



**HAL**  
open science

## Identification of Electric Multipolar Sources in Presence of Different Conducting Layers

Olivier Pinaud, Laure-Line Rouve, Olivier Chadebec, Cédric Goeau, Arnaud Guibert

► **To cite this version:**

Olivier Pinaud, Laure-Line Rouve, Olivier Chadebec, Cédric Goeau, Arnaud Guibert. Identification of Electric Multipolar Sources in Presence of Different Conducting Layers. Compumag 2019:22nd International Conference on the Computation of Electromagnetic Fields, Jul 2019, Paris, France. hal-02280315

**HAL Id: hal-02280315**

**<https://hal.science/hal-02280315>**

Submitted on 10 Nov 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Identification of Electric Multipolar Sources in Presence of Different Conducting Layers

O. Pinaud<sup>1</sup>, LL. Rouve<sup>1</sup>, O. Chadebec<sup>1</sup>, C. Goeau<sup>2</sup>, A. Guibert<sup>2</sup>

<sup>1</sup>Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab, F-38000 Grenoble, France

<sup>2</sup>DGA Techniques Navales, F-29200 Brest, France

This paper deals with the multipole expansion identification of complex electric sources generating a static current flowing in a conductive medium bounded by layers with different conductivities. The expansion series in homogeneous medium is enriched by additional terms resulting from the generalization of the image method applied to multipoles. From close electric measurements, the solving of an inverse problem allows determining the multipolar sources, whatever the conductive environment. The efficiency of the approach is demonstrated through a numerical test.

*Index Terms*— Electric Multipole Expansion Identification, Method of Images, Inverse Problems.

## I. INTRODUCTION

To prevent a ship hull from being corroded, currents are usually sent in the conductive sea water through active anodes, placing hull damaged areas in their corrosion immunity domain. This method called impressed current cathodic protection (ICCP) creates complex currents lines flowing from anodes to other parts of the ship (propellers, corroded zones).

This current flow in seawater induces a measurable electrical field that may endanger a military ship, if detected by mine sensor. The electrical signature of a ship is very difficult to estimate by forward modelling because of the lack of knowledge of important parameters such as the state of the corroded area. Thus, it is fundamental to determine from near measurements an equivalent model of electric field sources that allows then the ship far field signature evaluation, independently of its environment.

To achieve this, the multipole expansion identification is a very efficient mathematical method for field extrapolation (see [1] for magnetic field application). Multipole expansion, here a spherical harmonic (SH) decomposition, is a series of weighted functions which are solutions of the Laplace equation. The series can be truncated to represent the far field with a chosen accuracy [2]. From a limited set of field measurements, the solving of an inverse linear problem enables the determination of the weighting coefficients.

The use of this method in the context of electric multipole identification is more complex than for the magnetic case. Magnetic field produced by intrinsic ship sources is not modified by the non-magnetic surrounding free space (air, seawater). On the contrary, the electric field due to intrinsic ship electric sources is strongly influenced by the air/water interface or by the sea floor that present very different conductivities.

This paper proposes to extend the multipole expansion identification method to electric fields flowing in a domain composed of layers with different conductivities. The classical expansion series is enriched by additional terms built through the generalization of the method of images to multipoles. These new terms are introduced in a linear inverse problem in order to

determine the proper source coefficients. The efficiency of the approach and its accuracy are validated by a numerical test case.

## II. SPHERICAL HARMONIC EXPANSION AND IMAGES METHOD

In a homogeneous linear isotropic conductive medium  $\Omega_1$  (conductivity  $\sigma_1$ ), the static electric field  $\mathbf{E}_s$  derives from a potential  $V_s$  that verifies the Laplace equation. In spherical coordinates (center  $O_1$ ) and at a point  $P(r_1, \theta_1, \varphi_1)$  outside the sources, a solution for  $V_s$  is the spherical harmonic expansion:

$$V_s(r_1, \theta_1, \varphi_1) = \frac{1}{4\pi\sigma_1} \sum_{n=0}^{\infty} \sum_{m=-n}^n a_n^m \cdot \frac{Y_n^m(\theta_1, \varphi_1)}{r_1^{n+1}} \quad (1)$$

where  $a_n^m$  are the SH weighting coefficients associated to the electrical sources (which are assumed to be independent of the environment) and  $Y_n^m$  are the Legendre functions. Equation (1) corresponds to an electric field flowing in the free space  $\Omega_1$ .

We introduce now a second homogeneous linear isotropic conductive medium  $\Omega_2$  (conductivity  $\sigma_2$ ) separated from  $\Omega_1$  by an interface. The electric sources are still located in  $\Omega_1$ . The total potential  $V_{tot}$  in  $\Omega_1$  is the sum of the contribution of (1)  $V_s$  and of an image potential  $V_i$ .

$$V_{tot} = V_s + V_i \quad (2)$$

$$V_i(r_2, \theta_2, \varphi_2) = \frac{-A}{4\pi\sigma_1} \sum_{n=0}^{\infty} \sum_{m=-n}^n (-1)^{n+|m|} \cdot a_n^m \cdot \frac{Y_n^m(\theta_2, \varphi_2)}{r_2^{n+1}} \quad (3)$$

The factor “ $A$ ” only depends on conductivities of both media:

$$A = (\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2) \quad (4)$$

A full paper will develop the demonstration for obtaining (3) and (4).

Finally, potential  $V_s$  is linked to the spherical reference centered on  $O_1$  while  $V_i$  is associated to a new spherical reference, whose center  $O_2$  is the symmetrical of  $O_1$  with respect to the interface, and where P coordinates are  $(r_2, \theta_2, \varphi_2)$ . Let us notice that the  $V_{tot}$  expression is only valid for a point inside  $\Omega_1$ . Consequently,  $V_i$  can be seen as the SH expansion in  $\Omega_1$  of a multipolar source located in  $\Omega_2$ , image of the one located in  $\Omega_1$ , named the “sources” and characterized by SH coefficients  $a_n^m$ .

### III. MULTIPOLE IDENTIFICATION

Due to the decreasing law of spherical functions, the truncation of the harmonic expansion at a maximum order  $n = N_{\max}$  gives a good enough approximation.

Let us assume that  $K$  tri-axial sensors are located in  $\Omega_1$  (the seawater). By applying the gradient operator to (1) and (2), we get the forward linear mathematical function linking SH coefficients to the measured electric field. Written in a matrix format, this gives:

$$\mathbf{M} \cdot \mathbf{X} = \mathbf{E}_{\text{mes}} \quad (5)$$

where  $\mathbf{M} \in \mathbb{R}^{3K \times (N_{\max}^2 + 2N_{\max})}$ ,  $\mathbf{X} \in \mathbb{R}^{N_{\max}^2 + 2N_{\max}}$ ,  $\mathbf{E}_{\text{mes}} \in \mathbb{R}^{3K}$ .  $\mathbf{X}$  represents the vector of unknowns  $a_n^m$  and  $\mathbf{E}_{\text{mes}}$  is associated to measurements.

To properly take into account the interface effect on the electric field distribution in  $\Omega_1$ , the matrix  $\mathbf{M}$  is simply iteratively built, for example with only one interface :

$$\mathbf{M} = \mathbf{M}_s + \mathbf{M}_i \cdot \mathbf{W}_{si} \quad (6)$$

where  $\mathbf{M}_s$  and  $\mathbf{M}_i$  are both harmonic matrices, respectively referenced to “sources” centre  $O_1$  and “images” centre  $O_2$ , as introduced by (2). Matrix  $\mathbf{W}_{si}$  is a diagonal matrix linking images to sources SH coefficients:

$$W_{si} = -A \cdot \begin{bmatrix} \ddots & & & \\ & (-1)^{n+|m|} & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \quad (7)$$

Finally, solving (5) using a simple singular matrix decomposition technique leads to the SH coefficients, allowing then a post-processing of the field on other points and even with other environmental conditions (conductivities, interfaces, etc.).

### IV. NUMERICAL EXAMPLE

#### A. Forward modelling

We consider a cylinder immersed in seawater at a given depth from the surface (Fig.1). Two opposite current densities are imposed at its both extremities generating a current flowing in the half part of the problem and acting as a “long” electric dipole (in fact it is a more complex source firstly because the interface distorts the current lines and secondly because at this close distance of measurements, higher harmonic terms cannot be neglected). A classical Boundary Element Method (BEM) coupled with the method of image in order to take into account the air/water interface [3] models the problem (Fig.2). Numerical measurements are computed on a plane located below the cylinder as an input of the inverse problem (measurements plane, Fig. 3) allowing us to generate numerical measurements.

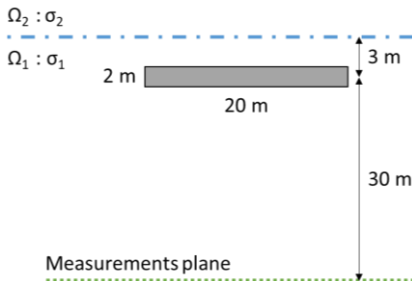


Fig. 1. Representative schematic of the cylinder in its environment

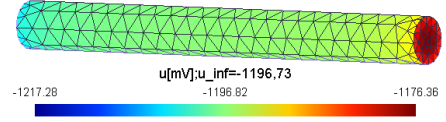


Fig. 2. Meshed cylinder with electric potential solved by BEM.

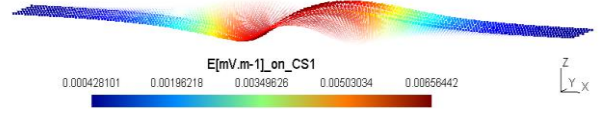


Fig. 3. Electric field computed on the plane (measurement plane).

#### B. SH identification and field a posteriori extrapolation

The inverse problem is solved from the virtual electric field measurements and SH coefficients are obtained. These coefficients are the same as the one obtained with the same source but with no interface (in free water): this demonstrates that the developed method of images leads to the good source. The field is then extrapolated on other points at least as far as those used for identification. Furthermore, it is even possible to extrapolate the field with other environment conditions. Fig. 4 compares in free water, the electric field components computed by BEM, with the ones extrapolated from the previously identified SH coefficients (with one interface) at order  $N_{\max} = 3$ . These comparison points are located along a line, parallel to the cylinder and placed 40 m under the cylinder (verification line).

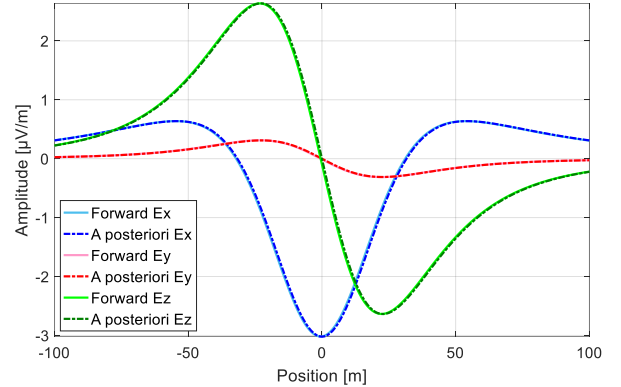


Fig. 4. Forward computation and a posteriori harmonic extrapolation in free water of the electric field components on a verification line

### V. CONCLUSION

This short paper briefly presents the images method applied to spherical harmonic expansion in the context of electric sources identification in a layered conductive environment. The numerical example clearly shows the efficiency of the method. Mathematical development and some more complex numerical cases will be presented and discussed in a full paper.

### REFERENCES

- [1] L-L. Rouve, L. Schmerber, O. Chadebec, A. Foggia, A., “Optimal magnetic sensor location for spherical harmonic identification applied to radiated electrical devices”, IEEE Trans. on Magn., 42, 4, 2006.
- [2] J. P. Wikswo Jr. and K. R. Swinney, “Scalar multipole expansions and their dipole equivalents,” J. Appl. Phys., vol. 56, pp. 3039–3049, 1984.
- [3] N. G. Zamani, J. M. Chuang, and J. F. Porter, “BEM simulation of cathodic protection systems employed in infinite electrolytes,” Int. J. for Numer. Methods Eng., vol. 24, pp. 605–620, 1987.