An Inverse Integral Formulation for Steel Shell Magnetization Identification

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An algorithm for the reconstruction of unknown magnetization in an iron sheet is proposed. The magnetization distribution is reconstructed from magnetic measurements made on sensors placed in the surrounded air region. The problem is solved thanks to the inversion of an integral formulation, based on the face interpolation of the flux density in the sheet and leading to a linear matrix system. The final system is solved with a balanced singular value decomposition in order to stabilize the solution. The efficiency of the method is demonstrated with both numerical and experimental test cases.

Index Terms— **magnetization identification, magnetostatic inverse problem, volume integral method, thin magnetic shell.**

I. Introduction

A magnetic ferromagnetic sheet has two kinds of magnetization. The induced one is the reversible magnetic reaction of the material to an inductor field (i.e. the earth magnetic field) and can easily be computed. The case of the permanent one, which depends on the magnetic history of the sheet, is more problematic. Its determination needs the use of external magnetic measurement made on sensors in the surrounding air region and the solving of an inverse problem. Different algorithms have already been proposed in the literature for volume region [1]-[2]. In this paper, we focus on a new method, dedicated to the sheet geometry and more robust and accurate than our previous works presented few years ago [3]. It can be applied to the evaluation of magnetic anomaly created by navy ships or submarines in order to reduce the risk of detection or destruction by magnetic mines.

II. Forward integral formulation

Let us consider a ferromagnetic thin region Ω with a reversible linear reluctivity ν placed in a low level inductor magnetic field $\boldsymbol{H_0}$(the earth magnetic field for instance). We consider that the magnetic material has a permanent magnetic state which can be represented by a coercive field $\boldsymbol{H_c}$ such as:

$$\boldsymbol{H} = \nu\boldsymbol{B} - \boldsymbol{H_C} \quad (1)$$

where $\boldsymbol{H}$ is the total magnetic field and $\boldsymbol{B}$ is the induction. The total magnetic field is the sum of the inductor field and the field created by the total magnetization of the shell. The problem is governed by the magnetostatic integral equation with $\boldsymbol{B}$ as a state variable [4]:

$$\nu\boldsymbol{B} + \boldsymbol{\nabla}\int_{\Omega}(\nu_0 - \nu)\boldsymbol{B}.\nabla \mathrm{G}\,\mathrm{d}\Omega = \boldsymbol{H_0} + \boldsymbol{H_C} \quad (2)$$

where G is the standard 3D Green's function (i.e. the inverse of the distance between the point where the induction is expressed and the integration point). For magnetic sheets, the approximation of thin element simplifies a volume problem to a surface one by considering that the magnetic induction in the active region is uniform according to the thickness and is tangential to the sheet. In [4], it has been shown that it is appropriate to interpolate $\boldsymbol{B}$ with Withney 2-form shape functions (also known as face shape functions) and expressed by a linear combination of magnetic flux Φ_{B} flowing through the equivalent faces of the meshed surface Ω_s (i.e the edge of the mesh elements in our case). In order to ensure the free-divergence of the flux, the resolution is performed in the basis of independent fluxes Φ_{BI} [5]. Applying a Galerkin method, a linear system is obtained:

$$\mathrm{M}\Phi_{\mathrm{BI}} = \mathrm{S}_{\mathrm{H_0}} + \mathrm{S}_{\mathrm{H}_C} \quad (3)$$

where, $\mathrm{M} \in \mathbb{R}^{\mathrm{N_{BI}}\times\mathrm{N_{BI}}}, \Phi_{\mathrm{BI}} \in \mathbb{R}^{\mathrm{N_{BI}}\times 1}, \mathrm{S}_{\mathrm{H_0}} \in \mathbb{R}^{\mathrm{N_{BI}}\times 1}$ and $\mathrm{S}_{\mathrm{H}_C} \in \mathbb{R}^{\mathrm{N_{BI}}\times 1}$, $\mathrm{N_{BI}}$ being the number of independent fluxes. The expression for each matrix can be found in [4].

In a classical forward problem, coercive and inductor fields are known as well as the reversible reluctivity of the material. However, in the context of an inverse problem, the point of view is different. The inductor field as well as the reversible reaction to it are known but the magnetic state in a null inductor field is unknown. In other words $\boldsymbol{H_c}$ has to be determined for each element of the mesh. This is why magnetic external measurements have to be added in order to provide additional information.

III. Inverse problem formulation

An equation linking the flux distribution in the shell to the induction measured on external magnetic sensors has to be added. The magnetic field in air is expressed with the following integral expression [5]:

$$\boldsymbol{B_{mes}} = \nabla\int_{\Omega_S}(1 - \nu/\nu_0)\nabla_S\boldsymbol{B}.\mathrm{G}\,d\Omega_s - \boldsymbol{B_0} \quad (4)$$

where $\boldsymbol{B_{mes}}$ is the induction measured on a magnetic sensor located in the vicinity of the ferromagnetic body. Equation (4) can also be discretized considering the independent fluxes flowing in the shell as unknowns and leading to the following matrix system:

$$\mathrm{B_{mes}} = \mathrm{A}\,\Phi_{\mathrm{BI}} \quad (5)$$

where $\mathrm{B_{mes}} \in \mathbb{R}^{\mathrm{N_{Bmes}}\times 1}$ and $\mathrm{A} \in \mathbb{R}^{\mathrm{N_{Bmes}}\times\mathrm{N_{BI}}}$ and $\mathrm{N_{Bmes}}$ being the number of measurements. Let us remember that in context of an inverse problem, $\mathbf{H_c}$ is an unknown of the problem and has to be identified. To determine it numerically, an interpolated function space has to be selected. A good choice can be also the face shape functions. Thus, the last term in (3) can be replaced by a matrix N product keeping unknowns Φ_{Hc} associated to $\boldsymbol{H_c}$ explicit, which leads to a new matrix system:

$$M\Phi_{BI} - N\,\Phi_{Hc} = S_{H0} \quad (6)$$

By combining (5) and (6) after some algebra, the final matrix system with Φ_{Hc} as unknown is obtained:

$$-A\,M^{-1}N\Phi_{Hc} = B_{mes} - A\,M^{-1}S_{H0} \quad (7)$$

To solve (7), a classical balanced singular values decomposition (SVD) is used to improve the condition number of the final matrix system. Let us notice that the solution of the problem is not unique and the use of the SVD leads to the solution with the minimal norm. However, we have to notice that to minimize $\boldsymbol{H_c}$ fluxes flowing through the shell does not make sense physically. It is more consistent to favour a solution which ensures the flux regularity. This is why it is preferable to replace the problem unknowns by a fictive scalar distribution which is the surface divergence of $\boldsymbol{H_c}$. This resolution leads to a regular and physical distribution.

IV. RESULTS

A. Numerical validation

In order to validate the algorithm, a test configuration is proposed. A square ferromagnetic plate with a linear anhysteretic reluctivity ν_a and a null coercive field is placed in a homogenous inductor field. Its magnetic state $(\boldsymbol{H}, \boldsymbol{B})$ is computed by solving (3). Then, for each element, equation (1) is solved locally, considering now the reversible reluctivity ν in order to compute the distribution $\boldsymbol{H_c}$ in each element of the plate. In a second step, (3) is solved again with a new inductor field in order to get a complex magnetic state mixing permanent and reversible magnetizations. The numerical measurements are generated with (4) on four tri-axis sensors. The inverse problem is then solved and the identified coercive field is compared to the initial one (Fig.1). Both distributions are coherent and the extrapolated signature very accurate (Table 1).

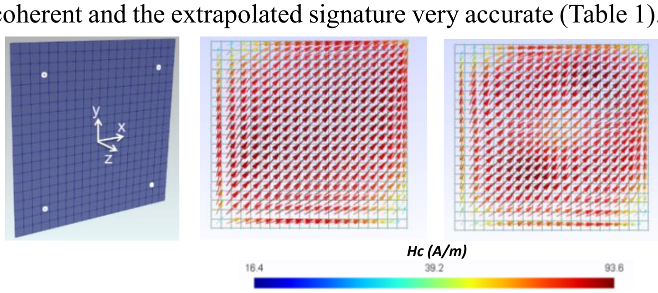

Fig 1: Iron sheet mesh and inversion test configuration (dimensions: 1m², number of elements: 440, number of unknowns: 760, thickness: 2 mm, 4 tri-axis sensors at 5 cm from the sheet). Left: mesh and sensors location. Middle: reference ***Hc****. Right: Identified* ***Hc.*** *The same range is provided for both* ***Hc.***

Induction components	**Bx**	**By**	**Bz**
Mean error %	0,49	1,35	0,54
Max error %	2	2,03	1,42

Table 1 : Error computed between the reference and the identified state on a computation line located at 0.5 meter from the sheet.

B. Test of the algorithm with experimental data

An experimental set-up (Fig. 2.) has been developed. It is based on a hollow cylinder which is filled by oil and can be pressurized thanks to a pump. The algorithm can be used in order to study magneto-elastic effects i.e. the variation of the permanent magnetization versus the combined effect of a low magnetic inductor field and a high-pressure acting on the shell [6]. This study would be relevant to predict magnetic anomaly created by submerged submarines for instance. Eight tri-axes fluxgate magnetic sensors are placed around the device for the external magnetic field measurement.

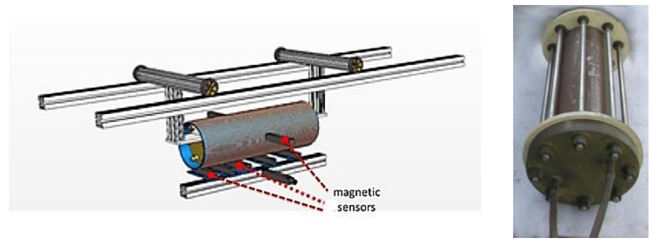

Fig 2: Plans of the experimental set up (left), pressurized cylinder (right).

The cylinder is initially demagnetized. Then it is placed in an axial uniform inductor magnetic field of 40 μT and an applied internal pressure of 10 MPa. After such a process, it gets a permanent magnetization which can be determined using our algorithm (Fig.3.). In order to check if the state of the shell has been correctly identified, the magnetic field is then computed on a line and compared with others measurements. Both curves match, the error being less than 1%. It demonstrates the efficiency of the method.

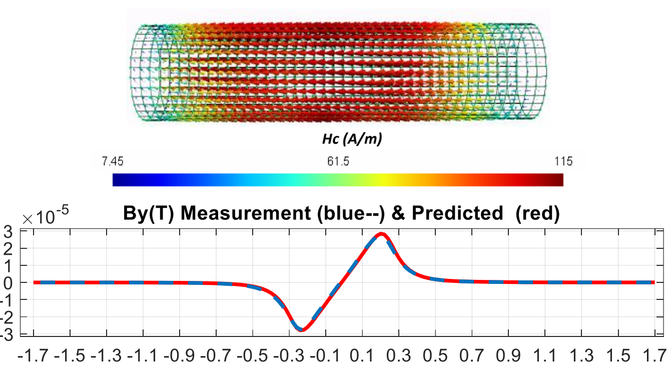

Fig 3: Distribution of magnetic coercive field on the cylinder (top), vertical magnetic induction on a line under the device (blue -- predicted / red measured) (bottom).

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