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# Influence of spectrum spreading on the SEAREV wave energy converter

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## ABSTRACT.

This paper describes how the production matrix of the SEAREV wave energy device is modified by the directional spreading of the wave spectrum and the resulting influence on the annual production at a given test site.

A cosine power '2s' function applied to Pierson-Moskowitz spectrum is chosen to model the spreading. The linearized equations of the device, including the motion of the internal pendulum wheel are presented. Numerical results are presented for the power recovered by the device in random sea. Influence of the sea state on the recovered energy was evaluated using an increasing spreading parameter. We show that the results obtained when accounting for the spectrum spreading may be very different from those obtained when considering a mono-directional wave train.

**KEY WORDS :** Wave energy; point absorber; spreading function; Pierson Moskowitz spectrum; directional wave spectrum.

## INTRODUCTION.

Directional spreading is a fundamental property of a random wave field and is a key element for modelling the behaviour of a body interacting with ocean waves. Information about the directional distribution of wave energy is required for the proper prediction of various oceanographic and geomorphologic phenomena. However, most of the reported numerical simulations in wave energy only take into account a mono-directional wave train. Here, we want to show that directional spreading may have a large influence on the mean annual production of wave energy converters (WECs).

The power recovered by our device was calculated by using three-dimensional linear time-domain program. This program was based on linear theory using three-dimensional hydrodynamic coefficients and excitation forces computed by the seakeeping computer program AQUAPLUS.

Several methods to simulate random waves with a spreading function can be found in the related literature : the cosine power-models, the wrapped-around Gaussian model, the Von Mises formula, the hyperbolic type model, the double peak spreading model ; among them,

the most commonly used is the cosine power-model.

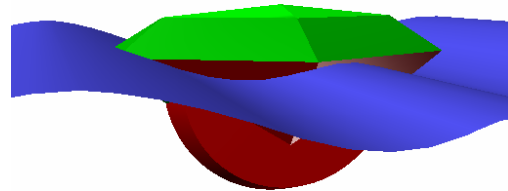


Figure 1 : The SEAREV device.

In our study, we have considered Yeu Island, on the French Atlantic coast as a test site. This place can be considered like a case of fully developed sea, in consequence the Pierson Moskowitz spectrum was used. The significant wave height and the peak period were the only sea states statistics available for this site in the meteorological data bases. This data are not sufficient to simulate a real random sea accounting for the effects of wave directional spreading. So, to model this effect, we are chosen a numerical method implementing the cosine power-models. This method uses a spreading parameter  $s$  which corresponds to a specific sea state in term of directional spreading. Generally the spreading parameter should be estimated from measured data. There are several ways to do so. Cartwright (1963) represented the spreading function into a Fourier series, Mitsuyasu et al (1975) showed that maximum spreading parameter corresponding to the peak frequency of wave spectrum can be determined from the nondimensional parameter of wave age, which is the ratio of wave phase speed to wind speed. Goda (1965) recommended for  $s$  value of 10 for wind waves, 25 for swell with short decay and 75 for swell with long decay distance. Here we have chosen to compute the recovered energy for 6 different values for  $s$ .

In the following section we develop the complete set of equations governing the interacting motions of the floating device and the on-board moving mass.

## GOVERNING EQUATIONS

The SEAREV device, currently under development at Ecole Centrale de Nantes, is a second generation WEC. It is a floating device,

completely closed, featuring an embarked pendulum wheel. Under the action of the waves, the floating hull and the internal pendulum start to move, each one with its own motion. The relative motion between them is then used to drive a electrical generator via a hydraulic Power Take Off (PTO) system.

### Assumptions and notations.

The SEAREV device is assumed to have two vertical planes of symmetry. We assume that the axis of the internal cylinder, around which it rotates, is perpendicular to the main symmetry plane of the floating body. We assume that the mean direction of wave spectrum is aligned with the plane of symmetry along the abscise direction Ox. Under these assumptions, in a 3D incident wave train, the floating body moves in surge, heave, swell, pitch, roll and yaw mode.

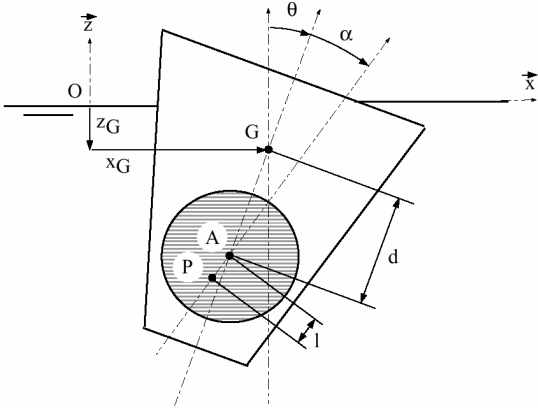


Figure 2 : SEAREV notations.

Let  $x_G$  the surge motion,  $z_G$  the heave motion of the centre of gravity  $G$  of the floating body,  $\varphi$  the roll motion,  $\theta$  the pitch motion,  $\psi$  the yaw motion and  $\alpha$  the relative motion between the floating body and the inner pendulum. (the Power Take Off -PTO- system works this relative motion a). Let  $m_b$  the floating body mass,  $I_b$  its inertia,  $m_p$  the inner pendulum mass,  $I_y$  its inertia. Let  $l=AP$  the distance between the center of rotation  $A$  of the inner pendulum and his own center of gravity  $P$ ,  $d=GA$  the distance between the center of gravity of the floating body and the center of rotation of the pendulum. We assume the amplitude of the waves and the resulting motion of the body to be sufficiently small to validate the classical linear theory approach.

### Equations.

Let  $Z = (x_G, y_G, z_G, \varphi, \theta, \psi, \alpha)^T$  the position vector. In linearized theory and in frequency domain, one can write the equations of motion as :

$$\begin{bmatrix} -(M + C_M(w))w^2 \\ + i(B + C_A(w))w + (K + K_H) \end{bmatrix} Z = F_{ex}(w) \quad (1)$$

With :

$F_{ex}$  : the hydrodynamic excitation force due to the incoming and diffracted wave.

$C_M$  : the added mass matrix.

$C_A$  : the hydrodynamic damping matrix resulting from the radiation of waves by the body.

$B$  : the external damping matrix

$M, K$  : the proper mass matrix and the stiffness matrix of the device respectively.

$K_H$  : the matrix of hydrostatic stiffness.

We can note that the SEAREV device has two symmetry axes in the vertical plane, so the equations of motion for the heave mode and the yaw mode are uncoupled from the others and do not contribute to energy absorption in linear theory. So we will keep only in the future  $x_G, y_G, \varphi, \theta, \alpha$  as unknowns in the forthcoming equations.

The last two matrices  $M$  and  $K$  contain coupling terms between the pendulum and the floating body which are given by :

$$M = \begin{pmatrix} m_b + m_p & 0 & & & \\ 0 & m_b + m_p & & & \\ 0 & m_p(l-d) & \dots & & \\ m_p d & 0 & & & \\ -m_p l & 0 & & & \\ & 0 & m_p(d-l) & & -m_p l \\ & m_p(l-d) & 0 & & 0 \\ \dots I_{by} + I_{py} + m_p d(l-d) & & 0 & & 0 \\ & 0 & I_{py} + I_{py} - m_p d(l-d) & & -m_p d l + m_p l^2 + I_{py} \\ & 0 & I_{py} + m_p l(l-d) & & I_{py} + m_p l^2 \end{pmatrix}$$

$$K = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_p g d & 0 & 0 \\ 0 & 0 & 0 & 0 - m_p g d & 0 \\ 0 & 0 & 0 & m_p g l & m_p g l \end{pmatrix}$$

The matrix  $B$  represents the energy absorption system. In this simulation, the power take off is modelled by a simple linear damping torque applied at the center of rotation of the cylinder. It is given by the expression:  $-B_{PTO} \dot{\alpha}$ . So the external damping matrix  $B$  reads :

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B_{PTO} \end{pmatrix}$$

In the frequency domain, the mean power absorbed by the device during a wave period is given by :

$$\hat{P}(w) = \frac{1}{2} B_{PTO} |w \alpha(w)|^2 \quad (2)$$

In the time domain, the equations of motion are given by :

$$(M + C_M(\infty)) \ddot{Z} = F_{ex}(t) - B \dot{Z} - \int_0^t H(t-\tau) \dot{Z}(\tau) d\tau - (K + K_H) Z \quad (3)$$

Where the radiation forces have been written as a convolution product according to the well-known Cummins' decomposition. The mean absorbed power over a time T is now given by :

$$\hat{p} = \frac{1}{T} \int_0^T B_{PTO} \dot{\alpha}^2(t) dt \quad (4)$$

Computation of the hydrodynamic coefficients  $C_M$ ,  $C_A$ ,  $F_{ex}$  are performed by using the frequency domain seakeeping computer code AQUADYN [6], and the computation of the memory function:  $H(t)$  by using the time-domain seakeeping code ACHIL3D [5].

In time domain, a direct integration of the equation of motion (Eq. 3) can be done by using a fourth order Runge-Kutta algorithm, for example, or, alternatively, it can first be transformed in order to remove the convolution product by using Prony's method. This method has been implemented by Daubisse [6] and further developed by Clément [7] for the computation of impulse response of radiation forces.

This method compute couples of variables  $(\alpha_i, \beta_i)$  defining the following approximation of the real function  $H$  of the equation 3 :

$$H_{ij}(t) \approx \sum_{m=1}^{N_{ij}} \alpha_{ijm} e^{\beta_{ijm} t} \quad (5)$$

$H$  being a real function, either  $(\alpha_i, \beta_i)$  are real, either they are complex and systematically associated with their complex conjugates. So, if :

$$I_{ij}(t) = \int_0^t H_{ij}(t-\tau) \dot{Z}(\tau) d\tau \quad (6)$$

the computation of the convolution product in the equation 3 gives the following result :

$$\begin{cases} M \ddot{Z}(t) + B \dot{Z}(t) + I(t) + K Z(t) = F_{ex}(t) \\ \dot{I}(t) = \beta I(t) + \alpha \dot{Z}(t) \end{cases} \quad (7)$$

With,

$$I(t) = \begin{pmatrix} \sum_{m=1}^{N_{11}} I_{11}(t) + \sum_{m=1}^{N_{15}} I_{15}(t) \\ \sum_{m=1}^{N_{22}} I_{22}(t) + \sum_{m=1}^{N_{24}} I_{24}(t) \\ \sum_{m=1}^{N_{42}} I_{42}(t) + \sum_{m=1}^{N_{45}} I_{44}(t) \\ \sum_{m=1}^{N_{51}} I_{51}(t) + \sum_{m=1}^{N_{55}} I_{55}(t) \\ 0 \end{pmatrix}$$

$$\dot{I}_{ijm} = \beta_{ijm} I_{ijm}(t) + \alpha_{ijm} \dot{Z}_j(t)$$

Finally, if  $X = (Z^T \dot{Z}^T I^T)$  is the new state vector including the hydrodynamic states  $I$ , the generalized state equation of the system becomes :

$$\dot{X} = AX + C \quad (8)$$

With,

$$A = \begin{pmatrix} I_d & 0 & 0 \\ 0 & (M + C_M(\infty))^{-1} & 0 \\ 0 & 0 & I_d \end{pmatrix} \begin{pmatrix} 0 & I_d & 0 \\ -(K + K_H) & -B & \delta \\ 0 & \alpha & \beta \end{pmatrix}$$

$$C = \begin{pmatrix} I_d & 0 & 0 \\ 0 & (M + C_M(\infty))^{-1} & 0 \\ 0 & 0 & I_d \end{pmatrix} \begin{pmatrix} 0 \\ F_{ex}^t \\ 0 \end{pmatrix}$$

## RANDOM SEA MODELLING.

The basic premise of all parametric spreading function is that the single peak directional spectrum is described by :

$$E(f, \theta) = E(f) \cdot D(f, \theta) \quad (9)$$

where  $D(f, \theta)$  is directional spectral density function,  $E(f)$  one-dimensional energy spectral density function,  $D(f, \theta)$  angular spreading function  $f$  and  $\theta$  being the frequency and the direction.

## Frequency spread.

To compute an annual energy production, we needed a fully developed wave spectra which to its natural stability corresponds to a sea developed over a long period of time. The energy spectrum model used here is the Pierson Moskowitz spectrum, given by :

$$E(f) = \frac{A}{f^5} \exp\left[-\frac{B}{f^4}\right] \quad (10)$$

$$\text{with } A = \frac{5}{16} \frac{H_{1/3}^2}{T_p^4}$$

$$B = \frac{5}{4} \frac{1}{T_p^4}$$

and parameterised by the significant wave height  $H_{1/3}$  and the peak period  $T_p$ .

## Directional spread.

The direction spreading function can be modeled using a variety of parametric models (Pierson & al Cote & al, Longuet-Higgins & al, Donelan & al) [10]. Due to specifications of sites, no single model is universally accepted. Here the angular spreading function used is the cosine power '2s' model describes as follows :

$$D(f, \theta) = G(s) \cos^{2s} \left[ \frac{(\theta - \theta_m)}{2} \right] \quad (11)$$

where,

$$G(s) = \frac{2^{2s-1} \Gamma^2(s+1)}{\pi \Gamma(2s+1)}$$

is defined such as

$$\int_{-\pi}^{\pi} D(f, \theta) d\theta = 1$$

$D(f, \theta)$  being the directional spreading function,  $f$  the wave frequency,  $\theta$  the wave direction.  $\Gamma$  is the Gamma function,  $\theta_m$  the mean wind direction and  $s$  the spreading parameter which is, a priori, a function of frequency and wind speed.

This spreading function was proposed by Longuet-Higgins & al. (1963). Here the value of the parameter 's' controls directional spreading around the mean wave direction

After sampling by equal angular step  $\Delta\theta$  and by equal frequency step  $\Delta f$  the amplitude of each component of the wave of angular direction  $\theta$  and frequency  $f$  is given by :

$$a(f, \theta) = \sqrt{2E(f, \theta) \Delta f \Delta \theta} \quad (12)$$

and one can synthesize a representative sample of the waves for a sea state defined by the couple  $(H_{1/3}, T_1)$  by using :

$$\eta(x, y, t) = \sum_{i=1}^{N_\theta} \sum_{j=1}^{N_w} a\left(\frac{w_j}{2\pi}, \theta_i\right) \cos\left((k_x \cos \theta_i + k_y \sin \theta_i)x - w_j t + \varphi_{ij}\right) \quad (13)$$

where  $\eta$  is the free surface elevation  $w_j$  the circular frequency of the  $j$ -th wave component for one angular direction and phases  $\varphi_{ij}$  are randomly determined.

## THE TEST CASE SITE.

### Geographic situation.

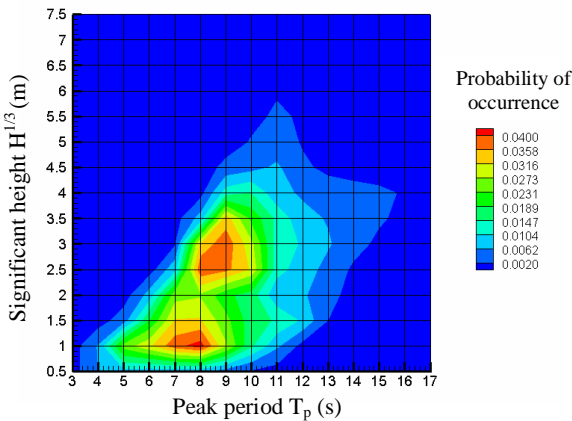


Figure 3 : Scatter diagram of the resource of Yeu island.

In our study, Yeu island, on the west coast of France, was considered as test case site. Two years of sea states statistics were available for this site (Figure 3).

### Energy and power.

For a unidirectional wave system, the power transported per meter of crest length is :

$$P_{available} = \rho g \int_0^{\infty} C_g(f) E(f) df \quad (14)$$

With  $C_g$  the group velocity and  $\rho, g$  respectively the volumic mass of the fluid and the gravity.

If we consider a directional spectrum, then this formula gives the power being transported across a fixed circle of unit diameter. It was not exactly the goal of our study. So, we assumed that our device went and stood naturally to the same direction of the primary wave , consequently the waves could spread out around the device symmetrically in relation to its direction (Figure 4).

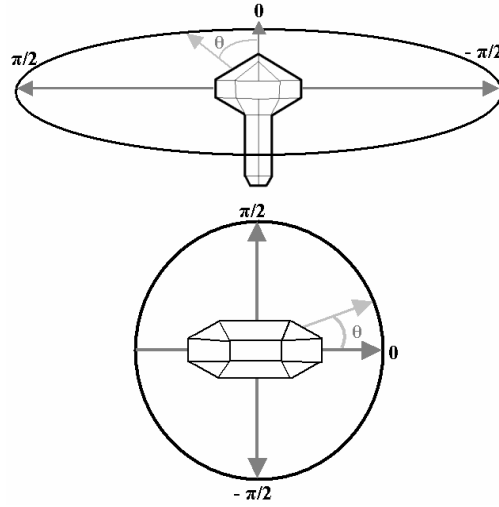


Figure 4 : Representation of wave direction in relation to the SEAREV device position.

Then the mean wave direction  $\theta_m$  became equal to  $0^\circ$  in the equation 3. We assumed that the waves distribution was over an angular range of  $\pm 90^\circ$  around  $\theta_m$ , which is a relatively good assumption for the idealized angular distribution when  $s > 5$  (Figure 5).

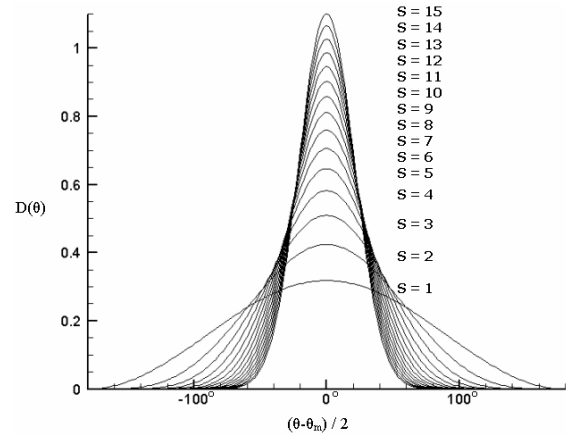


Figure 5 : Idealized angular distribution.

Finally, we obtained the following relation :

$$P_{available}^+ = \rho g \int_0^{\infty} df \int_{-\pi/2}^{\pi/2} C_g(f) E(f, \theta) \cos(\theta) d\theta \quad (15)$$

To calculate the annual available energy of this site, one must weight the available power for all possible sea states by their probability of occurrence C given by the previous scatter diagram (Figure 3).

#### MEAN ANNUAL EXTRACTED POWER.

In linearized theory, the mean extracted power for a given sea state is :

$$\hat{P} = \sum_{j=1}^{N_w} \hat{p}(w_j) \left[ a \left( \frac{w_j}{2\pi}, \theta_i \right) \right]^2 \quad (16)$$

where the amplitudes  $a$  only depends on the sea state and  $\hat{p}(w_j)$  is given by equation 4. Like in the previous section, to calculate the total absorbed energy E over a full year, one must sum the power absorbed for all possible sea states weighted by their probability of occurrence C given by the previous scatter diagram (Figure 3). We have :

$$E = \sum_{H_{1/3}} \sum_{T_p} C(H_{1/3}, T_p) \times \sum_{j=1}^{N_w} \hat{p}(w_j) a \left( \frac{w_j}{2\pi}, \theta_i \right) \quad (17)$$

#### RESULTS.

##### Analytical study.

Analytical study has been realized on the available energy of a multi-directional wave field. The infinite depth assumption was applied to Eq. (15), and the spectral density  $E(f, \theta)$  was described in Eq. (10). The equation was then written as :

$$P^+ = 410,3.H^{1/3^2}.T_p.G(s).Q(s) \quad (18)$$

With,

$$Q(s) = 2 \int_0^{\pi/2} \cos^{2s} \left( \frac{\theta}{2} \right) \cos(\theta) d\theta \quad (19)$$

The superscript + means that the cosine function is positive. Finally we had :

$$P^+ = 0,410.H^2_{1/3}.T_p.H(s) \quad (20)$$

The behaviours of the functions  $G(s)$ ,  $Q(s)$  and  $H(s)$ , defined previously, in relation with the spreading parameter  $s$  are shown in the figure 6. An energy loss of 10% was observed for values of  $s$  around 10. This loss becomes smaller and smaller while  $s$  becomes greater (Table 1) and converges to 0.

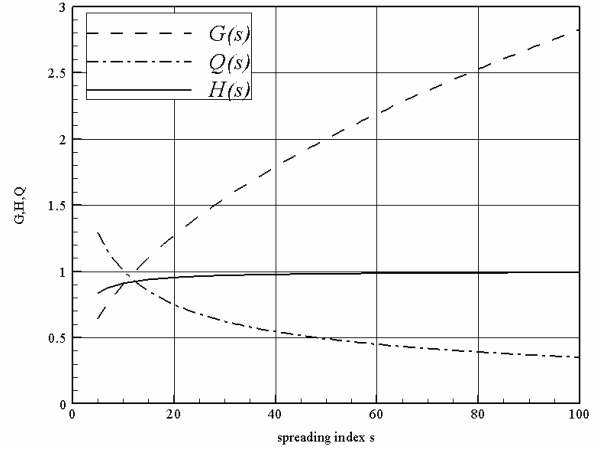


Figure 6 : Behavior of  $G$ ,  $Q$  and  $H$  functions in relation to  $s$ .

We can conclude that spreading has not significant impact on the available energy.

Table 1. Influence of  $s$  on the function  $H(s)$ .

s	H(s)
5	0.83434
10	0.90910
30	0.95238
50	0.98039
100	0.99010

##### Numerical results.

The numerical results in the following are based on the floating body SEAREV2595. Its width was 15.3 meters and its volume cubic root was 11m. We must note that in this study, no body motion control was applied on the relative motion of the inner pendulum.

We computed annual energy production with six values of the spreading parameter  $s$  : 2 and 5 to represent wide distribution spectrum, 10 for the wind waves, 30 and 50 for different short crest waves and 100 to be near to the mono-directional case.

The table 2 compares the mean annual capture width and the energy ratio for each value of the spreading parameter  $s$ . The capture width of a device for a particular sea state was calculated as the absorbed power resulting from the numerical simulation divided by the incident wave power for that simulation. The capture ratio for a particular sea state was calculated by the capture with divided by the volume cubic root of the device.

The results for the current case  $s=10$  showed a capture width of 4,28m, that means that the device absorbs around 39% of the energy passing through its own width. The case near the mono-directional wave train,  $s=100$ , presents a capture width of 4,65m and energy ratio of 42%. These results show us a weak influence of the directional spectrum with an absorption rate of a current sea state which just allows a benefit of 3% in relation to the idealized case of mono-directional wave train.

Table 2. Variations of mean annual capture width and absorbed energy with spreading parameter.

S	Mean annual capture width (m)	Capture ratio.
2	3.22	29%
5	4.11	37,4%
10	4.28	38.9%
30	4.52	41.1%
50	4.59	41.7%
100	4.65	42.2%

The figure 6 shows the influence of the spreading parameter on the capture ratio.

However, the effect of the directional spectrum is undeniable. We see easily that more  $s$  increases to  $s=100$  and more capture ratio increases, so consequently more the sea state is near the mono-directional case and more the SEAREV device absorbs energy. For example, there is a factor 1,4 between the extreme cases  $s=2$  and  $s=100$ . We can notice that the loss of energy between spreading parameter value is less important when  $s$  is more and more close to 100.

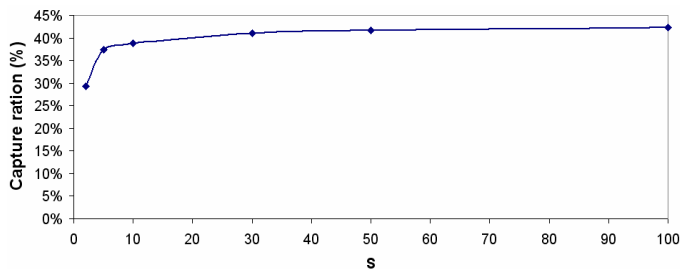


Figure 4 : Variation of capture ratio with spreading parameter.

## CONCLUSION.

The study of the available energy of a random sea in relation to the spreading parameter  $s$  showed that the contribution of the spreading in the formulation is negligible. The numerical results showed that we can't neglect the directional spectrum when we want to correctly predict the mean annual recovered power of a WEC in extreme cases. It is evident that a reliable estimation of the directional wave properties at a site is necessary prerequisite to correctly predict the annual energy production but a study like this present work can give some important information about the effects of the directionality of the sea.

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