of C1 because $\eta_2 > \eta_1$. As a consequence, the replacement of C2 is considered to be more effective in improving the system reliability than replacement of C1.

Case 2 Both components are working at time t and their degradation levels are measured, i.e., $\mathfrak{F}_t^1 = \mathfrak{F}_t^2 = 2$. Since both components are subject to stochastic degradation processes, the degradation level of each component at time $t = 20$ can be random. As an example, two experiments, namely cases 2a et 2b, are carried out to simulate the degradation process evolution of two components. The degradation levels of both components at time $t = 20$ for each experiment are reported in Table 1. Based on the components' degradation levels at time t, the RIM measure of each component is evaluated by using Eq.(9) and Eq.(8). In the first experiment (case 2a), $RIM^2(30, 20) > RIM^1(30, 20)$, consequently C1 is more important than C2 in improving the system reliability. However, in the second experiment (case 2b), $RIM^2(30, 20) < RIM^1(30, 20)$, C1 becomes more important than C2.

	Information available at $t = 20$				$u = 30$		Ranking	
	$\mathfrak{F}_t^1$	X_t^1	$\mathfrak{F}_t^2$	X_t^2	$RIM^1(u,t)$	$RIM^2(u,t)$	C1	C2
Case 1	1	-	1	-	0.1844	0.3182	2	1
Case 2a	2	44.32	2	34.56	0.3818	0.1445	1	2
Case 2b	2	33.66	2	34.21	0.1098	0.4291	2	1

Table 1: RIM measure and importance ranking of a two-component system.

Figure 5 shows the components' degradation trajectories and the corresponding RIM measures. It is clear that the RIM values and the importance ranking of the components may change regarding to their degradation trajectories.

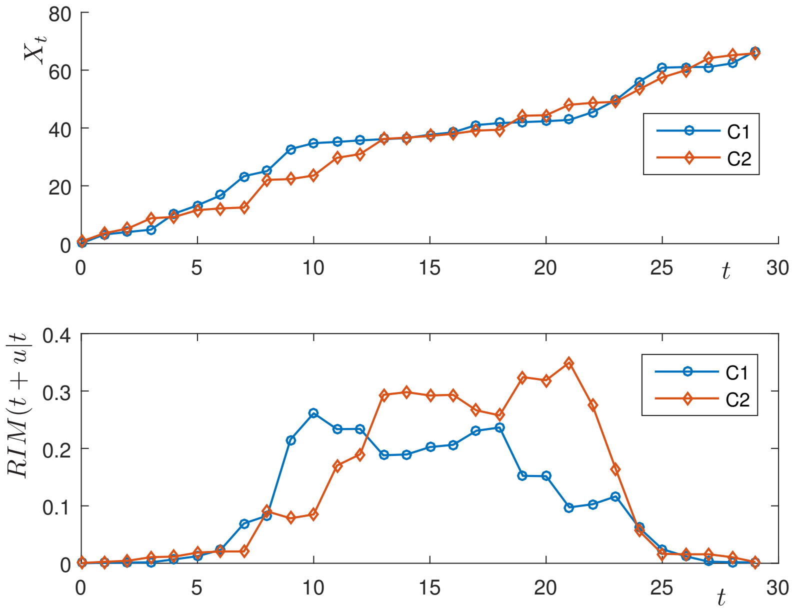

Figure 5: Components' degradation trajectories and corresponding RIM measures with $u = 30$.

The numerical results show that RIM measure allows taking into account efficiently not only the degradation processes of all the components, but also their conditions (states or degradation levels) at given time t, which is crucial to determine the most appropriate component to be selected for preventive maintenance. Consequently, the proposed RIM measure turns to be an efficient decision-aiding indicator for the selection of the most appropriate component to be maintained in order to improve the system reliability on a given mission horizon $[t, t + u]$.

3.3 RIM of a group of components

The RIM definition of an individual component can be extended to a group of components. Indeed, RIM measure of a group G containing k components $\{j_1, ..j_k\}$ with $k = 2, 3, ...$ can be written as follows

$$\begin{aligned} RIM^{\{j_1,..jk\}}(u,t) = & R(t+u|\mathfrak{F}_t^1, \mathfrak{F}_t^2, ..., \{\mathfrak{F}_t^{j_1} = 3, \mathfrak{F}_t^{j_2} = 3, \ldots, \mathfrak{F}_t^{j_k} = 3\}, ..., \mathfrak{F}_t^n) \\ & - R(t+u|\mathfrak{F}_t^1, \mathfrak{F}_t^2, ..., \mathfrak{F}_t^n). \end{aligned} \tag{11}$$

$RIM^{\{j_1,..jk\}}(u,t)$ gives the potential improvement in the conditional reliability of the system when all the components within group G are replaced at time t. By definition, RIM of a group components allows taking into account the joint effect in the system reliability improvement when the group components are preventively replaced together. As a consequence, RIM is not additive, i.e.,

$$RIM^{\{j_1,..jk\}}(u,t) \neq \sum_{l=1}^{k} RIM^{\{j_1\}}(u,t). \tag{12}$$

The uses of RIM measure for ranking the importance of a component/group of several components will be discussed in more detail in section 5.

4 An extension of RIM measure for maintenance decision-making

In the framework of maintenance optimization, different kinds of criteria (such as reliability/safety, productivity, maintenance cost, etc.) have been used for evaluating and/or determining an optimal maintenance policy. It is also shown in the literature that that maintenance cost is the most popular criterion for optimization process and should be integrated in the decision rules. [18, 25, 13]. In that way, we propose herein to integrate both the system reliability improvement and the maintenance cost into a single indicator for maintenance decision-making.

4.1 Maintenance costs and economic dependence

For a multi-component system with complex structure, the system may still operate even when maintenance operations are carried out on some of its components. On the other hand, maintenance operations on some others of its components may lead the entire system to shutdown, which may incur an additional cost due to for example production loss caused by maintenance. In a general way, the preventive maintenance cost of component i, denoted $C_p^{(i)}$, can thus be divided into three parts as follows:

$$C_p^i = c_p^i + c_0^i + d^i \cdot C_d \cdot \mathcal{I}_{\{i \text{ is critical component}\}}, \tag{13}$$

where:

- a specific component cost c_p^i depending on the component characteristics and maintenance labor costs;
- a setup-cost c_0^i indicates the preparation cost (or logistic cost) associated with component i and can be shared when several maintenance activities are performed together. Different setup-cost model have been discussed in [18, 13]. For example, it is pointed out in [25, 9] that execution of a group of maintenance activities usually requires only one set-up;
- an unavailability cost $d^i \cdot C_d$ is incurred if component i is critical, i.e. the system is stopped during d^i time units for executing the preventive maintenance of component i (C_d is the unavailability cost rate of the system). This additional cost can be also shared when several maintenance activities are simultaneously executed by different maintenance teams [7];
- $\mathcal{I}_{\{.\}}$ denotes the indicator function with $\mathcal{I}_{\{.\}} = 1$ if the event $\{.\}$ is true; otherwise $\mathcal{I}_{\{.\}} = 0$.

From a practical point of view, several components may be preventively replaced together due (i) economic raisons regarding to the economic dependence between components; and (ii) technical reasons relying on some specific constraints such as reliability/availability, limited repairmen [7]. In that way, assume that several components, considered as a group denoted G, are preventively replaced together, the corresponding preventive cost can be expressed as

$$C_p^G = \sum_{i \in G} c_p^i + c_0^G + d^G \cdot C_d \cdot \mathcal{I}_{\{G \text{ is a cut set}\}} \tag{14}$$

with,

- c_0^G is the setup-cost for executing the maintenance of group G. c_0^G may depend on both technical factors (e.g. proximity between components), and economic factors (e.g. sales strategies of components' suppliers). c_0^G is bounded:

$$\max_{i \in G} c_0^i \leq c_0^G \leq \sum_{i \in G} c_0^i$$

For a simple model, c_0^G is constant, e.g., $c_0^G = c_0^i = c_0^j$ with $i, j \in G$, see [25, 9].

- d_G is the total maintenance duration of group G and may depend on both the number of maintenance team and the structural dependence [7, 13]. d_G is also bounded:

$$\max_{i \in G} d^i \leq d^G \leq \sum_{i \in G} d^i.$$

To better highlight the economic dependence, Eq.(14) can be written as follows:

$$C_p^G = \sum_{i \in G} C_p^i - \underbrace{(\sum_{i \in G} c_0^i - c_0^G)}_{\text{saved setup-cost: positive impact}} \underbrace{- C_d \cdot \Big(\sum_{i \in G} d^i \cdot \mathcal{I}_{\{i \text{ is critical}\}} - d^G \cdot \mathcal{I}_{\{G \text{ is a cut set}\}} \Big)}_{\text{saved shutdown cost: positive or negative impacts}}, \tag{15}$$

The second part of (15) represents the positive impact of economic dependence since it is always positive. The third part could be zero (when group G is not a cut set, consequently all components of the group are not critical), negative (when group G is a cut set but any component of the group is critical) or positive (when at least one component is critical, group G becomes a cut set and the total maintenance duration of its critical components is higher than the joint maintenance duration of the group).

4.2 An extension of RIM measure

To integrate both the improvement reliability and associated maintenance cost, an extension of RIM measure for component i, denoted RIM_c, can be defined as follows

$$RIM_c^i(u,t) = \frac{h(RIM^i(u,t))}{C_P^i}, \tag{16}$$

where $h(RIM^i(u,t))$ is a benefit function with respect to the improvement quantity in the system reliability, $RIM^i(u,t)$, given by the replacement of component i at time t. It should be emphasized that $h(RIM^i(u,t))$ can be seen as the benefit in terms of cost associated with the system reliability improvement.

By definition, as $RIM_c^i(u,t)$ shows the ratio between the benefit given by the maintenance of component i and the maintenance cost of component i, it can help to better select a component to be preventively maintained. Indeed, two cases can be herein specified:

- $RIM_c^i(u,t) \geq 1$, i.e., the benefit is higher than the maintenance cost, component i is then considered as cost-effective one at time t, i.e., component i should be an admissible candidate for preventive maintenance;

- $0 \leq RIM_c^i(u,t) < 1$ means that the benefit given by the maintenance of component i is lower than its maintenance cost. As a consequence, component i is not cost-effective at time t, i.e., it should not be selected for preventive maintenance at least from a financial point of view.

It is shown in the literature that the investment cost is usually an exponential function of the reliability, see [15]. In that way, as an example, the following benefit model is used in this paper:

$$h(RIM^i(u,t)) = a \cdot e^{b \cdot RIM^i(u,t)}, \tag{17}$$

with a and b are positive real numbers characterizing a mount of reliability improvement in terms of cost. The values of a and b may depend on various factors such as components/system characteristics, technology, etc.

From (16) and (17), $RIM_c^i(u,t)$ can be written as

$$RIM_c^i(u,t) = \frac{a \cdot e^{b \cdot RIM^i(u,t)}}{C_P^i}. \tag{18}$$

For a group of several components, RIM_c can also defined as follows

$$RIM_c^{\{j_1,..j_k\}}(u,t) = \frac{a \cdot e^{b \cdot RIM^{\{j_1,..j_k\}}(u,t)}}{C_P^{\{j_1,..j_k\}}}. \tag{19}$$

It is clear that RIM_c allows taking into account both the joint effect on the system reliability improvement and the economic dependence when the group's components are preventively replaced together.

The uses of RIM_c for ranking the importance of a component/group of several components and selecting "optimal" components in condition based maintenance decision-making are illustrated through an numerical system and presented in the next section.

5 Numerical example

We re-consider the system composed of five non repairable components which is shown in Figure 1. It should be noticed that the system studied has been chosen such that basic structures (series, parallel) are investigated. It is not directly connected to a real cases study, but real systems can be studied using the same method, starting from any reliability block diagrams. From a methodological point of view, this example is complete enough to illustrate the different steps of the modeling approach, to have meaningful and relevant numerical results and to get more insight into the components importance ranking for decision-making resulting from the application of the proposed RIM measures.

It is assumed that the reliability behavior of three components C1, C4, C5 is described by homogeneous gamma stochastic process (see Appendix A) with shape and scale parameters γ_i & η_i $(i = 1, 4, 5)$ respectively which are reported in Table 2. For components C2 and C3, their failure behavior are assumed to be described by Weibull distribution (see Appendix B) with scale parameter $\lambda_i > 0$, and shape parameter $\beta_i > 1$ $(i = 2, 3)$ which are also shown in Table 2. It should be noticed that gamma stochastic process and Weibull distribution are herein used as an example, other stochastic processes (such as the Wiener process, the inversed Gaussian process, etc.) can be also used depending on the degradation behavior of components.

Component	α_i	β_i	γ_i	η_i	L_i
C1	-	-	1	2	100
C2	88	2.2	-	-	-
C3	75	3.25	-	-	-
C4	-	-	1	300	100
C5	-	-	1	2.1	100

Table 2: Parameters of a 5-component system.

The system reliability function as with respect to the components' reliability is given in Eq.(3).

The system is required to serve a mission from t to $t + u$. It is assumed that, in the aim to improve the system reliability, several components should be replaced by the new ones at time t. For this purpose, one interesting question arising is which components should be chosen to be preventively replaced. This can be solved by using the proposed RIM measures.

5.1 RIM : importance ranking and maintenance decision-making

In this section, it is assumed that maintenance costs are not considered due to whatever reason, e.g., they are not available. To illustrate the use of RIM importance measure we consider in this study that only one or two components can be replaced at time t. Of course, RIM can be used to select any group of more than two components.

5.1.1 RIM of individual components

First, inspection operations are carried at time $t = 20$. The information about the components' condition is then provided. It is assumed components C2 and C3 are in working state ($\mathfrak{F}^2_{t=20} = \mathfrak{F}^3_{t=20} = 1$). Components C1, C4 and C5 are also in working state and their degradation levels are measured, i.e., $\mathfrak{F}^1_{t=20} = \mathfrak{F}^4_{t=20} = \mathfrak{F}^5_{t=20} = 2$. By executing a numerical experiment to simulate the degradation process of these components with the parameters shown in Table 2, their degradation levels at $t = 20$ are reported in Table 3. Based on the available information, RIM of individual components are evaluated. To illustrate the use of RIM importance measure, two missions are considered with $u = 20$ and $u = 30$. The obtained results are reported in Table 3. According to the RIM values, an importance ranking is provided.

Component	At time $t = 20$		Mission 1: $u = 20$		Mission 2: $u = 30$	
	$\mathfrak{F}^1_t$	X^i	$RIM^i(u,t)$	Ranking	$RIM^i(u,t)$	Ranking
C1	2	37.21	0.010	5	0.124	2
C2	1	-	0.038	3	0.036	5
C3	1	-	0.040	2	0.045	4
C4	2	40.60	0.105	1	0.096	3
C5	2	39.62	0.018	4	0.176	1

Table 3: RIM measure and ranking of individual components in two missions.

The numerical results show that given the components' condition (states or degradation levels) at time t, the RIM value of each component depends on the mission duration u. As a consequence, the RIM importance of a component may change with the considered mission horizon u. For example, when $u = 20$ (mission 1) the most important component in improving the system reliability improvement for the mission interval [20 40] is C4. It should be noticed however that C4 may be no longer the most important one another mission interval, e.g., C5 becomes the most important component when $u = 30$.

5.1.2 RIM of two-component groups

It is assumed now that two components can be replaced together at time t. To select the "optimal" group which can provide the highest improvement in the system reliability,

RIM measure is applied for all groups of two components. Table 4 reports the RIM value of two-component groups and their importance ranking for two missions considered.

	Mission 1: $t = 20, u = 20$		Mission 2: $t = 20, u = 30$	
Group	$RIM^G(u,t)$	Ranking	$RIM^G(u,t)$	Ranking
(C1,C2)	0.049	9	0.178	7
(C1,C3)	0.051	8	0.196	6
(C1,C4)	0.117	2	0.278	3
(C1,C5)	0.029	10	0.406	1
(C2,C3)	0.082	5	0.086	10
(C2,C4)	0.105	3	0.105	9
(C2,C5)	0.057	7	0.238	5
(C3,C4)	0.104	4	0.108	8
(C3,C5)	0.058	6	0.259	4
(C4,C5)	0.125	1	0.353	2

Table 4: RIM measure and ranking of 2-component groups.

It is shown again that RIM measure depends on the mission duration u. Based on the RIM value, the importance ranking for groups of two components is given. The results show also that at a given time t, the importance ranking may be changed regarding to the mission duration u. Group (C4,C5) is the most important group for a mission interval [20 40] ($t = 20, u = 20$), i.e., group (C4,C5) should be selected to be preventively replaced in order to improve the system reliability. However, group (C4,C5) is no longer the most important one in interval [20 50] ($t = 20, u = 30$), otherwise, group (C1,C5) becomes the most important and should be selected for preventive maintenance activities.

It should be noticed that the importance ranking of a group can not be deduced from the importance ranking of its individual components. Indeed, looking at the results reported in Table 3, for mission 1 ($t = 20$ et $u = 20$), the importance ranking of C3 is higher than that of C5 but at the group level, group (C3,C4) is less important than group (C4,C5). This can be explained by the joint effect on the system reliability when two components are replaced together which is not additive.

5.2 RIM extension with consideration of costs: RIM_c

In this section, the costs are assumed to be known. Table 5 reports the data related to maintenance costs and durations of the system. It should be noticed that, in this study, all parameters are given in arbitrary units, e.g., arbitrary cost unit (acu) and arbitrary time unit (atu). The down time cost rate is $c_d = 20$ acu. The cost coefficients are $a = 40$ and $b = 10$. It is also assumed all components are economically dependent. More precisely, when two components (Ci,Cj) with $i \neq j$ are preventively replaced together, the total maintenance duration is equal to $d^{(Ci,Cj)} = max(d_i, d_j)$ and its setup cost is $c_0^{(Ci,Cj)} = max(c_0^i, c_0^j)$.

Unit	C1	C2	C3	C4	C5
c_p^i	100	50	50	130	120
c_0^i	15	8	8	10	15
d^i	2	1.5	1.5	1.5	2

Table 5: Cost parameters of a 5-component system.

To integrate the maintenance costs, the variant of RIM with consideration of costs, RIM_c, is applied for individual components and for groups of two components.

5.2.1 RIM_c of individual components

The total preventive maintenance cost for each component is firstly calculated by using Eq.(13). RIM_c of individual components are then evaluated from Eq.(18), the obtained results are reported in Table 6.

	Mission 1: $t = 20, u = 20$		Mission 2: $t = 20, t = 30$	
Component	RIM_c^i	Ranking	RIM_c^i	Ranking
C1	0.287	4	0.892	4
C2	**1.012**	2	0.964	3
C3	**1.024**	1	**1.078**	2
C4	0.816	3	0.744	5
C5	0.273	5	**1.322**	1

Table 6: RIM_c indicator and ranking of individual components.

It is shown again that RIM_c depends on the mission duration u. It is important to note that the importance ranking based on RIM_c is not the same as the one provided by RIM (see again Table 3). This can be explained by the fact that RIM is related to the improvement ability on the system reliability when a component is replaced. While RIM_c indicator focuses on the balance between the cost benefit thanks to the replacement of a component and its maintenance cost. More precisely, in the first case ($u = 20, t = 20$), according to the RIM measure, C4 is the most important component. However, regarding to the RIM_c, C3 is the most important one. In addition, $RIM_c^i(20, 20) < 1$ with $i = 1, 4, 5$ mean that, from a financial point of view, components C1, C4 and C5 are not cost-effective for preventive maintenance at time $t = 20$ with $u = 20$. While in the second case ($t = 20$, $u = 30$), C5 becomes cost-effective and ranks the first place.

5.2.2 RIM_c of groups of two components

RIM_c indicator is herein applied for all two-component groups and the obtained results are reported in Table 7.

	Mission 1: $t = 20, u = 20$		Mission 2: $t = 20, u = 30$	
Group	$RIM_c^G(u, t)$	Ranking	$RIM_c^G(u, t)$	Ranking
(C1,C2)	0.320	7	**1.168**	7
(C1,C3)	0.324	6	**1.382**	6
(C1,C4)	0.451	5	**2.255**	4
(C1,C5)	0.194	10	**8.418**	1
(C2,C3)	0.841	1	0.873	8
(C2,C4)	0.521	3	0.517	10
(C2,C5)	0.301	9	**1.836**	5
(C3,C4)	0.522	2	0.533	9
(C3,C5)	0.318	8	**2.358**	3
(C4,C5)	0.457	4	**4.486**	2

Table 7: RIM_c indicator and ranking of 2-component groups.

It is not surprising again that the importance ranking based on RIM_c differs from the one provided by RIM (see again Table 7). As an example of comparison, when $t = 20, u = 20$, group (C1,C5) is the most important group according to the RIM , while, regarding to the RIM_c, group (C2,C3) becomes the most important one. It should be however noticed that any group of two components is cost-effective when $t = 20, u = 20$.

5.3 Joint consideration of RIM and RIM_c

Both RIM and RIM_c can be considered jointly to find the most appropriate component. Figure 6 shows the RIM vs RIM_c of individual components for two cases (t=20, u=20) and (t=20, u=30).

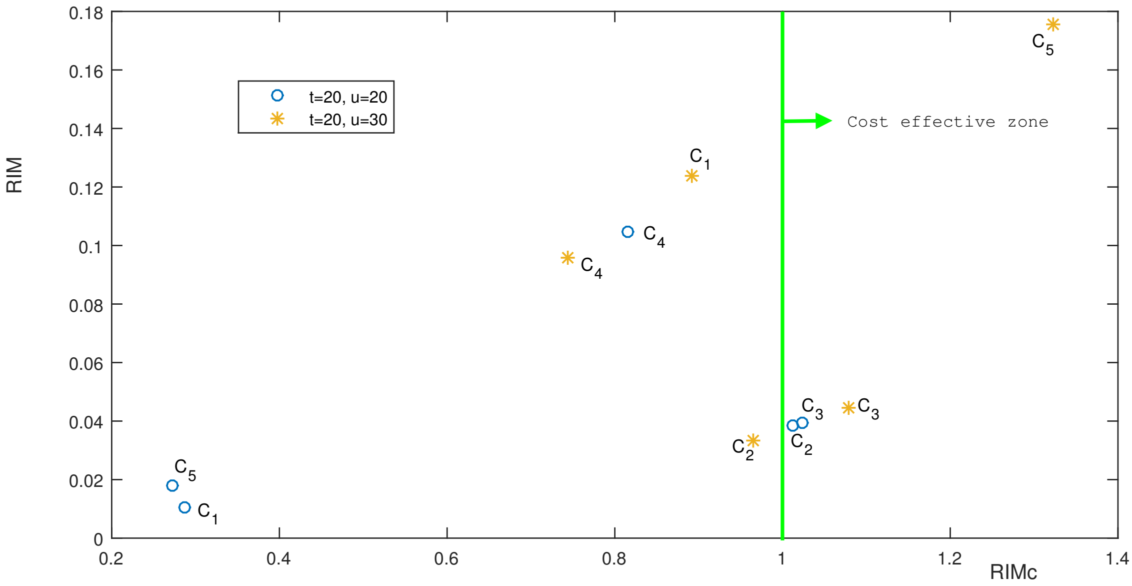

Figure 6: Joint consideration of RIM and RIM_c for ranking individual components.

The most important components are given in the top right corner of the figure and the components in the bottom left corner are the less important. It is clear that C4 is the most important component for the period [20 ; 50] (t = 20, u = 30). When looking at the period [20 ; 40] (t = 20, u = 20), it is difficult to find the most important component. However, if the decision maker judges that the reliability is a priority criterion, C4 should be then selected for preventive maintenance. Otherwise, if an improvement of 0.04 in the system reliability is enough, C3 should be the best choice.

In the same manner, Figure 7 presents the RIM vs RIM_c of two-component groups for t=20, u=30. The results show that group (C1,C5) is the most important regarding to both RIM and RIM_c criteria.

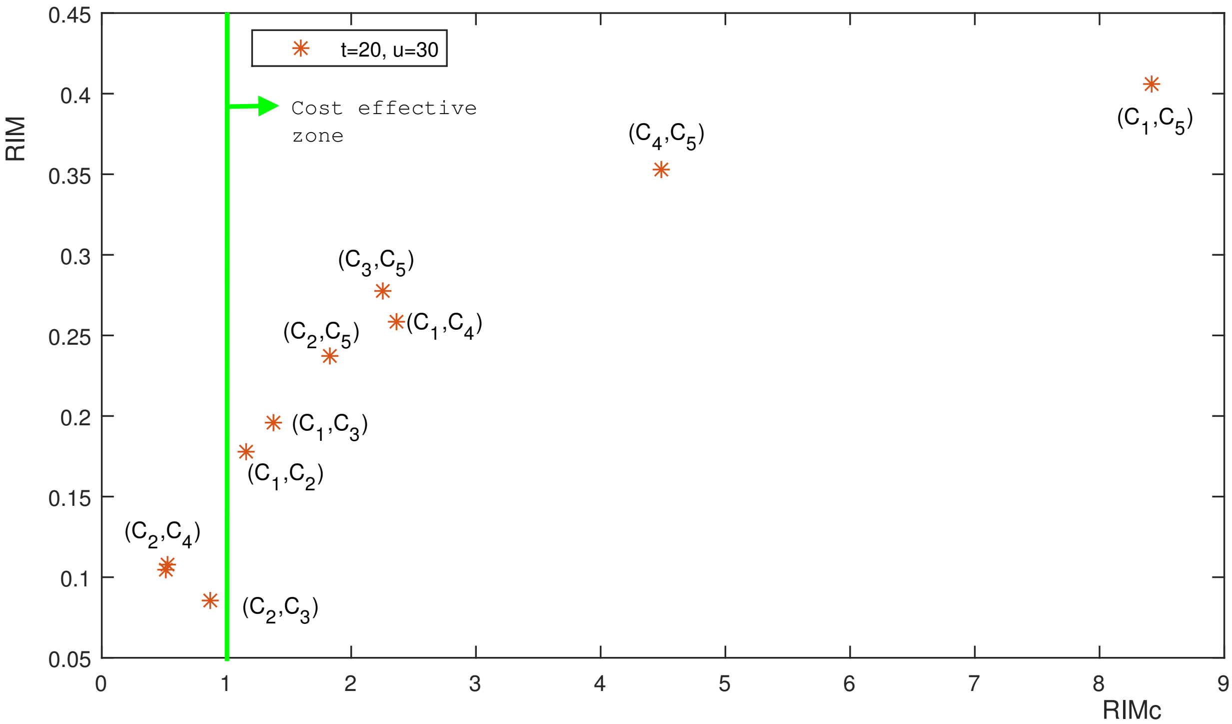

Figure 7: Joint consideration of RIM and RIM_c for ranking two-component groups.

6 Conclusions

In this work, a time-dependent importance measure, namely RIM , is introduced for multi-component systems. RIM provides the improvement ability on the system reliability considering the current components' condition (monitoring information on the states or degradation levels) for a given mission horizon when a component/group components is maintained. This measure could be used as an indicator for condition based maintenance decision-making for a given mission to minimize the probability of system failure during the mission. Moreover, an extension of RIM measure, called RIM_c, which allows taking into account both the improvement ability and different costs structures (maintenance and benefit costs), is proposed. In that way, economic dependence between components is considered and integrated to rank the importance of a group components. RIM_c can help to find the most cost-effective component/group of components. Both RIM and RIM_c are time-dependent. In addition, the current components' condition may have also significant impacts on RIM measure and and its extension (RIM_c). The use and advantages of both RIM and RIM_c measure are then illustrated through an numerical example of 5-

non repairable components system. The numerical results show that, for a given mission, RIM and RIM_c may provide two different importance rankings. From a practical point of view, RIM measure and its extension RIM_c are complementary and should be jointly considered in order to determine the most appropriate component/group of components for maintenance decision-making.

Our future research works will focus on the investigation of RIM measures with consideration of the stochastic dependence between components. Another perspective should be the development of RIM measures in the link with remaining useful life (RUL).

Appendix

Appendix A. Reliability evaluation with gamma deterioration process

Assume that the degradation process of component i is described by a gamma processes $(\tilde{X}_t)_{t\geq 0}$ which has the following characteristics:

- $(\tilde{X}_t)_{t\geq 0}$ has independent increments;
- for all $0 \leq l < t$, the random increment $\tilde{X}_t - \tilde{X}_l$ follows a gamma probability density (pdf) with shape parameter $\gamma_i(t-l)$ and scale parameter η_i:

$$f_{X_t^i}(x) = \frac{1}{\Gamma(\gamma_i(t-l))}\eta_i^{\gamma_i(t-l)}x^{\gamma_i(t-l)-1}e^{-\eta_i x}\mathcal{I}_{\{x\geq 0\}},$$

where, $\mathcal{I}_{\{x\geq 0\}}$ is an indicator function. $\mathcal{I}_{\{x\geq 0\}} = 1$ if $x \geq 0, \mathcal{I}_{\{x\geq 0\}} = 0$ and otherwise;

The mean deterioration speed and its variance are γ_i/η_i and γ_i/η_i^2 respectively. Various deterioration behaviors can be modeled by changing the couple of parameters γ_i, η_i.

The reliability of component i can be evaluated as follows

$$R^i(t) = \int_{x_0^i}^{Z_i} \frac{1}{\Gamma(\gamma_i t)}\eta_i^{\gamma_i t}x^{\gamma_i t-1}e^{-\eta_i x}dx,$$

where x_0^i is the degradation level of component i at time 0.

Appendix B. Reliability evaluation with Weibull lifetime distribution

If the failure behavior of component i is described by Weibull stochastic process, its lifetime follows then a probability density function (pdf) given by

$$f_A^i(x) = \frac{\beta_i}{\alpha_i}\Big(\frac{t}{\alpha_i}\Big)^{(\beta_i-1)} e^{-\left(\frac{t}{\alpha_i}\right)^{\beta_i}},$$

where, parameters $\alpha_i > 0$ (scale parameter) and $\beta_i > 1$ (shape or form parameter). The reliability of component i is expressed as

$$R^i(t) = e^{-(\frac{t}{\alpha_i})^{\beta_i}}$$

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