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ACOUSTIC SIMULATION OF BACH'S PERFORMING FORCES IN THE THOMASKIRCHE

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ABSTRACT

Since room acoustics profoundly affect musicians' performance style, renderings of early music should account for historical acoustics as well as instrument design and ensemble size. This paper describes the recording process for the project *Hearing Bach's Music as Bach Heard It*, which uses acoustic measurements and geometric acoustic modeling to render the soundscape of Bach's Thomaskirche in Leipzig, Germany in 1539 and 1723, the year Bach arrived. This historical model was used to calculate section-to-section binaural impulse responses for each section of musicians on a Bach cantata. Using real-time convolution and close-miking, the musicians were recorded while performing in the virtual church at both time periods, with audible differences between the two. Some discussion follows as to the optimal method for arranging dry multitrack recordings of historical works when separate anechoic chambers are not available for each musician.

1. INTRODUCTION

The study of cultural heritage sites is often undervalued when the objects of their study are well-preserved: indeed the very preservation of historically significant sites often leads us to take them for granted, assuming they will always be with us. This cultural tendency was briefly arrested after the colossal fire at Notre Dame de Paris on 15 April 2019. Because of the specific musicological importance of Notre Dame's reverberant acoustic, some feared that the church's soundscape might be permanently lost [1]. However, due to the extensive measurement and simulation work of Postma, Katz, and others, the historical acoustics can be rebuilt including the contributions of individual surfaces and materials [2]. One hopes that this

tragedy will give greater urgency to the task of acoustically cataloging historically significant sites which have not been as extensively studied as Notre Dame.

Besides applications in gaming and virtual reality, acoustic simulation technology also has potential to transform historical musicology by rendering audible lost soundscapes from the past [3–8]. This in turn may allow the role of historical spaces to be explored along with historical instruments and ensembles [9]. This requires individual recordings of each instrument being simulated, which involves many performers in the case of much of the Western classical repertoire. Simulation is also complicated by the fact that trained performers naturally alter their tempo and articulation in different acoustic environments [10].

Another era in European classical music that is distinctly tied to a specific place is the mature compositional period of J.S. Bach (1685–1750) in Leipzig, Germany. This paper describes the project *Hearing Bach's Music as Bach Heard It*, which addresses the acoustic sound field presented by the Thomaskirche in Leipzig, Germany in 1723, the start of J.S. Bach's tenure as cantor. In addition, alterations to the virtual model also allow an investigation of the acoustics before the alterations to the church by the Lutheran Reformers in the 16th century. An entire Bach cantata, *Herz und Mund und Tat und Leben* (BWV 147), has been recorded to virtually place the musicians in the virtual church at these two points in the space's history.

2. BACKGROUND

2.1 Bach and Acoustics

Much of the historically informed performance movement has focused on the work of Bach, including the construction of historical instruments more similar to what Bach's musicians would have used [11]. In addition, musicologists disagree over whether Bach used a small (1 voice per part) or large (3–4 voices per part) chorus for his cantatas in the Leipzig Thomaskirche and other churches [12–15]. One recent work arguing for a much larger vocal ensemble relies almost exclusively on general acoustical arguments but produces no quantitative evidence in support of this conclusion [16].



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Recordings of Bach’s choral works tend to ignore the issue of room acoustics, or implicitly manipulate the reverberation present to favor their preferred conclusion about ensemble size: compare Joshua Rifkin’s (relatively dry) 1987 recording of BWV 106 to Masaaki Suzuki’s 1996 version, which instead features longer (artificial) reverberation and slower tempos. Surely this is begging the question: certainly smaller ensembles benefit from a more intimate acoustic, while larger choirs can make better use of a long reverberant tail, but what acoustic conditions was Bach really working with? Room acoustic conditions profoundly affect composers and performers: for instance, Bach’s role model Dietrich Buxtehude composed for the much larger church of St. Mary’s in Lubeck, where his choir was forced to sing in unison to guide the congregation through the space’s much longer reverberation time [17].

Bach was an organist and consultant with decades of experience in carefully listening to the acoustics of different churches. He spent the last 27 years of his life in Leipzig, composing and leading music for all the major liturgical and civil spaces of Leipzig. Bach had a strong preference for the Thomaskirche, possibly because he deemed it superior for choral music [18]. The combination of Bach’s keen ear and his close association with the Thomaskirche make this church one of the most significant performance spaces in the history of Western music. However, the church has been altered many times, and its current interior is quite different from Bach’s time [19].

In 1930 English acoustician Hope Bagenal, along with Bach scholar C.S. Terry, put forth the hypothesis that the relatively clear acoustics of the Thomaskirche were established by renovations during the Lutheran Reformation in the 16th century [20, 21]. Their primary argument was that the Lutherans’ removal of screens (ensuring clear sight lines) and addition of seating galleries would have increased the musical clarity and reduced the church’s reverberation time, creating a more clear acoustic channel for Bach’s complex polyphonic music. Bagenal’s 1930 article provided some rudimentary calculations (based on Sabine’s formula) to argue for his conclusions. This study aims to put his hypothesis to the test by simulating the effect of the Reformation alterations on performers singing in the Bach-era church through a real-time auralization.

2.2 History of the Thomaskirche

Though Bagenal’s narrative of the church’s history was a bit simplistic, he did correctly name the 16th century alterations as the most relevant to the soundscape during Bach’s tenure there. The earliest known construction on the site was a late-Gothic church dating from the mid-12th century, which experienced several different alterations from 1200-1500 [19]. Though the majority of the galleries were added after the Reformation, a smaller choir gallery existed at the west end of the nave as early as 1498 [22]. After Martin Luther preached in the church in 1539, early bouts of iconoclasm resulted in the removal of the rood screen from the chancel and the destruction of all but one

of the church’s fifteen altars. Due to the increased importance of the sermon in the Protestant service, the church required additional seating (note that the city did not state any acoustical motivation for this change, only the desire, common to many concert halls, to cram in as many listeners as possible). Thus in 1570-71 Hieronymus Lotter supervised the expansion of the west gallery (in blue on Fig. 1) for the Thomanerchor to occupy, and built the “Renaissance” galleries on the south and north of the nave, highlighted on Fig. 1 in brown [23].

These stone galleries exist today in part, but drawings such as that of Lilo Häring (Fig. 2) show that these were originally double-level galleries, similar to those still present in Leipzig’s Nicholairkirche today [23]. The upper-level galleries were constructed entirely from wood while providing wooden pews full of congregants [24], which would have certainly increased the acoustic absorption compared to the pre-Reformation church with much lower seating capacity.

By Bach’s time the upper galleries themselves stretched around to the sides of the west choir gallery, elevated “town piper galleries,” (Fig. 3)¹ built in 1632, elevated above both side of the vocalists [23, 24]. The strings played from the south gallery, and the winds from the north gallery, while the continuo, trumpet, and drums (if any) were positioned around the organ’s *Rückpositiv* [24]. Bach himself generally led from the south gallery, playing the violin [16].

3. METHOD

3.1 Acoustic Measurement and Simulation

The current project uses a combination of acoustic measurements in the current church and calibrated geometric acoustical (GA) modeling to simulate the acoustics of the church today and as it existed in 1723 (the beginning of Bach’s tenure there) as well as the 1539 church with its lower seating capacity, rood screen, and intact altars. The church’s room impulse response was measured at the source and receiver positions shown in Fig. 1 using a custom-built dodecahedral loudspeaker, a CoreSound TetraMic, and an exponential swept sinusoid [25]. Full details of these measurements will be addressed more fully in a future publication, but the empty church’s mid-frequency T30 value was about 3.7s, similar to that measured before the church’s renovations in the 1960s [26].

A GA model of the empty present-day church was constructed in CATT-Acoustic v.9.1 [27, 28]. The acoustic measurements were used to calibrate the material scattering and absorption coefficients within known surface values [29, 30]. Using this calibrated present-day model, two additional models were created: one for the pre-Reformation era church (c. 1539), and one for the post-Reformation church (c. 1723), corresponding to the state of the space when Bach assumed the position of cantor

¹ In private conversation Christoph Wolff has confided to the authors that he is revising this image to ensure that the lowest part of the piper galleries were even with the top of the lower gallery, unlike in Figure 3.

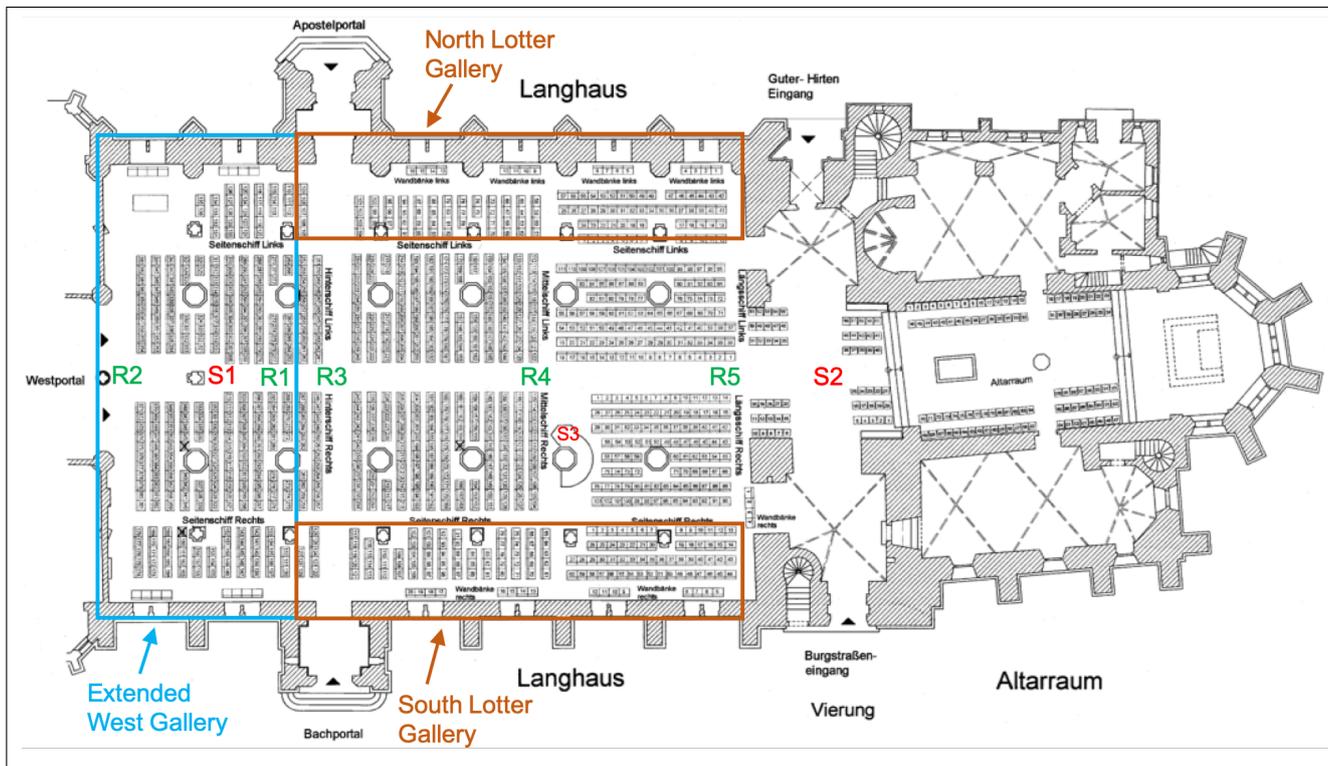


Figure 1. Plan of Thomaskirche, with positions of Lotter galleries and source/receiver positions during impulse response measurements.

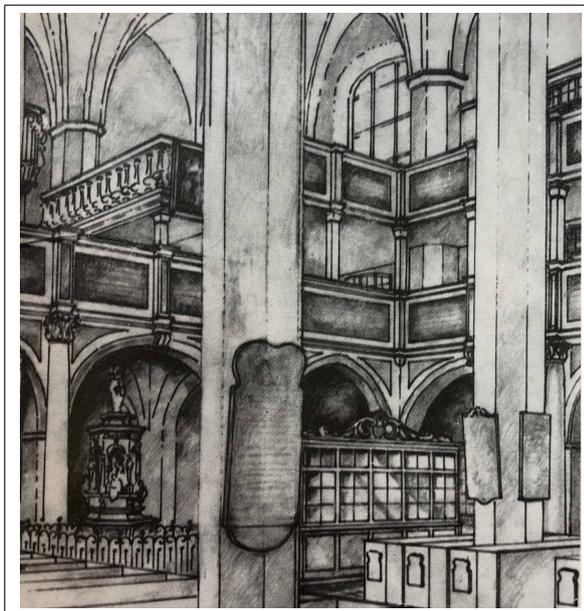


Figure 2. Reconstruction of the second level of galleries that existed in Bach's time [23].



Figure 3. Wolff's reconstruction of the town piper galleries around the west choir gallery [24]. The strings (and Bach) would have been in the south gallery (on the left), the woodwinds would have played from the north gallery opposite, with the choir one level down on the larger west gallery.

there.

To create this calibrated GA model of the church in its present form, we started by modeling the church's outer geometry directly in CATT. This geometry is based on measurements taken from drawings of the church which we then verified on site with laser-based distance measurements. With the outer geometry of the church constructed, we added the surrounding galleries and pews, mapping seating areas as audience planes. Lastly, we assigned preliminary absorption and scattering coefficients to all surfaces. While some surfaces were common and had known scattering and absorption coefficients, others were unknown and required estimation and calibration. For the absorption coefficients of unknown materials, we began by using absorption data of similar materials with known absorption. Scattering coefficients were calculated using CATT's estimate function [30], given by

$$\text{scatt}(f) \begin{cases} \leq 0.99 \\ \geq 0.10 \end{cases} = 0.5 \sqrt{\frac{d}{\lambda}}, \quad (1)$$

where λ is wavelength and d is the characteristic depth of the surface in meters.

3.2 Cantata Selection and Forces

The cantata *Herz und Mund und Tat und Leben*, BWV 147 ("Heart and Mouth, and Deed, and Living") was selected as the initial test piece for the process. Chosen for its variety of instrumental and solo vocal forces, the work features a highly contrapuntal *da capo* opening chorus, plus iterations of the well-known chorale setting – commonly known as "Jesu Joy of Man's Desiring" – as the sixth movement and with additional verses set as the final tenth movement of the work. It is a fairly rare work within the Bach oeuvre in its inclusion of solo arias for all four voices types (soprano, alto, tenor, and bass). Additionally, the solo movements and recitatives feature an astounding array of obbligati instruments including oboe d'amore, violin, organ, trumpet, and two oboe da caccia.

Originally composed for Weimar in 1716 (BWV 147a), the work was adapted and expanded for use during the Feast of the Visitation of the Blessed Virgin Mary in 1723 as part of Bach's first annual cantata cycle for Leipzig. The majority of movements are in simple or compound triple meter with the opening chorus notably consisting of 66 measures; the structures allude to the Holy Trinity and illustrate Bach's interest in numerology and his interest to connecting musical form to theology.

The recording was made with early-music specialists all performing on period instruments and bows. For the recording process, the choruses and chorales were recorded both following the "traditional choir" approach using four singers to a part as well as the "OVPP" (one vocalist per part) understanding alongside varied string forces with a larger tutti ensemble of three violin I, three violin II, two violas, two violoncelli, and one violone (eleven strings) and a smaller string configuration of 2-2-1-1-1 (seven strings) respectively. The opening chorus and

chorales movements do not lend themselves easily to concertist and ripienist contrasts and additionally only a single set of solo vocal parts survive, eliminating the necessity to consider this kind of deployment.² Solo movements, when involving strings (Nos. 2 and 9) were recorded using the smaller string configuration.

3.3 Cantata Recording

Using this historical model, we have produced virtual sources for the primary musician groups present in Bach's time, including strings, woodwinds, continuo, trumpet, organ, and the choir. These sources have been placed in the GA model of the 1723 Thomaskirche's double galleries (which no longer exist) based on historical evidence of their locations during Bach's cantata performances. Each of these 6 sources was also used as a receiver, and the GA was used to generate binaural impulse responses for the 6x6 matrix of source-receiver combinations. In addition, three congregational source positions were simulated at different distances from the musicians along the nave of the church.

Since isolation was crucial for the accurate generation of the simulated acoustic environment, the microphones used were mainly directional condensers, and placed for the most optimal compromise between isolation, proximity effect and comb-filtering.

Miking the instruments individually also added to the overall complexity of the setup and utilization of the available physical space. For this reason the main control room in Kreeger Studios was used as an iso booth for the singers, with a video feed from the live room. The session was then recorded from a smaller API1608-based control room, with the addition of eight Grace Design preamps, for a total track count of 34 channels. The other instruments were placed in the live room with the conductor.

The software used was Pro Tools Ultimate, with HDX/HD I/O converters set to a sample rate of 48kHz and 24-bit. Higher sample rates were not used in order to allow for sends to multiple instances of convolution reverb plugins with minimum latency and CPU-processing. The simulated binaural impulse responses for each section-to-section path were then sent out from the reverb channels in Pro Tools to the musicians' headphones.

The recording sessions took place in August 2019 with the main ensemble in a dry recording studio, with additional absorbing surfaces placed between sections, reducing but not eliminating bleed entirely. Using this setup, different musicological variables (i.e. small versus larger instrumental and vocal forces) and acoustical variables (i.e. the BRIR from 1723 or 1539) could be tested during multiple recordings of the same chorale. This method allows the final auralization to incorporate the effect of the church's acoustics on the performance as well as the response at the audience's position.

Further discussion is needed for an optimal method for

² See A. Parrott, The Essential Bach Choir [14], for a summary of the use of *concertists* and *ripienists* (ch. 4) and for a discussion of the various views on instrumental forces (ch. 9).

recording large ensembles dependent on the latency and bleed present in different setups, as well as the infrastructure needed to simultaneously record multiple performers in multiple rooms. In addition, as the project also involved the goal of creating an artistically sound final product, some tradeoffs had to be made between purely experimental virtual setups for shorter excerpts and finding stable positions where instrumentalists could hear each other clearly enough to play the entire cantata in the constrained studio time allowed.

4. DISCUSSION

Though it is well established that performers' tempo and articulation are shaped by the acoustics of their performance space [10], the question of how to recreate historical acoustical factors for ensemble performance has not yet been fully answered. Ideally we would like to have anechoic recordings of each instrument [31], but attempts to do this require a 'scratch track' that musicians play along with, which pre-determines the recording's tempo and reduces the feeling of realistic live performance for the musicians. The Virtual Haydn project, in contrast, provided virtual rooms according to historical simulations, but since only one (keyboard) player was needed, the recording could be conducted in a dry environment with no bleed [3]. In this case, though we did not have access to separate anechoic recording environments for each instrument, the use of a full recording studio allowed sufficient isolation for louder instruments, minimizing bleed while providing a very low noise floor. Close-miking each instrument allows each to be auralized separately, a technique which has provided good results even for fully reverberant environments [32].

While space constraints sometimes led to tradeoffs in player comfort during the recording process, this setup generally allowed the ensemble to experience the virtual space at two distinct moments in its history and adjust their performance accordingly. Although it would have been possible to simulate every individual instruments' binaural impulse response as heard at each musician's position, the amount of realtime convolutions necessary for this raised the risk of increased latency, as well as the possibility of crashing the ProTools setup; as a result the "section by section" approach was deemed good enough to render the macro-changes in the church's acoustics while still accounting for the spatial distances between the sections. Closed-back headphones (though not the preference of most working classical musicians) allowed the performers to hear the virtual space rather than the recording studio, while also minimizing bleed.

5. CONCLUSIONS AND FUTURE WORK

While the use of virtual models cannot replace documentary evidence in support of a musicological conclusion, so-called Computational Musicology (and the Digital Humanities more generally) can provide new kinds of evidence which must be interpreted within a larger musicological

framework. The twisting historical path of Bach's influences, from religion to architecture to music can only be investigated through an acoustical lens, which may bring to light new aspects of the composer's practical limitations and aesthetic decisions. As VR technology and room modeling become more widespread, it is hoped that such techniques will also inform the standard practice in early music performance and recording.

The recordings from these sessions will have final reverb filters applied directly in CATT for a series of source-receiver auralizations at different listening points in the virtual Thomaskirche. Working with web and UX designers, the next step in the project will involve a user interface allowing listeners to experience the cantata from a series of listening points on the floor or in the galleries.

Future investigations will include other spaces in Bach's performance career and various genres of concerted pieces. Some acoustical aspects, such as diffraction effects, which are not adequately rendered by the GA model, can be modeled using a hybrid system employing Finite Difference modeling at low frequencies and ray tracing at high frequencies. In addition, future recordings may be conducted in a dry sound stage with more options for reconfiguring the ensemble to allow better isolation of each instrument while still rendering the real-time convolutions in such a way as to create a realistic feeling that the musicians are playing together in the present, but also in the past.

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