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An accurate and fast post-processing method for computing the magnetic flux through a coil is discussed and applied to the analysis of inductive power transfer (IPT) systems. This post-processing method is well adapted to various numerical methods. In this paper, a volume integral method based on magnetic vector potential interpolation is focused to solve the computational magnetostatic problem for the field quantities. It does not require the discretization of the predominant air domain so is perfectly suited for the modeling of IPT couplers. In magnetostatic analysis, the magnetic flux is expressed as a summation of two terms. The first one is created by source coils and is computed by integral expressions. The second one is obtained by a light integration only on the magnetic region. The method is accurate and associated to a reduced computation time. It is compared with both finite element method and experimental results to check its performance.

***Index Terms*—Inductive power transfer (IPT), magnetic flux computation, magnetic vector potential, mutual-inductance, volume integral method (VIM).**

I. Introduction

MAGNETIC flux computation is important for the design and the optimization of electromagnetic devices. It is a useful quantity to evaluate the magnetic coupling in transformers, electric machines or various novel applications like inductive power transfer (IPT) which is one of emerging technologies to achieve the contactless power transmission [1].

Nowadays, IPT technology has been gradually employed from wireless charging for consumer electronics to higher power transmission applications, such as the underwater charging and the online charging of electric vehicles [2]. As illustrated in Fig. 1, a mobile IPT system is principally composed of three parts: 1) The primary side includes several power converters to generate a high-frequency current; 2) The secondary side includes several power converters to regulate the high-frequency induced voltage to feed the load; 3) Both sides are wirelessly connected by an electromagnetic coupler, which is made up of a primary coil, a secondary coil and a ferrite structure to enhance the capacity of energy transfer.

The mutual-inductance between both coils in the coupler drives the capacity and stability of the power transmission. Stationary IPT system operates in a constant coupling condition to maintain a stable and efficient power transmission [3]. However, in the context of mobile applications, the external mechanical disturbances can produce an unavoidable instability to the system due to a bad positioning of the coupler [4]. Thus, in the design and optimization of couplers, it is essential to accurately and quickly evaluate the sensitivity of the mutual-inductance versus its degrees of freedom [5].

Analytical approaches, such as [6], have been proposed to calculate the mutual-inductance between two coils with canonical geometries. However, this method is not general and cannot deal with ferrite core structure.

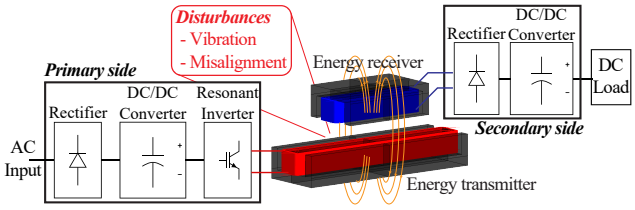

Fig. 1. Illustration of a mobile IPT system.

Numerical approaches are more general in the study of engineering problems. Many electromagnetic simulation software based on finite element method (FEM) could be convenient for the analysis of the coupler. However, the FEM requires to completely mesh both the active domain (i.e magnetic regions and conductive electrical region) and the inactive domain (i.e. large air volume region). In the context of IPT couplers, the predominant air region leads to an undesirable increase of both mesh size and computation times. Moreover, with the FEM, a new mesh has to be generated for each new relative positioning of both parts of the coupler.

Alternatively, the volume integral method (VIM) does not require the mesh of the air region and can deal with non-linear magnetic materials [7], [8]. For instance, a VIM based on the edge elements interpolation of magnetic vector potential has been proposed in [9]. It has shown a remarkable efficiency for solving problems with predominant air region. It is thus very suitable to model IPT couplers with this method. In this paper, the VIM will be used to solve the magnetostatic problem to get the magnetic state (i.e. the magnetic vector potential interpolated on the mesh of the problem).

Once the magnetic state obtained, a new post-processing method is proposed in this paper to compute magnetic flux in coils and is implemented to calculate the mutual-inductance between both coils in an IPT coupler. The main idea is that the magnetic flux can be computed by summing two terms. The first one is generated by source coils in the domain without

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magnetic region (i.e. flux generated by coils in vacuum) and the second one is generated by the magnetization in the ferrite core. The computation of both terms will be described in the next sections. Even the proposed method has been developed and validated in the VIM context, it should be pointed out that this approach is general and can be applied to the FEM as well, especially for magnetic scalar potential formulations [10].

II. Integral Formulation

Figure 2 illustrates an EE-type coupler which is often using in the online charging of electric vehicles. This coupler has three degrees of freedom for translation (e.g. the vertical distance between both parts, the movement of the car and the misalignment along the direction of movement). In magnetostatic analysis, it can be abstracted as a computational problem in a whole domain Ω, where coil regions are denoted by Ω_{0_k} and non-conducting magnetic regions are denoted by Ω_{m}. The boundary between magnetic regions and the free space is denoted by Γ_{m}.

A. Magnetostatic Integral Equation

Without any massive conductive region, the problem is governing by magnetostatic equations

$$\begin{cases} \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J} \end{cases} \tag{1}$$

where $\mathbf{B}$ is the magnetic flux density, $\mathbf{H}$ is the magnetic field intensity and $\mathbf{J}$ is the current density. The constitutive relation of magnetic materials is

$$\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M} \right) \tag{2}$$

where $\mathbf{M}$ means the magnetization. The relation between $\mathbf{M}$ and $\mathbf{B}$ in materials is

$$\mathbf{M} = \left(\nu_0 - \nu_{\mathrm{m}} \right) \mathbf{B} \tag{3}$$

where ν_0 is the reluctivity of free space and ν_{m} is the reluctivity of ferromagnetic materials. The ν_{m} can be expressed by $\nu_{\mathrm{m}} = \nu\left(\mathbf{B}\right)$ to represent the non-linear characteristic of materials. Besides, $\mathbf{H}$ can be decomposed into the sum of the source magnetic field $\mathbf{H}_0$ and the reduced magnetic field $\mathbf{H}_{\mathrm{r}} = \nabla \varphi_{\mathrm{r}}$ as

$$\mathbf{H} = \mathbf{H}_0 - \nabla \varphi_{\mathrm{r}} \tag{4}$$

According to [11], the reduced scalar potential φ_{r} is

$$\varphi_{\mathrm{r}} = \frac{1}{4\pi} \int_{\Omega} \left(\nu_0 - \nu_{\mathrm{m}} \right) \mathbf{B} \cdot \nabla \left(\frac{1}{r} \right) \mathrm{d}\Omega \tag{5}$$

where the r means the distance between the point where the reduced potential is expressed and the integration point.

B. Magnetic Vector Potential Formulation

By introducing the magnetic vector potential $\mathbf{A}$ that satisfies $\mathbf{B} = \nabla \times \mathbf{A}$, (4) becomes

$$\nu_{\mathrm{m}} \left(\nabla \times \mathbf{A} \right) + \nabla \varphi_{\mathrm{r}} = \mathbf{H}_0 \tag{6}$$

The magnetic vector potential $\mathbf{A}$ can be interpolated only on magnetic regions Ω_{m} with first order edge elements functions

$$\mathbf{A} = \sum_{j}^{N_{\mathrm{e}}} \mathbf{w}_j A_j \tag{7}$$

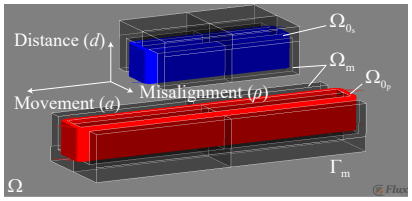

Fig. 2. A magnetostatic problem with conductive regions and magnetic regions, illustrating by a EE-type coupler of IPT system.

where $\mathbf{w}_j$ is the function associated to edge j, $A_j = \int_j \mathbf{A}\ \mathrm{dl}$ is the integration of the tangent component of magnetic vector potential $\mathbf{A}$ on the edge, and N_{e} is the total number of edges in the magnetic regions mesh. Using a Galerkin method with $\nabla \times \mathbf{w}_i$ as test functions, the projection of (6) on magnetic regions Ω_{m} leads to

$$\begin{aligned} &\int_{\Omega_{\mathrm{m}}} \nu_{\mathrm{m}} \left(\nabla \times \mathbf{w}_i \right) \cdot \left(\nabla \times \mathbf{A} \right)\ \mathrm{d}\Omega \\ &+ \int_{\Omega_{\mathrm{m}}} \nabla \times \mathbf{w}_i \cdot \nabla \varphi_{\mathrm{r}}\ \mathrm{d}\Omega = \int_{\Omega_{\mathrm{m}}} \nabla \times \mathbf{w}_i \cdot \mathbf{H}_0\ \mathrm{d}\Omega \end{aligned} \tag{8}$$

C. System Resolution

The previous equation (8) can be rewritten as a matrix system:

$$\left([\mathbf{R}] + [\mathbf{L}] \right) \{A\} = \{\mathbf{U}_0\} \tag{9}$$

where the matrices $\mathbf{R}$, $\mathbf{L}$ and $\mathbf{U}_0$ are discussed in [7], as:

$$\begin{cases} \mathbf{R}_{ij} = \displaystyle\int_{\Omega_{\mathrm{m}}} \nu_{\mathrm{m}} \left(\nabla \times \mathbf{w}_i \right) \cdot \left(\nabla \times \mathbf{w}_j \right) \mathrm{d}\Omega \\ \mathbf{L}_{ij} = \displaystyle\frac{1}{4\pi} \int_{\Gamma_{\mathrm{m}}} \frac{1}{S_i} \left(\sum_{f_i} \delta \upsilon_{f_i} \frac{1}{S_j} \int_{f_j} \frac{1}{r}\ \mathrm{d}f \right)\ \mathrm{d}\Gamma \\ \mathbf{U}_{0i} = \displaystyle\int_{\Omega_{\mathrm{m}}} \nabla \times \mathbf{w}_i \cdot \mathbf{H}_0\ \mathrm{d}\Omega \end{cases} \tag{10}$$

where the S_i is the area of face f_i shared between two adjacent volume elements and $\delta \upsilon_{f_i}$ is the reluctivity jump. A Newton-Raphson method is used to solve the problem with the non-linear material and to get magnetic vector potential $\mathbf{A}$ [7].

III. Magnetic Flux Computation

The magnetic flux through a coil k associated to region Ω_{0_k}, can be expressed as

$$\Phi_k = \int_{\Omega_{0_k}} \mathbf{j}_{0_k} \cdot \mathbf{A}\ \mathrm{d}\Omega \tag{11}$$

where the $\mathbf{j}_{0_k}$ is a function space describing the normalized current density in each point of the coil domain. For the problem contained coil regions Ω_{0_k} and non-conducting magnetic regions Ω_{m}, the magnetic vector $\mathbf{A}$ can be firstly separated in two terms, as $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_{\mathrm{m}}$, where $\mathbf{A}_0$ and $\mathbf{A}_{\mathrm{m}}$ are generated respectively by coils in vacuum and the magnetic regions. Therefore, the contribution of the total magnetic flux Φ_k through a coil k is represented by

$$\Phi_k = \Phi_{0_k} + \Phi_{\mathrm{m}_k} \tag{12}$$

where Φ_{0_k} and Φ_{m_k} are contributions of all the coils and of the magnetization of magnetic regions.

A. Computation of Φ_0

First, a general expression for the magnetic vector potential $\mathbf{A}_0$ which is generated by all coils is

$$\mathbf{A}_0 = \frac{\mu_0}{4\pi} \sum_l \left(\int_{\Omega_{0_l}} \frac{\mathbf{j}_{0_l}}{r} \, \mathrm{d}\Omega \right) I_l \tag{13}$$

where the I_l is the current flowing in the coil l and r is the distance between the integration point in coils and the point where the vector potential is calculated. Thus, for the coil k, the first part of magnetic flux Φ_{0_k} generated by all the coils can be calculated by

$$\Phi_{0_k} = \frac{\mu_0}{4\pi} \int_{\Omega_{0_k}} \mathbf{j}_{0_k} \cdot \left(\sum_l I_l \int_{\Omega_{0_l}} \frac{\mathbf{j}_{0_l}}{r} \, \mathrm{d}\Omega \right) \mathrm{d}\Omega \tag{14}$$

A semi-analytical integration method based on the discretization of the coils can be used to compute the double integral (14). The first integral on the coil region Ω_{0_l} can be evaluated by the analytical expression like proposed in [12]. The second integral on the coil region Ω_{0_k} can be computed by quadratic Gauss points integration.

B. Computation of Φ_m

The contribution of vector potential $\mathbf{A}_\mathrm{m}$, generated by magnetic regions Ω_m, can be calculated from the magnetization field $\mathbf{M}$ as

$$\mathbf{A}_\mathrm{m} = \frac{\mu_0}{4\pi} \int_{\Omega_\mathrm{m}} \mathbf{M} \times \nabla \left(\frac{1}{r} \right) \mathrm{d}\Omega_\mathrm{m} \tag{15}$$

Therefore, the magnetic flux Φ_{m_k} can be computed by

$$\Phi_{\mathrm{m}_k} = \frac{\mu_0}{4\pi} \int_{\Omega_{0_k}} \mathbf{j}_{0_k} \left[\int_{\Omega_\mathrm{m}} \mathbf{M} \times \nabla \left(\frac{1}{r} \right) \mathrm{d}\Omega_\mathrm{m} \right] \mathrm{d}\Omega_0 \tag{16}$$

However, in order to get an accurate solution of the integral over the conductive regions Ω_{0_k}, the coil mesh should be adapted to the variation of the magnetization field in these regions. Since a great number of elements will increase the computation time, it may be not efficient to compute directly the integral (16), especially when dealing with the complex geometries. An alternative method is thus proposed without the need of a fine discretization for the coil. By applying the partial integration theorem to the equation (11), it yields

$$\begin{aligned} &\int_{\Omega_{0_k}} \mathbf{j}_{0_k} \cdot \mathbf{A}_\mathrm{m} \, \mathrm{d}\Omega = \int_\Omega (\nabla \times \mathbf{h}_{0_k}) \cdot \mathbf{A}_\mathrm{m} \, \mathrm{d}\Omega \\ &= \int_\Omega \mathbf{h}_{0_k} \cdot (\nabla \times \mathbf{A}_\mathrm{m}) \, \mathrm{d}\Omega + \int_\Omega \nabla \cdot (\mathbf{h}_{0_k} \times \mathbf{A}_\mathrm{m}) \, \mathrm{d}\Omega \end{aligned} \tag{17}$$

where $\mathbf{h}_{0_k}$ represents the magnetic field generated by the coil k with 1 A. According to the divergence theorem, the first term $\int_\Omega \nabla \cdot (\mathbf{h}_{0_k} \times \mathbf{A}_\mathrm{m}) \, \mathrm{d}\Omega$ is equal to the integral on the boundary by $\int_\Gamma (\mathbf{h}_{0_k} \times \mathbf{A}_\mathrm{m}) \cdot \mathbf{n} \, \mathrm{d}\Gamma$. Thanks to continuity in medium, the term $\mathbf{h}_0$ and $\mathbf{A}_\mathrm{m}$ can be expanded until the infinity boundary Γ_∞ where $\mathbf{h}_{0_k} = 0$, thus

$$\int_\Omega \nabla \cdot (\mathbf{h}_{0_k} \times \mathbf{A}_\mathrm{m}) \, \mathrm{d}\Omega = \int_{\Gamma_\infty} (\mathbf{h}_{0_k} \times \mathbf{A}_\mathrm{m}) \cdot \mathbf{n} \, \mathrm{d}\Gamma = 0 \tag{18}$$

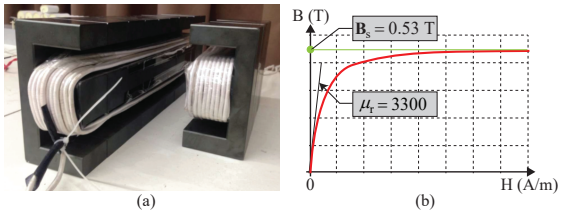

Fig. 3. Experimental coupler prototype. (a) Photo of the coupler. (b) Magnetization curve of the ferromagnetic material.

Therefore, the magnetic flux Φ_{m_k} generated by magnetization can be computed by the second term:

$$\Phi_{\mathrm{m}_k} = \int_\Omega \mathbf{h}_{0_k} \cdot (\nabla \times \mathbf{A}_\mathrm{m}) \, \mathrm{d}\Omega \tag{19}$$

From (2), (4) and the decomposition of $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_\mathrm{m}$, the field $\mathbf{B}$ can be rewritten as

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A}_0 + \nabla \times \mathbf{A}_\mathrm{m} \\ &= \mu_0 \left(\mathbf{H}_0 - \nabla \varphi_\mathrm{r} + \mathbf{M} \right) \end{aligned} \tag{20}$$

By notice that $\nabla \times \mathbf{A}_0 = \mu_0 \mathbf{H}_0$, the previous equation allows to express $\mathbf{A}_\mathrm{m}$ such as

$$\nabla \times \mathbf{A}_\mathrm{m} = \mu_0 \left(-\nabla \varphi_\mathrm{r} + \mathbf{M} \right) \tag{21}$$

Substituting (21) into (19), the flux Φ_{m_k} becomes

$$\Phi_{\mathrm{m}_k} = \mu_0 \int_\Omega \mathbf{h}_{0_k} \left(-\nabla \varphi_\mathrm{r} + \mathbf{M} \right) \mathrm{d}\Omega \tag{22}$$

Applying the partial integration theorem to the integral $\int_\Omega \mathbf{h}_{0_k} (\nabla \varphi_\mathrm{r}) \, \mathrm{d}\Omega$, it yields

$$\int_\Omega \nabla \cdot (\varphi_\mathrm{r} \mathbf{h}_{0_k}) \, \mathrm{d}\Omega - \int_\Omega \varphi_\mathrm{r} \nabla \cdot \mathbf{h}_{0_k} \mathrm{d}\Omega \tag{23}$$

Actually, both terms are equal to zero. For the first term, because of the theorem of divergence, we get

$$\int_\Omega \nabla \cdot (\varphi_\mathrm{r} \mathbf{h}_{0_k}) \, \mathrm{d}\Omega = \int_\Gamma \varphi_\mathrm{r} \mathbf{h}_{0_k} \mathrm{d}\Gamma = \int_{\Gamma_\infty} \varphi_\mathrm{r} \mathbf{h}_{0_k} \mathrm{d}\Gamma \tag{24}$$

where the $\varphi_\mathrm{r} = 0$ and $\mathbf{h}_{0_k} = 0$ at the infinity boundary. For the second term, the divergence of $\mathbf{h}_{0_k}$ is also equal to zero.

Finally, the term Φ_{m_k} which is the contribution of ferromagnetic core, can be expressed through a simple integral over only magnetic regions Ω_m, as

$$\Phi_{\mathrm{m}_k} = \mu_0 \int_{\Omega_\mathrm{m}} \mathbf{h}_{0_k} \cdot \mathbf{M} \, \mathrm{d}\Omega \tag{25}$$

where $\mathbf{M} = (\nu_0 - \nu_\mathrm{m}) (\nabla \times \mathbf{A})$.

IV. Application to IPT System

In IPT system, the mutual-inductance of the coupler between both coils (the primary coil p caring a current I_p and the secondary coil s with zero current) can be calculated by

$$M_\mathrm{sp} = \frac{\Phi_\mathrm{s}}{I_\mathrm{p}} = \frac{\Phi_{\mathrm{c_s}} + \Phi_{\mathrm{m_s}}}{I_\mathrm{p}} \tag{26}$$

Figure 3 presents an experimental EE-type coupler used in IPT system. The ferromagnetic material used in the coupler is the TDK PC95 [13], with a initial relative permeability $\mu_\mathrm{r} = 3300$ and a saturation magnetic flux density $B_\mathrm{s} = 0.53$ T.

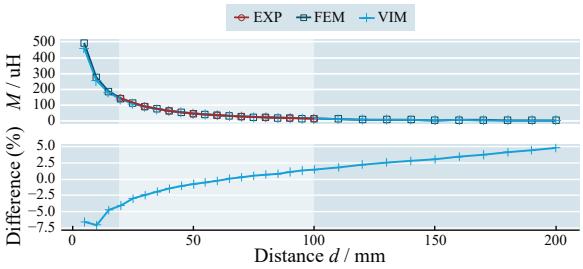

Fig. 4. Sensitivity analysis of mutual-inductance versus distance and the relative difference between VIM and FEM.

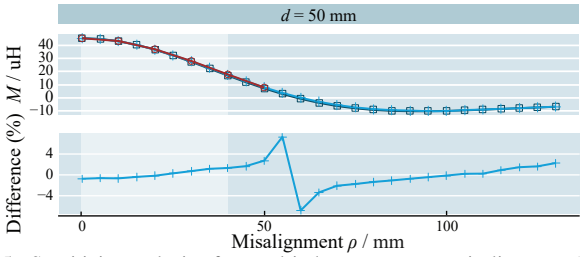

Fig. 5. Sensitivity analysis of mutual-inductance versus misalignment (nominal distance $d = 50$ mm) and the relative difference between VIM and FEM.

After solving the computational magnetostatic problem by VIM and calculating (26), the mutual-inductance variation characteristics versus degrees of freedom of the coupler are presented in Fig. 4, Fig. 5 and Fig. 6. All the computational results (denoted by "VIM") have been checked by experimental measurements (denoted by "EXP") with a very good agreement. Moreover, the comparison of the results obtained with our approach versus those obtained in a commercial finite element analysis software (denoted by "FEM") shows a difference of less than 2% in the operation zone.

In the comprehensive analysis of the sensitivity versus the positioning geometrical parameters, the mutual-inductance decreases with the distance in Fig. 4, due to the increase of the magnetic flux leakage in the air. Thus, a nominal distance setting to 50 mm can enable the coupler to operate with a sufficient value of mutual-inductance (50 μH) and with a relatively small variation. In the sensitivity analysis of misalignments and movements with a nominal distance $d = 50$ mm, the mutual-inductance reaches the maximum value when both parts perfectly face each other and the decline tendency are in symmetry for the EE-type coupler. The sensitivity analysis can enable to evaluate the disturbance range of mutual-inductance for the design and optimization.

Furthermore, as illustrated in Fig. 7, the convergence of both algorithms (VIM and FEM) versus the number of elements in the magnetic region has been studied to analyze the efficiency of the proposed method. With much less elements to reach the convergence (6042 for the VIM and 161501 for the FEM), the VIM formulation and the propose flux computation technique is an effective method for the study of electromagnetic devices like the coupler in IPT system.

V. Conclusion

To fulfill a more efficient post-processing method of flux computation, an integral formulation has been proposed to compute magnetic flux and it has been applied to calculate the mutual-inductances of the coupler in the IPT system. The magnetic vector potential VIM formulation is used to solve the magnetostatic problem without considering the predominant inactive region. Then, an original magnetic flux computation method with a good efficiency has been proposed. Let us notice that this flux computation method is general and can be applied whatever the numerical method used to solve the problem (VIM but also FEM based formulation).

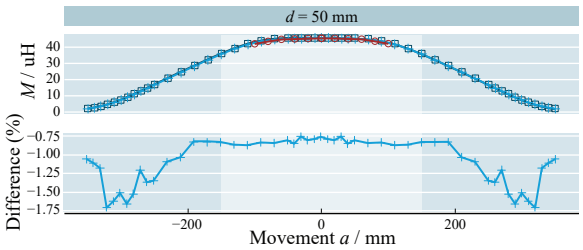

Fig. 6. Sensitivity analysis of mutual-inductance versus movement (nominal distance $d = 50$ mm) and the relative difference between VIM and FEM.

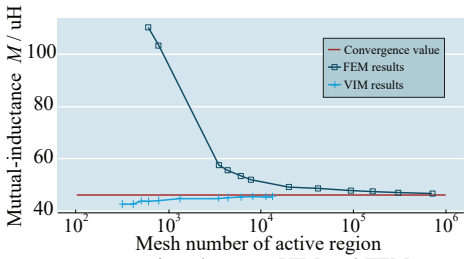

Fig. 7. Convergence comparison between VIM and FEM.

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