



# A Hierarchical Approach For Splitting Truck Platoons Near Network Discontinuities

Aurélien Duret, Meng Wang, Andres Ladino

## ► To cite this version:

Aurélien Duret, Meng Wang, Andres Ladino. A Hierarchical Approach For Splitting Truck Platoons Near Network Discontinuities. *Transportation Research Procedia*, 2019, 38, pp.627-646. 10.1016/j.trpro.2019.05.033 . hal-02283196v2

**HAL Id: hal-02283196**

**<https://hal.science/hal-02283196v2>**

Submitted on 13 Aug 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

23rd International Symposium on Transportation and Traffic Theory, ISTTT 23, 24–26 July 2019,  
Lausanne, Switzerland

## A Hierarchical Approach For Splitting Truck Platoons Near Network Discontinuities

Aurelien Duret<sup>a</sup>, Meng Wang<sup>b,\*</sup>, Andres Ladino<sup>a</sup>

<sup>a</sup>*IFSTTAR, ENTPE, LICIT UMR-T9401, 25 avenue Francois Mitterrand, 69625 Bron cedex, France*

<sup>b</sup>*Department of Transport & Planning, Delft University of Technology, Stevinweg 1, 2628 CN Delft, Netherlands*

---

### Abstract

Truck platooning has attracted substantial attention due to its pronounced benefits in saving energy and promising business model in freight transportation. However, one prominent challenge for the successful implementation of truck platooning is the safe and efficient interaction with surrounding traffic, especially at network discontinuities where mandatory lane changes may lead to the decoupling of truck platoons. This contribution puts forward an efficient method for splitting a platoon of vehicles near network merges. A model-based bi-level control strategy is proposed. A supervisory tactical strategy based on a first-order car-following model with bounded acceleration is designed to maximize the flow at merge discontinuities. The decisions taken at this level include optimal vehicle order after the merge, new equilibrium gaps of automated trucks at the merging point, and anticipation horizon that the platoon members start to track the new equilibrium gaps. The lower-level operational layer uses a third-order longitudinal dynamics model to compute the optimal truck accelerations so that new equilibrium gaps are created when merging vehicles start to change lane and the transient maneuvers are efficient, safe and comfortable. The tactical decisions are derived from an analytic car-following model and the operational accelerations are controlled via model predictive control with guaranteed stability. Simulation experiments are provided in order to test the feasibility and demonstrate the performance and robustness of the proposed strategy.

© 2017 The Authors. Published by Elsevier B.V.

Peer-review under responsibility of the scientific committee of the 23rd International Symposium on Transportation and Traffic Theory.

**Keywords:** Truck platooning, cooperative merging, hierarchical control, traffic flow model, model predictive control, cooperative systems.

---

### 1. Introduction

Automated driving have attracted considerable attention in the recent years, because of the fundamental changes they bring to transportation systems and new services enabled by them. The literature has shown that individual automation can hardly enhance traffic operations while it is commonly agreed that connected/cooperative automated

---

\* Corresponding author. Tel.: +31 (15) 27 84030.

E-mail address: [m.wang@tudelft.nl](mailto:m.wang@tudelft.nl)

vehicles (CAVs) possess great potential in increasing roadway capacity and traffic flow stability [12, 25, 27, 28, 33]. Vehicle platooning is one of multiple applications that stands out in the domain, characterized by a string of CAVs respecting a specified equilibrium spacing policy [3, 16, 26, 28]. The reduction of the equilibrium spacing breaks the capacity limits of today's network and enhances fuel economy.

Truck platooning is expected to be deployed earlier than passenger cars due to the pronounced benefits in terms of fuel saving [1] and promising business models [4, 18]. Several on-road pilots have identified problems that truck platooning brings to traffic at freeway entrances and exits [4, 18, 29], which is of paramount importance to traffic safety and throughput of the road network. Therefore, coordination and control at network discontinuities are important challenges for vehicle platooning in the real world. This was well recognized early in Automated Highway Systems studies in the 1990s [31, 32]. Often, a hierarchical framework where a platoon coordination layer is placed between the link traffic control layer and the vehicle control layer is adopted. The platoon coordination layer is primarily concerned with platoon-level maneuvers such as platoon formation, split, merge, and exit, which is the focus of this paper.

A few active platooning strategies have been proposed in the literature under within a full CAV environment. A set of protocols for platoon maneuvers on highways was proposed in [10], including merge, split, and lane change. The design of the protocol is based on a finite state description of platoon maneuvers under the command of an upper-level traffic control layer. Despite the pioneering role of this work, the design of the split protocol did not address important decisions regarding where/when to split the platoon at highway entrance and optimal trajectories for vehicles. In [8], entry and exit platoon maneuvers on highway were discussed. The proposed strategies were applicable in cases where dedicated automated vehicle lanes and transitional lanes are disposed near highway entries and exits. The design was based on the assumption that the merging vehicle is a CAV and it required infrastructure changes, e.g. a parallel transition lane or dedicated ramps, raising concerns over the applicability in reality. In [9], two basic platoon maneuver strategies, merge and split, were proposed to facilitate a CAV merge into an existing platoon and a platoon member leaving the formation respectively. The split strategy was further elaborated by sub-tasks of initiating split request, creating safe gap and changing lane. A similar design using finite state machine approach was also reported in [2] for platoon maneuver protocols. The protocols were combined with cooperative adaptive cruise control (CACC) logic used to represent longitudinal behavior. However, the optimal moment to start the gap creation process and how the transient maneuver looks like were not formulated, leaving the operational strategies and corresponding algorithms for platoon maneuvers unanswered.

Cooperative automated maneuvering protocols were designed for a highway lane drop scenario and an unsignalized T-intersection in [20]. Field tests by 9 teams with CAVs under a real network but restricting normal traffic show the performance of the maneuver protocols. These protocols determine largely the efficiency of the resulting traffic operations. However, this relation is not taken into account explicitly in the design and it does not handle mixed traffic conditions. In [15], a decision algorithm that computes a target reference path for each vehicle and a fuzzy longitudinal controller that guarantees the merge for a vehicle approaching from the minor road tracks were proposed, but the design was restricted to autonomous vehicle systems rather than CAVs. In a more recent work [19] proposed an optimal vehicle trajectory design for cooperative merging, where gap policies are imposed at the initial and final time of the maneuver according to a specified merging sequence. In this approach, trajectories are first designed via an optimal control problem and then applied to the vehicles, before applying the decision the controller selects the most restrictive acceleration. A different vehicle string modeling approach was presented in [3], where a spring-mass-damper analogy was adopted to describe platooning dynamics. This modeling approach allows one to model platoon dynamics near highway entry and exit by controlling the spring constant and damping coefficient, where a CAV joins or leaves a platoon, but does not generate optimal merging decisions and trajectories. More recently, the cooperative merging problem was treated in [24], in this case the optimal control problem aims to optimize the fuel efficiency of the system as CAVs approach the merging zone. Finally, [11] formulates a stochastic switched system model in which is analyzed how platoon-induced congestion varies with the fraction of platooned vehicles at merge, yet the decisions on when and where to split the platoon is not addressed.

Notice that another body of literature focused on platoon formation strategies [9, 26, 30], for a more general review on coordinated control on vehicles at intersections we invite the reader to explore [23]. We restrict the discussion on platoon formation since this paper concentrates on how to split a platoon rather than forming a platoon. Literature shows that quite some effort in defining platoon maneuver protocols at highway entry and exit. These studies focus on

dynamics and interactions between platoons or between a platoon and an individual CAV, which implies communication between interacting platoons/vehicles. Some even require additional changes in the infrastructure, which may impede the near-term application of the strategies. In addition, there is a gap between finite state description of the platoon maneuvers and the detailed operational truck platooning strategies for the transition between states. In case of truck platooning, it is likely that the platoon has to be detached to facilitate merging vehicles from on-ramp sections. A decision-making strategy to support when and where to split the truck platoon before the merge section and the corresponding operational algorithm to execute the longitudinal motion of trucks remains as a scientific challenge. Although an attempt was made in [7], it assumes a single merging vehicle and the question of how to control the continuous trajectories of interacting vehicles remains unanswered.

This contribution proposes a hierarchical decision and control framework for automated truck platoons to facilitate lane-changing maneuvers of surrounding vehicles near on-ramps and off-ramps. The tactical layer uses a first-order traffic flow model to generate decisions about when and where to yield a safe and comfortable gap that maximizes throughput. The operational layer uses a third-order longitudinal dynamics model to control truck accelerations such the gap has been created when the merging vehicle starts the lane change and the transient maneuvers are efficient, safe and comfortable. The tactical layer considers limited acceleration and deceleration capabilities and the operational layer takes into account safety constraints in addition to admissible acceleration and speed with guaranteed stability under receding horizon optimal control approach. We remark that although the contribution is motivated by truck platooning, the proposed design is generic and is applicable in all CAV platoons.

The remainder of the paper is organized as follows. Section 2 presents the general hierarchical decision and control framework. Sections 3 and 4 present the mathematical formulations of tactical and operational levels, respectively. Section 5 illustrates the performance of the proposed control framework under various connectivity assumptions and network configurations. Finally, Section 6 concludes the paper.

## 2. Merging problem and model-based hierarchical decision framework

Let us consider an existing platoon of CAVs in an equilibrium condition approaching the merging section (See Fig. 1a). At some point in the road network a lane reduction situation forces vehicles to merge into the formation. The following issues and questions appear as part of the problem formulation:

- How to design a maneuver strategy for the platoon so that it leads to the safe and efficient interaction between the CAV platoon and the merging traffic?
- How to perform the maneuver considering interaction between the CAV acceleration controller and the platoon maneuver strategy?

The framework proposed here, is composed of two levels (See. Fig. 1b). At the tactical level, the layer takes information from V2V communication regarding positions and speeds of vehicles in the platoon, and positions and speeds of merging vehicles via V2I communication when they pass a fixed road-based sensor. A model-based strategy is used to determine the optimal vehicle indexes in the platoon to yield gaps for merging vehicles and time instants they start the yielding process, given a speed drop they accept compared to the equilibrium speed. It also outputs the desired state parameters for the interacting vehicles after merge, notably the desired time gap of the yielding trucks. The tactical decisions are then sent to the operational layer, where a model predictive controller determines the command accelerations that regulate the speeds and positions of CAVs in the platoon. Before the merging vehicles reach the end of the merging section, the CAVs open a sufficient gap and they will change lane to the main carriage way (See. Fig. 1b). The tactical layer updates its decisions at low frequencies, e.g. every 5-10 seconds, while the operational layer updates vehicle accelerations at high frequencies, e.g.  $dt \leq 0.1s$ . When the operational layer fails to find feasible solutions that respect system constraints, it requests re-evaluation of the tactical decisions at irregular times and may even overrule the tactical decisions to find safe and comfortable trajectories.

The hierarchical design follows the architecture proposed earlier in [32] for automated highway systems. The tactical layer pertains to the coordination layer while the operational layer corresponds to the vehicle layer in [32]. In the ensuing, we formulate the two layers respectively.

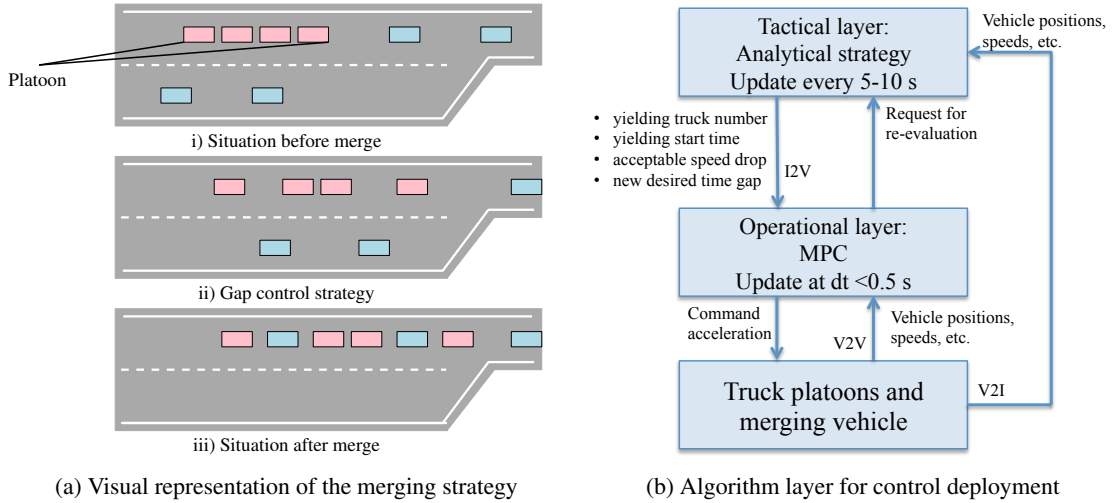


Fig. 1: Multiple merging problem and our approach to solution

### 3. Tactical level: analytical car-following model with merging process

When one or several vehicles have to merge into a platoon of CAVs with short spacing, the platoon should anticipate and split to open sufficient gaps. Two problems arise : (1) How to find optimal ordering in the final formation, i.e. which members of the platoon should decelerate and yield gaps for the merging vehicles; (2) How to find the anticipation times, or equivalently the time instants that the gap creation process should start for each yielding vehicle. The tactical layer proposes a solution for both, based on the analytical Newell car-following (CF) model with bounded acceleration and deceleration.

Newell has proposed a simplified theory to describe the follow-the-leader behavior of vehicles. It describes equilibrium (safe and comfortable) states that maximize the flux, which can be summarized as follows: when the leading vehicle is very distant the following vehicle has a free-flow speed  $u$ . Under car-following situations, the following vehicle  $n$  replicates its leader  $n - 1$  trajectory with a shift in time  $\tau$  and space  $d$ . Under stationary conditions with vehicles traveling at the same speed  $v$ , vehicle spacing  $s$  becomes:

$$s = d + v\tau \quad (1)$$

This original formulation of the model allows for infinite acceleration. For a more realistic description, we consider that acceleration rate is comprised in a bounded interval denoted  $a^-$  and  $a^+$  respectively. These bounds ensure feasible dynamics, in particular for trucks whose acceleration and braking capacities are limited. They may vary from one vehicle to another to replicate heterogeneous behavior.

**Assumption 1.** Car-following parameters. *For the sake of simplicity in the remainder of the paper, it is assumed that merging vehicles can be divided into 2 categories: connected automated vehicle and human driver vehicle (HDV). Both follow the Newell theory: CAVs have parameters  $[d^p, \tau^p]$ , designed with respect to the constant (time or space) gap policy under platooning operation, and HDVs with parameters  $[d, \tau]$ . It is also assumed that the maximum wave speed is the same for both CAVs and HDVs:  $w = d/\tau = d^p/\tau^p$ . It should be noted that this assumption can be easily relaxed by considering individual parameters per vehicle instead of vehicle class.*

#### 3.1. Vehicle ordering process

The merging procedure can be modeled as an arrival process of vehicles from two different lanes  $\ell-, \ell+$  to a particular point in the space  $X_m$ . The situation could involve the arrival of multiple types of vehicles in both lanes,

CAVs or HDVs. The strategy follows the principle that CAVs actively adapt their dynamics to reach new equilibrium situations to maximize the flow at the merging position.

Let us consider the situation of multiple vehicles driving in the lane  $\ell-$  and willing to merge into the lane  $\ell+$ , where platoon CAVs are located. The initial conditions of the problem can be formulated as follow :  $\mathcal{I}_{\ell+} = \{i_0, i_1, \dots, i_n\}$  and  $\mathcal{J}_{\ell-} = \{j_1, \dots, j_m\}$  denote the maps of discrete values containing ordered indexes of vehicles traveling in the target and the original lanes respectively, and  $g_k^\ell = [x_k^\ell \ t_k^\ell]^T$  is the position-time of the vehicle  $k$  in the lane  $\ell$ .

**Assumption 2.** Boundary conditions. All vehicles belonging to the sets  $\mathcal{J}_{\ell-}, \mathcal{I}_{\ell+}$  maintain their (free-flow) speed  $u$  between their initial positions and the merging position  $X_m$ . The leader trajectory establishes the boundary condition of the ordering problem. Moreover, HDVs that need to merge cannot be controlled and hence their trajectories are considered as (internal) boundary conditions of the problem.

**Assumption 3.** Final states. It is assumed that the platoon settles down to equilibrium at the position  $X_m$  with all vehicles following Newell equilibrium conditions, with parameters  $[d^p, \tau^p]$  for CAVs following CAVs, and with parameters  $[d, \tau]$  otherwise. The ordering of the final formation is given by an ordered sequence  $\mathcal{O} = \{\sigma_0, \sigma_1, \dots, \sigma_{n+m}\}$  where the leader of the final formation is the leader of the initial formation:  $\sigma_0 = i_0$ .

We consider two main scenarios: the first one where vehicles willing to merge are all CAVs (see Fig. 2); and the second one where mixed vehicles are arriving (see Fig. 3). The new equilibrium conditions imply an ordering strategy which leads to a  $np$ -hard problem with combinatorial characteristics.

### 3.1.1. Homogeneous CAV merging

The set of problem is illustrated in Fig. 2a: where the green solid line is the leader's trajectory (boundary condition); blue dashed lines denote CAVs following a specific platoon formation in lane  $\ell+$  at some initial position  $X_b$ ; and brown dashed lines starting with  $j_1$  and  $j_2$  depict the detection of CAVs (via e.g. loop detectors, camera, GPS) moving along the lane  $\ell-$  and willing to merge into  $\ell+$  at  $X_m$ . With assumptions 2 and 3, the natural ordered sequence can be

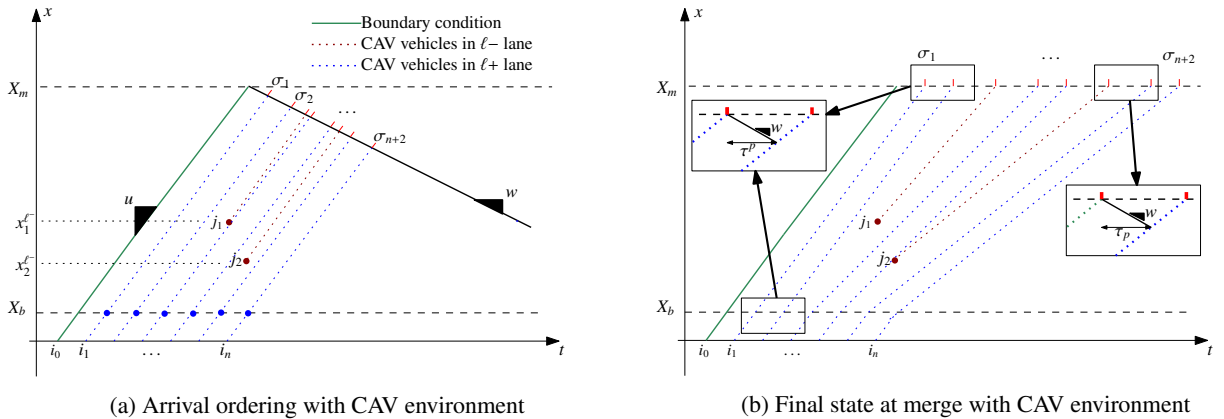


Fig. 2: Ordering and final state with CAV environment

achieved by projecting initial conditions  $g_k^\ell$  with the maximum speed  $u$  along a shock wave  $w$ , as illustrated in Fig. 2a. Let  $p(g_k^\ell)$  be the projection of a particular point  $g_k^\ell = [x_k^\ell \ t_k^\ell]^T$  into the shock wave starting from the leader's position  $[X_m \ T_m^0]$  and propagating backward with a constant speed  $-w$ . The projection is the intersection of two planes, which can be formulated as a linear system:

$$\begin{aligned} P_1 : \quad x &= -wt + X_m + wT_m^0, \\ P_2 : \quad x &= ut + x_k^\ell - ut_k^\ell. \end{aligned} \quad (2)$$

The solution is given by:

$$\begin{bmatrix} p_x(g_k^\ell) \\ p_t(g_k^\ell) \end{bmatrix} = \frac{1}{u+w} \begin{bmatrix} 1 & u \\ -1 & w \end{bmatrix} \cdot \begin{bmatrix} X_m \\ x_k^\ell \end{bmatrix} + \begin{bmatrix} w & 0 \\ 0 & -u \end{bmatrix} \begin{bmatrix} T_m^0 \\ t_k^\ell \end{bmatrix} \quad (3)$$

The *a-priori* order  $O^*$  is given by the sequence organizing the full set of projections  $\mathcal{P} = \{p(g_1^{\ell+}), \dots, p(g_{n+m}^{\ell+})\}$  in a time ordered set (see Fig. 2a) such that:

$$p_t(g_k^{\ell+}) < p_t(g_{k+1}^{\ell+}), \forall k \in [1 : m+n] \quad (4)$$

At this stage, the optimal ordering sequence  $O^*$  is established. Let us consider the vehicle with indexes  $[i_x, \sigma_x]$  in the initial and final platoon respectively. Far upstream the merge position, it replicates the trajectory of the leader of the platoon with a time shift  $T_{i_x}^0$  (See Fig. 4). As it approaches the merging position, a transient period occurs and the vehicle opens a gap by increasing its time shift until it reaches the final value  $T_{\sigma_x}^f$ .

$$\begin{cases} T_{\sigma_x}^0 = \sum_{k=i_1:i_x} \tau_k & \text{initial shift in time (baseline = leader of the platoon)} \\ T_{\sigma_x}^f = \sum_{k=\sigma_1:\sigma_x} \tau_k & \text{final shift in time (baseline = leader of the platoon)} \end{cases} \quad (5)$$

### 3.1.2. Mixed HDVs/CAVs merging

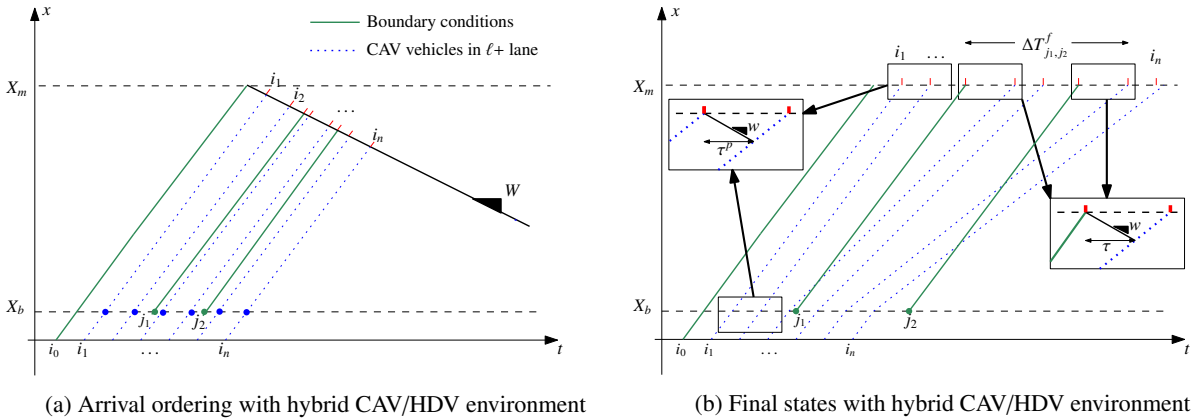


Fig. 3: Ordering and final state with hybrid CAV/HDV environment

A hybrid situation of merge can be found when HDVs traveling along lane  $\ell-$  are willing to merge into  $\ell+$ . In this particular case the ordering process is not flexible anymore as Eq. (4) due to internal boundary conditions imposed by (uncontrolled) HDVs' trajectories. This particular scenario is detailed in Fig. 3a, where green solid lines denote internal boundary conditions of the problem. In this case, the *ordering problem* can be cast as a *resource allocation problem* where the objective of the controller is to allocate the maximum amount of CAVs between two consecutive internal conditions, i.e. HDVs' trajectories).

$$\eta \leq \left\lfloor \frac{\Delta \tau_{jk,jk+1}^f - 2\tau}{\tau^p} \right\rfloor + 1 \quad (6)$$
$$\begin{cases} T_{\sigma_x}^0 = \tau^p & \text{baseline = CAV leader} \\ T_{\sigma_x}^f = \tau + \sum_{k=1:\sigma_x} \tau^p & \text{baseline = HDV leader,} \end{cases} \quad (7)$$

### 3.2. Yielding dynamic

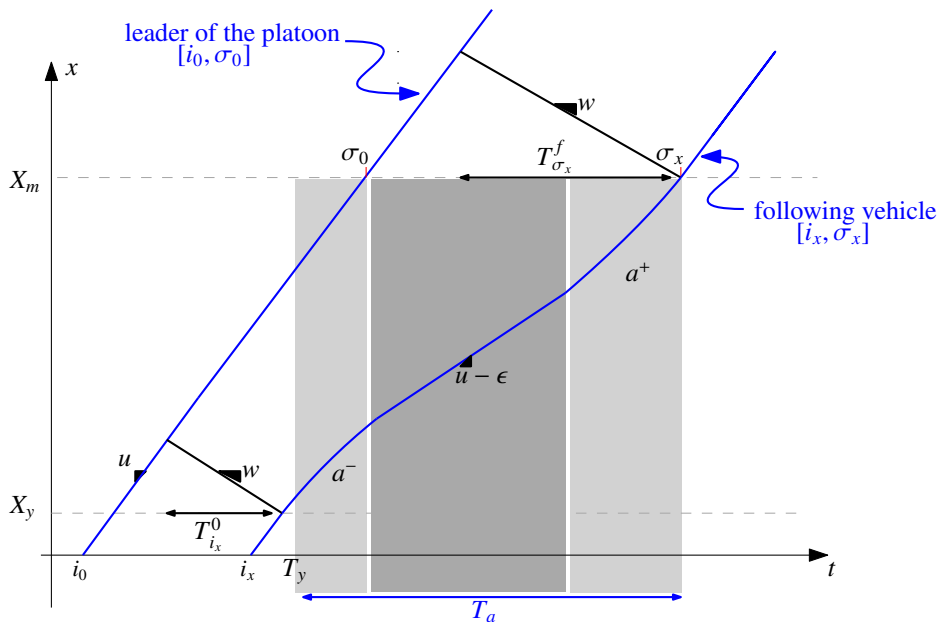


Fig. 4: Split dynamic merge: Vehicle separation process from its reference leader.



The duration of the transient period  $T_a$  for the vehicle  $[i_x, \sigma_x]$  is a key decision variable to estimate. The problem is presented in Fig. 4 with the following assumptions.

**Assumption 4.** Transient dynamic. *During the transient period, the yielding vehicle follows a dynamic in three successive phases:*

- deceleration period with a (negative) acceleration rate  $a^-$
- speed drop period, with a constant speed  $v = u - \epsilon$
- acceleration period with an acceleration rate  $a^+$

The problem can be formulated as a linear problem where the anticipation time  $T_a$  is unknown, as illustrated graphically in Fig. 4. The anticipation time involving three phases is formulated as follows [7]:

$$T_a = \frac{\epsilon}{2} \cdot \left( \frac{1}{a^+} - \frac{1}{a^-} \right) + \frac{u + w}{\epsilon} \cdot (T_{\sigma_x}^f - T_{\sigma_x}^0) \quad (8)$$

Now consider the current position-time of CAV indexed  $i_x$  in the lane  $\ell+$ , denoted  $g_{i_x}^{\ell+} = [x_{i_x}^{\ell+} \ t_{i_x}^{\ell+}]$ , one can demonstrate that the time at which the vehicle should start the yielding maneuvers writes, after simplification:

$$T_y = \left( t_{i_x}^{\ell+} + \frac{X_m - x_{i_x}^{\ell+}}{u} \right) + (u + w) \left( \frac{1}{u} - \frac{1}{\epsilon} \right) (T_{\sigma_x}^f - T_{\sigma_x}^0) - \frac{\epsilon}{2} \cdot \left( \frac{1}{a^+} - \frac{1}{a^-} \right) \quad (9)$$

In Eq. (8) and 9, the speed drop  $\epsilon$  is assumed to be known *a priori*. However, if the yielding vehicle is too close to the merging position,  $\epsilon$  may be too low for opening a sufficient gap. From Eq. (8), the problem can be inversed and the speed difference  $\epsilon$  becomes the new decision variable function of the anticipation time:

$$\epsilon = \frac{T_a}{2 \cdot (1/a^+ - 1/a^-)} \left( 1 + \sqrt{1 - 2 \cdot \frac{(1/a^+ - 1/a^-) \cdot (u + w) (T_{\sigma_x}^f - T_{\sigma_x}^0)}{T_a^2}} \right) \quad (10)$$

Note that the position at which HDVs are detected, i.e.  $X_b$  in Figure 2 and 3, should be far enough from the downstream position  $X_m$  so once the vehicle  $i_n$  has been detected the control maneuver can perform. This can be bounded by  $X_m - uT_a$  where  $T_a$  is given in Eq. (8).

#### 4. Operational Layer: Model Predictive Control Approach

This section describes the model predictive controller at the operational layer and the control algorithm taking into account tactical decisions during the operation. The goal is to control the trajectories of the CAVs so that they follow the preceding vehicle with the constant time gap (CTG) policy [17, 21, 22], or equivalently a constant shift in time  $\tau^p$ .

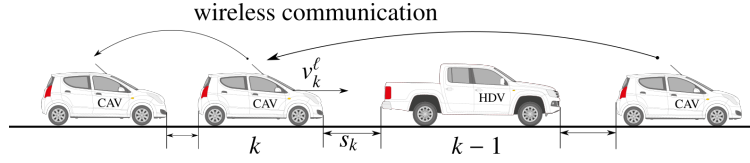


Fig. 5: Mixed fleet of CAV and HDV vehicles

#### 4.1. System dynamics for CAV platoon

We consider the longitudinal dynamics described by the following linear vehicle model:

$$\begin{pmatrix} \dot{x}_k(t) \\ \dot{v}_k(t) \\ \dot{a}_k(t) \end{pmatrix} = \begin{pmatrix} v_k(t) \\ a_k(t) \\ \frac{-a_k(t) + \alpha_k(t)}{T_e} \end{pmatrix} \quad (11)$$

where  $\alpha_k(t)$  denotes the control input for acceleration of vehicle  $k$ . Note that the third-order linear model is derived from a nonlinear vehicle dynamic model based on the so-called *exact linearization* approach [33].  $T_e$  represents the *engine time constant* in the vehicle propulsion/braking systems, e.g. it takes some time  $T_e > 0$  for the vehicle system to drive the actual acceleration  $a_k$  to the desired acceleration  $\alpha_k$ .

Given that one of the key objectives is to regulate the headway  $s_k$  (see Fig. 5) of two consecutive vehicles in the platoon toward some desired value, the system state from the perspective of the whole platoon with  $n$  CAVs (including the leader with index  $k = 1$ ) can be described by the headway errors  $\mathbf{e}_s = (e_{s,1} \ e_{s,2} \ \dots \ e_{s,n})^T$ , speed errors  $\mathbf{e}_v = (e_{v,1} \ e_{v,2} \ \dots \ e_{v,n})^T$  and accelerations  $\mathbf{a} = (a_1 \ a_2 \ \dots \ a_n)^T$  between consecutive vehicles, e.g.

$$e_{s,k}(t) = s_k(t) - s_k^r(t), \quad e_{v,k}(t) = v_{k-1}(t) - v_k(t), \quad k = \{1, \dots, n\} \quad (12)$$

$s_k^r(t)$  denotes the reference/desired headway, which is a linear function of vehicle speed:  $s_k^r = v_k h_k^r + d^p$  with  $h_k^r = \tau^p$  by default and  $d^p$  the minimum headway vehicle  $k$  maintains.

The full state vector represented by  $\mathbf{x} = (e_{s,1}, e_{v,1}, a_1, e_{s,2}, e_{v,2}, a_2, \dots, e_{s,n}, e_{v,n}, a_n)^T$ . The control vector is the accelerations of all CAVs in the platoon: e.g.  $\mathbf{u} = (\alpha_1, \alpha_2, \dots, \alpha_n)^T$ . The system dynamics model in state-space form is described by:

$$\frac{d}{dt} \mathbf{x} = \frac{d}{dt} (e_{s,1}, e_{v,1}, a_1, e_{s,2}, e_{v,2}, a_2, \dots, e_{s,n}, e_{v,n}, a_n)^T = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{d}) = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{C}\mathbf{d} \quad (13)$$

with

$$\mathbf{A}^{3n \times 3n} = \begin{pmatrix} A_1 & & & \\ A_0 & A_2 & & \\ & \ddots & \ddots & \\ & & A_0 & A_n \end{pmatrix}, \quad A_k = \begin{pmatrix} 0 & 1 & h_k^r \\ 0 & 0 & -1 \\ 0 & 0 & -1/T_e \end{pmatrix}, \quad A_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad \mathbf{B}^{3n \times n} = \begin{pmatrix} B_1 & & \\ & \ddots & \\ & & B_n \end{pmatrix}, \quad B_k = \begin{pmatrix} 0 \\ 0 \\ 1/T_e \end{pmatrix}, \quad k = \{1, \dots, n\};$$

$$\mathbf{C}^{3n \times 1} = (0, 0, 1, 0, \dots, 0)^T; \quad \mathbf{d} = a_0$$

$\mathbf{d} = a_0$  denotes system disturbance, which is the acceleration of the uncontrolled vehicle preceding the first truck in the platoon.

#### 4.2. Optimal platooning control problem

The platooning system seeks an optimal control trajectory  $\mathbf{u}(\cdot)$  in the finite prediction horizon  $[t_0, t_0 + T_p)$  that minimizes a cost function [34], which can be formulated as the following mathematical optimization programme:

$$\begin{aligned} J_{T_p} = \min_{\mathbf{u}_{[t_0, t_0+T_p)}} & \int_{t_0}^{t_0+T_p} \mathcal{L}(\mathbf{x}(t), \mathbf{u}(t)) dt = \min_{\mathbf{u}_{[t_0, t_0+T_p)}} \int_{t_0}^{t_0+T_p} (\mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t)) dt \\ \text{s.t.} & \quad \dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} + \mathbf{C} \mathbf{d} = f(\mathbf{x}, \mathbf{u}) \\ & \quad \mathbf{x}(t_0) = \mathbf{x}_0 \\ & \quad \mathbf{x}(t_0 + T_p) = \mathbf{0} \\ & \quad \mathbf{x} \in \mathcal{X} \\ & \quad \mathbf{u} \in \mathcal{U} \end{aligned} \quad (14)$$

where  $\mathcal{L}$  denotes the so-called running cost or stage cost, and  $\mathbf{Q}$  and  $\mathbf{R}$  are positive definite weight matrices defined as:

$$\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_s^{n \times n} & \mathbf{0}^{n \times n} & \mathbf{0}^{n \times n} \\ \mathbf{0}^{n \times n} & \mathbf{Q}_v^{n \times n} & \mathbf{0}^{n \times n} \\ \mathbf{0}^{n \times n} & \mathbf{0}^{n \times n} & \mathbf{0}^{n \times n} \end{pmatrix}, \mathbf{Q}_s^{n \times n} = \begin{pmatrix} c_1 & & \\ & \ddots & \\ & & c_1 \end{pmatrix}, \mathbf{Q}_v^{n \times n} = \begin{pmatrix} c_2 & & \\ & \ddots & \\ & & c_2 \end{pmatrix}, \mathbf{R}^{n \times n} = \begin{pmatrix} c_3 & & \\ & \ddots & \\ & & c_3 \end{pmatrix} \quad (15)$$

The optimal control problem in Eq. (14) shows the controller minimizes gap and speed errors, in addition to accelerations/decelerations. The optimization is subject to the system dynamics model of Eq. (13) initial condition of  $\mathbf{x}(t_0) = \mathbf{x}_0$ , terminal constraint  $\mathbf{x}(t_0 + T_p) = \mathbf{0}$  and the admissible constraints on state and control variables:  $\mathbf{x}(t) \in \mathcal{X}$ ,  $\mathbf{u}(t) \in \mathcal{U}$ . The constraints on the state variable  $\mathcal{X}, \mathcal{U}$  are specified as:

$$\mathcal{X} := \{s_k(t) > d^p; v_k(t) \in [v^-, v^+], \forall k \in \{1, \dots, n\} \quad (16)$$

$$\mathcal{U} := \{\alpha_k(t) \in [a^-, a^+]\} \quad (17)$$

This implies that the headway should be no less than the minimum headway  $d^p$  and the controlled vehicles travel within a speed range of  $[v^-, v^+]$ . By default,  $v^- = 0$  and  $v^+ = u$ .

#### 4.3. Solution to the optimal control problem

At each time instant  $t_0$ , the problem Eqs. (14) to (17) is solved online with an efficient algorithm based on Pontryagin's Principle[34], which entails defining the Hamiltonian  $\mathcal{H}$  as follows:

$$\mathcal{H}(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}) = \mathcal{L}(\mathbf{x}, \mathbf{u}) + \boldsymbol{\lambda}^T \cdot \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad (18)$$

where  $\boldsymbol{\lambda} = (\lambda_{s,1}, \lambda_{v,1}, \dots, \lambda_{s,n}, \lambda_{v,n})^T$  denotes the so-called *co-state* or *marginal cost* of the state  $\mathbf{x}$ . Using the Hamiltonian, we can derive the *necessary condition* for optimality with:  $\mathbf{u}^* = \arg \min_{\mathbf{u}} \mathcal{H}(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda})$ . Furthermore, the co-state

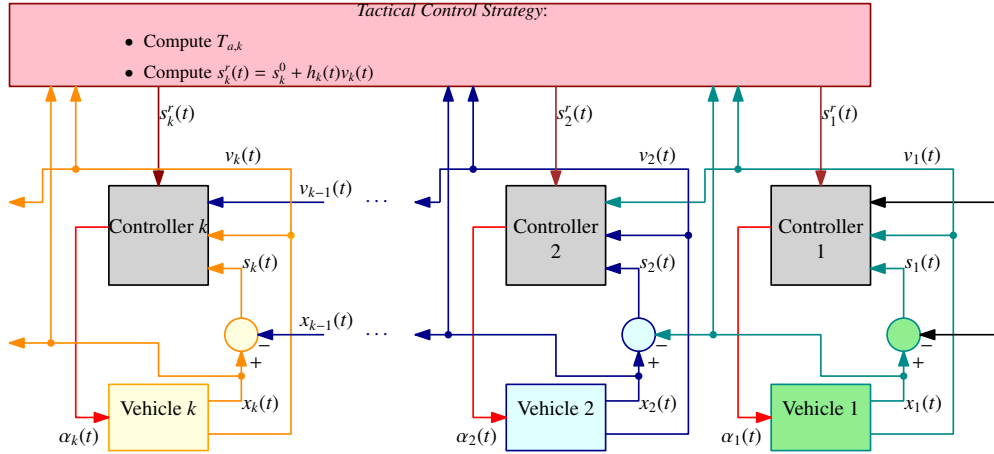


Fig. 6: Full control strategy deployment

has to satisfy the following dynamic equation:

$$-\frac{d}{dt}\lambda = \frac{\partial \mathcal{H}}{\partial \mathbf{x}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}} + \lambda^T \cdot \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \quad (19)$$

subject to the terminal conditions of  $\lambda(t_0 + T_p)$  [34]. In particular, in this study the running cost in Eq. (20) was implemented. This cost promotes smooth acceleration profiles for all vehicles while ensuring the desired performance.

$$\mathcal{L} = \sum_{k=1}^n \left( c_1 (s_k - s'_k)^2 + c_2 (v_{k-1} - v_k)^2 + c_3 \alpha_k^2 \right), \quad \forall k \in \mathcal{I}_{CAV} \quad (20)$$

where we omit the  $t$  dependence for the sake of readability. The optimal control problem is transcribed to a set of coupled ordinary differential equations of (13) and (19). An iterative algorithm is proposed to solve the problem efficiently. Algorithm 1 illustrates the full control strategy solved each time step  $t$ .

#### 4.4. Stability of operational layer

One of the important properties of closed loop systems is stability. Under model predictive control setting, this is not a trivial task. An initial work and a general review on stability of model predictive control strategies has been addressed in [13]. In this case we focus on the adaptation of the proof to the current platooning control problem. In the ensuing, we prove that the proposed operational layer controller guarantees the stability of the closed loop system.

**Proposition 1.** *The closed loop longitudinal platooning control system is stable.*

*Proof.* The main idea of the proof is to use the value function  $J_{T_p}$  as the Lyapunov function and show that it monotonically decreases with time  $t$  [14], i.e. let  $\delta t > 0$  denote the infinitesimal time duration, one wishes to show

```

Data: Initial condition:  $[x_k^{\ell+}(0), v_k^{\ell+}(0)] \quad \forall k \in \mathcal{I}_{\text{CAV}}, \quad [x_k^{\ell-}(0), v_k^{\ell-}(0)] \quad \forall k \in \mathcal{I}_{\text{HDV}}$ 
Result: Control input:  $\alpha_k^{\ell+}(t)$ 
1 , Gap:  $s_k^{\ell+}(t)$  begin
2   Tactical layer: at detection time  $t = t_k^{\ell}$ 
3   begin
4     Find projections:  $\mathcal{P} = \{p(g_1^{\ell+}), \dots, p(g_n^{\ell+}), p(g_1^{\ell-}), \dots, p(g_m^{\ell-})\}$  according to (3)
5     Determine ordering:  $O^*$  as (4)
6     Determine equilibrium gap  $\mathbf{h} = \gamma(O^*)$  via (24)
7     Compute anticipation times  $T_{a,k}$  using (5), (7), (8)
8   end
9   Operational layer: each time step  $t_i$ 
10  for each  $t_i$  do
11    for  $k \in \mathcal{I}_{\text{CAV}}$  do
12      Measure:  $s_k(t_i), v_k(t_i), v_{k-1}(t_i), a_k(t_i)$ 
13      Initialization:  $\Lambda_k^{(0)}(t) = (\Lambda_{s,k}^{(0)}(t) \quad \Lambda_{v,k}^{(0)}(t))^T = (0 \ 0)^T, t \in T_{\Delta} = [t_i, t_i + T_p]$ 
14      iteration number  $i = 1$ , initial convergence error  $e^{(1)} = e_0$ , and stopping error threshold  $e_{\text{stop}}$ 
15      while  $e^{(i)} \geq e_{\text{stop}}$  do
16         $\alpha_k^{*(i)}(t) = \arg \min_u \mathcal{H}(v_k(t), v_{k-1}(t), \alpha_k(t), \Lambda^{(i-1)}) = -\Lambda_{v,k}^{(i-1)}(t)/2c_3T_e$ 
17        Project  $\alpha_k^{*(i)}(t)$  into  $\mathcal{U}$ 
18        Solve forward dynamics with initial condition  $\mathbf{x}_k(t_i) = (e_{s,k}(t_i), e_{v,k}(t_i), e_{a,k}(t_i))^T$ 
19         $\dot{\mathbf{x}}_k(t) = f_k(\mathbf{x}_k(t), \alpha_k^{*(i)}(t)) \Rightarrow \mathbf{x}_k^{(i)}(t), \quad t \in T_{\Delta}$ 
20        Solve backward co state dynamics with final condition  $\lambda_k(t_i + T_p) = 0$ 
21         $-\dot{\lambda}_k(t) = \frac{\partial \mathcal{H}(\mathbf{x}_k(t), \mathbf{x}_{k-1}(t), \alpha_k^{*(i)}, \lambda(t))}{\partial \mathbf{x}} \Rightarrow \lambda_k^{(i)}(t), \quad t \in T_{\Delta}$ 
22        Update the costate:  $\Lambda_k^{(i)}(t) = (1 - \beta)\Lambda_k^{(i-1)}(t) + \beta\lambda_k^{(i)}(t)$ 
23        Compute error:  $e^{(i)} = \|\Lambda_k^{(i)}(t) - \lambda^{(i)}(t)\|_2$ 
24        Augment iteration:  $i = i + 1$ 
25      end
26      Output: Select first sample from  $\alpha_k^{*(i)}(t) \Rightarrow \mathbf{u}^{*(i)}(t_i)$ 
27    end
28    Apply control  $\mathbf{u}^{*(i)}(t_i)$  to system (13)
29  end

```

**Algorithm 1:** Closed loop operation for the *tactical* and *operational* layer

$J_{T_p}(\mathbf{x}(t_0)) > J_{T_p}(\mathbf{x}(t_0 + \delta t))$  for  $\forall t_0$ . Note that:

$$\begin{aligned}
 J_{T_p}(\mathbf{x}(t_0)) &= \min_{\mathbf{u}(\cdot)} \int_{t_0}^{t_0+T_p} (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)) dt \\
 &= \min_{\mathbf{u}(\cdot)} \int_{t_0+\delta t}^{t_0+T_p} (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)) dt + \int_{t_0}^{t_0+\delta t} (\mathbf{x}^{*T}(t)\mathbf{Q}\mathbf{x}^*(t) + \mathbf{u}^{*T}(t)\mathbf{R}\mathbf{u}^*(t)) dt \\
 &= J_{T_p-\delta t}(\mathbf{x}(t_0 + \delta t)) + \int_{t_0}^{t_0+\delta t} (\mathbf{x}^{*T}(t)\mathbf{Q}\mathbf{x}^*(t) + \mathbf{u}^{*T}(t)\mathbf{R}\mathbf{u}^*(t)) dt
 \end{aligned} \tag{21}$$

where  $\mathbf{u}^*$  and  $\mathbf{x}^*$  denote the optimal control and state trajectories respectively. Taking the limit of the right-hand-side of the above equation when  $\lim \delta t \rightarrow 0$  yields:

$$J_{T_p}(\mathbf{x}(t_0)) = J_{T_p-\delta t}(\mathbf{x}(t_0 + \delta t)) + \underbrace{(\mathbf{x}^{*T}(t)\mathbf{Q}\mathbf{x}^*(t) + \mathbf{u}^{*T}(t)\mathbf{R}\mathbf{u}^*(t))}_{\text{running cost}} \delta t \quad (22)$$

Hence

$$\begin{aligned} J_{T_p}(\mathbf{x}(t_0)) - J_{T_p}(\mathbf{x}(t_0 + \delta t)) &= (J_{T_p-\delta t}(\mathbf{x}(t_0 + \delta t)) - J_{T_p}(\mathbf{x}(t_0 + \delta t))) + (\mathbf{x}^{*T}(t)\mathbf{Q}\mathbf{x}^*(t) + \mathbf{u}^{*T}(t)\mathbf{R}\mathbf{u}^*(t)) \delta t \\ &= \min_{\mathbf{u}(\cdot)} \int_{t_0+\delta t}^{t_0+T_p} (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)) dt - \min_{\mathbf{u}(\cdot)} \int_{t_0+\delta t}^{t_0+T_p+\delta t} (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)) dt \\ &\quad + (\mathbf{x}^{*T}(t)\mathbf{Q}\mathbf{x}^*(t) + \mathbf{u}^{*T}(t)\mathbf{R}\mathbf{u}^*(t)) \delta t \end{aligned} \quad (23)$$

The positive definite nature of  $\mathbf{Q}$  and  $\mathbf{R}$  guarantees the positiveness of the last term on the right hand of Eq. (23). For the first two terms, since we have the terminal constraint  $\mathbf{x}(t_0+T_p) = 0$ , no cost will incur after the terminal time  $t_0+T_p$ . The non-increasing nature of the value function as a function of the prediction horizon  $T_p$  is guaranteed. Therefore, the first two terms in the right hand side are non-negative. This qualifies  $J_{T_p}(\mathbf{x}(t))$  as a monotonically decreasing function of time and thus a *Lyapunov function*. This completes the proof.  $\square$

#### 4.5. Casting tactical decisions into the platooning problem

In order to achieve to align the maneuvers and control, the decisions of vehicle order in the final formation after merge and the time window for trucks that need to yield a larger gap for the merging vehicles from the tactical layer (See Section 3) need to be integrated into the lower level operations. The vehicle order determines the reference time gap  $h_k^r$  for truck  $k$  with respect to truck  $k-1$ . The changed reference depends on the vehicle type of the merging vehicle. Consider the case in which  $m$  vehicles are willing to merge into the  $\ell+$  lane. In this sense, the tactical strategy will find the final order of the combined vehicles according to the ordering process detailed in Section 3.1. The final order corresponds to a mixture of CAV within the platoon  $\mathcal{I}_{\text{CAV}}$  and new vehicles. In general, let us consider a reference headway function determining the equilibrium headway between consecutive vehicles given by a map  $\mathbf{h} = \gamma(\mathcal{O})$ ,  $\mathbf{h} = (h_1^r(t) \ h_2^r(t) \ \dots \ h_n^r(t))^T$  which can be detailed as

$$h_k^r(t) = \begin{cases} \tau^p & \text{if } \sigma_{k-1} \in \mathcal{I}_{\ell+} \\ \tau + \tau^p & \text{if } \sigma_{k-1} \in \mathcal{J}_{\ell-} \wedge \sigma_{k-1} \in \mathcal{I}_{\text{HDV}} \\ 2\tau^p & \text{if } \sigma_{k-1} \in \mathcal{J}_{\ell-} \wedge \sigma_{k-1} \in \mathcal{I}_{\text{CAV}} \end{cases} \quad \forall \quad t \in [T_k - T_{a,k}, T_k] \quad (24)$$

In order to open a sufficient gap at the moment of the merge the tactical layer decides the time should augment the gap between its predecessor and the current vehicle. This action maneuver should be coordinated in some specific time previous to the merging time  $T_k$  for a specific vehicle.

Given that at the moment of merge the desired gap  $s_k^r(T_k) = s_k^0 + h_k^r(T_k)v_k(T_k)$  should be achieved, this task is achieved by modifying the value  $h_k^r(t)$  so that Eq. (24) is held. The amount of time in order to perform the maneuver is  $T_{a,k}$  given by Eq. (8), so that  $s_k^r(T_k - T_{a,k}) = s_k^0 + h_k^r(T_k)v_k(T_k - T_{a,k})$ . This value is considered as an input within the solution of the optimal control problem Eq. (14).

In addition, a soft reference change is introduced given by a sigmoid function:

$$h_k^r(t) = \frac{\alpha}{1 + e^{-\beta(t-(T_y+T_a/2))}}, \forall t \in [T_k - T_{a,k}, T_k]$$

where the parameters  $\alpha, \beta$  are calibrated so that the rise time of the sigmoid function is approximately  $T_a$  and the change corresponds to the amount of change from the equilibrium  $T_{\sigma_x}^f - T_{\sigma_x}^0$ . The control policy Eq. (14) was applied over CAV vehicles only. The results lead to the dynamic time response for the time-shift reference as shown in Fig. 7a. As it can be seen the controller at low level accomplishes its objective by taking the time-shift to the desired value. In this case the setting time for the controller is around 25.5 s.

And finally, the maneuver considers the constraints in the new space of speeds  $\mathcal{X}_c = \mathcal{X} \cup \{v_k(t) \geq v^+ - \epsilon\}$ . This maneuver is detailed in Fig. 6 where the interaction between the *tactical* layer and the *operational* layer is depicted.

Note that both the tactical layer and the operational layer are re-evaluated at regular intervals and under special conditions the operational layer can request the re-evaluation of the tactical decisions at irregular intervals. In addition, the merging vehicles can use the full length of the acceleration lane as buffer zone to find acceptable gaps. These make it unlikely that the control framework leads to unfeasible solutions given the feasible initial condition.

## 5. Simulation study case

To illustrate the performance of hierarchical control framework, a multiple merging scenario has been implemented and tested in a in-house microscopic traffic flow simulator, Symuvia<sup>1</sup>. The scenario considers a platoon of 8 CAVs driving along a long single-lane freeway at the free-flow speed  $v=25$  m/s and moving toward an on-ramp with  $\tau = 1.0$  s. In the same time, two vehicles are coming from the on-ramp: they are detected over a loop sensor located far upstream the on-ramp section and their passage times and speeds are sent to the tactical layer. The merging times are predicted at  $T_m = 42.5$  s and 46.5 s respectively, while the merging positions has been fixed at  $X_m = 0$  m for the sake of readability.

The performance of the hierarchical framework is presented at multiple stages. First, we analyze the behavior of the operational layer taking into account the tactical decisions when both merging vehicles are CAVs. Second, we compare the traffic performance depending on whether merging vehicles are CAVs or HDVs.

### 5.1. Operational performance in fully CAV environment

The performance of the acceleration controller can be examined by executing the gap opening command from the tactical layer. In this experiment, the speed drop parameter  $\epsilon$  is set to 3m/s. Here, acceleration boundaries have been fixed as [5] in between  $(-1.5, 1.5)$  m/s<sup>2</sup>. The tactical layer generates the following decisions presented in Table 1 (full CAV scenario): vehicles  $i_3$  and  $i_5$  will starts their yielding maneuvers at times 17.1s and 19.6s respectively.

Fig. 7a shows the reference time shift and the simulated time shift dynamics. It can be shown that vehicles 3, 5 start to open gap (as reflected by the increase of time shift) from the yielding start time and finally settle down to the new equilibrium time shift (2.11s, 2.59s) at the end of the simulation. Here values are referred to the CAV leader time shift. This shows the proposed controller stabilizes the jump in reference commanded by the tactical layer.

Fig. 7b shows that the computed control input does not reach saturation meaning that the maneuver provides also comfort for the driver traveling in a CAV. For vehicles where the time-shift difference  $T_{\sigma_x}^f - T_{\sigma_x}^0$  is larger it is expected to have stronger decelerations/accelerations as it is the case for the vehicle  $i_5$  with a top input control around  $-0.96$  m/s<sup>2</sup> at 22.5s.

<sup>1</sup> All results for visualization and reproduction of the results are available at <https://github.com/research-licit/ISTTT2019>

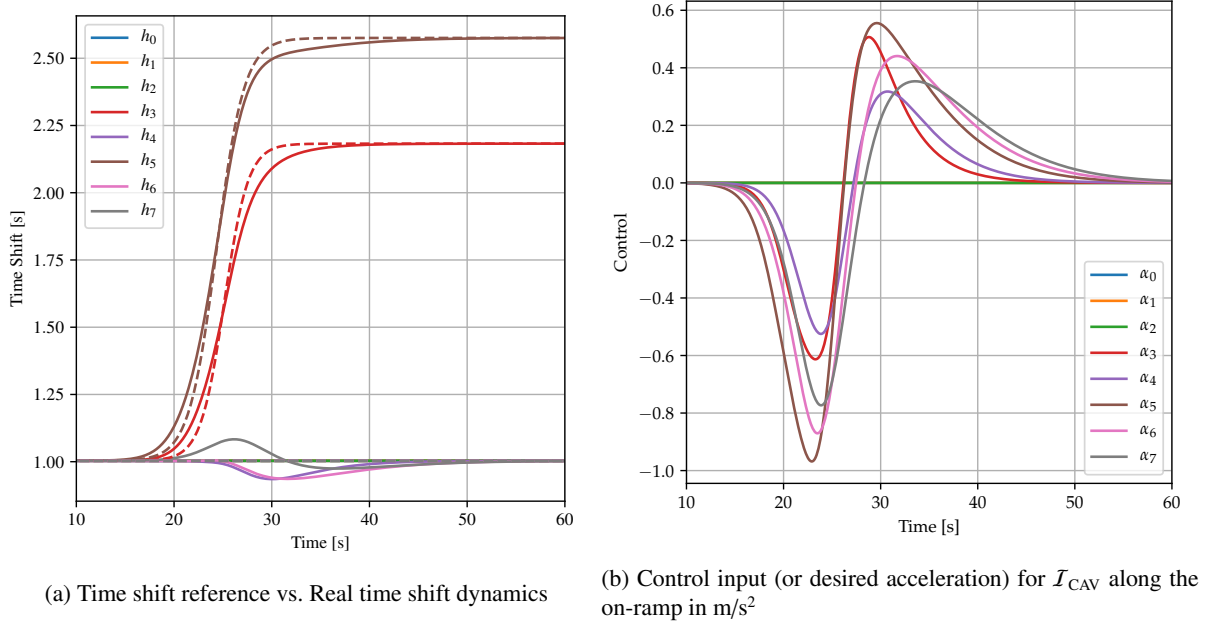


Fig. 7: Operational layer performance

### 5.2. Robustness of the operational layer

In reality there are always uncertainties or disturbances in the closed loop system. These can stem from the mismatch between the system dynamics model used by the MPC controller and the real system behavior. The uncertainties can be explicitly controlled via a robust control [6, 36]. Although the proposed operational layer controller is deterministic, but the predictive and feedback nature of the MPC scheme enables it to reject quite some disturbances. In this section, we illustrate the robust performance of the proposed strategy to model mismatch and delays.

In order to test the robustness of the operational strategy we conducted multiple tests. In particular, we consider two types of disturbance: one due to the imperfection implementation of the desired acceleration, and the other due to the state feedback delay. As a general test we considered the second vehicle in the formation yielding to open a specified space headway as seen in Fig. 8, and the right part of Fig. 9. The test has been performed for different values of engine time constant  $T_e$ . As it can be seen the dynamic response of the controller tends to be faster when the time constant increases. Although these dynamic variations do not have a strong impact on the performance of the strategy given that the equilibrium is achieved within the specified time, it is important to consider these variations as part of future control designs. The parameters in the model may differ with respect to the real ones, and it is important to achieve good performance in such cases. In Fig. 9 the operational layer has been tested in cases where the engine time constant  $T_e$  does not match the value known by the predictive controller. In particular the parameter has been perturbed 100ms from its nominal value. As it can be seen the dynamic response in the parameter mismatch case is close to the perfect match one. Finally we tested the effects of introducing feedback delays in the control loop. The nature of these delays can be understood from the communication network between vehicles. Fig. 10 illustrates the effects of introducing delays from 100ms up to 1400ms. As it can be seen the controller is robust to delay up to 600ms and the stability of the controller only degrades for delays over 600ms. It is important to highlight that the introduction of delays is a problem that has been studied already in the literature and its effect can be compensated up to certain level [35].



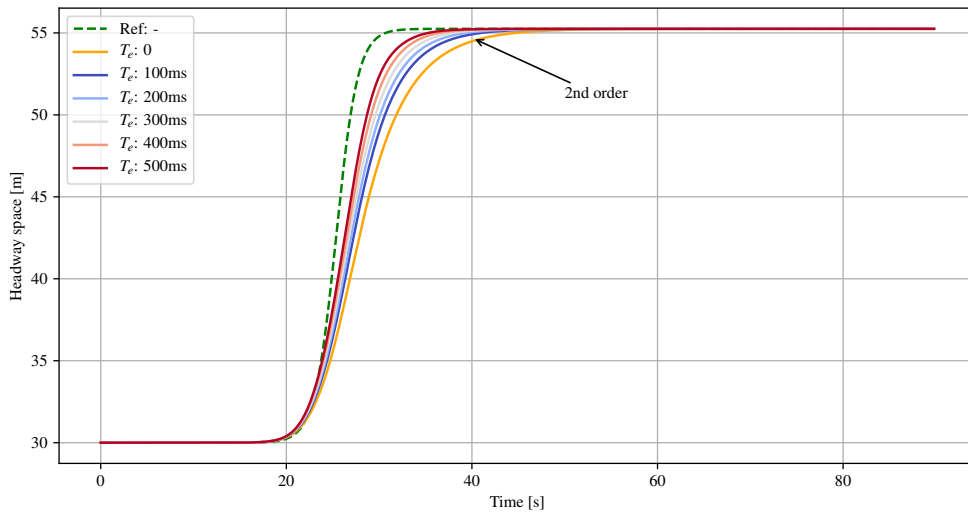


Fig. 8: Variation of dynamic response of the controller of under model parameter uncertainty

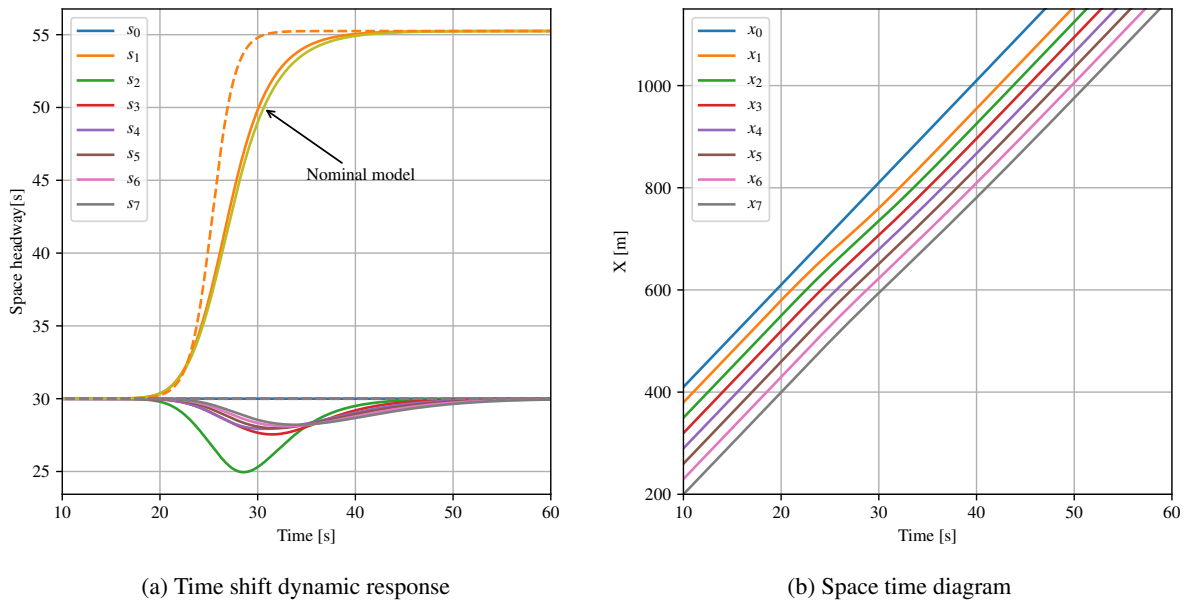


Fig. 9: Model mismatch performance

### 5.3. Comparison between mixed traffic and full CAV merges

Now two types of vehicles are considered: CAVs which have a time-shift  $\tau^p = 1.0$  s; and HDVs which have a time-shift  $\tau = 1.8$  s. For both types of vehicles, the maximum wave speed is  $w = 6.25$  m/s.

*Tactical layer results.* At the tactical layer, solution provides the yielding times  $T_y$  and corresponding time shift  $T_{\sigma_x}^f - T_{\sigma_x}^0$  as presented in table Table 1. In this case two vehicles are inserted approximately at 7.9 s and 11.9 s.

In the first situation vehicles are considered to be HDVs, so in this case vehicles will work as constraints for the system, creating *internal boundary conditions*. Yielding times are computed according to Eq. (7) according to the

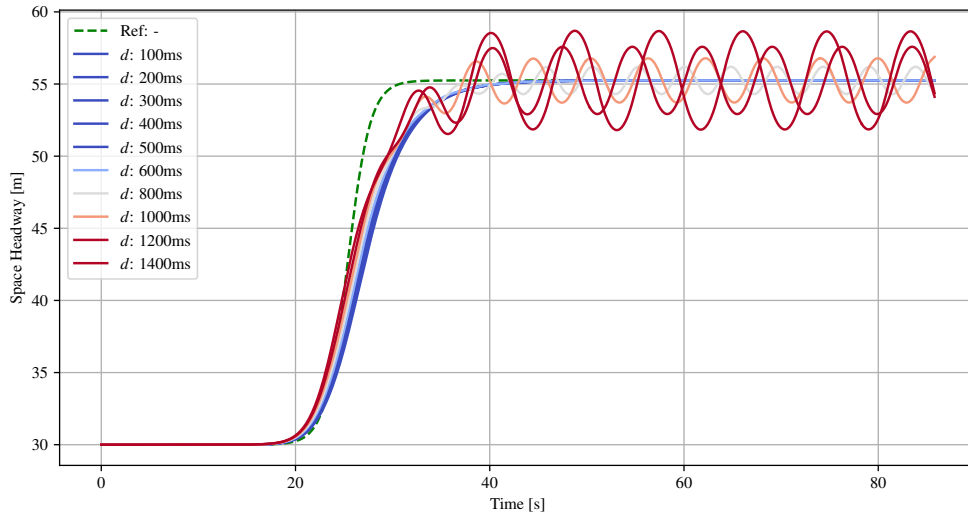


Fig. 10: Effect of the delayed measurements in the stability of the controller.

allocation rule established in Eq. (6). In a second scenario vehicles are inserted at the same point in time and space but they are CAVs, which are allowed to modify their equilibrium conditions as depicted in Fig. 4.

The analytical strategy at the tactical layer yields the decisions as: vehicles of the platoon ( $i_{k=3,5,7}$ ) have to open a gap, and the anticipation time for each one is around  $T_a \leq 13$  seconds, equivalent to the yielding start time of  $T_y = T_m - T_a$  s for the operational layer. The amount of time shift a vehicle should yield from the equilibrium condition is given by the difference  $T_{\sigma_x}^f - T_{\sigma_x}^0$  as pointed out in Fig. 4. In this case according to the policy allocation

Vehicle index $i_k$	Full CAV			Mixed scenario		
	$T_y[s]$	$T_a[s]$	$T_{\sigma_x}^f - T_{\sigma_x}^0[s]$	$T_y[s]$	$T_a[s]$	$T_{\sigma_x}^f - T_{\sigma_x}^0[s]$
$i_3$	37.1	9.16	1.0	37.3	8.83	0.91
$i_5$	39.6	9.18	0.0	36.4	13.77	1.18
$j_1$ (merging)	34.4	8.60	0.86	N/A	N/A	N/A
$i_7$	37.9	13.33	1.0	N/A	N/A	N/A
$j_2$ (merging)	38.0	11.99	0.81	N/A	N/A	N/A

Table 1: Vehicle index to be relaxed - Anticipation times

for the mixed scenario it can be seen that the conditions impose 3 internal boundary conditions where vehicles can be allocated upstream each internal boundary condition, in this case, 2 vehicles are allocated upstream the 2 first boundary conditions while the rest is allocated behind the 3rd internal boundary condition.

**Trajectories.** A graphical illustration of the trajectories is presented in Fig. 11. First, in Fig. 11a the mixed traffic scenario is observed. It can be seen that only vehicles travelling in the on ramp modify their equilibrium in order to receive vehicles traveling in the on ramp. Vehicles in the platoon anticipate and open two gaps to offer sufficient space for merging maneuvers. In Fig. 11b a fully CAV scenario is deployed. In this case, vehicles travelling along the on-ramp also anticipate and modify their trajectories in order to adapt to the optimal time-shift at the moment of merge.

**Performance indicators.** In order to measure the traffic performance of the strategy, we compare the outflow downstream the merge for three different situations. Here the outflow is measured downstream the merge, and is defined

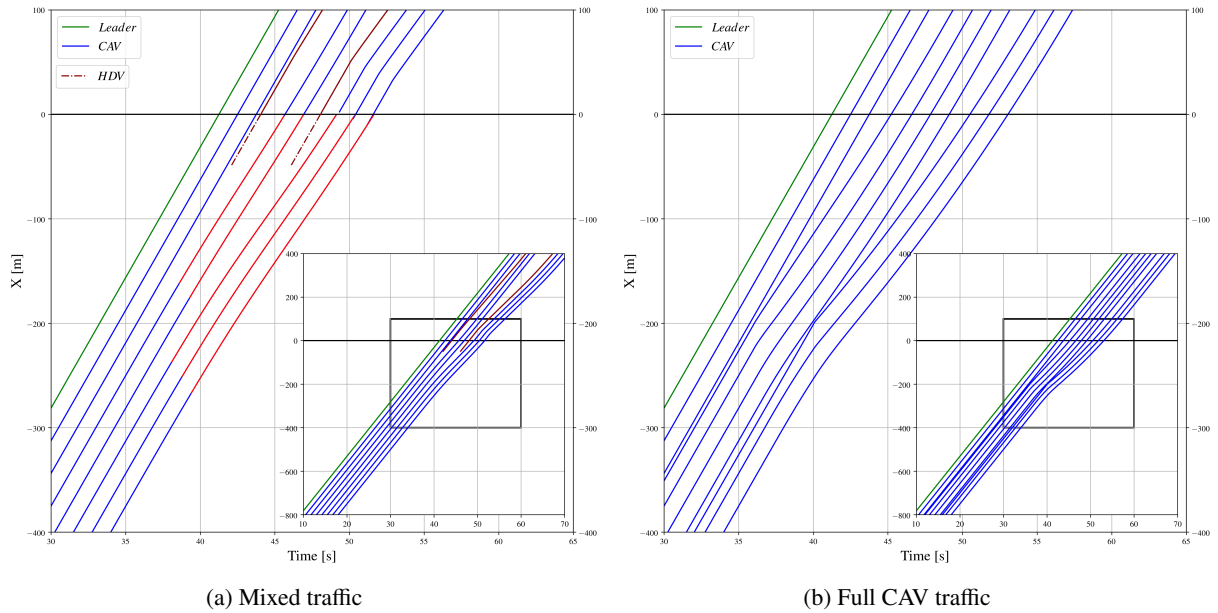


Fig. 11: Trajectories around the merging position for platooning vehicles (blue) and merging vehicles (brown)

as the ratio between the total number of vehicles downstream the merge (here  $8+2$ ) and the time interval between the first and the last vehicle the measurement position. The first situation with no anticipation: merging vehicles force the merging maneuvers and platooning vehicles are forced to react and have to decrease their speed downstream the merging point in order to reach a new equilibrium. The second situation considers mixed traffic with HDVs on the on-ramp. The third situation considers CAVs on the on-ramps. The results are summarized in Table 2, which show clear benefits of the control strategy on the maximum throughput of the merge, while preserving safe and comfortable merging conditions.

	No control	Mixed traffic	Full CAVs
Platoon outflow [veh/s]	0.71(–)	0.91 (+28%)	1.05 (+48%)

Table 2: Platoon outflow for three levels of connectivity

## 6. Conclusion

**Main findings.** Platooning strategies are expected to improve network capacities while minimizing fuel consumption. The first item requires a careful management of conflicts situations between platoons and surrounding conditions. The proposition of the paper contributes minimizing this drawback, by providing a rigorous and comprehensive framework for managing platoons near network discontinuities. It proposes a generic hierarchical approach to split platoons of trucks approaching an on-ramp. The hierarchical framework entails using an analytical car-following model to decide optimal tactical decisions and using a more detailed model to predict and control operational acceleration dynamics of trucks. The tactical layer generates the optimal vehicle indexes in the platoon to yield gaps for merging vehicles and time instants they should start the yielding process, given a speed drop that they accept compared to the equilibrium speed. It also outputs the desired state parameters for the interacting vehicles after merge, notably the desired time gaps of the yielding trucks. At the operational layer, CAV platoon uses the new desired time gaps of yielding trucks as the new reference and start to switch the reference as commanded by the tactical layer. It follows a model predictive control approach where the controller regulates vehicle accelerations to follow the desired time gaps,

under admissible gap, speed and acceleration constraints . The acceptable speed drop is formulated as an additional constraint in speed. The solution guarantees the possibility for the merging vehicles to join the platoon under safe and comfortable conditions, with a limited impact on mainline traffic and merge capacity.

In fact, the proposed framework is generic because: first, it can be extended to any type of connected and automated vehicle in presence of mixed surrounding vehicles; second, it addresses any conflicting situation between mandatory lane-changing maneuvers and platooning systems. Consequently, it can be deployed near on-/off-ramps, lane drops and weaving sections, where long platoon of trucks may become a problem for vehicles that must undertake mandatory lane-changing maneuvers. The solution can be adapted to variant length of platoon, merging vehicle speed, number of vehicle(s) to merge, to passenger car platoons. It can also address the cooperative merging of CAVs by treating the merging CAVs as part of a virtual platoon.

The results of the paper can shed some light on the benefits on splitting strategies. It anticipates and controls traffic surrounding merging maneuvers, which smooths traffic dynamics during the fuel consuming transient period, and which avoids unsafe over-reactive deceleration/acceleration maneuvers. Also, it optimizes outflows at network discontinuities, which determine the global network capacities.

*Observations & future research.* The proposed framework paves the way for further research. We have presented a fully deployable hierarchical control strategy with the main objective to treat the problem of active platoon maneuver near merges under both CAV and mixed traffic conditions. The strategy accounts full observability of the system, and a centralized approach is used to solve a bi-level control strategy under ideal communication and perfect feedback information assumption. Future research in this line includes the analysis on performance of the framework under perturbations due to the presence of noise affecting the sensors in vehicles or the network infrastructure. In general, parameter uncertainty within the system can also introduce alterations in the performance of the strategy. Here, computations such as merging time, location of merging vehicles and performance of the controller are subjects of further analysis regarding impact of uncertainties and the necessity of using robust control approaches instead of deterministic control. Subsequent research can extend the present work by considering the fuel consumption as part of the control strategy. The addition of fuel consumption and emission minimization objectives can be considered in both the tactical and the operational levels, and the hierarchical framework can be strengthened with more realistic car following models. Finally, the proposed control strategy remains local while vehicle fleet managers approach the platooning issues from a routing point of view at the network scale. Integration and coordination of local maneuver and global control will become a major challenge for upcoming years.

## Acknowledgements

The activity presented in the paper is part of the research project ENSEMBLE co-funded by the European Union under the Horizon 2020 (H2020) Research and Innovation Program (grant agreement No 769115).

## References

- [1] Alam, A.A., Gattami, A., Johansson, K.H., 2010. An experimental study on the fuel reduction potential of heavy duty vehicle platooning, in: 13th International IEEE Conference on Intelligent Transportation Systems, IEEE. pp. 306–311. [10.1109/ITSC.2010.5625054](#).
- [2] Amoozadeh, M., Deng, H., Chuah, C.N., Zhang, H.M., Ghosal, D., 2015. Platoon management with cooperative adaptive cruise control enabled by VANET. *Vehicular Communications* 2, 110–123. [10.1016/j.vehcom.2015.03.004](#).
- [3] Bang, S., Ahn, S., 2017. Platooning Strategy for Connected and Autonomous Vehicles: Transition from Light Traffic. *Transportation Research Record: Journal of the Transportation Research Board* 2623, 73–81. [10.3141/2623-08](#).
- [4] Bhoopalam, A.K., Agatz, N., Zuidwijk, R., 2018. Planning of truck platoons: A literature review and directions for future research. *Transportation Research Part B: Methodological* 107, 212–228. [10.1016/j.trb.2017.10.016](#).
- [5] Bokare, P.S., Maurya, A.K., 2017. Acceleration-Deceleration Behaviour of Various Vehicle Types. *Transportation Research Procedia* 25, 4737–4753. [10.1016/j.trpro.2017.05.486](#), [arXiv:arXiv:1011.1669v3](#).
- [6] Chen, N., Wang, M., Alkim, T., Van Arem, B., 2018. A Robust Longitudinal Control Strategy of Platoons under Model Uncertainties and Time Delays. *Journal of Advanced Transportation* 2018, 15–17. [10.1155/2018/9852721](#).
- [7] Duret, A., Wang, M., Leclercq, L., 2018. Truck platooning strategy near merge : Heuristic-based solution and optimality conditions, in: *Transportation Research Board 97th Annual Meeting*, Transportation Research Board, Washington.
- [8] Godbole, D., Eskafi, F., Singh, E., Varaiya, P., 1995. Design of entry and exit maneuvers for IVHS, in: *Proceedings of 1995 American Control Conference - ACC'95*, American Autom Control Council. pp. 3576–3580. [10.1109/ACC.1995.533803](#).

- [9] Halle, S., Laumonier, J., Chaib-Draa, B., 2004. A decentralized approach to collaborative driving coordination, in: Proceedings. The 7th International IEEE Conference on Intelligent Transportation Systems (IEEE Cat. No.04TH8749), IEEE. pp. 453–458. [10.1109/ITSC.2004.1398942](#).
- [10] Hsu, A., Eskafi, F., Sachs, S., Varaiya, P., 1993. Protocol design for an automated highway system. *Discrete Event Dynamic Systems* 2, 183–206. [10.1007/BF01797158](#).
- [11] Jin, L., Čičić, M., Amin, S., Johansson, K.H., 2018. Modeling the Impact of Vehicle Platooning on Highway Congestion, in: Proceedings of the 21st International Conference on Hybrid Systems: Computation and Control (part of CPS Week) - HSCC '18, ACM Press, New York, New York, USA. pp. 237–246. [10.1145/3178126.3178146](#).
- [12] Mahmassani, H.S., 2016. 50th Anniversary Invited Article Autonomous Vehicles and Connected Vehicle Systems: Flow and Operations Considerations. *Transportation Science* 50, 1140–1162. [10.1287/trsc.2016.0712](#).
- [13] Mayne, D., Rawlings, J., Rao, C., Scokaert, P., 2000. Constrained model predictive control: Stability and optimality. *Automatica* 36, 789–814. [10.1016/S0005-1098\(99\)00214-9](#).
- [14] Mayne, D.Q., 2014. Model predictive control: Recent developments and future promise. *Automatica* 50, 2967–2986. [10.1016/j.automatica.2014.10.128](#).
- [15] Milanés, V., Godoy, J., Villagra, J., Perez, J., 2011. Automated On-Ramp Merging System for Congested Traffic Situations. *IEEE Transactions on Intelligent Transportation Systems* 12, 500–508. [10.1109/TITS.2010.2096812](#).
- [16] Milanés, V., Shladover, S.E., Spring, J., Nowakowski, C., Kawazoe, H., Nakamura, M., 2014. Cooperative Adaptive Cruise Control in Real Traffic Situations. *IEEE Transactions on Intelligent Transportation Systems* 15, 296–305. [10.1109/TITS.2013.2278494](#).
- [17] Naus, G.J., Vugts, R.P., Ploeg, J., Van De Molengraft, M.J., Steinbuch, M., 2010. String-stable CACC design and experimental validation: A frequency-domain approach. *IEEE Transactions on Vehicular Technology* 59, 4268–4279. [10.1109/TVT.2010.2076320](#).
- [18] Nowakowski, C., Thompson, D., Shladover, S.E., Kailas, A., Lu, X.Y., 2016. Operational Concepts for Truck Maneuvers with Cooperative Adaptive Cruise Control. *Transportation Research Record: Journal of the Transportation Research Board* 2559, 57–64. [10.3141/2559-07](#).
- [19] Ntousakis, I.A., Nikolos, I.K., Papageorgiou, M., 2016. Optimal vehicle trajectory planning in the context of cooperative merging on highways. *Transportation Research Part C: Emerging Technologies* 71, 464–488. [10.1016/j.trc.2016.08.007](#).
- [20] Ploeg, J., Semsar-Kazerooni, E., Morales Medina, A.I., de Jongh, J.F., van de Sluis, J., Voronov, A., Englund, C., Bril, R.J., Salunkhe, H., Arrue, A., Ruano, A., Garcia-Sol, L., van Nunen, E., van de Wouw, N., 2017. Cooperative Automated Maneuvering at the 2016 Grand Cooperative Driving Challenge. *IEEE Transactions on Intelligent Transportation Systems*, 1–14. [10.1109/TITS.2017.2765669](#).
- [21] Ploeg, J., van de Wouw, N., Nijmeijer, H., .  $\mathcal{L}_p$  string stability of cascaded systems: Application to vehicle platooning. *IEEE Transactions on Control Systems Technology*, 786–793. [10.1109/TCST.2013.2258346](#).
- [22] Rajamani, R., Zhu, C., 2002. Semi-autonomous adaptive cruise control systems. *IEEE Transactions on Vehicular Technology* 51, 1186–1192. [10.1109/TVT.2002.800617](#).
- [23] Rios-Torres, J., Malikopoulos, A.A., 2017a. A Survey on the Coordination of Connected and Automated Vehicles at Intersections and Merging at Highway On-Ramps. *IEEE Transactions on Intelligent Transportation Systems* 18, 1066–1077. [10.1109/TITS.2016.2600504](#).
- [24] Rios-Torres, J., Malikopoulos, A.A., 2017b. Automated and Cooperative Vehicle Merging at Highway On-Ramps. *IEEE Transactions on Intelligent Transportation Systems* 18, 780–789. [10.1109/TITS.2016.2587582](#).
- [25] Roncoli, C., Papamichail, I., Papageorgiou, M., 2016. Hierarchical model predictive control for multi-lane motorways in presence of Vehicle Automation and Communication Systems. *Transportation Research Part C: Emerging Technologies* 62, 117–132. [10.1016/j.trc.2015.11.008](#).
- [26] Saeednia, M., Menendez, M., 2016. Analysis of Strategies for Truck Platooning. *Transportation Research Record: Journal of the Transportation Research Board* 2547, 41–48. [10.3141/2547-07](#).
- [27] Shladover, S.E., 2005. Automated vehicles for highway operations (automated highway systems). *Proceedings of the Institution of Mechanical Engineers. Part I: Journal of Systems and Control Engineering* 219, 53–75. [10.1243/095440705X9407](#).
- [28] Shladover, S.E., Nowakowski, C., Lu, X.Y., Ferlis, R., 2015. Cooperative Adaptive Cruise Control. *Transportation Research Record: Journal of the Transportation Research Board* 2489, 145–152. [10.3141/2489-17](#).
- [29] Tsugawa, S., Jeschke, S., Shladover, S.E., 2016. A Review of Truck Platooning Projects for Energy Savings. *IEEE Transactions on Intelligent Vehicles* 1, 68–77. [10.1109/TIV.2016.2577499](#).
- [30] Tuchner, A., Haddad, J., 2017. Vehicle platoon formation using interpolating control: A laboratory experimental analysis. *Transportation Research Part C: Emerging Technologies* 84, 21–47. [10.1016/j.trc.2017.06.019](#).
- [31] Varaiya, P., 1993. Smart Cars on Smart Roads: Problems of Control. *IEEE Transactions on Automatic Control* 38, 195–207. [10.1109/9.250509](#).
- [32] Varaiya, P., Shladover, S.E., 1991. Sketch of an IVHS systems architecture, in: *Vehicle Navigation and Information Systems Conference*, 1991, IEEE. pp. 909–922. [10.1109/VNIS.1991.205835](#).
- [33] Wang, M., 2018. Infrastructure assisted adaptive driving to stabilise heterogeneous vehicle strings. *Transportation Research Part C: Emerging Technologies* 91, 276–295. [10.1016/j.trc.2018.04.010](#).
- [34] Wang, M., Hoogendoorn, S.P., Daamen, W., van Arem, B., Happee, R., 2015. Game theoretic approach for predictive lane-changing and car-following control. *Transportation Research Part C: Emerging Technologies* 58, 73–92. [10.1016/j.trc.2015.07.009](#).
- [35] Wang, M., Hoogendoorn, S.P., Daamen, W., van Arem, B., Shyrokau, B., Happee, R., 2018. Delay-compensating strategy to enhance string stability of adaptive cruise controlled vehicles. *Transportmetrica B* 6, 211–229. [10.1080/21680566.2016.1266973](#).
- [36] Zhou, K., Doyle, J.C., 1998. *Essentials of robust control*. Prentice Hall.