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► To cite this version:

Nils Krah, Jean Michel Létang, Simon Rit. An analytical method to sample proton trajectories under the influence of multiple Coulomb scattering. International Conference on the Use of Computers in Radiation Therapy (ICCR), Jun 2019, Montreal, Canada. hal-02283827

HAL Id: hal-02283827

<https://hal.science/hal-02283827v1>

Submitted on 31 Jan 2025

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An analytical method to sample proton trajectories under the influence of multiple Coulomb scattering

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Introduction

Multiple Coulomb scattering (MCS) is the most frequent process when Monte Carlo simulating the transport of therapeutic protons (<300 MeV) through a medium. It leads to a succession of many small angle deflections and causes the protons to follow curved trajectories. As part of a work in the context of proton computed tomography, we developed a method to analytically sample proton trajectories in a homogeneous medium based on an energy dependent likelihood function which we think is of interest to the community of Monte Carlo users and developers, e.g. to accelerate simulation of MCS.

Materials & Methods

We define a proton trajectory in a medium as a function t of depth z and its log-likelihood as:

$$- \log L[t] = \int_{z_{in}}^{z_{end}} f(z) \left(\frac{d^2 t(z)}{dz^2} \right)^2 dz, \quad (1)$$

where $f(z)$ is a weighting function that depends on the protons' energy and thus on depth. It can be parametrized by a polynomial fit based on a Monte Carlo simulation of protons in water [1]. We approximate the trajectories as polynomials $t(z) = \sum_{n=0}^N a_n z^n$ and solve the integral above analytically (demonstration out of scope in this supporting document). This leads to a Gaussian distribution of the coefficients a_n which can then be easily sampled.

Results

Figure 1 left shows an example of 20 proton trajectories in water simulated with Gate/Geant4 [2] at 200 MeV and using only electromagnetic interactions (*emstandard* physics list, max. step size 1 mm). Polynomials were fitted to the trajectories and the average root mean square error (RMSE) over the trajectories between data and fit was calculated. The right panel shows the RMSE as a function of polynomial degree based on 1e6 trajectories. Error bars indicate variation of RMSE among the sample.

Figure 1: *Example proton trajectories in water generated with Gate/Geant4 (left). Average RMSE between Monte Carlo trajectories and fit as a function of polynomial degree (right).*

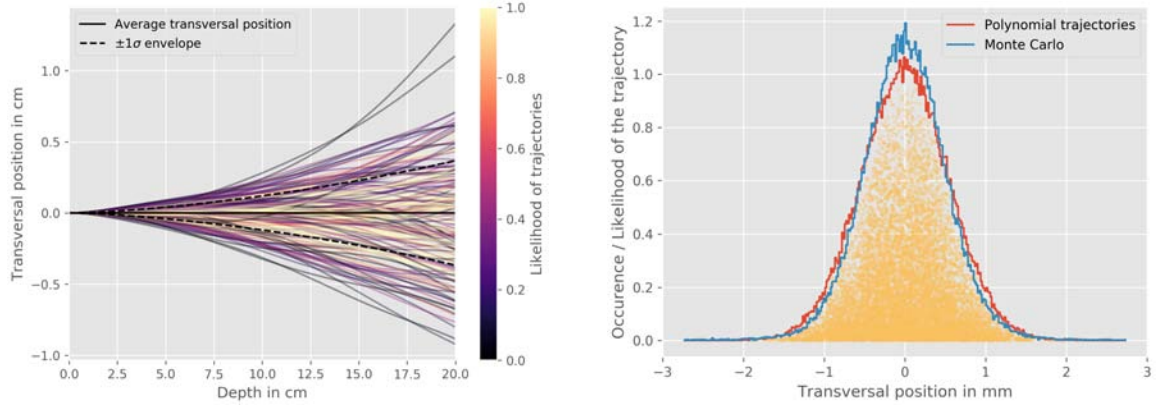


Figure 2: Analytically sampled proton (200 MeV) trajectories in water with the color-coded likelihood (left). Transversal distribution of proton positions at 10 cm depth (right).

Figure 2 left shows analytically sampled proton trajectories in water at 200 MeV with their likelihood calculated through eq. 1, i.e. a straight line would have $L=1$. The right panel shows the distribution of transversal positions at 10 cm depth. Each yellow data point represents one sampled trajectory with its likelihood.

Discussion & Conclusions

Approximating proton trajectories under the impact of MCS by polynomials of degree 4 or higher seems to be sufficiently accurate for medical physics applications where simulation grids are rarely finer than 1 mm. The spatial distributions of analytically sampled and Monte Carlo simulated trajectories agree to better than 5%. Currently, the formalism only works in media without material heterogeneities in the direction transversal to the proton beam. Discrete nuclear scattering events are not modeled. With the analytical method, millions of trajectories can be generated in a matter of seconds on a single CPU. The method could be a starting point for variance reduction techniques or useful as forward projector when simulating proton tomographic images.

References

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- [2] Sarrut, D. et al. (2014). A review of the use and potential of the GATE Monte Carlo simulation code for radiation therapy and dosimetry applications. *Medical Physics*, 41. DOI: 10.1118/1.4871617

Acknowledgements

The work of Nils Krah was supported by funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 753370.

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