Simplified model for droplet transient heating

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1. Introduction

Transient heating occurs in rocket engines, in particular in the following cases :

1. After injection, the fuel or oxidizer liquid droplets are not stabilized, and one can observe a relaxation period,

2. For stabilized droplets, acoustic waves generated by the engine cause departure from the stabilized regime and can originate high frequency instability.

It seems that thermal exchange inside the droplet has an important effect on transient behavior and on the vaporization stability.

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There is often a problem with numerical codes which are more and more complex. Then one needs simple analytical models, able to describe transient heating.

We will present here:

- First: a discrete (two-layer) model for heat transfer inside the droplet

- Secondly: a study of the response of an evaporating droplet to an acoustic field

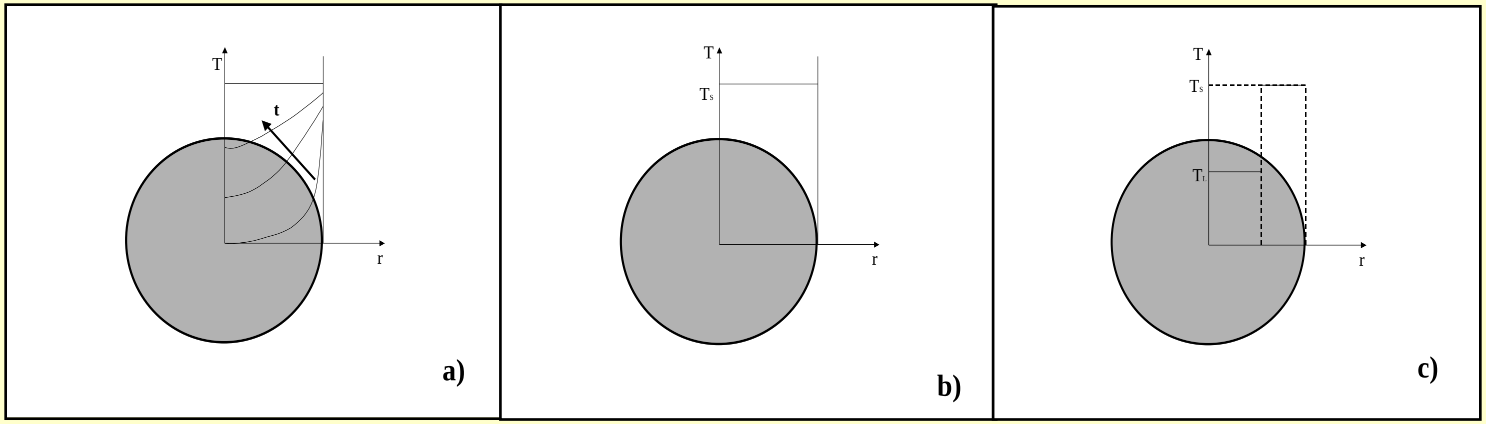
Basic considerations on transient heating and dynamic response of an evaporating droplet are discussed.

The two-layer model is proposed here to take into account heat transfer inside the droplet (assuming a finite heat exchange coefficient between both layers).

High frequency combustion instability in liquid rocket engines is a result of coupling between combustion processes and the chamber acoustics. Droplet evaporation is one possible driving mechanism of combustion instability.

In order to investigate this possibility, we consider an evaporating droplet submitted to an acoustic field. The objective of the study is to determine droplet response.

2. Two-layer model for droplet transient heating



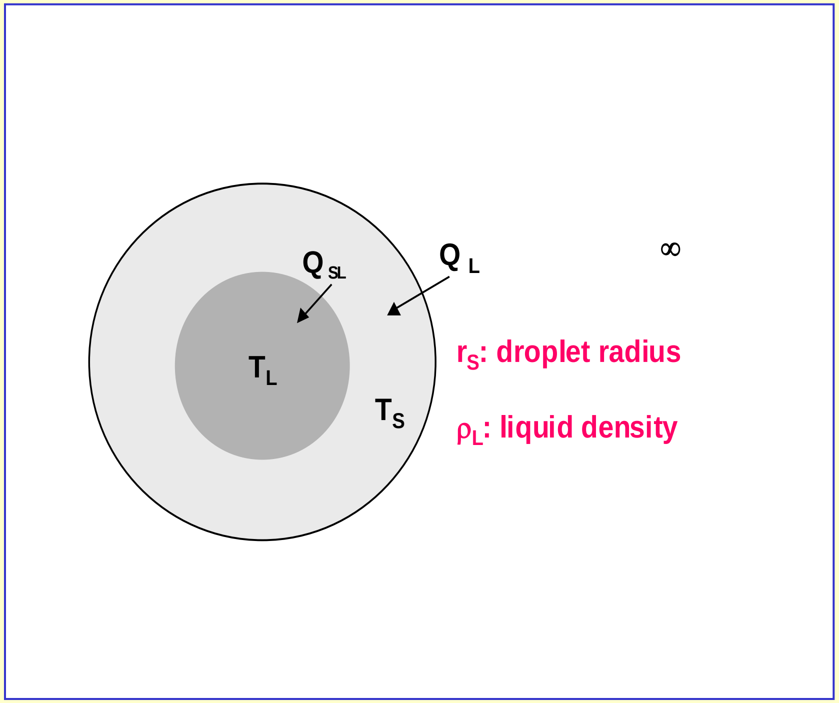
Temperature profiles inside the liquid droplet for three models

a) Conductive heat exchange (Law & Sirignano, 1977)

b) Infinite thermal conductivity (cf. Chin & Lefebvre, 1985)

Two-layer model (Present work): an alternative to a) and b)

Two-layer model: a discrete system with two temperatures to describe the droplet evolution



$$M_L \, c_L \frac{dT_L}{dt} = Q_{SL} \;, \qquad M_S \, c_L \frac{dT_S}{dt} = -Q_{SL} + Q_L$$

Heat fluxes and energy equation of the liquid phase

We suppose an ideal mixture of perfect gases and the quasi-steady hypothesis in the gaseous phase.

$$M = \frac{4}{3}\pi r_S^3 \rho_L, \quad M_L = \frac{4}{3}\pi r_L^3 \rho_L, \quad M_S = \frac{4}{3}\pi \left(r_S^3 - r_L^3\right)\rho_L$$

$$\dot{M} = 2\pi \frac{k}{c_p} r_S \, Nu \ln\left(1 + B_T\right), \quad \dot{M} = 2\pi \rho D r_S \, Sh \ln\left(1 + B_M\right)$$

$$Q_{SL} = 2\pi k_L r_L Nu_L \left(T_S - T_L\right), \quad Q_L = \dot{M}\,\ell\left(\frac{B'_T}{B_T} - 1\right)$$

$$B_T = \frac{c_p\left(T_\infty - T_S\right)}{\ell + Q_L / \dot{M}},$$

$$B'_T = \frac{c_p\left(T_\infty - T_S\right)}{\ell} \qquad \ell: \textit{latent heat}$$

At rest: Nu=2, Nu_L=2, Sh=2.

Here: B_T (and B_M) are not constant. If Le=1, B_T=B_M

$$\beta = \frac{3k\,Nu}{2\rho_L\,c_L\,r_S^2}\;,\;\beta_S = \frac{\beta}{1-\varepsilon}\;,\;\beta_L = \frac{3k_L\,Nu_L}{2\rho_L\,c_L\,r_L^2}\;,\;\beta' = \frac{\varepsilon\,\beta_L}{1-\varepsilon}\;,\;\varepsilon = \left(\frac{r_L}{r_S}\right)^3$$

$\beta\,,\,\beta_S\,,\,\beta_L\,,\,\beta'$ *heat exchange coefficients* *volume ratio*

Droplet energy equations:

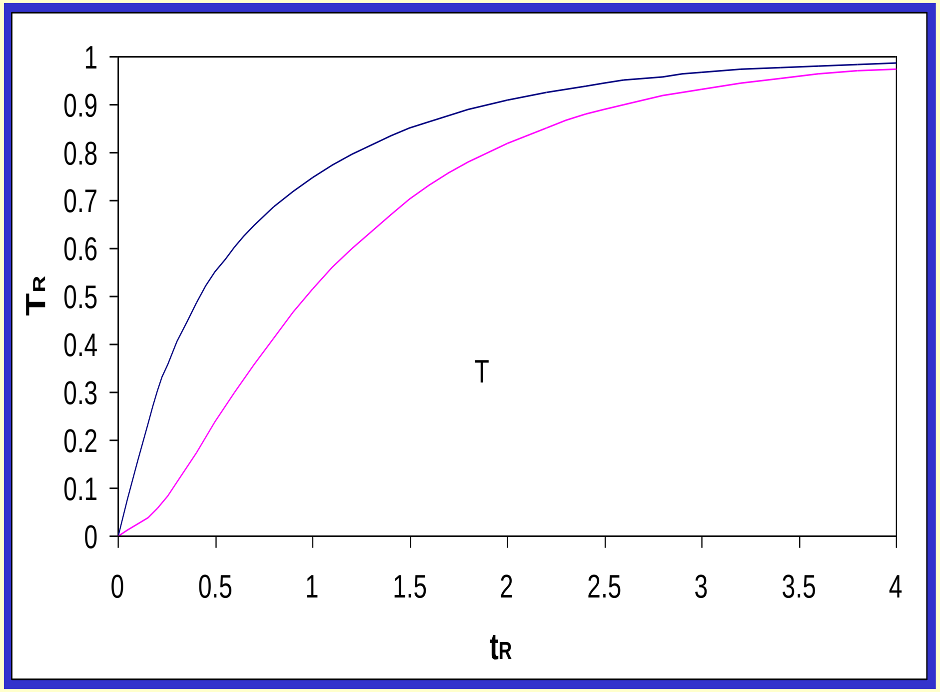
$$\begin{cases} \dfrac{dT_L}{dt} = \beta_L\left(T_S - T_L\right) \\ \dfrac{dT_S}{dt} = -\beta'\left(T_S - T_L\right) + \beta_S\,\dfrac{\ln\left(1+B_T\right)}{B_T}\left(T_\infty - T_S\right) - \beta_S\,\dfrac{\ell}{c_P}\ln\left(1+B_T\right) \end{cases}$$

Case of constant coefficients

If we assume coefficients $\boxed{\beta', \beta_L, \beta_S}$ and the Spalding parameter B_T to be constant (The parameter B_T associated with heat exchange can be deduced from the mass Spalding parameter B_M. In the particular case Le=1, $B_T = B_M$), we obtain

$$\boxed{\frac{d^2 T_L}{dt^2} + \left(\beta_L + \beta' + \beta_S \frac{\ln(1+B_T)}{B_T} \right) \frac{dT_L}{dt} + \beta_S \, \beta_L \frac{\ln(1+B_T)}{B_T} \left(T_L - T_\infty + \frac{\ell}{c_p} B_T \right) = 0}$$

Case of constant coefficients



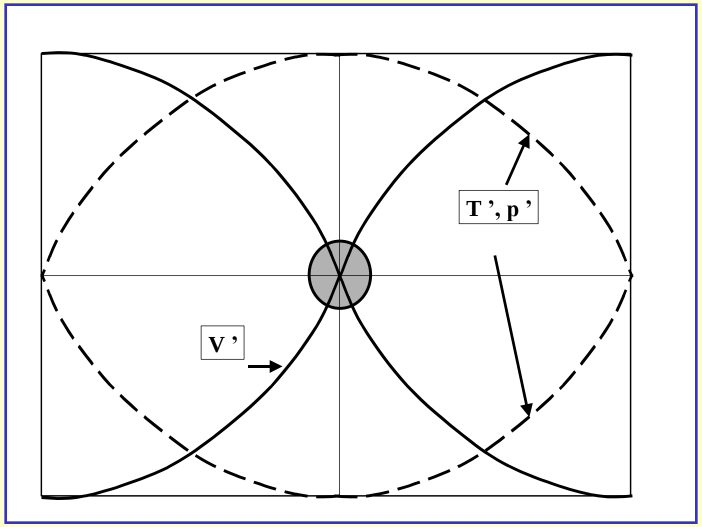
$$T_{RL} = (T_L - T_i)/(T_\infty - T_i - \ell B_T/c_p)$$
$$T_{SL} = (T_S - T_i)/(T_\infty - T_i - \ell B_T/c_p)$$

T_i : *initial temperature*

$t = 0: T_L = T_S = T_i$

Evolutions of reduced temperatures T_{RL} of the inner sphere ——, and T_{RS} of the outer sphere (surface) —— of an evaporating droplet, as functions of reduced time $t_R = \beta' t$, with the two-layer model for heat exchange

3. Analysis of frequency response characteristics



Droplet at a velocity node in a closed cavity

Response factor:

$$N = \iint_{V,t} q'(V,t)\, p'(V,t)\, dt\, dV \Big/ \iint_{V,t} \left(p'(V,t)\right)^2 dt\, dV$$

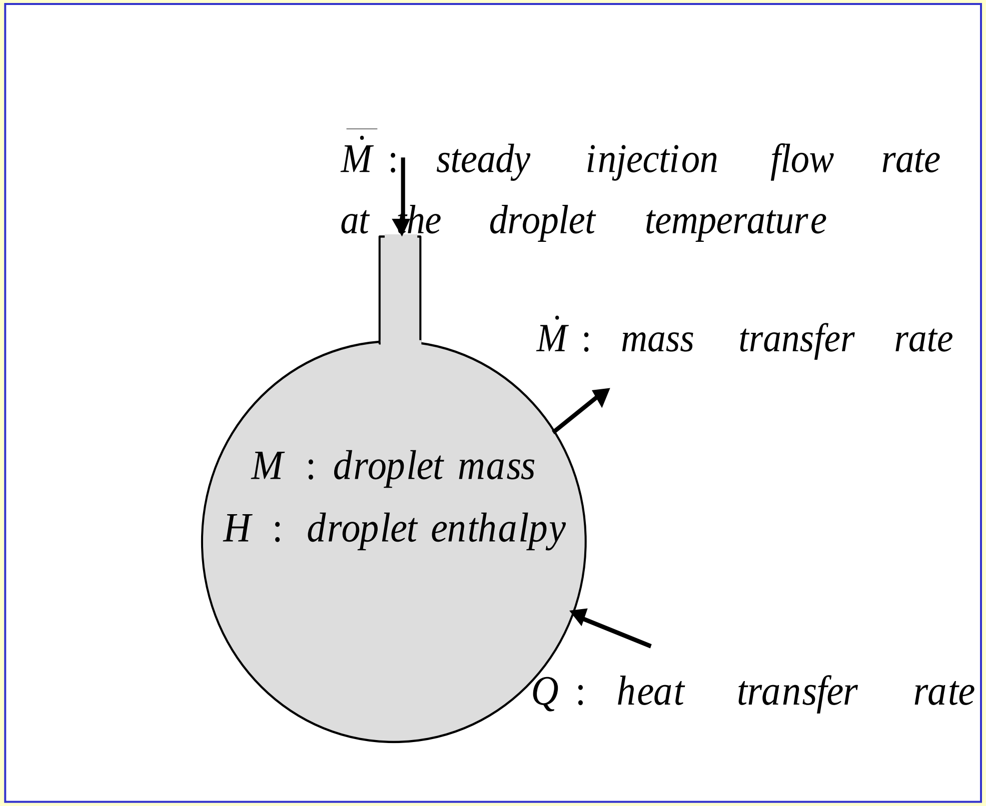
For sinusoidal oscillations with the same period

$N = \left(|\hat{q}|/|\hat{p}|\right) \cos\theta$, where $\boxed{|\hat{q}|, |\hat{p}|}$ are modulus

and $\boxed{\theta}$ the phase difference between *q'* and *p'.*

pressure reduced fluctuation $\boxed{p' = (p - \overline{p})/\overline{p}}$

$\boxed{q' = (q - \overline{q})/\overline{q} = \dot{M}'}$ *mass flow rate reduced fluctuation*



A vaporizing drop of radius r_S, continuously supplied by a steady flow rate $\bar{\dot{M}}$.

The mean droplet of Heidmann and Wieber

Linearized theory:

$$\boxed{f' = \left(f - \bar{f}\right) / \bar{f}}$$

$$\boxed{f' = \hat{f}\, e^{i\,\omega\, t}}$$

- Linearized equations are written for small perturbations
- Equations in $\hat{f}$ are deduced
- $$N = \mathrm{Re}\left(\hat{\dot{M}} / \hat{p}\right) = \mathrm{Re}\left(\alpha\, Z\right)$$
  $$\theta = Arg\left(\hat{\dot{M}} / \hat{p}\right) = Arg\left(\alpha\, Z\right)$$

Droplet with a temperature field (without convection): two-layer model

$$\bar{\tau}_v \frac{dM'}{dt} = -\dot{M}' , \qquad \dot{M}' = \frac{1}{3} M' + \alpha \left(p'_L - p'_C \right), \qquad p'_L = \bar{b}\ T'_S , \ p_L = \exp\left(a - \frac{b}{T_S - c} \right)$$

$$\begin{cases} \dfrac{dT'_L}{dt} = \beta_L \left(T'_S - T'_L \right) , \quad \lambda \bar{\tau}_v \left[\varepsilon \dfrac{dT'_L}{dt} + (1-\varepsilon) \dfrac{dT'_S}{dt} \right] + \mu T'_S - \bar{a}\ p'_C = 0 \\ \text{with} \quad \lambda = \dfrac{c_L \bar{T}_S}{\ell} , \quad \mu = \dfrac{\bar{T}_S}{\bar{T}_C - \bar{T}_S} - \dfrac{2c}{\bar{T}_S - c} + \bar{b} \varphi , \quad \bar{a} = \dfrac{\bar{T}_C}{\bar{T}_C - \bar{T}_S} \dfrac{\gamma - 1}{\gamma} + \varphi , \\ \varphi = \left(\dfrac{\bar{B}_M}{Le} \dfrac{\left(1 + \bar{B}_M\right)^{\frac{1}{Le} - 1}}{\left(1 + \bar{B}_M\right)^{\frac{1}{Le}} - 1} \dfrac{\bar{Y}_{AC} \bar{Y}_{FS}}{\bar{Y}_{AS} \left(\bar{Y}_{FS} - \bar{Y}_{FC} \right)} + \dfrac{\left(M_A - M_F \right) \bar{Y}_{FS}}{M_F \bar{Y}_{AS} + M_A \bar{Y}_{FS}} \right) \dfrac{M_A}{M_F \bar{X}_{FS} + M_A \bar{X}_{AS}} \end{cases}$$

$$u = 3\omega \bar{\tau}_v , \quad A = 3\left(\bar{a}\bar{b} - \mu \right) / \lambda , \ \Lambda = 3\beta_L \bar{\tau}_v , \quad B = 3\mu / \lambda , \quad \varepsilon = \left(r_L / r_S \right)^3$$

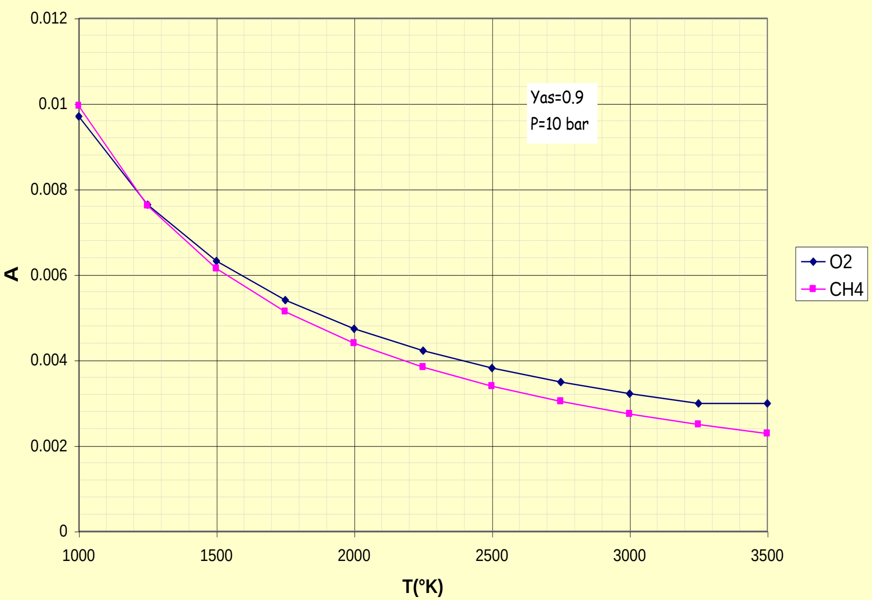
$A, B, \varepsilon, \Lambda$ are constant coefficients.

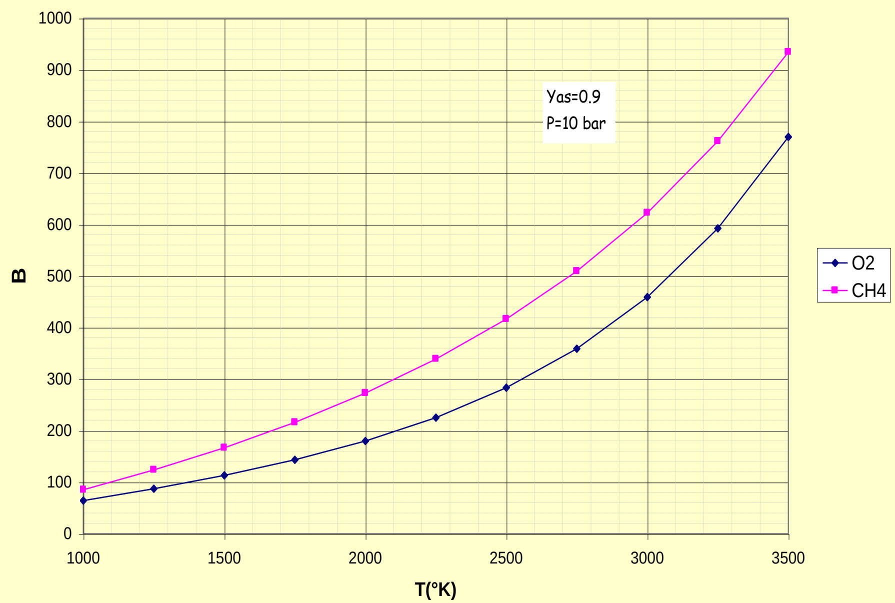
Droplet with a temperature field : two-layer model

Transfer function:

$$Z = \frac{1}{\alpha}\frac{\hat{\dot{M}}}{\hat{p}_C} = \frac{iu}{1+iu}\,\frac{(\Lambda+iu)(A-iu)-\varepsilon u^2}{(\Lambda+iu)(B+iu)+\varepsilon u^2}$$

$$u = 3\omega\bar{\tau}_v, \qquad \Lambda = 3\beta_L\bar{\tau}_v \qquad =\bar{\tau}_v/\bar{\tau}_T\,, \qquad \varepsilon = (r_L/r_S)^3$$





Evolution of coefficients A and B versus temperature

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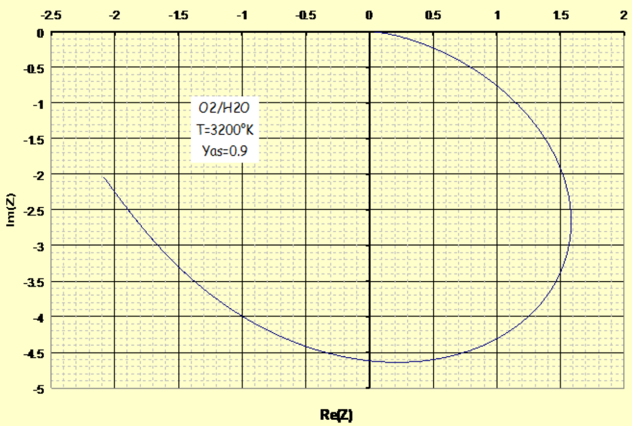
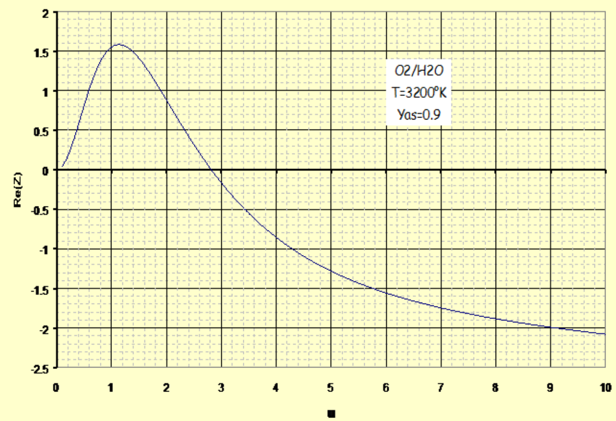
The case of a droplet at uniform temperature without external flow $\left(\varepsilon = 0 \text{ or } \Lambda = \infty\right)$

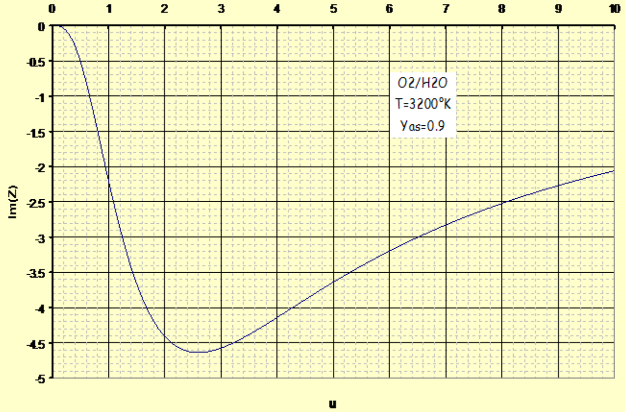
$$Z_0 = \frac{1}{\alpha}\frac{\hat{\dot{M}}}{\hat{p}_C} = \frac{iu}{1+iu}\,\frac{A-iu}{B+iu}$$

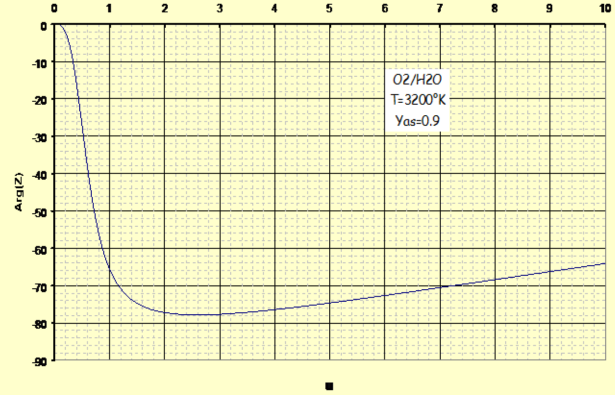




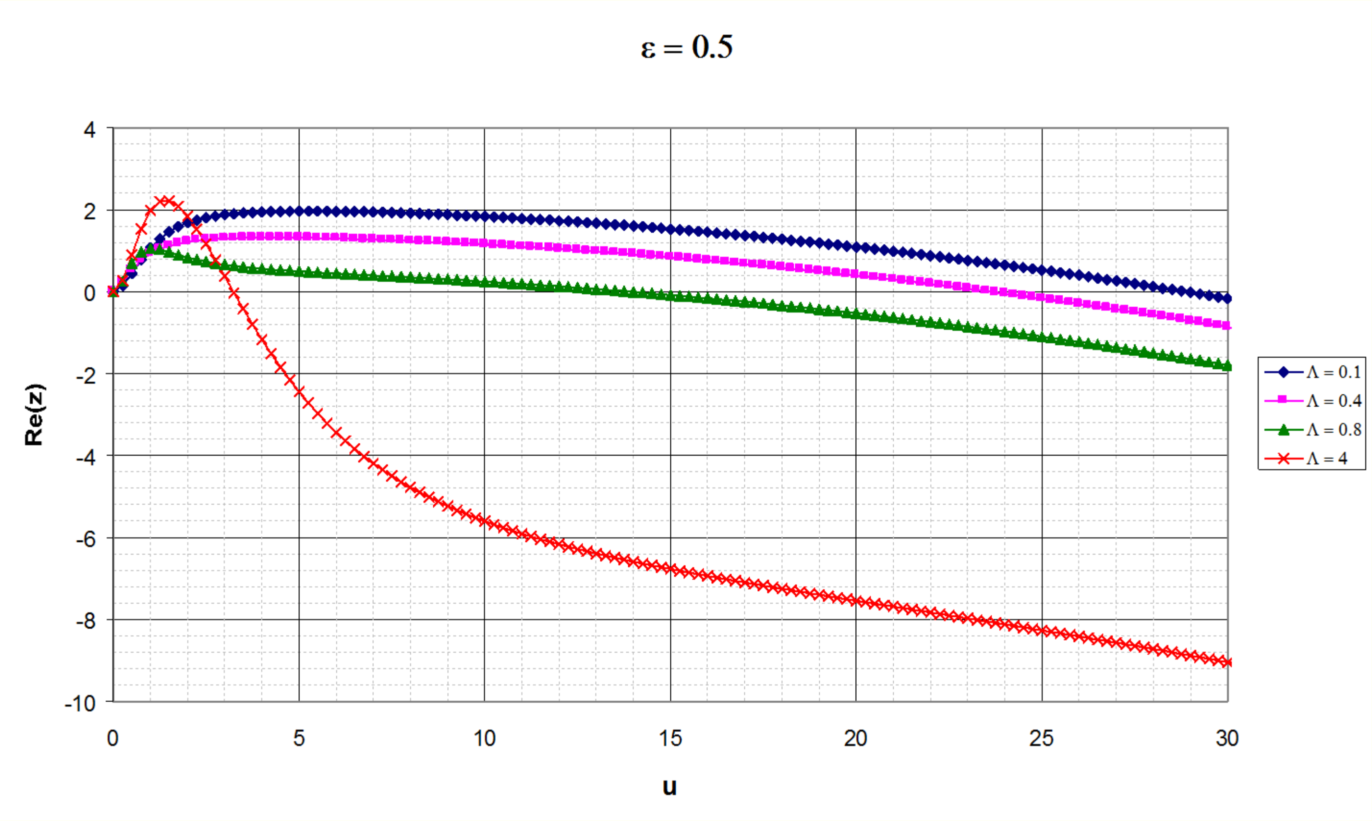
$$N_0/\alpha = \mathrm{Re}\left(Z_0\right)$$

Oxygen in water vapor

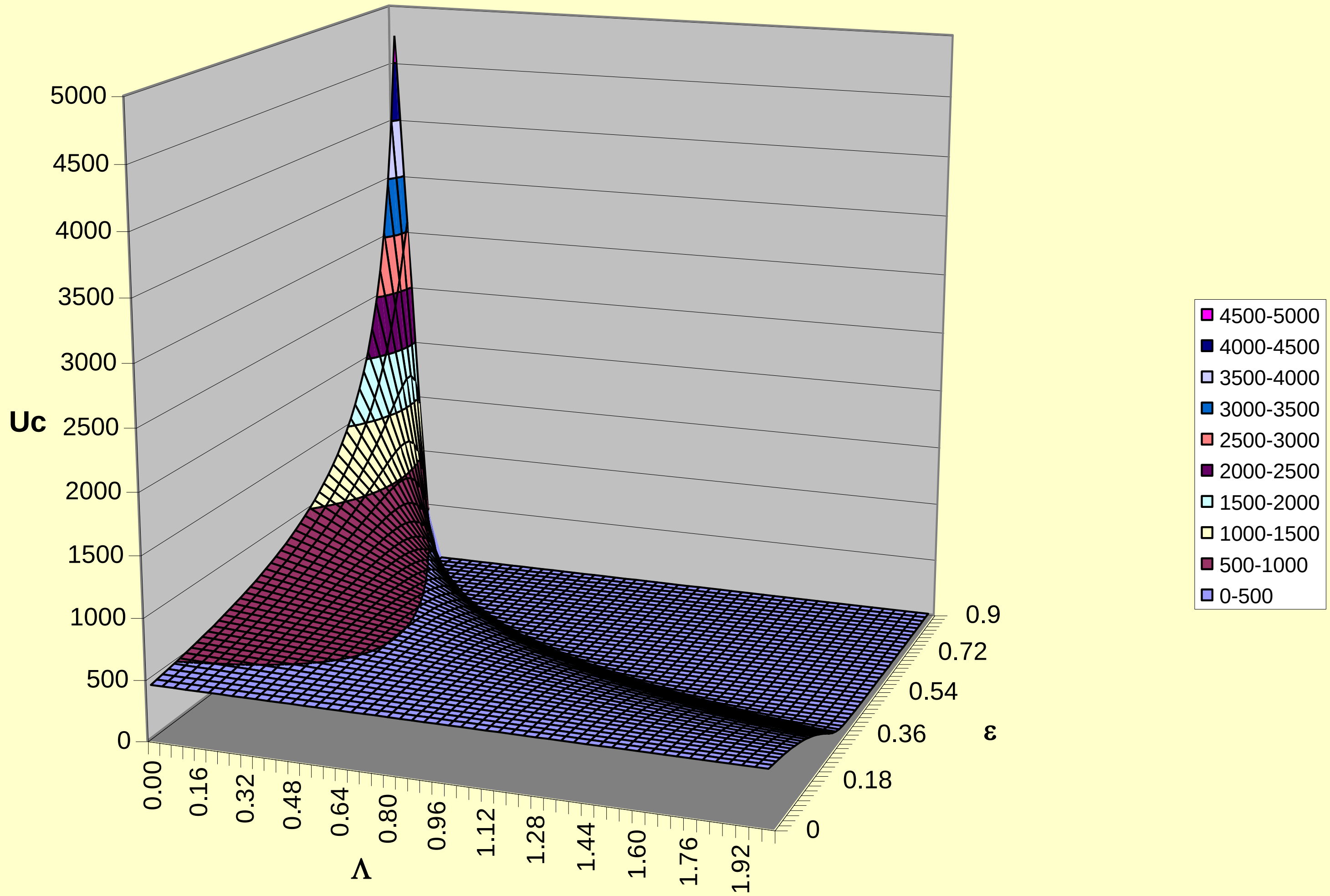




$$\theta_0 = Arg\left(Z_0\right)$$



Influence of the reduced heat exchange coefficient between the two liquid layers, on the reduced response factor N/α , for $\varepsilon = (r_L/r_S)^3 = 0.5$, T=3000K, p=10b, Y_{AC}=0.9



Effects of ε and Λ on the reduced cut-off frequency u_c

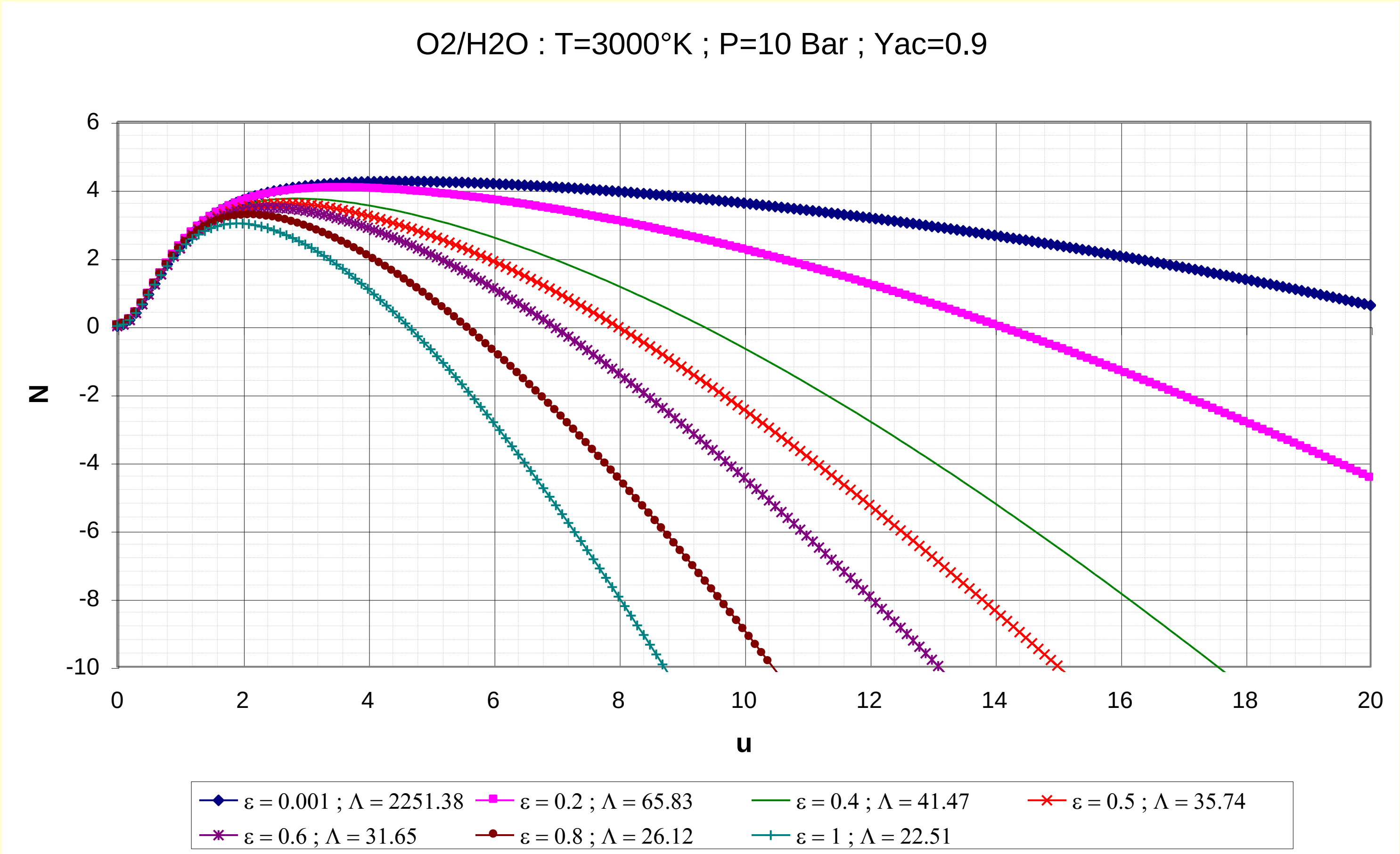
Influence of ε for liquid Nusselt number equal to 2:

$$\bar{\tau}_v = \frac{\overline{M}}{\overline{\dot{M}}} = \frac{(4/3)\pi\,\rho_L\,\bar{r}_S^3}{4\pi\,\overline{\rho D}\,\bar{r}_S\ln\left(1+\overline{B}_M\right)} = \frac{\rho_L\,\bar{r}_S^2}{3\overline{\rho D}\ln\left(1+\overline{B}_M\right)}$$

$$\bar{\tau}_T = \frac{2\rho_L\,c_L\,\bar{r}_L^2}{9k_L\;Nu_L}$$

$$\Lambda = \frac{\bar{\tau}_v}{\bar{\tau}_T} = \frac{3k_L\;Nu_L}{2c_L\;\overline{\rho D}\ln\left(1+\overline{B}_M\right)}\varepsilon^{-2/3}$$

$$\Lambda\;\varepsilon^{2/3} = 3k_L/c_L\;\bar{\rho}\;\;\overline{D}\ln\left(1+\overline{B}_M\right)$$



Response factor as a function of reduced frequency for several values of ε and Λ

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4. Conclusions and prospects:

• **A two-layer model is used for internal droplet heat exchange.**

• **A linearized model is developed on the basis of Heidmann and Wieber theory A dynamic model is investigated on the basis of the two previous models, and is then used in the stability analysis to determine the complex transfer function and the response factor of a vaporizing droplet submitted to small periodic perturbations. Only the case of a pressure perturbation is considered. In every case the results shows the important effect of thermal exchange inside the droplet on the vaporization stability.**

• **To continue these investigations, it should be interesting:**

- To conceive a N-layer model to compare with the two-layer model and find the volume fraction ε

- To study the effect of convection due to relative velocity gas/droplet

- For the future, to validate numerically the results

This work has been supported by CNES and ONERA. The authors are indebted to Richard PHIBEL who has performed numerical calculation for this report and to Yves MAURIOT who has verified equations