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### ► **To cite this version:**

Tristan Bonargent, Tomas Menard, Eric Pigeon, Olivier Gehan. Observer and first-order low-pass filter based attitude estimation for rigid bodies subject to external acceleration. 2019 IEEE 58th Conference on Decision and Control (CDC), Dec 2019, Nice, France. pp.629-634, <10.1109/CDC40024.2019.9030202>. <hal-02276192>

**HAL Id: hal-02276192**

**<https://hal.science/hal-02276192v1>**

Submitted on 23 Mar 2020

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# Observer and first-order low-pass filter based attitude estimation for rigid bodies subject to external acceleration

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**Abstract**—In this paper, we consider the problem of estimating the attitude of a rigid body who is subject to non-negligible external acceleration, based on the measurements provided by a classical IMU unit: gyroscope, accelerometer and magnetometer. Two algorithms are proposed, based on the combination of a low-pass filter in the fixed inertial frame and an observer. The stability of the proposed schemes are proved, based on a Lyapunov approach. Simulations are provided in order to illustrate the performances of the proposed observers.

## I. INTRODUCTION

The estimation of attitude for a rigid body has been a great subject of interest for the last decades. An estimation of the attitude is indeed needed in many fields such as robotics [27], bio-logging [11], indoor positioning [30] or UAV [10]. Nowadays, the estimation of orientation is usually based on the measurement of two vectors in some body fixed frame whose reference value in an inertial fixed frame (usually earth fixed frame) is known, together with the angular velocity in body fixed frame. This usual configuration is mainly due to the development of low-cost systems equipped with MEMS inertial measurement unit, which contains an accelerometer, a magnetometer and a gyroscope.

Many approaches have been developed for the reconstruction of the attitude, using these measurements. They can be divided into three main approaches (surveys can be found in [9] and [37] for example). The first category contains optimization based approaches and is usually referred to as the Wahba's problem [36], [7]. For this approach, no filtering is performed and only the measurements at time  $t$  is used for the estimation of the orientation at time  $t$ . The second approach is based on stochastic filtering and contains different types of Kalman filters, such as EKF [20], [38], UKF [8] or particle filter [28]. Though providing interesting results, the main drawback is that convergence cannot be ensured in general and the tuning of the gains might not be easy. The third approach is based on nonlinear observers. The main advantage of this approach is that a large domain of convergence can be ensured and the tuning of the gains can be done through a systematic procedure. These observers can be designed directly on  $SO(3)$  [18], [5], [35], [22] or on  $\mathbb{R}^{3 \times 3}$  (forgetting the underlying geometry structure) [1], [2], [25].

These approaches give satisfactory results when the considered body is subject to only low magnitude external acceleration. Indeed, it is usually assumed that the accelerometer provide a measurement of the gravity only, neglecting external acceleration. But this is generally not the case when medium to high acceleration are considered, which can occur when considering mobile robots such as UAV [13] or even when a smartphone is carried by a pedestrian [26].

In order to alleviate this problem, different solutions have been considered. Additional measurements have been considered, such as position provided by a GPS [13], [15], or velocity either in inertial frame [21], [12], [31] or in body fixed frame [4], [34], [14]. This allows to obtain an estimation of the external acceleration and thereby correcting the fact that the magnetometer does not measure only the gravity in the body fixed frame. A second approach consists in using a model for the acceleration, but this can be used only for specific cases where such a model exists and is known such as in some robotic vehicles, for example for quadrotors [23], [24]. Another approach consists in using the fact that the norm of the accelerometer is known when there is no external acceleration, the difference  $\mu$  between the norm of the measured acceleration and the gravity constant should actually be equal to zero. In [29], [32], [17], the accelerometer are discarded in the update phase if the value of  $\mu$  is too high. Residual errors are used in [33], [19] to detect external acceleration. In [30], the update phase of the Kalman filter is only performed during periods considered as quasi static field, which is defined as a period of low variance of measurements.

No observer-based approach, with proof of stability, taking explicitly into account non negligible external acceleration without extra measurements or extra knowledge on the external acceleration model exists. An approach is then proposed here based on the assumption that the acceleration in the inertial frame can be decomposed into a low frequency component equal to the known constant gravity vector and a high frequency component equal to the external acceleration. It should be noted that this assumption is not true for the measured acceleration, since the measurements are done in

the body fixed frame, which means that one cannot directly apply a low pass filter to the measured acceleration unless the angular velocity is very low. The proposed attitude estimator is composed of a first order low pass filter together with an observer. Furthermore the considered estimator forgets the underlying geometry of  $SO(3)$ , which means that the estimates belong to  $\mathbb{R}^{3 \times 3}$ . The drawback is that the dimension of the observer increases, but the advantage is that there is no topological limitation, unwinding phenomena or singularities for achieving global stability due the structure of  $SO(3)$  [3], [6]. Two observers are proposed here. A first observer is based on a coordinate transformation which allows to obtain a linear error system, but the computation of the observer gain implies computing the inverse of a time-varying matrix. A second observer is then proposed with a different gain, which does not involve computing the inverse of this matrix.

The paper is organized as follows. The model used to design the observer is depicted in section II, together with some notations. Section III contains the proposed observers. The considered low-pass filter is given in subsection III-A. The two observers are presented in subsections III-B and III-C respectively. Some simulations are provided in section IV in order to illustrate the performances of the proposed approach. Finally, section V concludes the paper.

## II. PRELIMINARIES

### A. Notations

Throughout the paper, a block diagonal matrix is represented as  $\text{diag}(A_1, \dots, A_n)$ . The identity matrix and the square zero matrix, of dimension  $n \in \mathbb{N}$ , are respectively denoted  $I_n$  and  $0_n$ . For  $x, y \in \mathbb{R}^3$ ,  $x \times y$  represents the cross product. For a vector  $x = (x_1 \ x_2 \ x_3)^T \in \mathbb{R}^3$ ,  $x_\times \in \mathbb{R}^{3 \times 3}$  is the associated skew-symmetric matrix, that is

$$x_\times = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix} \quad (1)$$

The lower and upper eigenvalues of a matrix  $M$  are denoted  $\lambda_{\min}(M)$  and  $\lambda_{\max}(M)$  respectively.

### B. Physical model

Consider an inertial reference frame  $\{I\}$  and a body fixed frame  $\{B\}$ . The rotation matrix from  $\{B\}$  to  $\{I\}$ , of the considered moving body, is denoted  $R(t) \in SO(3)$  and verifies the following differential equation

$$\dot{R}(t) = R(t)(\omega(t))_\times, \quad t \geq 0 \quad (2)$$

where  $\omega(t) \in \mathbb{R}^3$  is the angular velocity of the moving body in the frame  $\{B\}$ .

One assumes that the rate gyro measurements are available without bias. In addition to the rate gyro, one has access to the measurements of an accelerometer and a magnetometer.

The output  $a$  of the accelerometer is in the following form

$$a(t) = R^T(t)(a_e(t) - g_0) \quad (3)$$

where  $g_0 = (0 \ 0 \ G)$ , with  $G \approx 9.81$  the gravitational constant and  $a_e(t)$  is the external acceleration, that is all

other accelerations applied to the body.

The output  $m$  of the magnetometer is in the form

$$m(t) = R^T(t)m_I \quad (4)$$

where  $m_I$  is the earth magnetic field (expressed in the inertial frame) and is constant.

In the remainder of the paper the following assumption is supposed to hold true.

**Assumption 1.** *The vectors  $a(t)$  and  $m(t)$  are non-collinear for all  $t \geq 0$ , more precisely, there exists  $\alpha > 0$  such that  $\|a(t) \times m(t)\| \geq \alpha, \forall t \geq 0$ .*

In a more general way, one assumes that 3 independent vectors  $v_i(t) \in \mathbb{R}^3$  are measured in the body fixed frame, and are in the following form

$$v_i(t) = R^T(t)(b_i + p_i(t)) \quad (5)$$

where  $b_i \in \mathbb{R}^3$  is a known constant reference vector and  $p_i(t) \in \mathbb{R}^3$  is a time varying unknown perturbation. This is not restrictive since one can take  $a \times m$  for the third vector, in order to obtain three linearly independent vectors, when only accelerometer and magnetometer measurements are available. The perturbations  $p_i, i = 1, 2, 3$ , and their derivative will be supposed to be bounded in the following.

**Assumption 2.** *There exist  $\gamma_1, \gamma_2 > 0$  such that*

$$\|p_i(t)\| \leq \gamma_1 \text{ and } \|\dot{p}_i(t)\| \leq \gamma_2 \text{ for } i = 1, 2, 3 \text{ and all } t \geq 0. \quad (6)$$

### C. Design model

Since the observer is designed directly on  $\mathbb{R}^{3 \times 3}$ , forgetting the  $SO(3)$  structure, one now writes the rotation matrix  $R$  in vector form. Similarly as in [1], denoting

$$R(t) = (z_1(t) \ z_2(t) \ z_3(t))^T, \quad z_i(t) \in \mathbb{R}^3, \quad i = 1, 2, 3 \quad (7)$$

one can write  $R(t)$  as a vector, as follows

$$x_2(t) = (z_1^T(t) \ z_2^T(t) \ z_3^T(t))^T \in \mathbb{R}^9 \quad (8)$$

It is then straightforward to show that

$$\dot{x}_2(t) = -S_3(\omega(t))x_2(t) \quad (9)$$

where

$$S_3(x) = \text{diag}(x_\times, x_\times, x_\times) \in \mathbb{R}^{9 \times 9} \quad (10)$$

Following equation (5), the 3 measurements can be written in a general way as follows

$$b_i + p_i(t) = W_i(t)x_2(t), \quad i = 1, 2, 3 \quad (11)$$

with

$$W_i(t) = \begin{pmatrix} v_i^T(t) & 0_{1 \times 3} & 0_{1 \times 3} \\ 0_{1 \times 3} & v_i^T(t) & 0_{1 \times 3} \\ 0_{1 \times 3} & 0_{1 \times 3} & v_i^T(t) \end{pmatrix} \in \mathbb{R}^{3 \times 9} \quad (12)$$

One then obtains, in compact form, the relation

$$B + P(t) = W(t)x_2(t) \quad (13)$$

with  $B = (b_1^T \ b_2^T \ b_3^T)^T \in \mathbb{R}^9$ ,

$P(t) = (p_1^T(t) \ p_2^T(t) \ p_3^T(t))^T \in \mathbb{R}^9$

and  $W(t) = (W_1^T(t) \ W_2^T(t) \ W_3^T(t))^T \in \mathbb{R}^{9 \times 9}$

It should be noted that by assumptions 1 and 2, the matrix  $W(t)$  is non singular and bounded for all  $t \geq 0$ , more

precisely, one can further say that there exist  $\nu_1, \nu_2 > 0$  such that

$$\nu_1 I_9 \leq W(t) \leq \nu_2 I_9, \quad \forall t \geq 0. \quad (14)$$

### III. OBSERVER DESIGN

#### A. Low-pass filter design

One considers the linear time-varying system (9)-(13):

$$\dot{x}_2(t) = -S_3(\omega(t))x_2(t) \quad (15)$$

$$B + P(t) = W(t)x_2(t) \quad (16)$$

The aim is to reconstruct  $x_2$ , but the problem is that the output  $B + P(t)$  is unknown since only  $B$  is known. The idea is based on the assumption that the perturbations are not low frequency, then the output of the following augmented system

$$\dot{x}_1(t) = \frac{1}{\tau}(-x_1(t) + W(t)x_2(t)) \quad (17)$$

$$\dot{x}_2(t) = -S_3(\omega(t))x_2(t) \quad (18)$$

$$y(t) = x_1(t) \quad (19)$$

is approximately equal to  $B$ , indeed  $P(t)$  is filtered out by the first stage of system (17)-(18), which is simply a first order low pass filter, with time constant  $\tau > 0$ , whose transfer function is given by  $G(s) = \frac{1}{\tau s + 1}$ .

It should be noted that the augmented system (17)-(18)-(19) is uniformly observable since the matrix  $W(t)$  is non singular for all  $t \geq 0$  by assumption 1. Then the design of an observer is possible.

#### B. First observer

The first proposed observer is given by

$$\dot{\hat{x}}_1(t) = \frac{1}{\tau}(-\hat{x}_1(t) + W(t)\hat{x}_2(t)) + k_1(B - \hat{x}_1(t)) \quad (20)$$

$$\dot{\hat{x}}_2(t) = -S_3(\omega(t))\hat{x}_2(t) + k_2 W^{-1}(t)(B - \hat{x}_1(t))$$

where  $K = \begin{pmatrix} k_1 & k_2 \end{pmatrix}^T \in \mathbb{R}^2$  is the gain of the observer and is chosen in such a way that the matrix

$$\bar{A} \triangleq A - KC = \begin{pmatrix} -\frac{1}{\tau} - k_1 & \frac{1}{\tau} \\ -k_2 & 0 \end{pmatrix} \quad (21)$$

is Hurwitz, where

$$A = \begin{pmatrix} -\frac{1}{\tau} & \frac{1}{\tau} \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (22)$$

One has the following convergence result in the perturbation free case.

**Theorem 1.** *Assume that  $P(t) \equiv 0$  and that the gains  $k_1, k_2$  are chosen such that the matrix  $\bar{A}$  is Hurwitz, then the state of observer (20) converges exponentially toward the state of system (17)-(18).*

*Proof.* First notice that in the perturbation free case, one has  $W(t)x_2(t) = B$ , for all  $t \geq 0$ , then  $\varepsilon(t) = x_1(t) - B$  converges exponentially to zero, since the first-order low-pass filter  $G(s) = \frac{1}{\tau s + 1}$  has a static gain equal to 1. Thus, there exists  $\alpha, \beta > 0$  such that  $\|\varepsilon(t)\| \leq \beta e^{-\alpha t}$ .

Consider the error state  $e = \begin{pmatrix} e_1^T & e_2^T \end{pmatrix}^T$ , with  $e_1 = (x_1 - \hat{x}_1)$ ,

$e_2 = W\tilde{x}_2$  and  $\tilde{x}_2 = (x_2 - \hat{x}_2)$ . Since  $P(t) \equiv 0$ , one can show that

$$\dot{W}(t) = W(t)S_3(\omega(t)) \quad (23)$$

And one can obtain that

$$\dot{e}(t) = \bar{A}e(t) + K\varepsilon(t) \quad (24)$$

The error signal  $e(t)$  is thus given by

$$e(t) = e^{\bar{A}t}e(0) + \int_0^t e^{\bar{A}(t-s)}K\varepsilon(s)ds \quad (25)$$

since  $\bar{A}$  is assumed to be Hurwitz, there exists  $\gamma, \varphi > 0$  such that  $\|e^{\bar{A}t}\| \leq \varphi e^{-\gamma t}$  (one can further assume without loss of generality that  $\gamma \neq \alpha$ ), and so

$$\|e(t)\| \leq \varphi e^{-\gamma t}\|e(0)\| + \beta\varphi \frac{e^{-\alpha t} - e^{-\gamma t}}{\gamma - \alpha} \quad (26)$$

This shows the exponential convergence to zero of  $e_1(t)$  and  $e_2(t)$ . Since  $W(t)$  verifies (14),  $\tilde{x}_2(t)$  also converges exponentially to zero.  $\square$

The next corollary provides a practical stability result in the case of bounded external accelerations.

**Corollary 1.** *Assume that the external acceleration satisfies assumption 2 and that  $k_1, k_2$  are chosen such that the matrix  $\bar{A}$  is Hurwitz, then the estimation error of observer (20)  $\tilde{x} = \begin{pmatrix} \tilde{x}_1^T & \tilde{x}_2^T \end{pmatrix}^T$ , with  $\tilde{x}_1 = x_1 - \hat{x}_1$  and  $\tilde{x}_2 = x_2 - \hat{x}_2$  is ultimately bounded, that is, there exists a bound  $K > 0$  such that*

$$\limsup_{t \rightarrow +\infty} \|\tilde{x}(t)\| \leq K \quad (27)$$

*Proof.* In the case where the perturbation  $P(t)$  is non zero, the dynamics of  $W(t)x_2(t)$  is given by

$$\frac{d}{dt}(W(t)x_2(t)) = \dot{P}(t) \quad (28)$$

and the output of the first order filter by  $x_1(t) = B + \varepsilon(t)$ . The signal  $\varepsilon(t)$  is bounded since it corresponds to the sum of two signals, the first one is the filtered version of  $P(t)$  by the first order filter and the second one is a signal converging exponentially to zero, which is due to the possibly incorrect initial condition when filtering  $B$ .

Let us denote  $e(t) = \begin{pmatrix} e_1^T & e_2^T \end{pmatrix}^T$ , with  $e_1 = \tilde{x}_1$ ,  $e_2 = W\tilde{x}_2$ , then, one has

$$\dot{e}(t) = \bar{A}e(t) + K\varepsilon(t) + D\dot{P}(t) \quad (29)$$

with  $D = \begin{bmatrix} 0 & I_9 \end{bmatrix}^T$ . It is direct to see that the error  $e(t)$  is practically bounded since the matrix  $\bar{A}$  is Hurwitz, and  $\varepsilon(t)$  and  $\dot{P}(t)$  are uniformly bounded. It directly follows that  $\tilde{x}_1$  and  $\tilde{x}_2$  are ultimately bounded because of (14).  $\square$

**Remark 1.** *Corollary 1 only provides practical convergence. Indeed, there are two sources of error for the reconstruction of the attitude, first the output of system (17)-(18) is not perfectly known, since one assumes that it is equal to  $B$  in the observer, and the model of  $W(t)x_2(t)$  is uncertain, due to the non zero derivative of  $P(t)$ . It is then not possible to obtain an arbitrarily small error. Nevertheless, the estimate given by the proposed observer is very satisfactory, as illustrated with the simulations in section IV. A more precise effect of these two sources of error shall be done in future works in*

order to determine the optimal gains, but is beyond the scope of this article, due to the limited space.

### C. Second observer

The second proposed observer is given by

$$\begin{aligned}\dot{\hat{x}}_1(t) &= \frac{1}{\tau}(-\hat{x}_1(t) + W(t)\hat{x}_2(t)) \\ &\quad + k_1 W(t)W^T(t)(B - \hat{x}_1(t)) \\ \dot{\hat{x}}_2(t) &= -S_3(\omega(t))\hat{x}_2(t) + k_2 W^T(t)(B - \hat{x}_1(t))\end{aligned}\quad (30)$$

where the gains are chosen as  $k_1 > 0$  and  $k_2 = \frac{k_1}{2}$ . One has the following convergence result in the perturbation free case.

**Theorem 2.** Assume that  $P(t) \equiv 0$  and that  $k_1 > 0$  and  $k_2 = \frac{k_1}{2}$ , then the state of observer (30) converges exponentially toward the state of system (17)-(18).

*Proof.* One denotes  $e = (e_1^T \ e_2^T)^T$ ,  $e_1 = x_1 - \hat{x}_1$ ,  $e_2 = W(t)\tilde{x}_2$  and  $\tilde{x}_2 = x_2 - \hat{x}_2$ . The error equation is given by

$$\dot{e}_1 = -\frac{1}{\tau}e_1 + \frac{1}{\tau}e_2 - k_1 W W^T e_1 + k_1 W W^T \varepsilon \quad (31)$$

$$\dot{e}_2 = -\frac{k_1}{2} W W^T e_1 + \frac{k_1}{2} W W^T \varepsilon \quad (32)$$

Consider the following candidate Lyapunov function

$$V(e) = e^T M e, \quad M = \begin{pmatrix} \frac{1}{4}I_9 & -\frac{1}{4}I_9 \\ -\frac{1}{4}I_9 & \frac{1}{2}I_9 \end{pmatrix} \quad (33)$$

This is a valid candidate Lyapunov function since  $M$  is definite positive and thus there exists  $\varrho_1, \varrho_2 > 0$  such that  $\varrho_1 \|e\| \leq \sqrt{V(e)} \leq \varrho_2 \|e\|$ .

One has

$$\dot{V}(e) \leq -e^T N e + 2\sigma_2 \sqrt{V(e)} \|\varepsilon\| \quad (34)$$

where

$$N = \begin{pmatrix} \frac{1}{2\tau} + \frac{k_1 \lambda_1}{4} & -\frac{1}{2\tau} \\ -\frac{1}{2\tau} & \frac{1}{2\tau} \end{pmatrix} \quad (35)$$

with  $\lambda_1 = \inf_{t \geq 0} \lambda_{\min}(W(t)W^T(t))$ ,  $\lambda_2 = \sup_{t \geq 0} \lambda_{\max}(W(t)W^T(t))$  and  $\sigma_2 = \frac{k_1 \lambda_2}{8\rho_1}$ . One can note that  $\lambda_1 > 0$  because of inequality (14) and  $\lambda_2 < +\infty$  because the perturbation  $P(t)$  is assumed to be uniformly bounded.

Furthermore, since  $N$  is positive definite as soon as  $k_1 \lambda_1 > 0$ , there exists  $\sigma_1 > 0$  such that

$$\dot{V}(e) \leq -2\sigma_1 V(e) + 2\sigma_2 \sqrt{V(e)} \|\varepsilon\| \quad (36)$$

Then

$$\frac{d}{dt}(\sqrt{V(e)}) \leq -\sigma_1 \sqrt{V(e)} + \sigma_2 \|\varepsilon\| \quad (37)$$

Applying the comparison lemma (lemma 3.4 p.102 [16]) and following the same lines as in the end of the proof of Theorem 1 gives the result.  $\square$

**Corollary 2.** Assume that the external acceleration satisfies assumption 2 and the gains are chosen as  $k_1 > 0$  and  $k_2 = \frac{k_1}{2}$ , then the estimation error of observer (30)  $\tilde{x} = (\tilde{x}_1^T \ \tilde{x}_2^T)^T$ , with  $\tilde{x}_1 = x_1 - \hat{x}_1$  and  $\tilde{x}_2 = x_2 - \hat{x}_2$  is ultimately bounded, that is, there exists a bound  $K > 0$  such that

$$\limsup_{t \rightarrow +\infty} \|\tilde{x}(t)\| \leq K \quad (38)$$

The proof of Corollary 2 combines the same ideas as the ones of corollary 1 and Theorem 2 and is then omitted.

## IV. SIMULATIONS

The behavior of the proposed observers are now illustrated through simulations.

For all the simulations, the rotation dynamic is given by equation (2) and the angular velocity is depicted on figure 1. The rotation is initialized in such a way that the Euler angles Yaw, Pitch, Roll are equal to  $[0.4383 \ 0.9902 \ 1.0074]$  at time  $t = 0$ . The measurements given by the accelerometer and the magnetometer are taken as:

$$a(t) = R^T(t)(a_e(t) - g_0) + \varepsilon_a(t) \quad (39)$$

$$m(t) = R^T(t)m_I + \varepsilon_m(t) \quad (40)$$

where  $a_e$  is the external acceleration and depends on the simulation,  $\varepsilon_a$  and  $\varepsilon_m$  are white noise with mean equal to 0 and standard deviation equal to 1 and 0.1 respectively,  $g_0 = [0 \ 0 \ 9.81]$  and  $m_I = [0.434 \ -0.0091 \ 0.9008]$ .

The observers presented in this paper will be compared with the classical observer from [18] with explicit error formulation, without bias estimation, since no bias is considered here. The tuning parameters of the Mahony observer are taken as  $k_1 = k_2 = k_p = 1$ . The two proposed observers are implemented with the same gains  $k_1 = 1$  and  $k_2 = 1/2$ . The time constant of the low-pass filter is taken as  $\tau = 2s$ .

All the implemented observers are initialized at  $\hat{R}(0) = I_3$ . When external acceleration is considered, it starts only after time  $t = 10s$  in order for the observers to converge before. Furthermore, the mean square error (MSE) will be computed in order to compare the different observers, but between  $t = 10s$  and  $t = 100s$  (the end of the simulation), in order to take into account only the effect, on the error, of the external acceleration and not the transient behavior.

Three different simulations are considered here. First, the external acceleration is equal to zero, only noise on the measurements is considered. The Euler angles with their reconstructed version and the error  $\tilde{R} = \|R - \hat{R}\|$  are reported on figure 2. All three observers perform quite well, even if the observers proposed in this paper filter the noise better, which has a direct effect on the estimation. It can be seen that the second proposed observer convergence is slower than the first one. This is due to the fact that the error dynamics depend on  $W(t)W^T(t)$  and then can become slower if the eigenvalues of this matrix are low, while for the first observer the error dynamics are independent from  $W(t)$ . The MSE of observer 1, observer 2 and the Mahony observer are respectively equal to  $1.00^\circ$ ,  $2.47^\circ$  and  $4.60^\circ$ .

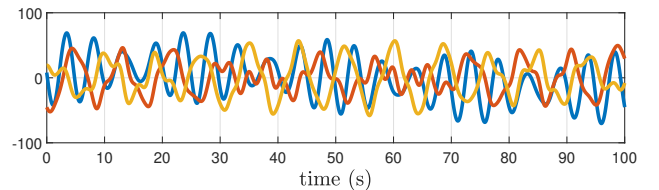


Fig. 1: Angular velocity  $\omega(t)$

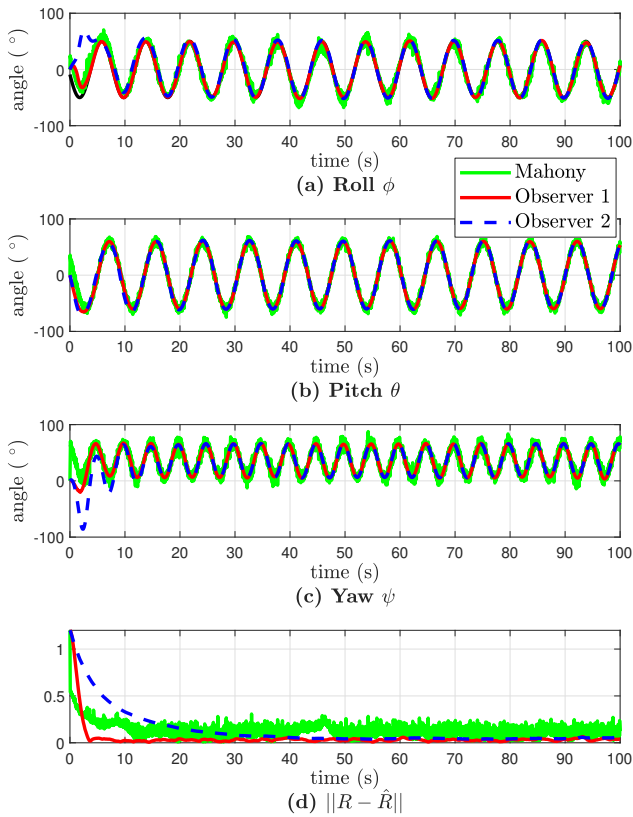


Fig. 2: Euler angles and error  $\tilde{R}$  with no external acceleration. A second simulation is considered with medium acceleration. The external acceleration applied on the rigid body is given on figure 3 with its filtered version. The external acceleration is well filtered, since its magnitude is divided by ten, which means that the time constant of the low-pass filter is correctly chosen. The Fourier transform of the applied external acceleration is depicted on figure 4. The norm of the error matrix  $\|R - \hat{R}\|$  are reported on figure 5. The two proposed observer performed very well, while the impact of the external acceleration on the Mahony observer is quite high. The MSE of observer 1, observer 2 and the Mahony observer are respectively equal to  $2.41^\circ$ ,  $3.80^\circ$  and  $11.99^\circ$ .

In the third simulation, the external acceleration is taken as twice the acceleration in the second simulation and is then quite high (the same order of magnitude as the gravitational constant). The reconstructed Euler angle and the error  $\|R - \hat{R}\|$  are given on figure 6. Observer 1 still performs very well, observer 2 estimations are pretty good, but the Mahony observer is completely off. The MSE of observer 1, observer 2 and the Mahony observer are respectively equal to  $2.95^\circ$ ,  $7.88^\circ$  and  $33.53^\circ$ .

## V. CONCLUSION

We have presented two observers who provide an estimation of the attitude of a moving rigid body subject to external acceleration. The stability of these two observers are guaranteed through a Lyapunov approach. The good performances of the proposed approach have been illustrated through several simulations.

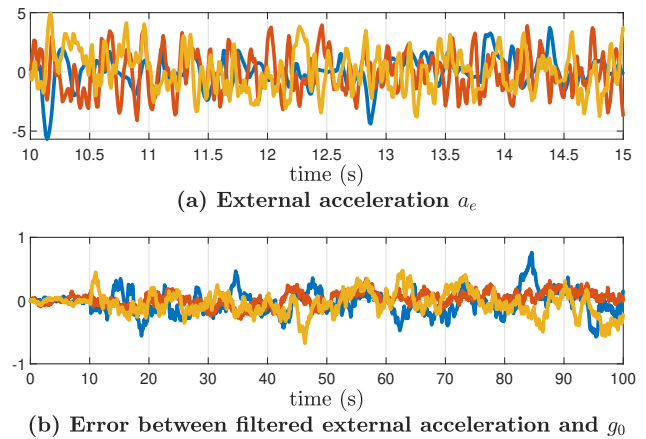


Fig. 3: External acceleration and difference between the filtered acceleration and  $g_0$  in the inertial frame for medium acceleration

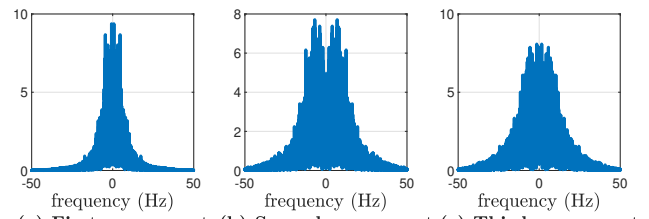


Fig. 4: Frequency representation of the external acceleration

The main drawback of the proposed approaches is the fact that the dimension of the proposed observer is higher than the classical  $SO(3)$  observers, due to their geometry free structure. But the provided simulations show that the external acceleration are well filtered and the performances are very promising.

In future works, the authors will try to transform the observers so that their estimate evolve on  $SO(3)$  in order to reduce the dimension of the proposed observers. Furthermore, a thorough analysis of the effect of the external acceleration on the estimation error would be interesting in order to tune the gains in an optimal way.

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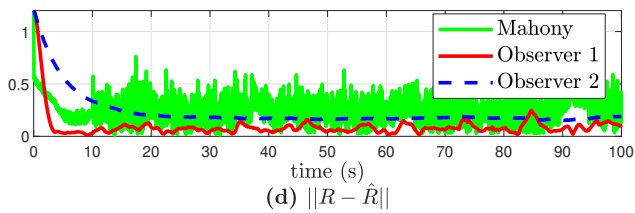


Fig. 5: Error  $\tilde{R}$  with medium external acceleration

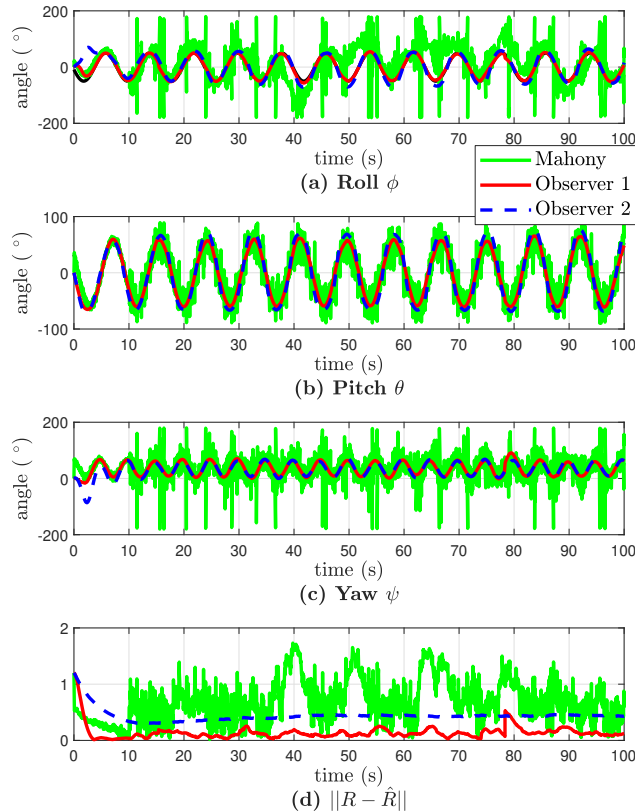


Fig. 6: Euler angles and error  $\tilde{R}$  with high external acceleration

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