



European Research in Mathematics Education I, volume 3

Konrad Krainer, Fred Goffree, Peter Berger

► To cite this version:

Konrad Krainer, Fred Goffree, Peter Berger. European Research in Mathematics Education I, volume 3. Krainer, K.; Goffree, F.; Berger, P. First Conference of the European Society for Research in Mathematics Education, Osnabrück, Germany. Volume 1, Osnabrück, Germany: Forschungsinstitut für Mathematikdidaktik and ERME, 1999, European Research in Mathematics Education I: Proceedings of the First Conference of the European Society for Research in Mathematics Education (CERME 1, August 1998), 3-925386-52-1. hal-02270714

HAL Id: hal-02270714

<https://hal.science/hal-02270714>

Submitted on 29 Aug 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

European Research in Mathematics Education I.III

**From a Study of Teaching
Practices to Issues in
Teacher Education**

**Konrad Krainer
Fred Goffree
Peter Berger
(Editors)**

European Research in Mathematics Education I.III

From a Study of Teaching Practices to Issues in Teacher Education

Proceedings of the First Conference of the
European Society for Research in Mathematics Education
Vol. 3

Konrad Krainer · Fred Goffree · Peter Berger
(Editors)

Forschungsinstitut für Mathematikdidaktik, Osnabrück 1999
ISBN: 3-925386-52-1

Die Deutsche Bibliothek – CIP-Einheitsaufnahme

Krainer, Konrad; Goffree, Fred; Berger, Peter:
Research in Mathematics Teacher Education III. From a Study
of Teaching Practices to Issues in Teacher Education.
– Proceedings of the First Conference of the European Society
in Mathematics Education, Vol. 3 –
ISBN: 3-925386-52-1

Printed versions:

Libri Books on Demand

Vol. 1: ISBN 3-925386-53-X

Vol. 2: ISBN 3-925386-54-8

Vol. 3: ISBN 3-925386-55-6

Internet versions (pdf-files, html-overviews):

[http://www.fmd.uni-osnabrueck.de/ebooks/
erme/cerme1-proceedings/cerme1-proceedings.html](http://www.fmd.uni-osnabrueck.de/ebooks/erme/cerme1-proceedings/cerme1-proceedings.html)

Vol. 1: ISBN 3-925386-50-5

Vol. 2: ISBN 3-925386-51-3

Vol. 3: ISBN 3-925386-52-1

Contents

<i>Preface</i>	1
<i>List of Contributors</i>	11
1. Teacher Education and Investigations into Teacher Education: A Conference as a Learning Environment	13
<i>Konrad Krainer</i>	
2. Teacher Education and Investigations into Teachers' Beliefs	41
2.1 Introduction: Teachers' Beliefs and Conceptions as a Fundamental Topic in Teacher Education	43
<i>João Pedro da Ponte</i>	
2.2 Teachers' Beliefs on Problem Solving and Mathematics Education	51
<i>Luis Carlos Contreras, Nuria Climent, José Carrillo</i>	
2.3 Affective Components of Teachers' Computer Beliefs: Role Specific Aspects	63
<i>Peter Berger</i>	
2.4 Research on Teachers' Beliefs: Empirical Work and Methodological Challenges	79
<i>João Pedro da Ponte, Peter Berger, Lucilla Cannizzaro, Luis Carlos Contreras, Ildar Safuanov</i>	
3. Teacher Education and Investigations into Teachers' Knowledge	99
<i>José Carrillo, Moisés Coriat, Hélia Oliveira</i>	
3.1 Background: On Teachers and on Knowledge	101
3.2 Different Approaches to a Characterisation of Teachers' Knowledge	109

3.3	Ways of Promoting Mathematics Teachers' Knowledge	125
3.4	Synthesis and Future Work	137
4.	Teacher Education and Investigations into Teachers' Practice(s)	147
4.1	Good practice	149
	<i>Fred Goffree, Hélia Oliveira, Maria de Lurdes Serrazina, Julianna Szendrei</i>	
4.2	A Study of Teachers' Practices: Organisation of Contents and of Students' Work.	171
	<i>Marie-Jeanne Perrin-Glorian</i>	
4.3	A Digital Representation of 'Full Practice' in Teacher Education: The MILE Project	187
	<i>Fred Goffree and Wil Oonk</i>	
5.	Teacher Education Through Teachers' Investigation into Their Own Practice	201
	<i>Barbara Jaworski</i>	
6.	Investigations into Teacher Education: Trends, Future Research, and Collaboration	223
	<i>Konrad Krainer and Fred Goffree</i>	
	<i>Name Index</i>	243

Preface

This book is a product of *Thematic Group 3* “From a Study of Teaching Practices to Issues in Teacher Education” of the First Conference of the European Society for Research in Mathematics Education (CERME 1), held in Osnabrück, Germany, from 27th till 31st August, 1998. The editors gratefully thank *Elmar Cohors-Fresenborg* and *Inge Schwank* from the University of Osnabrück for their collaboration.

Some remarks on ‘practice’, ‘teacher education’, and ‘research in teacher education’

A central point of discussion in teacher education is “*practice*”. We often speak about teaching practice(s), classroom practice(s), teachers’ practice(s), etc., sometimes meaning the same and sometimes seeing differences. Similarly, we have different understandings of “*teacher education*”. In both cases, our interpretation is influenced by our own working background: for example, it makes a difference whether someone mainly works in pre-service education for primary teachers (e.g. being a teacher educator in a country where much more emphasis is laid on pre-service education than on in-service) or someone mainly works in in-service education for secondary teachers (e.g. being involved in a national teacher professional development programme). Whereas, for example, in pre-service education the creation of learning environments where student teachers get involved with *practice* (in order to learn more than “the big theories” about their future practice), is very essential, the challenge in in-service education is often more to use classroom practice as a learning environment where the teachers get involved with *theories* (in order to learn a “language” to speak about their practice). Different links

between theory and practice will appear several times as a motif throughout this book. In the following, we briefly reflect on the terms “*practice*” and “*teacher education*” in order to make our understanding and usage of these terms explicit.

The most prominent part of teachers’ professional work is done in *classrooms*, designing, managing, and evaluating content-related and social learning processes of students, dealing with a broad range of interactions, communications, assessments, pedagogical situations, etc. For that, in most cases, the terms “*teaching practice*” and “*classroom practice*” are used. Sometimes the plural “teaching practices” is used to indicate that different teachers have different ways to design their teaching. In its singular form, both, “teaching practice” and “classroom practice” not only refer to an individual teacher’s way of teaching, but more generally indicate the *practical* field and context where teachers interact with their students – in contrast to the *theoretical* field and context where e.g. mathematics educators reflect on those practices. However, some people see an important difference between “teaching practice” and “classroom practice”: whereas “teaching practice” more clearly refers to the teacher as a *person* (maybe mainly expressing an interest in his/her teaching or an interest in studying different teaching styles), “classroom practice” more neutrally refers to the classroom as a *system* that includes more than the teacher’s actions (e.g. expressing an interest in the interaction process among students or between the teacher and the students). We tend to prefer to use “classroom practice” for two reasons: firstly, in the context of teacher education it is clear anyhow that the teacher plays an important role, and secondly, it expresses the point of view that we are more inclined to see the whole system in which learning and teaching processes are embedded. To some extent, this understanding expresses a shift from viewing classroom processes as mainly determined by the teacher to viewing classroom processes as complex features, where knowledge, meanings, norms, etc. are socially constructed and influenced by a variety of general conditions, as e.g. the importance of education in the society, the curriculum or the school climate.

This leads us to the term “*teachers’ practice*”. Although the most prominent part of teachers’ professional work is done in classrooms, or is essentially related to it (preparation, reflection, etc.), it includes more than that. Increasingly the quality of schools has become a permanent issue of public discussion and of scientific investigations in the field of organisational development of schools. There is more

and more awareness that a “good school” is more than the sum of “good classroom practices” of the individual teachers of a school, and that it is not only the principals’ responsibility to contribute to the further development of a school, but also the joint responsibility of teachers (and students, parents, etc.). Therefore, “teachers’ practice” cannot be confined to “classroom practice”.

In addition, there is another systemic interconnection that should not be underestimated: it is our pre-service and in-service education that has a big influence on the learning process of teachers. For this reason, also our “teacher education practice” should increasingly become an object of (self-) evaluation and investigation.

This leads us to “*teacher education*”. We understand teacher education as an *interaction process* (embedded in a social, organisational, cultural, ... context), mainly between teacher educators and (student) teachers, but also including systematic interactions among teachers aiming at professional growth. At the same time, we can see teacher education as a *learning environment* for all people involved in this interaction process.

The *overall goal* is the *improvement of teachers’ practice* (in the case of institutionalised in-service education or other forms of promoting teachers’ professional growth) or the adequate *preparation* for that practice (pre-service education). In both cases it is combined with an *improvement* of a complex network of (student) teachers’ knowledge, beliefs, etc. The most prominent part of mathematics teachers’ practice deals with interaction processes between the teacher and the students, focusing on students’ learning of mathematics. Therefore we see the goal of mathematics teacher education as *promoting (student) teachers’ efforts to establish or to improve their quality of teaching mathematics*, and the task of teacher educators as *designing adequate learning environments* to reach this goal in a joint effort with the (student) teachers. *Designing and evaluating* mathematics teacher education courses is one important part of mathematics educators’ professional work. However, all over the world there are big differences concerning mathematics educators’ responsibility for pre-service education, in-service education, administrative and management work or research activities.

Summing up, the title of *Thematic Group 3* can be interpreted in two ways. Firstly, it indicates a progress in the following sense: research in teacher education no longer confines itself to investigating teaching practice, but increasingly includes investigations on the broader complexity of teachers' practice and on investigations into our own teacher education practice.

Secondly, it indicates a progress in quite another sense: the study of teaching practice is no longer only a domain for academic researchers, but increasingly also one of student teachers and practising teachers; learning environments where they can investigate classroom practice are more and more seen as effective elements of teacher education. The new possibilities of multimedia systems support this development.

A major focus of this conference was *research*. Concerning teacher education one might ask the question: Should we e.g. speak about research *in* teacher education or about research *on* teacher education? The situation is even more complex than that as we will see below.

Research in teacher education can be interpreted as a general term for investigations carried out within the framework of teacher education or at least with the goal or a clear potential of using its results in teacher education.

A kind of research coupled relatively loosely with teacher education we call *research in the perspective of teacher education*. The investigations are not done in the context of teacher education and do not focus on interaction processes within teacher education. In the past, most research projects in teacher education, and even a considerable part of recent initiatives, fall into this category. One prominent example is the investigation of (student) teachers' beliefs, knowledge, and practice (see chapters 2, 3, and 4). There is much diversity concerning researchers' inclination to draw explicit conclusions for designs of teacher education. It has to be added that a variety of research – not explicitly aiming at drawing conclusions for teacher education – is often the basis for creating powerful learning environments in teacher education.

Generally closer to drawing conclusions in the field of teacher education is *research in the context of teacher education*. Here investigations (e.g. on teachers' beliefs, knowledge, practice, etc.) are done in the context of teacher education (e.g.

within the framework of an in-service education course) but they do not focus explicitly on the interaction process (within this course). In general, researchers are inclined to draw explicit conclusions for designs of teacher education, in particular concerning e.g. the course in which the investigations take place. Increasingly, (student) teachers are supported to do investigations on teachers' beliefs, knowledge, practice, etc. (see e.g. subchapter 4.3). We tend to subsume such investigations in the field of "research in teacher education" (here the question is less whether it fits to teacher education but more whether it can be titled as (traditional) "research", as "alternative research", as "action research", as carrying out "mini-research-tasks", etc.). For the purpose of this book, we use the term "investigations" as a generic term that includes all kinds of research, inquiry, systematic reflection, etc. However, it is a future challenge for teacher education to discuss this question in more detail.

Very close to drawing conclusions is *research on teacher education* courses, programmes or other forms of promoting teachers' professional growth. Here the focus is directly on teacher education and means investigations on the interaction between e.g. teacher educators and (student) teachers, the achievements of the participants, side-effects for the school, etc. One prominent reason to do that kind of research is *to evaluate the success* of teacher education courses or programmes, mostly with the intention to draw consequences from that internal or/and external evaluation (e.g. improving the course or stopping the programme). A special way of evaluation is self-evaluation on the basis of teacher educators' action research in order to improve their (teacher education) practice. Another reason is *to understand better the interactions* between teacher educators and (student) teachers, e.g. investigating which kind of teacher educators' interventions or teachers' beliefs influenced the process. In this latter case, the intention is – at least not explicitly – the improvement of one particular course or programme, but it is assumed that the results – because they really focus on teacher education – are a good basis for drawing conclusions for designing teacher education.

A direct connection to the improvement of teachers' practice is given in the case of *research as teacher education* (or teacher education *as* research, see chapter 6). A concrete form is (teachers') action research, understood e.g. as the systematic and self-critical reflection of practitioners into their own practice (see chapter 5). Research as teacher education means a close interconnection between *understanding*

and *improving* teachers' practice. The joint reflection on the learning process (e.g. interactions, improvement of practice, ...) plays a crucial role. In principle, teachers' action research can be done among professional teachers themselves, everyone being the teacher educator (and "critical friend") for the others. In most cases, however, action research is initiated and promoted in the context of a teacher education course or programme where teacher educators act as "facilitators" of action research but also have more traditional roles in their interaction with the participants (teacher education *with* research, see chapter 6). The borders between teacher education *with* research and teacher education *as* research are fluid.

A very special kind of research is *meta-research on teacher education* which means analyses of research activities or general conditions in the field of teacher education (or parts of it), for example working towards "state of the art"-reports.

All these kinds of investigations can be subsumed under *research in teacher education*. They also can be combined: for example, embedding investigations of (student) teachers in learning environments of teacher education courses might be a starting point to investigate how successful these investigations are concerning (student) teachers' beliefs, knowledge, practice, etc.

Some remarks on the work of *Thematic Group 3*

The *work of Thematic Group 3* (TG 3), "From a study of teaching practices to issues in teacher education", was characterised by a considerable amount of *diversity* that distinguished it from the other six thematic groups of the First Conference of the European Society for Research in Mathematics Education (CERME 1). We confine ourselves to sketch *four aspects*.

Firstly, the theme of the group covers a very *broad field of relevant issues*. As expressed by the title, teacher education includes much more than the study of teaching practices, for there are many factors influencing those practices, e.g. teachers' beliefs and knowledge, students' abilities and motivation, school climate and professional communication among teachers, general conditions of teaching (class size, space, time, ...), curriculum and assessment, structure and political orientation of the school system, internal and external support systems for the improvement of teaching (through different kinds of in-service education) or

teachers' socialisation process, starting from their own classroom experience, over participating in pre-service education and school practice to a fully responsible work as a practising teacher at a specific school.

Secondly, progress in teacher education not only means that we (as mathematics educators) confine ourselves to investigate more intensively teachers' practices (and beliefs, knowledge, ...) and make suggestions for its improvement but we also have to take *our own* (teaching, research, ...) *practice* etc. *as a matter of reflection and investigation*. Teacher education has to do with self-application of our theories, and therefore has another quality of challenge for our field. Our activities in teacher (pre- and in-service) education and our research activities are embedded into the whole system of further development of teachers' practice. We are not (only) external investigators and observers *of* this system, but we are (also) responsible actors *within* this system. We are not only trying to understand better teachers' practice (in order to give advise on how teachers, their practice, the curriculum, etc., could improve) but we also have the duty to understand our interventions into teachers' practice (e.g. through in-service education) or the implications of our pre-service education practice on student teachers' development (in order to improve our teacher education practice). Given this fact, teacher education – if we understand it as an interaction process between teacher educators and teachers – demands that the processes of understanding and improving (teachers' and our own practice) get more interwoven than in fields where we investigate situations and processes that do not include such a high personal involvement of ourselves.

Thirdly, teacher education has not the same *research tradition* as many other issues of mathematics education (like e.g. didactics of algebra or geometry). It is assumed that this is to a great extent a consequence of the aspect mentioned above: the high degree of involvement influences our approach to investigation and research. There are at least two approaches that play a prominent role in discussions in teacher education (conferences, journals, ...) all over the world – and also in our Thematic Group: One approach puts an emphasis on research on teachers (in a non teacher education context) in order to understand better teachers' practice (beliefs, knowledge, etc.), but often such research doesn't make clear connections to the improvement of (teachers' or teacher educators') practice. In this case the question is less whether we speak about research, but whether there is a close link to teacher

education. Another approach takes – to some extent – the other direction: the authors (mostly teacher educators) tell – more or less – “success stories” about pre- or in-service projects at their institutions. Here, the improvement of teachers’ practice, their professional growth etc. is in the foreground, more and more there are also reflections on teacher educators’ learning processes and consequences for improving the project. Often, the weak point here is that we cannot easily find a systematic reflection on the research question, the criteria for success and a presentation of data that helps the reader to duplicate the improvement (e.g. to understand the main relevant factors that are due to the project and less to other influences). Here, the topic is surely teacher education, but the challenge is really to understand and to describe theoretically the processes which lead to the improvement. Both approaches are important, but in both cases we need a closer relation between understanding and improving. It is clear that an emphasis on *research* demands an emphasis on understanding, and it is clear that *teacher education* mainly aims at improving (learners’ knowledge, practice, beliefs, ...). *Research in the field of teacher education* means to meet *both challenges*.

Fourthly, and this seems to be an outcome of the broadness of the field, the demand for self-applicability and the challenge of doing research in this field, *Thematic Group 3* not only had the largest number of participants, but also had the fewest number of accepted papers of all Thematic Groups of CERME 1. The group was characterised by a broad geographical and cultural diversity which was accompanied by a considerable heterogeneity of participants’ research and development traditions, mother languages and English abilities. This means a context where communication and collaboration during the conference and the process to achieve a joint product (in particular, this book) was a really tough task.

Given the time pressure to finish our chapter, we tried our best to find a compromise between realising the ERME-principles of communication, co-operation, and collaboration, aiming at scientific quality and coping with the deadline for publication. A lot of arguments could have been expressed more clearly and more interconnections could have been realised. Nevertheless, we are sure that our product marks a good starting point for deeper reflection on teacher education among mathematics educators in Europe and all over the world.

It would not have been possible to produce this chapter without the enormous motivation and self-discipline of all of its contributors, based on the extraordinary good working and social climate we jointly established at our meetings in Osnabrück. The following 22 people (in alphabetic order), coming from 13 different countries, participated in our Thematic Group, contributing to the quality of processes and products of our work: *Michele Artaud* (France), *Peter Berger* (Germany), *Lucilla Cannizzaro* (Italy), *José Carrillo* (Spain), *François Conne* (Switzerland), *Luis Carlos Contreras* (Spain), *Moisés Coriat* (Spain), *Fred Goffree* (The Netherlands), *Barbara Jaworski* (United Kingdom), *Konrad Krainer* (Austria), *Razia Fakir-Mohammed* (Pakistan, at present United Kingdom), *Maria Korcz* (Poland), *Hélia Oliveira* (Portugal), *Wil Oonk* (The Netherlands), *Marie-Jeanne Perrin-Glorian* (France), *João Pedro da Ponte* (Portugal), *Ildar Safuanov* (Russia), *Manuel Saraiva* (Portugal), *Andrei Semenov* (Russia), *Maria de Lurdes Serrazina* (Portugal), *Julianna Szendrei* (Hungary), and *Elisabeth Thoma* (Austria). It was a pleasure to co-ordinate efforts within this both professionally and personally fruitful learning community.

Yves Chevallard (France), *Fred Goffree* (The Netherlands), *Konrad Krainer* (Co-ordinator, Austria), and *Erkki Pehkonen* (Finland) were asked by the Programme Committee to lead the group. Unfortunately, Yves Chevallard and Erkki Pehkonen (whom we thank for his contributions during the planning process) had not been able to attend.

We gratefully thank *José Carrillo*, *Moisés Coriat*, *Barbara Jaworski*, and *João Pedro da Ponte* for their excellent preparation work for their meetings and the co-ordination of the corresponding chapters of the book.

Konrad Krainer, Fred Goffree, Peter Berger
December 1998

List of Contributors

Peter Berger

University of Duisburg (Germany)
berger@math.uni-duisburg.de

Lucilla Cannizzaro

University of La Sapienza, Rome (Italy)
mc3049@mclink.it

José Carrillo

University of Huelva (Spain)
carrillo@uhu.es

Nuria Climent

University of Huelva (Spain)
climent@uhu.es

Luis Carlos Contreras

University of Huelva (Spain)
lcarlos@uhu.es

Moisés Coriat

University of Granada (Spain)
mcoriat@platon.ugr.es

Fred Goffree

Freudenthal Institute (The Netherlands)
f.goffree@fi.ruu.nl

Barbara Jaworski

University of Oxford (United Kingdom)
barbara.jaworski@educational-studies.oxford.ac.uk

Konrad Krainer

University of Klagenfurt / IFF (Austria)
konrad.krainer@uni-klu.ac.at

Hélia Oliveira

University of Lisbon (Portugal)

helia.oliveira@fc.ul.pt

Will Oonk

Freudenthal Institute (The Netherlands)

w.oonk@fi.ruu.nl

Marie-Jeanne Perrin-Glorian

Université Paris 7 et Université d'Artois (France)

glorian@gauss.math.jussieu.fr

João Pedro da Ponte

University of Lisbon (Portugal)

jponte@fc.ul.pt

Ildar Safuanov

Pedagogical University of Naberezhnye Chelny (Russia)

safuanov@tlsoft.ru

Maria de Lurdes Serrazina

Escola Superior de Educação, Lisbon (Portugal)

lslm@mail.telepac.pt

Julianna Szendrei

Budapest Teacher Training College (Hungary)

100263.241@compuserve.com

CHAPTER 1

TEACHER EDUCATION AND INVESTIGATIONS INTO TEACHER EDUCATION: A CONFERENCE AS A LEARNING ENVIRONMENT

Konrad Krainer

University of Klagenfurt / IFF (Austria)

Introduction

Let us start with a quote (by Jaji, Nygara & Robson 1985, p. 153) related to an observation of an interaction within a teacher education course in Zimbabwe:

“Now what I am about to say is very important. It will almost certainly come up in the examinations, so I suggest that you write it down. (The group took out their pens ...) In the new, modern approach to teaching ... (They wrote it down as she spoke ...) we no longer dictate notes on children. Instead we arrange resources in such a way as to enable children to discover things themselves”.

Developed countries also seem(ed) to be concerned with similar challenges as the following analysis, provided by Kilpatrick (1982, p. 87), shows:

“If many of the preservice programs are reminiscent of medical school training – with their emphasis on an internship experience and their conscious building-up of group cohesiveness and loyalty – the inservice programs seem more like medical training clinics in developing countries, where native practitioners are

brought in for intensive instruction in new techniques and sent back home to spread the good word.”

Although it is clear that the field of mathematics teacher education is a more sophisticated field than 15 years ago (see e.g. Comiti & Ball 1996 for pre-service education, and Cooney & Krainer 1996 for in-service education), Kilpatrick’s analysis has merit even today and the Damoclean sword of not-applying the philosophies we preach is still accompanying teacher educators all over the world. The two examples show that “good practice” in mathematics teacher education is a relevant issue and deals, for example, not only with beliefs and knowledge of teachers, but is extremely influenced by our own beliefs and knowledge. The same holds true when we look at our investigations in the context of teacher education. This chapter shows that internationally there seems to be some progress – in the practice of teacher education as well as in our research in teacher education. But how does this relate to our conferences where teacher educators meet to discuss issues of teacher education? Is it possible – at least partially – to avoid lecturing and “telling” about issues where we try to argue that teachers should not have a “telling” role (see the two examples above)? At CERME 1 the organisers and participants of Thematic Group “From a study of teaching practices to issues in teacher education” (TG 3) tried to work in a design where joint work and discussion were more important than one-to-one presentations of prepared papers. A reflection on this challenging process, in particular putting an emphasis on the first meeting, is followed by a short overview of the written products of this group, as elaborated in chapters 2 to 6.

The international context and current needs

Teacher education, aiming at prospective and practising teachers’ personal and professional development, is seen as a major intervention to improve the *quality of education* on different (but closely interconnected) levels: the quality of students’ learning, the quality of (student) teachers’ preparation and anticipation of their future work, the quality of teachers’ practice, the quality of schools, the quality of an education system, or the quality of interaction between the education system and the society as a whole. However, due to different contexts, different people (students, student teachers, teachers, parents, mathematics educators, etc.), institutions (school

boards, political parties, unions, universities, ministries, etc.) and countries have different understandings on how to improve the quality of education. Therefore it is not surprising that *conflicting expectations* on teacher education are expressed by different sides.

Teacher education as a complex field

Teacher education is therefore a complex field dealing with enormous diversity characterised for example by elements such as regional circumstances, participants, designs and philosophies, topics and organising institutions (see e.g. Cooney & Krainer 1996).

Typical examples of *regional circumstances* are different general conditions for education and different needs of society, schools, and teachers, which lead to big differences in students' completion of secondary education (Nebres 1988) or in class size (Howson 1994). But it is also very relevant what kind of curriculum and system of assessment (e.g. national test or not) a country has, what status and standards the teaching profession of a country has and what sort of influence different groups (ministry, researchers, teachers, ...) have on these issues.

There is also diversity with regard to *participants*: courses can be confined to special groups of student teachers (e.g. participating in an introduction course to mathematics education, accompanied by a school practice at different primary schools) or practising teachers (e.g. all 8th grade mathematics teachers of a region, or all upper secondary mathematics and science teachers of a school), but there are also programs such as Family Math (see De la Cruz & Thompson 1992) in which parents and children work together in co-operative settings to solve problems and engage in mathematical explorations. Further projects exist, like MINERVA in Portugal, which generated a nation-wide community of teachers, trainers, and researchers that took as their task the "formation of teacher teams and the assertion of a project culture in schools" (Ponte 1994, p. 161).

Teacher education courses also have different *designs and philosophies*, for example, ranging from a strong emphasis on theory and subject matter to programmes where investigations into teachers' practice and teaching experiments play a crucial role. For example, a more traditional approach to in-service education

is for experts to come in from outside and tell teachers about new research results; by contrast, there are courses where teachers are seen as co-designers of in-service education in which they are increasingly motivated to take their further education into their own hands, for example, organising working groups at the end of the course, where the teacher educators are the participants (Krainer 1994). Such courses strive for joint learning of people coming from different institutions, an approach which seems to have become more prominent under the notion of co-reform (Frasier 1993).

Courses for prospective and practising mathematics teachers also demonstrate a broad diversity of *topics*, from dealing with mathematics content knowledge or with cross-curricular connections to considering assessment, new teaching methods or reflecting critically on new technology.

Diversity of *organising institutions* is shown when teacher education is organised by the school authority, or by institutions which are responsible for pre-service education as well as for in-service education, or by institutions where the connection to the participants is less strong but where other interests have to be negotiated, such as research interests or funding, or by self-organising groups of teachers such as the MUED in Germany (see e.g. Keitel 1992).

An additional issue which makes *teacher education* such a complex field is the fact that it *relates our research practice to our teaching practice* and therefore challenges us to apply the theoretical conceptions and philosophies we preach. Thus, it is also our beliefs – and not only those of the teachers – that have to be considered critically. Teacher education can be seen as *our big experiment* and as our continual struggle at the heart of our discipline.

Fundamental shifts in mathematics teacher education

Although the recent situation in mathematics teacher education and its related research is far from being a field with well-developed standards, both for theory and practice, the last thirty years seem to have brought considerable *progress*.

First, some brief general remarks on *literature* and *conference programs*: Research on teacher education developed from being “virtually non-existent in the 1960s and early 1970s” (Cooney 1994a, p. 618) to a field with increasing *literature*, for example, with the first Handbook of Research on Teacher Education published in

1990, with the section “Social Conditions and Perspectives on Professional Development” in the International Handbook on Mathematics Education (1996) or the start of the Journal of Mathematics Teacher Education (JMTE) in 1997. *Conference programs* reveal a similar picture. For example, at the International Congresses of Mathematical Education in Quebec (1992) and Sevilla (1996) there were a number of lectures, working groups, and topic groups that focused explicitly on teachers, their work and teacher education. Similar trends can be observed at PME-Conferences (see e.g. Hoyles 1992, p. 283, or the activities of the PME-Working Group “Research on the Psychology of Mathematics Teacher Development”, see e.g. Peter-Koop & Santos-Wagner 1998), at recent conferences in mathematics education held in German speaking or Nordic countries (see e.g. Kadunz et al. 1996; Hölzl & Neubrand 1996; Breiteig & Brekke 1998). Finally, the prominence of teacher education at CERME 1 demonstrate the value that is ascribed to this topic in recent discussions in mathematics education.

Second, two concrete examples of developments and progress from two parts of the world, outside Europe:

1. In the *United States* a lot of efforts towards mathematics teacher education have been made in the last decades. For example, Cooney (1994a) describes the change in teacher education in the last thirty years as a change of paradigm from *analytic perspectives* towards *humanist perspectives* (Mitroff & Kilmann 1978), from discovering reality to trying to understand the contexts that shape a person’s perception of his or her reality (Brown, Cooney & Jones 1990), having constructivism as an epistemological foundation for mathematics education. In its early stage, teacher education dealt primarily with updating teachers’ knowledge of mathematics. Research mostly focused on studying connections between student achievement and teachers’ characteristics, behaviours, and decisions, mostly on a quantitative basis, placing an emphasis on objectivity. Then the focus moved extensively towards interpretative studies describing teachers’ cognitions (beliefs, meaning-making processes, etc.) and the contexts that influence cognition. Cooney sees that progress has been made in *discarding false dichotomies* that pervaded teacher education, stating that we are now more aware of the necessity of blurring the distinction between theory and practice, content and pedagogy, researchers and teachers,

conceptualizing the latter as cognizant and reflective agents. He points out that teachers and teacher education have become focal points for research in mathematics education but that we need constructs that can meaningfully guide our research efforts. Grouws & Schultz (1996) describe a wide range of successful projects, studies, systemic initiatives, and collaboratives in mathematics teacher education, for example, stressing the importance of “reflective teaching partnerships”, and sketch some possible contributions to the development of useful theories.

2. Another area to examine in looking for progress in mathematics teacher education is the activities and future plans of institutes in *developing countries*. As an example, the Institute for Educational Development (IED) of the Aga Khan University in Karachi (Pakistan) describes its approach to teacher education as follows: “The IED was envisaged neither as a traditional ‘school of education’ nor ‘teacher training college’ – models of higher education that seem increasingly out of step with the real needs of teachers and schools, in both the industrial countries as well as in developing nations. The training that would be provided at IED will be guided by some crucial concerns. First, it will be field based, i.e. the training will take place within classrooms. The assumption behind this practice is that effective teaching skills are best acquired ‘on the job’. A second distinguishing feature of the training will be its reflective nature, i.e. the aim would be to make the IED students ‘reflective practitioners’, engaged in continual self-enquiry as practising teachers. A third major feature will be training in classroom based research.” (AKU/IED 1996). The IED establishes Professional Development Centres which focus on the improvement of teaching and learning in schools and classrooms in the region. The research policy of the IED promotes research projects which are realised in collaboration with partner academic institutions from all over the world.

There are also a lot of reports about promising initiatives and research in mathematics teacher education from other continents. Within this paper it is only possible to refer to a few of them. For example, in *Australia* the in-service teacher education programme ARTISM showed that teachers’ professional growth is a very individual and complex process with action and reflection as crucial elements (see e.g. Peter 1996), and the interactive tutorial program (aiming at facilitating)

“Learning about teaching” (Mousley, Sullivan & Mousely 1996) had been produced. *Africa* was the host of the 22th PME-Conference (Stellenbosch, South Africa) where the local organisers created a challenging design for a Plenary Panel on Teacher Education (see Breen 1998): The participants discussed the philosophy, quality, and evaluation of three projects – dealing with the same task, namely to offer a one-year in-service teacher education programme for all mathematics teachers of five schools in Cape Town – that were proposed to a (fictitious) commissioning body. Taipei (Taiwan), to mention again an example from *Asia* (see report on IED, Pakistan above), will be the host of an international conference on mathematics teacher education in 1999 in order to provoke discussion on this important issue. In *South America*, for example, the ethnomathematics approach (see e.g. D’Ambrosio 1994) has a long tradition and shows interesting links to teacher education. *Europe*, for example, has several times been the host of the Conferences on “Systematic cooperation between theory and practice in mathematics education” (see e.g. Seeger & Steinbring 1991, Bazzini 1994), standards for mathematics teacher education (see e.g. Goffree & Dolk 1995) have been worked out and books on pre-service education (see e.g. Giménez, Llinares & Sánchez 1996) and in-service education (see e.g. Krainer & Posch 1996) have been written.

Third, the progress in the field of mathematics teacher education might also be seen as a process of growing awareness of the *complexity of mathematics teaching* (Krainer 1993). In a first shift, recognising that teaching consists of more than presenting a pre-fabricated body of knowledge grounded in formalistic theories, research and development activities aimed at yielding a *broader sense of mathematical knowledge*. This included efforts to link mathematics with real life, to place an emphasis on the historical development of concepts and theories, to foster problem solving and to reflect on heuristic strategies, and to link contents with regard to specific and general educational objectives. The increased integration of pedagogical, psychological, social, historical, and epistemological aspects into the didactic discussion put the dominance of the subject matter into perspective. This shift might also be seen as the start of mathematics education’s struggle towards becoming a scientific discipline in its own right, i.e. a kind of emancipation from its most closely related science, namely mathematics (see e.g. the book “Didactics of Mathematics as a Scientific Discipline” by Biehler, Scholz, Strässer & Winkelmann

1994). The teacher's task was seen more and more as creatively engaging students in important mathematical activities like proving, problem solving, and modelling. However, very often a strong belief in the "manageability" of teaching through narrowly structured and covert guidance by the teacher remained. A second shift was caused by further research in mathematics education, for example, on students' thinking and on interaction in classrooms more and more integrating methods and results of related fields; the research showed that teaching cannot be seen as a simple transmission process resulting in pre-determinable learning by the students. This fundamentally *questioned the transferability of knowledge* and partially brought a shift of focus from teaching to learning, placing an emphasis on students' understanding. The students are seen less as consumers but more and more as producers and even as researchers. However, this increased awareness of the complexity of learning and teaching was also to have consequences in teacher education. The next shift, therefore, concerns again questioning the transferability of knowledge, this time from us as teacher educators to our prospective and practising teachers. It marks a step towards meeting demands which teachers formulate in the following sorts of questions: Why do mathematics educators propagate the active and investigative learner, although we the teachers have not been educated in that way, neither in pre-service nor in in-service education (with a few exceptions)? How and from whom do we get support in that direction? It is our task to find ways to take further steps in this direction, both theoretically and practically, and partially in collaboration with teachers.

Research foci and innovative forms of teacher education

The recent discussion in mathematics education shows an increasing interest in teachers' roles, challenges, beliefs, knowledge, practice(s) etc., in many cases emphasising the complexity of teachers' work: Doyle (1986), for example, characterises the demands of teaching with descriptors such as "multidimensionality", "simultaneity", "immediacy", "unpredictability", "publicness" and "history" (the accumulation of joint experiences). In parallel to this *quantitative shift* we can also observe a *qualitative shift*: recently more and more publications and conferences deal with topics like "teachers as professionals" (Stenhouse 1975), as "reflective practitioners" (Schön 1983, 1987), as "researchers" (e.g. Elliott 1991) or as "experts"

(Bromme 1992). Several approaches have characterised *basic elements of teachers' knowledge*. Shulman (1986), for example, proposed seven domains: knowledge of subject matter, pedagogical content knowledge, knowledge of other content, knowledge of the curriculum, knowledge of learners, knowledge of educational aims, and general pedagogical knowledge. Bromme (1992) created a topology of teachers' professional knowledge that attends to the nature of mathematics, pointing out that teaching is primarily a matter of taking "situation-appropriate" decisions based on available knowledge rather than generating additional perspectives for solving newly presented problems. Therefore the focus of teachers' work in the classroom primarily calls for a holistic and integrated view of knowledge rather than the existence of separate solutions to discrete problems. This perspective is supported by Berliner et al. (1988) who found that "expert teachers" are able to process a greater array of information about students and classroom situations than novice teachers and can therefore demonstrate a greater range of techniques for dealing with individual students. The conception of teachers' professional knowledge cannot be adequately described using the singular category of "knowledge", for their knowledge is a product of many types of knowledge created in quite diverse settings and often rooted in "local theories" (Brown & Cooney 1991) specific to their classroom situation. In order to put teachers' contribution to the quality of education in a broader context (than only teachers' work in their classrooms), Krainer (1994, 1998) describes four dimensions of teachers' professional practice which are general enough to be used in different situations and where both the competence and the attitudes of teachers are given equal consideration, namely *action, reflection, autonomy, and networking*. He claims that there is a need for more reflection and networking among teachers, in pre-service and in-service education as well as in their own practice at school, in order to promote teachers' professional communication and growth.

A similar motive, namely promoting teacher educators' professional communication and growth concerning our own beliefs, knowledge, and practice – as teachers in pre- and in-service courses – and our investigations into (student) teachers' beliefs, knowledge, and practice in order to learn from it for designing and evaluating teacher education, is the starting point for discussing teacher education at a conference like CERME 1.

The CERME 1 context as a learning environment in a Thematic Group

In May 1997, representatives from 16 European countries met in Osnabrück, Germany, in order to establish a new society, the European Society for Research in Mathematics Education (ERME) which aims at promoting “communication”, “co-operation” and “collaboration” in mathematics educational research in Europe. It was decided to organise in August 1998 its first Conference (CERME 1) at the same place and in the same spirit. The challenge for both, the new society and its conferences, is to build on Europe’s strong resources in research in mathematics education, on a common history which also shows interesting and rich diversities. Among other things, it means to foster and support co-operative activities on common themes, topics, and interests, aimed at (see ERME Manifesto):

- broadening understanding – going beyond local contexts to enhance knowledge and understanding in scientific areas;
- finding out – what research is going on in different places in Europe which might benefit from wider investigation;
- identifying key areas for research – areas of scientific interest or concern which should be a major focus for collaborative research on a wider scale, and encouraging the development of inter-European research;
- bringing together areas of expertise – drawing on expertise in related areas to broaden scientific knowledge.

It was the goal of CERME 1 to make some steps towards “communication” (learning about research in different countries), “co-operation” (starting to work in a way which recognises and draws on research in other places) and “collaboration” (working jointly in research projects). Given this philosophy, it was decided to establish seven Thematic Groups (with about 12 hours for joint work) as the kernel of the conference, based on the idea that co-operative work in Thematic Groups might provide the best opportunity for recognition and sharing to begin. The Programme Committee has tried to create an environment for discussion of ideas and issues in a genuine and collaborative way which at the same time allows a scientific programme presenting to the participants research papers which have been selected through a rigorous review process. It was intended that the accepted papers should not be

delivered orally in order to allow time in the groups for addressing and working on relevant ideas and issues. The group leaders and its co-ordinator were expected to play a key role in formulating the academic work on the group, structuring it according to the themes, and encouraging participation of all members.

One of the seven Thematic Groups at CERME 1 was the group “From a study of teaching practices to issues in teacher education”. It was tried to establish the review process in this group in an open and supportive way. Nearly all reviewers – in most cases also expected participants of the group – professionally acted in this way. Given the fact, that the focus of the conference was research, for example, some innovative descriptions of good practice in teacher education – where the research dimension was not elaborated clearly enough – were not taken into consideration. Finally, four research papers have been accepted.

Given the large number of participants – 22 people coming from 13 different countries with a diversity of cultural, theoretical, etc. backgrounds – and the fact that research in (the complex field of) teacher education is in the process of becoming a field of more systematic reflection and inquiry, TG 3 has to meet a particular challenge (see also preface).

The *main ideas and goals* of the 12 hours work within Thematic Group “From a study of teaching practices to issues in teacher education” were the following:

- Establishing a spirit of communication, co-operation, and collaboration in order to create an open and fruitful learning environment for all participants. Giving the participants (as individuals) and the group (as a social system) considerable scope of freedom to influence and actively contribute to the working process in order to give all the feeling of some kind of ownership of both, processes and results.
- Focusing on research from the very beginning in order to find bridges between theory and practice in teacher education.
- Making the group’s diversity of cultural, theoretical, etc. backgrounds and working priorities (pre- and in-service education, primary and secondary level, etc.) visible and open for discussion in order to get a feeling of the broad scope of approaches and traditions of European initiatives in teacher education.

- Continuously evaluating the group's working process during the conference in order to self-apply the idea of being "reflective practitioners".

This four *main ideas and goals* will be used later on in this chapter to structure our reflection on some processes of the work of *Thematic Group 3*. However, in order to make this reflection on the working process better understandable, it is necessary to sketch briefly the *working plan* for *TG 3* (indicating the planned time and the responsible leader/s for the meeting):

- Meeting 0: Informal start (1 hour; Krainer & Goffree)
- Meeting 1: Introduction and collection of research questions (2,5 hours; Krainer)
- Meeting 2: Teacher education and research on teachers' beliefs (1,5 hours; Ponte)
- Meeting 3a: Teacher education and research on teachers' practice (1,5 hours; Goffree)
- Meeting 3b: Teacher education as teacher research (1 hour; Jaworski)
- Meeting 4: Teacher education and research on teachers' knowledge (2,5 hours; Carrillo & Coriat)
- Meeting 5: Will be planned according to the progress of the working process, final evaluation (2,5 hours; Krainer).

Whereas meeting 0 and 1 mainly had the function to set the stage, atmospherically as well as concerning the content, meetings 2 to 4 were designed to get a closer look into some exemplary fields of investigations related to teacher education. Meeting 5 had not been planned in detail, however, it was intended to refer again to the research questions, to look back at the joint process and to look forward to further activities of the group. The *strategy* behind this plan can be visualised and commented on as shown in Figure 1.

The *reason for the choice* of the topics of meetings 2, 3a, 3b, and 4 – dealing with connections of teacher education with teachers' beliefs, knowledge, practice, and their own investigations into their practice – was mainly twofold. It marks the intention to integrate most teacher education related papers and posters submitted to

the conference as well as the intention to establish a structure that allows the building of bridges between the chosen topics. Of course, considerations of teachers' beliefs, knowledge, practice, and their own investigations into their practice are interconnected, and it was assumed that this could contribute to provoking discussion on similarities and differences as well as to help to focus the discussion around some key concepts of research in teacher education.

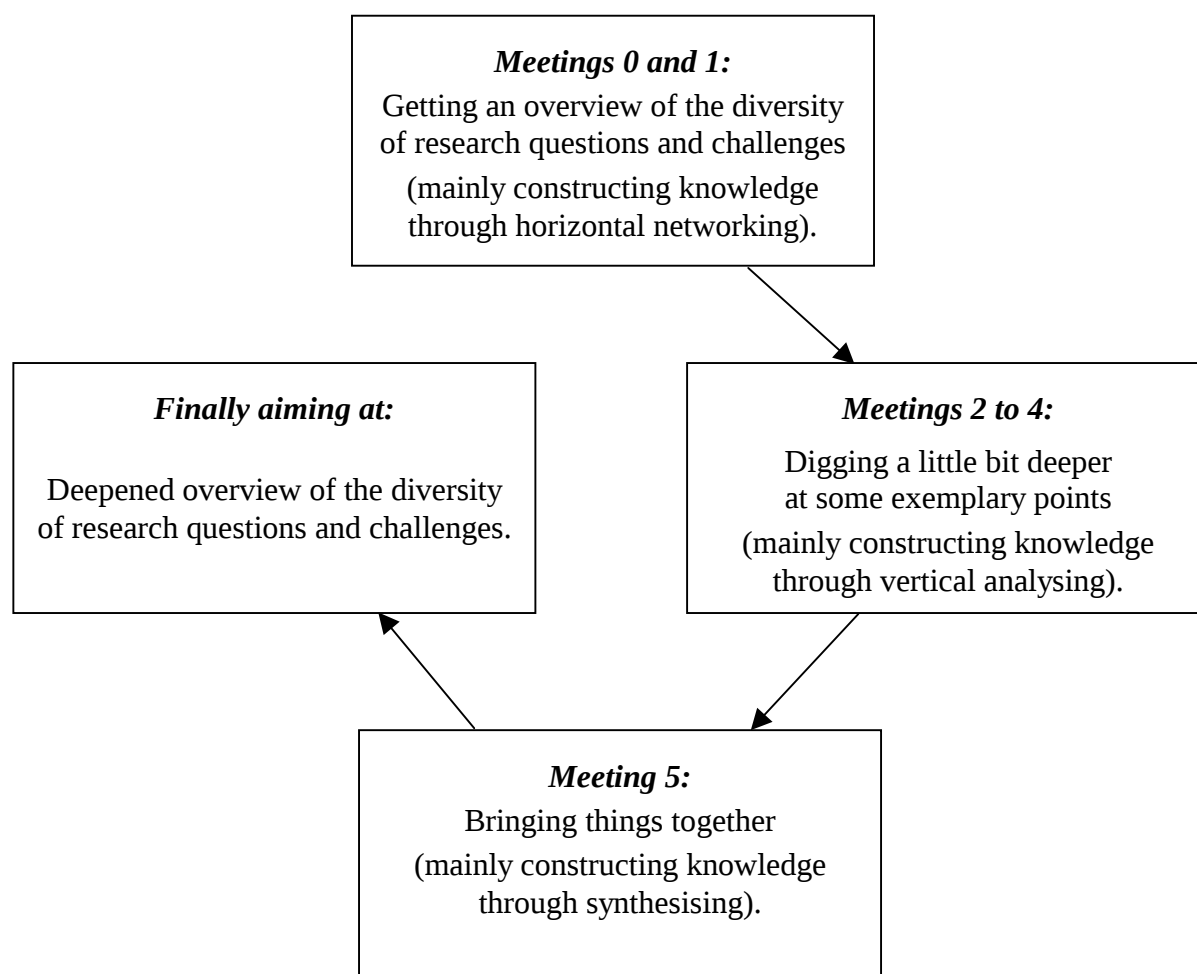


Fig. 1: Strategy for the meetings.

The “logic” behind these four topics can be visualised and commented on as depicted in Figure 2. We are interested in understanding better what, how, why, etc. (student) teachers act/do, know, believe, and reflect/investigate their own or other

teachers' classroom practice and how this knowledge can be used to design adequate learning environments in teacher education.

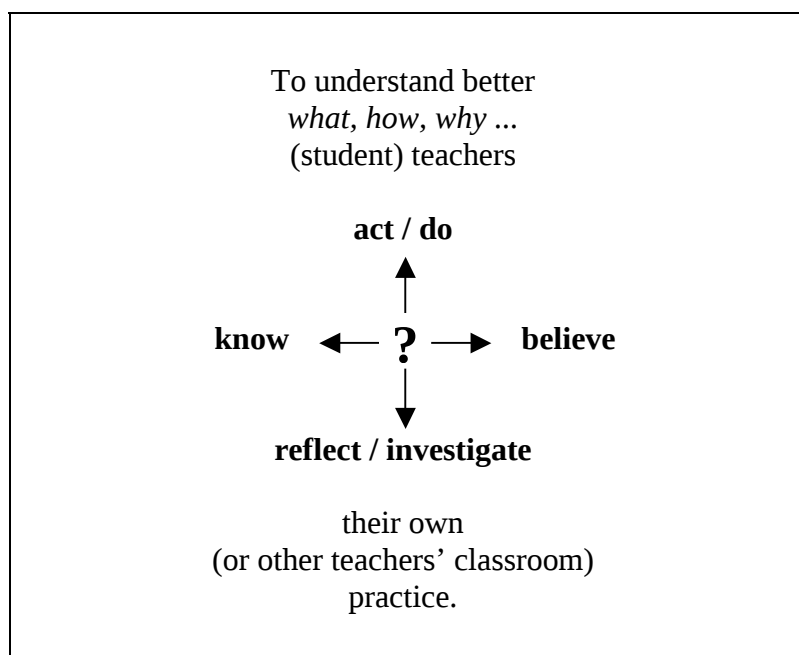


Fig. 2: Research interests.

The main goal of this better *understanding* is to contribute to an *improvement* of (student) teachers' ability to anticipate, design, analyse, evaluate, ... practice. Therefore, the promotion of (student) teachers' *understanding* and *improving* of (their anticipation of) practice can be seen as *major goals* of teacher education and the *development* of creative learning environments as a *major means* to achieve it. Whereas *understanding* is traditionally more the desire of *academic researchers* ("publish or perish"), (student) teachers are more interested in *improving* their practice ("act or perish"). However, for practising teachers seeing themselves as "reflective practitioners" (see e.g. Schön 1983 and 1987; see also chapter 5), it is crucial to connect the processes of understanding and improving: a better understanding of one's own beliefs, knowledge, actions, and reflections leads to an improvement of practice, which in turn evokes better understanding as well as new challenges that are the starting point for new interests to understand better ... It is

taken for granted that experiencing such processes should also be an important element in pre-service teacher education. This can be realised in two ways: teacher educators design learning environments where a) teacher students investigate (live or recorded) “classroom practice” (see e.g. subchapters 4.1 and 4.3), b) teacher students investigate their own learning, understanding, ... of mathematics, for example, concerning their personal achievement with regard to specific mathematical concepts (see e.g. D’Ambrosio & Campos 1992, Vollrath 1994), or they investigate their own learning, struggling, role-taking, ... during a teacher education course. Teacher in-service education has the advantage that teachers’ own “classroom practice” is available at first hand, but this is sometimes more a hindrance because established patterns and routines are not given up easily (“better known misfortune than unknown fortune”). In in-service as well as in pre-service teacher education, teacher educators are well advised to *develop* learning environments where (student) teachers’ *understanding* and *improving* of (their anticipation of) practice are promoted – taking into consideration one’s own (socially constructed) beliefs, knowledge, actions, and reflections. It is clear that teachers’ classroom practice – although surely the most prominent one – is not the only domain of teachers’ practice (e.g. collaboration with colleagues, parents; out-of-classroom activities; see also preface). However, when speaking about teachers’ practice, we are also well advised to speak about our practice, in particular our teacher educator’s practice. We also profit from *understanding* and *improving* our teaching, and we are also life-long learners – an attitude that we expect from our (student) teachers. In each pre- or in-service teacher education course we act as living examples of (less or more) “good practice” and influence (through our actions) the socialisation process of teachers. If we preach, for example, constructivist learning, we should design our learning environments in this way. In the past, teacher educators spent a lot of time discussing this issue. Now, when we speak about advanced approaches to teacher education, we need to meet the challenge, taking into consideration that (primary and secondary) students’, teacher students’ and teachers’ contexts, personal histories, perspectives of learning, beliefs and conceptions, etc. show a broad diversity. The challenge of self-application of our philosophy contributes to the complexity of teacher education.

Coming back to our conference, and our design of the learning environment, it seems to be valuable to reflect critically – at least in an exemplary way – on some

processes and products we achieved in *Thematic Group 3*. To some extent, the design of our activities (in teacher education, at conferences, ...) are always a mirror of our beliefs of “good” teacher education practice. Let’s take a chance to look at it and to learn from it!

On processes and products of *Thematic Group 3*

In most cases, conference proceedings contain papers which represent the outcomes of research or development projects. They are products which often do not reveal properly the processes which lead to its finalisation. However, often this “hidden dimension” would be very interesting to reconstruct; in a similar way we often argue that the historical development of concepts often is more important to learners than the ready-made mathematics they were told. Of course, this is always a matter of time and space. But shouldn’t we try to rediscover these processes – at least in the context of teacher education?

After this challenging introduction the reader shouldn’t expect too much. Our chapters and ideas are not so innovative and process-revealing as they probably could be. 12 hours at and 12 weeks after the conference are not enough to realise such high expectations. For this reason most of our contributions are written in a more or less traditional style, but always – in addition to referring to the international state of the art – with the intention to give as many participants of the conference a voice.

It is the intention of the *Thematic Group 3* contribution to the CERME 1 proceedings to give an insight into some processes that marked our co-operation during the conference. This is done with the idea in mind that we shouldn’t hesitate too much to make things visible from which we think they might be of interest to others, be they optimally reflected or not. Maybe, this provokes more discussion than totally polished papers.

In the following, some of these *processes* are briefly described (for more details see Thoma 1998) and reflected on, in particular focusing on meeting 0.

An exemplary case of the processes of Thematic Group 3

Given the *main ideas and goals* of Thematic Group 3, above all “Establishing a spirit of communication, co-operation, and collaboration in order to create an open and fruitful learning environment for all participants”: *How to start* (a total of 12 hours) work with a group of 22 mathematics educators from 13 different countries (with all its background of diversity) where some people know each other very well and others only know the names from the participant list, where some people did successfully pass the (open) review process and others did not (with reviewers and reviewees now in the same boat), with people with different intentions, expectations, fears, wishes, etc., and knowing from theory and practice very well that “starting situations” play a decisive role for further processes?

One possibility is to take a risk and to tempt the participants into activities which for many of them is assumedly an unexpected one (e.g. to leave their seats and to move around in order to find different “positions”), but giving them the feeling that (at least for the next 12 hours) we are a group aiming at a common goal, and at the same time making our different individual backgrounds visible and thus open for discussion in small groups and in the plenary.

The *design* of this meeting intended *four activities*. The first three of them had the same structure: The participants were told four or five criteria from which they had to choose the one which fitted best to their (professional, social, cultural, ...) situation. Then, standing in one of the corners (or in the middle) of the room, each participant was invited to discuss with his or her “neighbours” (having chosen the same criterion) for about ten minutes a specific question dealing with the criteria defined before. After that, the groups had to give a short feedback to the plenary, eventually followed by some questions or remarks.

The *four tasks* were:

Task 1: The participants were invited to find their “geographical” position within Europe (only the categories “North”, “East”, “South”, “West”, and “Middle” were allowed). In groups they then had the task to discuss the following question: “Is there a common ‘culture’ in teacher education in our region? What can we say about differences and similarities?” Here are some very brief comments:

- The “Middle” group, for example, found out that due to the geographical situation of Switzerland its teacher education system is much more complex than those of the other countries.
- The people of the “West” group all agreed in the constructivist approach but worked out different approaches to pre- and in-service education, for example, with The Netherlands putting much emphasis on pre-service education but less in in-service education.
- The “South” group compared their primary and secondary school system and found differences, for example, that the primary schools in France seem to be more scientific than those in Spain.
- “North” and “East” both had only one representative, and it was surprising for some people that the two participants from the UK went to different groups, namely one to “West” and one to “North”.

Task 2 (was not realised because of lack of time): The participants would have been invited to find their main position in a co-ordinate system with the axes primary-secondary school and pre- and in-service education.

Task 3: The participants were invited to find their position according to the research interest diagram mentioned above (I am mainly interested in *what, how, ... (student) teachers act/do, know, believe or reflect/investigate*). In groups they then had the task to discuss the following question: “What links do you see to the opposite research interest?” Again, some very brief comments:

- Group “Act/do”: The first thing we can see is the activity of the teacher. Looking at this acting/doing you can also see the three other parts.
- Group “Reflect/investigate”: Teachers have to make decisions quickly like referees have to do. Given this time pressure they need in teacher education a lot of time for reflecting on actions without that time pressure.
- Group “Know”: Teachers have to apply their knowledge under time pressure. Therefore it is essential to promote the knowledge of young teachers.
- Group “Believe”: Discussing beliefs is a necessary topic in teacher training. Researchers should change their role and beliefs: instead of trying to change

teachers they should promote their professional development. One participant added: I am a “believer” because the beliefs are the hidden constitutions for all the other things.

- Group “Middle”: You can’t reflect without acting! If you don’t know, you can’t believe! The middle represents the practice of teaching mathematics. One reason to join the group was to integrate all aspects.

The *fourth task* was the most open one of this meeting: Everyone was invited to take a glass of wine to drink and go around to talk with participants whom he or she had never met before. At the end of the meeting, the co-ordinator of the group recited a poem, wishing all an enjoyable climate and fruitful work in an open “learning community”.

Reflection on meeting 0

The participants were surprised by the unusual start of a conference meeting. But when the groups began to share their experiences during the first activity, the ice was broken. There was such intensive communication that the group leaders gave more time for this first activity and decided to miss out the second one. The feedback on meeting 0 – written anonymously, on principal – was very positive. However, the main point of reflecting on this meeting is not whether it was more or less successful, but the reasons for that.

In the following, each voice – which was properly identifiable as feedback to meeting 0 – is related to one of the four *main ideas and goals* of *Thematic Group 3* (in a few overlapping cases, the most fitting relationship was taken):

- Establishing a spirit of communication, co-operation, and collaboration in order to create an open and fruitful learning environment for all participants. Giving the participants (as individuals) and the group (as a social system) considerable scope of freedom to influence and actively contribute to the working process in order to give all the feeling of some kind of ownership of both processes and results. “*Good start for the group.*” “*I think, that meeting 0 provoked a desirable approach among people. It was its accomplished goal (very important to break distances).*” “*It was very good. Everybody spoke. It was a very good environment for the group. Congratulations. I think we are*

well guided.” “Informal meeting was very good and appropriate, and it was like a promise for a friendly climate.” “I think, it was a successful methodology. I liked it very much. Informal meeting – it was a good way to put all the people speaking.” “The great idea! Informal introduction was successful. Game-like environment released some initial stress.”

- Focusing on research from the very beginning in order to find bridges between theory and practice in teacher education. *“I liked the informal introduction to the group. It was a funny way to get acquainted to the research/professional interests of the others.”*
- Making the group’s diversity of cultural, theoretical, etc. backgrounds and working priorities (pre- and in-service education, primary and secondary level, etc.) visible and open for discussion in order to get a feeling of the broad scope of approaches and traditions of European initiatives in teacher education. *“Warming up the relationship across different people, the potential of teacher education, the ability of decision making, to understand different perceptions.” “Till now I don’t have critical remarks. It is good that all participants really have the opportunity to get engaged. It is interesting to hear which similar or different problems teacher educators from different countries have.”*
- Continuously evaluating the group’s working process during the conference in order to self-apply the idea of being “reflective practitioners”. *(No answer)*

The feedback shows that the meeting apparently contributed to the first three *main ideas and goals* of TG 3, in particular regarding the first one, “Establishing a spirit of communication, co-operation, and collaboration in order to create an open and fruitful learning environment for all participants”.

Let us use the above mentioned *four dimensions of teachers’ professional practice* – action, reflection, autonomy, and networking – to give one possible explanation for the reasons behind the feedback. The design of the meeting aimed at both, the promotion of *affective components* (attitudes) and *cognitive components* (competencies). In some comments, both components were used in a mixed way, for example, *“Warming up the relationship across different people, the potential of teacher education, the ability of decision making, to understand different perceptions.”* It should be stressed that the participants’ feedback shows once more

that each group – be it primary students or international experts in teacher education with high cognitive abilities – needs a certain kind of atmosphere with deep affective dimensions, for example, promoting processes that make it possible to “*break distances*”, to establish a “*friendly climate*”, to “*put all the people speaking*”, or to release “*initial stress*”. In addition to the balance of *affective* and *cognitive components*, there was a balance between experimental, constructive, and goal-directed work (action) on the one hand, and reflection on one’s own actions, situations, positions on the other hand. Also, the design promoted elements of self-initiative, self-organisation, and self-determined work on a personal level, but this was linked with communicative and co-operative work in groups and making its result also public for the whole group, thus combining individual and social learning processes. It is the interplay between these four dimensions and the two components that contributes to a powerful learning environment. The four dimensions can be used both, to *design* learning environments for pre- or in-service courses (or meetings like this) and/or to *analyse* such learning environments (for more details see e.g. Krainer 1998).

Written Products of Thematic Group 3

In the following we give a short preview of the chapters 2 to 5 that were elaborated by the leaders of the meetings 2 to 4 in a joint effort with some other participants of the Thematic Group. We finally refer to the concluding chapter which includes comments on some trends in research in teacher education and reflects on necessary research activities and collaboration in the future.

As mentioned before, as a result of the papers and posters submitted to *Thematic Group 3*, the meetings were structured along the following *research interests* in teacher education: *What, how, why, etc. (student) teachers believe, know, act/do, and reflect/investigate?* However, it was also intended to reflect on the corresponding results as being useful for teacher education practice. Therefore, *chapters 2 to 5* aim at finding a bridge between teacher education and investigations into teachers’ beliefs, knowledge, practice(s), and reflections. We use the term “investigations” – which is broader than research – in order to be able, for example, to include student teachers’ inquiry into teachers’ practice.

- *Chapter 2*, “Teacher education and investigations into teachers’ beliefs” (co-ordinated by João Pedro da Ponte, with contributions also from Peter Berger, Lucilla Cannizzaro, José Carrillo, Nuria Climent, Luis Carlos Contreras, and Ildar Safuanov), firstly gives an introduction to the broad field of research on teachers’ beliefs, highlighting “beliefs” and “conceptions” as foundational topics in teacher education. Then two empirical studies on teachers’ beliefs are presented. The first one looks at teacher’s beliefs about problem solving and its relation to beliefs about mathematics teaching and learning in general, indicating a clear interconnection. The second one investigates teachers’ beliefs concerning the computer, as a technical, personal, and pedagogical object, working out the importance of affective components of teachers’ beliefs. The chapter concludes with a brief survey of methodological approaches and necessary research tools and sketches some directions for future work in this field.
- *Chapter 3*, “Teacher education and investigations into teachers’ knowledge” (co-ordinated by José Carrillo and Moisés Coriat, with contributions also from Hélia Oliveira), firstly gives an introduction to the topic and some challenges, and then sketches different approaches to a characterisation of teachers’ knowledge (e.g. expert and prospective teachers knowledge, components of knowledge). This is followed by the question of how teachers’ knowledge can be promoted through teacher education, pointing out the importance of action research, situated learning, the use of narratives and the need for an integration of knowledge. The chapter concludes with open questions and a plea for viewing teacher education as an open process and argues for more communication, co-operation, and collaboration among teacher educators and researchers.
- *Chapter 4*, “Teacher education and investigations into teachers’ practice(s)” (co-ordinated by Fred Goffree, with contributions also from Marie-Jeanne Perrin-Glorian, Hélia Oliveira, Will Oonk, Maria de Lurdes Serrazina, and Julianna Szendrei), firstly gives an introduction to “good practice”, sketching different examples from Hungary and Portugal, where good practice in different contexts is realised or used for reflection. This is followed by a study of five teachers’ practices within the framework of the “theory of situations”

and the anthropologic approach of “didactic transposition”, investigating the organisation of content and the related didactical approach in secondary classrooms in France. Finally, the chapter presents a multimedial interactive learning environment (MILE) which is used in pre-service teacher education where student teachers construct practical and theoretical knowledge through investigating experienced teachers’ practice.

- *Chapter 5*, “Teacher education through teachers’ investigation into their own practice” (co-ordinated by Barbara Jaworski, with contributions also from Konrad Krainer, Razia Fakir-Mohammed, and Elisabeth Thoma), gives an insight into the work of *Thematic Group 3* in the meeting on teachers’ action research. Among others, it highlights the complexity of the teaching process and the variety of influences put on it (society, culture, ...) and presents answers of participants to questions, for example, concerning starting points, ways of involvement and contexts of action research, the theoretical background, or reflects on how action research fits with norms of established educational research. Some brief conclusions and directions for future work close the chapter.
- *Chapter 6*, “Investigations into teacher education: trends, future research, and collaboration” (written by Konrad Krainer and Fred Goffree), sketches some trends of investigations into mathematics teacher education, discusses the complexity of investigations into this field, reflects on learning from investigations, and points out some issues of future research and collaboration.

References

- AKU/IED (1996). *Mission Statement for the Institute for Educational Development (IED)*. Aga Khan University, and related material. Karachi (Pakistan): Aga Khan University.
- D’Ambrosio, B. & Campos, T. (1992). Pre-Service Teachers’ Representations of Childrens’ Understanding of Mathematical Concepts: Conflicts and Conflict Resolution. *Educational Studies in Mathematics*, Vol. 23, pp. 213–230.
- D’Ambrosio, U. (1994). Cultural framing of mathematics teaching and learning. In R. Biehler, R.W. Scholz, R. Strässer & B. Winkelmann (Eds.) (1994) *Didactics of Mathematics as a Scientific Discipline*. Dordrecht: Kluwer, pp. 443–455.
- Bazzini, L. (Ed.) (1994). *Theory and Practice in Mathematics Education*. Pavia: ISDAF.

- Berliner, D., Stein, P., Sabers, D., Clarridge, P., Cushing, K., & Pinnegar, S. (1988). Implications of Research on Pedagogical Expertise and Experience for Mathematics Teaching. In D. Grouws & T. Cooney (Eds.) *Effective Mathematics Teaching*. Hillsdale, NJ.: Erlbaum Associates & National Council of Teachers of Mathematics, pp. 67–95.
- Biehler, R., Scholz, R.W., Strässer, R. & Winkelmann, B. (Eds.) (1994). *Didactics of Mathematics as a Scientific Discipline*. Dordrecht: Kluwer.
- Breen, C. (1998). Plenary Panel: Diversity and Change in Mathematics Teacher Education. In A. Olivier & K. Newstead (Eds.) *Proceedings of the 22th Conference for the Psychology of Mathematics Education*, Vol. 1. University of Stellenbosch, South Africa, pp. 85–87.
- Breiteig, T. & Brekke, G. (Eds.) (1998). *Theory into practice in mathematics education. Proceedings from Norma 98*. Kristiansand: Agder College Research Series.
- Bromme, R. (1992). *Der Lehrer als Experte. Zur Psychologie des professionellen Wissens*. Bern-Göttingen-Toronto: Huber.
- Brown, S. & Cooney, T.J. (1991). Stalking the Dualism Between Theory and Practice. *Zentralblatt für Didaktik der Mathematik (ZDM)*, 23 (4), pp. 112–117.
- Brown, S., Cooney, T., & Jones, D. (1990). Mathematics Teacher Education. In W. Houston (Ed.) *Handbook of Research on Teacher Education*. New York: Macmillan, pp. 639–656.
- Cooney, T.J. (1994a). Research and Teacher Education: In Search of Common Ground. *Journal for Research in Mathematics Education*, 25, pp. 608–636.
- Cooney, T.J. (1994b). Teacher Education as an Exercise in Adaptation. In D. Aichele (Ed.) *Professional Development of Teachers of Mathematics*. Reston, VA: National Council of Teachers of Mathematics, pp. 9–22.
- Comitiy C. & Ball, D.L. (1996). Preparing Teachers to Teach Mathematics: A Comparative Perspective. In A. Bishop et al. (Eds.) *International Handbook of Mathematics Education*. Dordrecht: Kluwer, pp. 1123–1153.
- Cooney, T.J. & Krainer, K. (1996). Inservice Mathematics Teacher Education: The Importance of Listening. In A. Bishop et al. (Eds.) *International Handbook of Mathematics Education*. Dordrecht: Kluwer, pp. 1155–1185.
- De la Cruz, Y. & Thompson, V. (1992). Influencing Views of Mathematics Through Family Math Programs. Paper presented at ICME 7, Quebec, Canada. (Also cited in C. Gaulin, B. Hodgson, D. Wheeler, & J. Egsgard (Eds.) (1994) *Proceedings of the Seventh International Congress of Mathematical Education*. Sainte-Foy (Quebec): Les Presses de l'Université Laval, p. 223.)
- Doyle, W. (1986). Classroom Organization and Management. In M. Wittrock (Ed.) *Third Handbook of Research on Teaching*. New York: Macmillan, pp. 392–431.

- Elliott, J. (1991). *Action Research for Educational Change*. Milton Keynes and Philadelphia: Open University Press.
- Frasier, C. (1993) *A Shared Vision: Policy Recommendations for Linking Teacher Education to School Reform*. Denver: Education Commission of the States.
- Giménez, J., Llinares, S. & Sánchez, V. (Eds.) (1996). *Becoming a Primary Teacher. Issues from Mathematics Education*. Universities of Tarragona and Seville (Spain): Indugraphic.
- Goffree, F. & Dolk, M. (Eds.) (1995). *Standards for primary mathematics teacher education*. Enschede, Utrecht: SLO/NVORWO.
- Grouws, D.A. & Schultz, K. (1966). Mathematics Teacher Education. In J. Sikula (Ed.) *Handbook of Research on Teacher Education*. New York: Macmillan, pp. 442–458.
- Howson, G. (1994). Teachers of Mathematics. In C. Gaulin, B. Hodgson, D. Wheeler, & J. Egsgard (Eds.) *Proceedings of the Seventh International Congress of Mathematical Education*. Sainte-Foy (Quebec): Les Presses de l'Université Laval, pp. 9–26.
- Hoyles, C. (1992). Illuminations and Reflections: Teachers, Methodologies and Mathematics. In W. Geeslin & K. Graham (Eds.) *Proceedings of the 16th Conference for the Psychology of Mathematics Education*, Vol. 3. Durham, NH: University of New Hampshire, pp. 263–283.
- Hölzl, R. & Neubrand, M. (1996). Die Bedeutung des Zusammenhangs zwischen Forschung und Lehre in der Mathematikdidaktik für die Ausbildung der Mathematiklehrerinnen und -lehrer. Situationsanalyse, neue Ansätze und Erfahrungen. *Zentralblatt für Didaktik der Mathematik (ZDM)*, 28 (2), pp. 62–66.
- Jaji, G., Nygara, L. & Robson, M. (1985). Zimbabwe: Support for Tutors Utilising Video and Printed Modules. In Morris, R. (Ed.) *Studies in Mathematics Education. Vol. 4: The Education of Secondary School Teachers of Mathematics*. UNESCO, Paris, pp. 153–158.
- Kadunz, G., Kautschitsch, H., Ossimitz, G. & Schneider, E. (Eds.) (1996). *Trends und Perspektiven. Schriftenreihe Didaktik der Mathematik*, Band 23. Wien: Hölder-Pichler-Tempsky.
- Keitel, C. (1992). The Education of Teachers of Mathematics: An Overview. *Zentralblatt für Didaktik der Mathematik (ZDM)*, 24 (7), pp. 265–273.
- Kilpatrick, J. (1982). Casing the Case Studies: Concluding Remarks. *Journal of Research and Development in Education*, 15, 4, pp. 87–88.
- Krainer, K. (1993). Understanding Students' Understanding: On the Importance of Cooperation between Teachers and Researchers. In P. Bero (Ed.) *Proceedings of the 3rd Bratislava International Symposium on Mathematical Education*, Bratislava: Comenius University, pp. 1–22.
- Krainer, K. (1994). PFL-Mathematics: A Teacher Inservice Education Course as a Contribution to the Improvement of Professional Practice in Mathematics Instruction. In J. Ponte & J. Matos

- (Eds.) *Proceedings of the 18th Conference for the Psychology of Mathematics Education*, Vol. 3. University of Lisbon, Portugal, pp. 104–111.
- Krainer, K. & Posch, P. (Eds.) (1996). *Lehrerfortbildung zwischen Prozessen und Produkten*. Bad Heilbrunn: Klinkhardt. [See review in *Journal of Mathematics Teacher Education* 1 (1), pp. 113–116, 1998]
- Krainer, K. (1998). Dimensions of teacher's professional practice: Action, reflection, autonomy, and networking. In T. Breiteig & G. Brekke (Eds.) *Theory into practice in mathematics education. Proceedings from Norma 98*. Kristiansand: Agder College Research Series, pp. 36–44.
- Mitroff, I. & Kilmann, R. (1978). *Methodological Approaches to Social Sciences*. San Francisco: Jossey-Bass.
- Mousley, J., Sullivan, P. & Mousely, P. (1996). *An interactive tutorial program to facilitate learning about teaching*. Cat. No. LAT.101. Adelaide: Australian Association of Mathematics Teachers.
- Nebres, B. (1988). School Mathematics in the 1990's: The Challenge of Change Especially for Developing Countries. In A. Hirst & K. Hirst (Eds.) *Proceedings of the Sixth International Congress on Mathematical Education*. Budapest: Bolyai Mathematical Society, pp. 11–28.
- Peter, A. (1996). *Aktion und Reflexion. Lehrerfortbildung aus international vergleichender Perspektive*. Weinheim: Deutscher Studien Verlag.
- Peter-Koop, A. & Santos-Wagner, V. (1998). Research on the Psychology of Mathematics Teacher Development. In A. Olivier & K. Newstead (Eds.) *Proceedings of the 22nd Conference for the Psychology of Mathematics Education*, Vol. 1. University of Stellenbosch, South Africa, p. 272.
- Ponte, J.P. (1994). *MINERVA Project. Introducing NIT in Education Portugal*. Lisbon: Ministry of Education.
- Schön, D.A. (1983). *The Reflective Practitioner: How Professionals Think in Action*. New York: Basic Books.
- Schön, D.A. (1987). *Educating the Reflective Practitioner*. San Francisco: Jossey-Bass.
- Seeger, F. & Steinbring, H. (1991). Introduction. The theory-practice problem in mathematics education. *Zentralblatt für Didaktik der Mathematik (ZDM)*, 23 (4), pp. 109–111.
- Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), pp. 4–14.
- Stenhouse, L. (1975). *An Introduction to Curriculum Research and Development*. London: Heinemann.

- Thoma, E. (1998). Documentation of CERME 1. Unpublished report. Klagenfurt, Austria: University of Klagenfurt / IFF.
- Vollrath, H.J. (1994). Reflections on Mathematical Concepts as Starting Points for Didactical Thinking. In R. Biehler, R.W. Scholz, R. Strässer & B. Winkelmann (Eds.) (1994) *Didactics of Mathematics as a Scientific Discipline*. Dordrecht: Kluwer, pp. 61–72.

CHAPTER 2

TEACHER EDUCATION AND INVESTIGATIONS INTO TEACHERS' BELIEFS

2.1

INTRODUCTION

TEACHERS' BELIEFS AND CONCEPTIONS AS A FUNDAMENTAL TOPIC IN TEACHER EDUCATION

João Pedro da Ponte

University of Lisbon (Portugal)

In a memorable lecture delivered at ICME 2, René Thom (1973) expressed the view that behind any approach to mathematics teaching there is a philosophy of mathematics. This is hardly deniable. Any set of practices in a professional field is necessarily related to some perspective regarding the central objects in that field. Given the important role of the teacher in the educational process, it appears quite natural to study in-depth his or her personal philosophies about mathematics.

The argument may easily be expanded to other areas. The activity of the teacher is carried out within an educational system that has goals and objectives for the students' learning and developed a set of institutions and socially accepted practices to achieve it. Therefore, to have some insight into the way the teachers understand and carry out their job one needs to know their conceptions and beliefs about curriculum, learning, and teaching.

Beliefs and conceptions as key constructs in teacher research

Beginning with the seminal work of Thompson (1984) and Cooney (1985), the study of teachers' perspectives and personal philosophies constitutes an important strand of

work in mathematics education – in most cases using *beliefs* and *conceptions* as key constructs. Personal philosophies are related to phenomenology and mathematics educators tend to not be much versed in this area. In contrast, the notions of beliefs and conceptions are widely used in epistemological and psychological studies, some of them quite influential in mathematics education research. The key role of beliefs in human knowledge and behaviour is suggested by some epistemological and educational literature (reviewed, for example, in Thompson 1982)¹. Conceptions appear as another important construct to describe human thinking, and the term was used by Piaget in the title of several of his influential psychological studies².

However, the concepts ‘belief’ and ‘conception’ are difficult to define. They are used with different meanings and carry the label of “messy” constructs (Pajares 1992). For example, *beliefs* may be viewed as incontrovertible personal truths, that are idiosyncratic, have strong affective and evaluative components, and reside in episodic memory (Nespor 1987). Alternatively, they may be seen as dispositions to action and major determinants of behaviour, although being time and context specific (Brown & Cooney 1982). Most writers tend to agree that beliefs are not consensual and can be held with varying degrees of conviction (Thompson 1992). There are many issues that divide researchers: What is the relation between implicit beliefs and explicit beliefs? Does an implicit belief change its nature when becoming explicit? What is the structure of beliefs? Are there beliefs that play a stronger role influencing the practices than others? Are they the same for all teachers? Is it possible to espouse contradictory beliefs? Why? Are beliefs to be studied primarily from what one says or from what one does?

The notion of belief carries the idea of an inferior kind of knowledge. In everyday language “belief” is often synonymous with “religious belief”. Trying to avoid such caveats, some researchers decided to focus instead on *conceptions*. These may be seen as a conceptual *substratum* that plays a key role in thinking and action, providing ways of seeing the world and organising concepts (Ponte 1992, 1994) –

¹ The idea that beliefs are important to understand human behaviour is also well established in anthropology – the study of ancient tribes includes their beliefs about the nature of things, natural phenomena, and social relationships. However, this seemed to have little impact in this field of research in mathematics education.

² For example, *Children’s conception of number*, *Children’s conception of space*

reminding us of Brown and Cooney's view regarding beliefs. Other writers prefer to see conceptions as a conceptual umbrella. That is the case of Thompson (1992), who characterises them as "a general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (p. 130). Finally, it is possible to view conceptions as the set of positions that a teacher has about his/her practice about topics related to mathematics teaching and learning (Contreras 1998)³. Conceptions are rather difficult to study, since they are usually subconscious and rather elusive. And, of course, most questions that one may pose regarding beliefs have an equivalent frame in terms of conceptions.

Technical and common sense meanings

For some researchers (such as Thompson 1992), there is little point in making a fine technical distinctions between "beliefs" and "conceptions". These constructs are difficult to operationalise and they are largely overlapping. They have so many different *nuances* and meanings in everyday life (and in different languages) that a more precise characterisation may become artificial and, ultimately, a barrier to further progress of the research.

Other writers have argued that such a distinction is possible and useful (e.g., Ponte 1992). For example, beliefs would be statements about things that are regarded as true in some setting, whereas conceptions would be the main notions used to describe and pose questions regarding that setting. In this way, conceptions become a more general construct, that may be used to study areas in which the person does not appear to hold any strong beliefs. And a look at empirical studies suggests that we may differentiate between beliefs and myths – taking *myths* as commonplace notions about mathematics that are accepted without examining their consistency⁴ and involving, in some way, a general view of reality rather than a disciplinary one.

³ More precisely, Contreras (1998) regards conceptions as the set of positions that a researcher assumes that a teacher has about his/her practice about topics related to mathematics teaching and learning.

⁴ For example, identifying precision with uniqueness of results or assuming the existence of an answer for any question.

To make things even more complex, other constructs have been used with similar intentions. Personal philosophies and world views are concepts used in phenomenology and philosophy. In the educational literature – for example, in the research on personal practical knowledge of teachers – one also finds reference to implicit theories, personal theories, personal representations, views, perspectives, images, and metaphors. In studies related to psychological traditions – especially cognitive psychology and artificial intelligence – one finds, with similar meaning, frames, schemes, and rules of practice. And in sociological and psycho-linguistic studies one finds references to identity, self, and values.

Whatever the preference of researchers, beliefs and conceptions and similar constructs are not important in themselves but in their relation to other constructs such as attitudes, knowledge, and practices. The notion of attitude is central in social psychology. Attitudes are closely related to beliefs, especially when they are regarded as dispositions towards behaviour (as in Brown & Cooney 1982). And most writers tend to view beliefs as having an important affective side, suggesting preferences, inclinations, and lines of action. Thus, beliefs can be an important way of taking into account the affective side of the teacher's personality.

A necessary but difficult distinction to make is that between beliefs and knowledge (Thompson 1992). Some regard these as mutually exclusive: *knowledge* would be the beliefs that one is able to justify. The notion of knowledge as justified belief is appealing, but raises the issue of what counts as a justification. Even in scientific disciplines something that is regarded as a good justification today may be regarded a few years later as a gross mistake. And we do not want to restrict ourselves to scientific knowledge – in fact our main purpose is to study teachers' professional knowledge and professional practice. A way to solve this problem is to regard beliefs and conceptions as fundamental elements in structuring knowledge and, therefore, as part of knowledge. As any body of scientific knowledge always stands in a context of premises and primitive notions, also any domain of professional and practical knowledge would have its primitive notions (conceptions) and assumptions (beliefs).

And beliefs are certainly related to practice. That is the reason why the study of beliefs is important. However, one of the things that mathematics education researchers soon discovered (Thompson 1992), is that the relation between beliefs

and practices is indeed a complex one. One may hold some beliefs, for example, regarding mathematics teaching, and then act in different situations in ways that do not appear to be consistent with such beliefs (Cooney 1983). Is that because there are other subconscious beliefs that come into play in each concrete situation? Is that because of the constraints of the situation, that forces one to a continuing reframing of priorities and beliefs? What is the relation between beliefs/conceptions and practices? How do beliefs/conceptions influence practices? How do practices influence beliefs/conceptions? Which are the mediating factors?

The concepts used above provide a first approximation of the object one wants to study. Another is provided by the theoretical perspectives one holds regarding that object. Many studies carried out regarding teachers' beliefs and conceptions are done by researchers deeply interested in curriculum and educational innovation. The contrast between what teachers believe and think and the requirements of mathematics education reform, on the other hand, is sometimes striking. However, a closer look at mathematics teaching reveals that there are good reasons for teachers to act the way they do. If one wants to get a deeper understanding about the nature of teachers' beliefs and the ways they may change, a stronger theoretical background is required. Some references have already been made to social psychology, cognitive psychology (Brown & Borko 1992), phenomenology (Chapman 1997), and epistemology. One should expect that the growing interest of mathematics education researchers in anthropology and sociocultural psychology (see e.g. Crawford & Adler 1996) will also provide a specific way of looking into these ideas. Given the important role of the subconscious and of implicit aspects in these studies, one should also expect important contributions from the field of psychoanalysis.

There are two alternative positions among researchers in this field. For some, all (or most) of these terms have similar meanings. They assume that it is not useful (or it may be not even possible) to provide more technical meanings that will run against common sense uses. For others, some key terms should be selected and provided with more precise meanings. Therefore, on the conceptual side, we are left with two major challenges: (a) to decide which key constructs to use and how to characterise them; and (b) to choose other concepts to which to relate them, in the process of theory building. In other words, we need to decide whether we want to use beliefs and conceptions as theoretical or just common sense constructs, taking into

account the requirements for and the payoff from theoretical constructs; and we also need to ask what theories we have about teachers' beliefs and conceptions and how these are related to other major constructs such as professional practices, knowledge, and identity.

From the teacher education point of view, the issue regarding beliefs and conceptions is: how do teachers grow professionally? This issue is present in most of the work carried out in this area, both with pre-service and in-service teachers. What may trigger processes of professional growth? Which conditions foster processes? When are the changes merely superficial ones and when do we have fundamental changes? Is a change in beliefs and conceptions the first step to change the practices? If so, what are the next steps? What are "good" beliefs and practices? What is our role (as researchers and teacher educators) to promote teachers' professional growth?

This chapter presents two empirical studies focusing on the notion of belief. The first, by Contreras, Climent and Carrillo, looks at teachers' beliefs concerning problem solving and its relation to beliefs about mathematics teaching in general. The second, by Peter Berger, looks at teachers' beliefs regarding the computer, as a technical, personal, and pedagogical object. Following that, we review some key findings from previous literature and consider the methodological approaches and research tools (such as interviews, questionnaires, informal conversations, observations, diaries, narratives) necessary to study beliefs and conceptions. Finally, we sketch some directions for further work in this field.

References

- Brown, C.A. & Borko, H. (1992). Becoming a mathematics teacher. In D.A. Grouws (Ed.) *Handbook of research in mathematics teaching and learning*. New York: Macmillan, pp. 209–239.
- Brown, C.A. & Cooney, T.J. (1982). Research on teacher education: A philosophical orientation. *Journal of Research and Development in Education*, 15 (4), pp.13–18.
- Cooney, T.J. (1983). Espoused beliefs and beliefs in practice: The cases of Fred and Janice. In *Proceedings of the 5th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA)*. Montreal, Canada, pp. 162–169.

- Cooney, T.J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education*, 16 (5), pp. 324–336.
- Chapman, O. (1997). Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics*, 32 (3), pp. 201–228.
- Contreras, L.C. (1998). *Resolución de problemas: Un análisis exploratorio de las concepciones de los profesores acerca de su papel en el aula* (unpublished doctoral dissertation, University of Huelva).
- Crawford, K. & Adler, J. (1996). Teachers as researchers in mathematics education. In A.J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.) *International handbook of mathematics education*. Dordrecht: Kluwer, pp. 1187–1205.
- Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies*, 19 (4), pp. 317–328.
- Pajares, M.F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62 (3), 307–332.
- Ponte, J.P. (1992). Concepções dos professores de matemática e processos de formação. In M. Brown, D. Fernandes, J.F. Matos & J.P. Ponte, *Educação matemática: Temas de investigação*. Lisbon: IIE, pp. 185–239.
- Ponte, J.P. (1994). Mathematics teachers' professional knowledge. In J.P. da Ponte & J.F. Matos (Eds.) *Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lisbon, Portugal: University of Lisbon, Vol. I, pp. 195–210.
- Thom, R. (1973). Modern mathematics: Does it exist? In A.G. Howson (Ed.) *Developments in mathematics education*. Cambridge: Cambridge University Press, pp. 194–209.
- Thompson, A.G. (1982). *Teachers' conceptions of mathematics and mathematics teaching: Three case studies* (Unpublished doctoral dissertation, University of Georgia).
- Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15, pp. 105–127.
- Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.) *Handbook of research in mathematics teaching and learning*. New York: Macmillan, pp. 127–146.

2.2

TEACHERS' BELIEFS ON PROBLEM SOLVING AND MATHEMATICS EDUCATION

Luis Carlos Contreras, Nuria Climent, José Carrillo

University of Huelva (Spain)

Introduction

Research into beliefs on problem solving matches two major trends in mathematics education: On the one hand, it focuses on mathematical ideas and processes, on the other hand, it concentrates on teacher education. Current reforms ask teachers to apply new methods in their classrooms, a methodology based on problem solving. But, what do teachers think about that? What are their beliefs regarding the use of these new teaching concepts in the classroom?

Some years ago (Carrillo & Contreras 1994), we carried out a study about teachers' mathematics teaching conceptions, in which we highlighted some relationships between the teachers themselves and their mathematics conceptions. Later (Contreras, Carrillo & Guevara 1996), we studied teachers' problem solving modes (teachers performed as problem solvers themselves) and contributed some instruments to analyse their conceptions of problem solving in the classroom and Carrillo (1997) studied relationships between teachers' mathematics teaching and learning and mathematics conceptions and problem solving modes. At the moment, we are carrying out research in the field, considering teachers as teachers of problem solving not as problem solvers, and we are now able to share some ideas with respect

to the role that teachers give to problem solving in their classrooms (following Contreras, Carrillo & Guevara 1996). In addition, we wish to clarify the role that *beliefs on problem solving* (PSB) play within the set of *mathematics teaching and learning beliefs* (MTLB).

Methodology

In order to place this project within the context of research, we will use the terms described by Lincoln & Guba (1985), Bardin (1986), Wittrock (1989), Arnal et al. (1992) and fundamentally, Goetz & LeCompte (1988). We have carried out an ethnographic study by the help of which we can attempt to understand the events as they are conceived by the teachers analysed (Biddle 1989), immersing ourselves in the thoughts and actions of each one of them, during and after their teaching activity.

Beliefs are difficult to observe owing to their implicit character. Therefore we have opted for approaching teachers' thinking by inferring their beliefs through the analysis of their teaching actions, routines and action guidelines, which are easier to be observed (in this way the researcher gets the hypothetical constructs linked to every teacher).

This study includes a system of categories⁵ that we can use to highlight those constructions from the data which were made explicit. This will give us the descriptive, generative and constructive character. These categories have been subject to modifications due to the analysis of the information provided by the teachers.

The teaching of two secondary mathematics teachers has been analysed. Following Goetz and LeCompte (1988), they were chosen depending on relevance criteria (they were keen on being studied and being aware of their beliefs), and not randomly (however, they both have about 10 years of experience). Describing generalised guidelines of behaviour in the classroom is not an objective of this research, whereas an in-depth view of personal beliefs has been highlighted.

⁵ Tables II and III present different types of PSB divided into some categories (methodology, role in the subject, role in learning, student's role, teacher's role, role in assessment) and descriptors enumerated from 1 to 29.

The instruments which are used in this process belong to different levels, due to their characteristics and treatment⁶:

- *First-level instruments* (used for data gathering). They are like resources, in the sense that they are elaborated, individually, with regard to the profile which each teacher has at any given time. On the one hand, we have questionnaires⁷, semi-structured clinical interviews and classroom observations. As well as these, information has been taken from the remarks made by the teachers when delivering the questionnaires. The classroom observation sessions were complemented by memory stimulation sessions (Clark & Peterson 1989), carried out immediately following the development of the corresponding session. On the other hand, the analysis (structured) of a video recording of a teacher's performance (unconnected to the research project) in the classroom, carried out by each one of the teachers under study, and the artefacts (programming of units and tests done by the students) have also provided information about the teachers' beliefs.
- *Second-level instruments* (used for the data analysis). These are the categories and descriptors which allow us to organise and interpret the information in a detailed way (see appendix).
- *Third-level instruments* (classified presentation and interpretation of the data). Using the second level instruments, the corresponding units of information (Bardin 1986) are extracted from the first level instruments which are catalogued (interpreted) and presented as classified into categories.
- *Fourth-level instruments* (final presentation). These are the single reports which were made resulting from the descriptors obtained. In some way it is the profile (graphic representation of the set of hypothetical constructs) which has been sought which, in the final version, is preceded by a discussion session (the aim of which is to reach an agreement on those elements of the profile which are less clear). The researcher offers the subject the possibility of positioning

⁶ We use the same terminology as Carrillo (1997).

⁷ One questionnaire was closed, Likert type, and the other was semi-open-ended. The information coming from these has not been labelled. It was used in order to impulse the making of all the other instruments of first order.

himself in the different descriptions of a determined descriptor, in decreasing order of the closeness to the descriptor to which the researcher believes the subject corresponds. A description of the profile obtained with respect to the role given to problems in the classroom is included in this report.

While trying to develop the teachers' profiles, we want to highlight the instruments of the second level. We think that it is very important to provide qualitative research with instruments which allow to analyse information in depth.

TRADITIONAL TENDENCY (TR) is characterised by the use of lecturing style as usual technique and textbook as unique curricular material. Teachers give the subject an exclusively informative goal and suppose that learning is accomplished, using memory as the only resource, by superposition of units of information. Teachers conceive assessment as an activity to do at the end of each part into which the pupil's learning has been divided.

TECHNOLOGICAL TENDENCY (TE): Teachers do not show contents in their final stage; they simulate the process of construction. Teachers give the subject, apart from an informative goal, a practical character that allows its application. They suppose that learning is accomplished using memory, with an internal organisation. Teachers ask themselves (to use in an eventual future modification) about the learning process after the results obtained at the end of each part into which the pupil's learning has been divided.

SPONTANEOUS TENDENCY (S) is characterised by teachers' proposals of models manipulating activities, through which is expected the production, eventually, of unorganised knowledge. The subject has got a formative character, with the idea of being useful as an instrument for a pupil's change of attitude. Teachers think that one learns when the learning objective, that emerges randomly from the context, has got a meaning for pupils. They conceive assessment as a permanent sensor of learning that gives them the possibility of redirecting it in each moment.

INVESTIGATIVE TENDENCY (I) is characterised by teachers' organisation of the process that will lead to the acquisition by pupils of specific knowledge, through their investigations. The ultimate goal of the subject is giving the pupils some instruments that make autonomous learning possible. Teachers conceive assessment as a permanent sensor of learning that gives them the possibility of redirecting it in each moment, orienting teaching toward the foreseen learning through more appropriate contexts.

Tab. I: MTLB categories.

We started from a theoretical framework (tendencies and categories related to MTLB, following Carrillo & Contreras1994). Table I presents a brief description of these MTLB categories, based on the main MTLB descriptors.

Our above mentioned paper (Carrillo & Contreras1994) allowed us to analyse mathematics teaching and learning beliefs. But, at that moment, our scope became wider, in the sense that we wanted to go deeper into one descriptor of that instrument: the one related to the exercise-problem continuum.

The revision of related research literature (Kilpatrick 1985; Gaulin 1986; Carrillo 1995; Grouws, Good & Dougherty 1990; Chapman 1997) and the analysis of teachers' beliefs provided us with a long list of features that might characterise the use of problem solving by teachers in their classrooms (types of contexts, involved contents, required skills, mathematical goals, role and use of mistakes, ...). We thought it was possible to organise them through a categories system (as for MTLB). In this system the above mentioned features would be its descriptors. It led us to the tables shown in the appendix.

In order to provide precision and rigor, both tables and analyses have been developed by three researchers. Searching for consensus has been our method to discover the units of information and the corresponding descriptors. Repeated revision processes led us to the *definitive* (current) tables.

Results and conclusions

We can identify two types of results:

3. *Strictly methodological*. An instrument to analyse problem solving beliefs (see appendix), which theoretically answers the question about the role that beliefs on problem solving play within the set of mathematics teaching and learning beliefs.
4. *The teachers' profiles*. They are the practical side of the answer.

According to the tables shown in the appendix (methodological results already described), we will present their profiles with respect to mathematics teaching and

learning beliefs and to problem solving beliefs (the role that they give to problem solving in their classrooms) (MTLB and PSB).

Concerning the MTLB, the first of the teachers studied, Gema, shows a balanced profile between the Traditional (TR) and Technological (TE) tendencies. The other teacher, Pedro, also shows a MTLB profile predominantly TR and TE, with some traits belonging to the spontaneous (S) and investigative (I) tendencies. Basically, the two teachers could be characterised (as regards MTLB) by a rigid program, with the contents organised in isolated units; they do not make an initial diagnosis of their students who are evaluated exclusively by means of exams.

After the analysis of their conceptions on the role of problem solving in the classroom (PSB), Gema maintains her tendency between traditional and technological, the importance of the latter growing: She identifies problems with exercises, taken from an unorganised external list. They are monographic, standardised problems, with well defined wordings, and which demand a solid base of knowledge and, although at times they simulate real situations, they are formally resolved.

MTLB (Gema)			
TR		TE	
PSB (Gema)			
TR		TE	
MTLB (Pedro)			
Transkript	TE		S I
PSB (Pedro)			
TR	TE		S I

Fig. 1: MTLB and PSB of Gema and Pedro.

However, with Pedro an increase in the spontaneous character, as well as in the investigative one (to a lesser extent), can be observed in his PSB. Although Pedro shares the previous description of Gema, we must add humanist traits when, for example, he attempts to involve the students, using introductory problems or those

which encourage them to resolve and are related to reality. Moreover, whereas Gema values the capacity to identify the ideas to be applied, Pedro tends to value somewhat more the personal strategies. These results are represented in Figure 1. These graphs represent the percentage of descriptors in each tendency of the total obtained by each teacher. That represents, in the information given by Gema, 48% of the descriptors obtained are classified within the TR tendency and 52% within the TE (as regards the MTLB table). In the analysis of the PSB table, approximately 45 and 55% of the tendency descriptors have been obtained for the TR and TE tendencies respectively. Pedro's graphs have been interpreted in a similar way.

In the development of these profiles each descriptor for each tendency has been counted once in its respective table. The number of times which the same descriptor is obtained for each instrument of the first level has not been considered to be relevant. This might be a result of the fact that this data depends largely on the eventual repetitiveness of the teacher with regard to his utterances, and not only on the steadiness of his position. The same subject can (and tends to) present the same descriptor in different tendencies (for example, TR 1 and TE 1), even though at different times in their performance or utterances, or at a time which can not be catalogued with total accuracy in one of the two given tendencies. In this final case it has been counted as if there were both tendencies.

Despite the large similarity of the MTLB profiles, we observe some differences in the PSB profiles. This supports the hypothesis that the PSB profiles imply a clarification of the MTLB profiles, through the in-depth observation carried out by the PSB analysis instrument. The information provided by this instrument, focused in more concrete issues, seems to be clearer and sharper. Therefore, this instrument may be a preferable way to gather information about teachers' conceptions about teaching and learning mathematics.

We cannot generalise these results to other groups of teachers. However, we believe that the PSB tables present theoretical models of teachers' beliefs and, as well as that, facilitate the analysis of beliefs of specific subjects. In this sense, they facilitate the reproducibility, by means of replicas of this study, in which the tables will have to undergo the necessary changes (as in our study they have undergone modifications; for this reason we believe the instruments are valid, and this is a part of their extraordinary value).

The teachers in this study indicated their dissatisfaction with some aspects that they recognised in themselves and asked for help. This places the researchers in a privileged situation: the teachers want to change concrete aspects that they do not accept in their practices. The point is not to “provide” them with an alternative – as in typical teacher education activities – but stimulate them to design, construct, carry it out and reflect upon it.

In summary, we believe that any plan for teachers’ professional development should take into regard their beliefs regarding mathematics teaching – beliefs identified by the researcher, as well as by the teacher under study him- or herself, through verbalisation and reflection. The fact that the PSB seems to present less traditional positions than those of the MTLB, at the same time contributing to their clarification, suggests that it may well be a good starting point for teacher reflection.

On the other hand, problem solving is a subject normally well accepted by mathematics teachers – this is shown by many experiences in pre-service and in-service education based on problem solving (Carrillo 1995; Llinares 1996). This suggests that the PSB categories may also be a worthwhile methodological instrument to induce a change in the subjects’ beliefs.

References

- Arnal, J., Del Rincon, D. & Latorre, A. (1992). *Investigación educativa: Fundamentos y metodología*. Barcelona: Labor.
- Bardin, L. (1986). *El análisis del contenido*. Madrid: Akal.
- Biddle, B.J. & Anderson, D.S. (1989). Teoría, métodos, conocimiento e investigación sobre la enseñanza. In M.C. Wittrock (Ed.) *La investigación de la enseñanza: Enfoques, teorías y métodos*. Barcelona: Paidós, Vol. 1, pp. 95–148.
- Carrillo, J. (1995). La resolución de problemas en matemáticas: ¿Cómo abordar su evaluación. *Investigación en la Escuela*, 25, pp. 79–86.
- Carrillo, J. (1997). *Modos de resolver problemas y concepciones sobre la matemática y su enseñanza de profesores de alumnos de más de 14 años: Algunas aportaciones a la metodolog’ía de la investigación y estudio de posibles relaciones*. Huelva: University of Huelva Press.
- Carrillo, J. & Contreras, L.C. (1994). The relationship between the conceptions of mathematics and of mathematics teaching: A model using categories and descriptors for their analysis. In J.P.

- da Ponte & J.F. Matos (Eds.) *Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lisbon, Portugal: University of Lisbon, Vol. 2, pp. 152–159.
- Chapman, O. (1997). Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics*, 32, pp. 201–228.
- Clark, C. & Peterson, P. (1989). Procesos de pensamiento de los docentes. In M.C. Wittrock (Ed.) *La investigación de la enseñanza: Profesores y alumnos*. Barcelona: Paidós, Vol. 3, pp. 444–539.
- Contreras, L.C., Carrillo, J. & Guevara, F. (1996). The teacher as problem solver and his/her conception on problem solving in the classroom. In L. Puig & A. Gutiérrez (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Valencia, Spain: University of Valencia, Vol. I, p. 209.
- Gaulin, C. (1986). Tendencias actuales en la enseñanza de las matemáticas, I. *Números*, 14, pp. 11–18.
- Goetz, J.P. & Lecompte, M.D. (1988). *Etnografía y diseño cualitativo en investigación educativa*. Madrid: Morata.
- Grouws, D.A., Good, T.A. & Dougherty, B. (1990). Teacher conceptions about problem solving and problem solving instruction. In G. Booker, P. Cobb & T.N. Mendicutti (Eds.) *Proceedings of the 14th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Oaxtepec, Mexico: University of Mexico, Vol. 1, pp. 135–142.
- Kilpatrick, J. (1985). A retrospective account of the past 25 years of research on teaching mathematical problem-solving. In E.A. Silver (Ed.) *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, NJ.: Lawrence Erlbaum, pp. 1–15.
- Lincoln, Y.S. & Guba, E.G. (1985). *Naturalistic inquiry*. Beverly Hills, CA.: Sage.
- Llinares, S. (1996). Improving knowledge, professional growth and monitoring the development of mathematics teachers: A necessary integrating of theoretical frameworks. In L. Puig & A. Gutiérrez (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Valencia, Spain: University of Valencia, Addenda, pp. 23–31.
- Wittrock, M.C. (1989). *La investigación de la enseñanza* (translation of Gloria Vitale of the original in English 1986). Barcelona: Paidós.

Appendix: Second level instrument to analyse problem solving beliefs

Cat.	Description	Traditional	Technological	Spontaneous	Investigative
Methodology	1 <i>how problems are conceived</i>	Problems as exercises.	Problems as exercises; theoretical questions.	Problems as an increase in the potential of the activity of discovery.	Problems with room for awareness of what one is learning.
	2 <i>how they are chosen</i>	Unorganised external list.	Organised list according to the growing level of complexity of concepts taught.	Contingent selection of daily problems as a motivation and context of the class.	Organised collection in accordance with the objectives laid out.
	3 <i>how and when they are used</i>	At the end of the theme, as an application of the theory taught.	At the end of the theme, as an application of the theory taught.	As a vehicle to increase the potential of the spontaneous discovery of ideas.	During the whole process as training in a flexible framework of acquisition of conceptual and procedural knowledge.
	4 <i>how they are organised</i>	Unorganised exhaustive sequences	Structured sequences; conceptual spiral.	Contingent sequences depending on the context.	Procedural approach immersed in organised conceptual network.
Sense in the subject	5 <i>the why and wherefore</i>	To assimilate and reinforce theory, applying same.	To give the theory a pragmatic meaning; to introduce a theme, to explore and simulate the construction of knowledge.	To acquire procedures and encourage positive attitudes; to implicate the students in their learning.	Heuristic learning and process analysis for the construction and formalisation of concepts.
	6 <i>how they are resolved</i>	Formal resolution; mainly deductive route.	Formal resolution of real problems.	Intuitive approach to daily problems.	Mathematical resolution of problems. Induction and deduction.
	7 <i>type of problems</i>	Well defined problems. Resolution with 'heavy artillery', with unique processes and solution.	Well defined problems. Resolution with 'heavy artillery', with unique processes and solutions.	Problems attractive to resolve; valid for modelling; having no concrete conceptual aim; multiple process and solution.	Problems, including open ones. Initial condition susceptible to change; generating new problems; multiple process and solution
Role in learning	8 <i>are learned...</i>	Amplifying and reinforcing the conceptual field; monographic problems.	In their application concepts are structured; monographic problems.	Lending significance to the knowledge; polyvalent problems.	Contributing to the construction of semantic networks. Polyvalent problems.

<i>Role in learning</i>	9 <i>processes by means of...</i>	Training in formal test processes.	Identify the elements of the formal test processes.	Increase the potential of the intuitive processes.	Metacognitive aspects which favour the autonomous construction of knowledge.
	10 <i>by means of ...</i>	Initiation in the teacher's deductive styles. Standardisation.	Understanding of the teacher's resolution styles. Standardisation.	Be aware of the personal strategies.	Acquisition of heuristic styles.
	11 <i>by means of ...</i>	Individual resolution.	Individual resolution.	Group resolution.	Individual and collective. Final negotiation in the whole group.
	12 <i>mathemat. aptitude</i>	The resolution capacities are defined.	The resolution capacities are defined.	The resolution capacities can be increased.	The resolution capacities can be increased.
	13 <i>mathemat. attitude</i>	One likes problem solving or not	Occasionally, the context attracts more students to get involved.	When the student believes that he is capable of creating, he will get involved.	When the student believes that he is capable of creating, he will get involved
<i>Student's role</i>	14	Tries to identify concepts and algorithms to apply.	Tries to assimilate the theoretical concepts applying them; reconstructs processes.	Develops a trial-error activity.	Approaches the problem as an investigation.
	15 <i>what does he/she do?</i>	Captures and repeats styles.	Captures and repeats styles.	Test; maintains an empirical attitude.	Analyses and perfects his personal resolution style.
	16	Accepts processes and results.	Accepts processes and results.	His opinion on the events is considered.	Discusses the contributions of the rest and his own.
	17 <i>how he designates responsibility</i>	Initiates and protagonises the process in an exclusive way.	States and contextualises the problem giving some protagonism to the students.	Suggests problems.	Generates problems and implicates the students.
<i>Teacher's role</i>	18	Provides explicit semantic keys.	Provides implicit and explicit semantic keys	There are no explicit semantic keys.	Does not provide semantic keys; suggests heuristic ones.
	19 <i>interactions</i>	Waits for and corrects responses from the students.	Waits for and corrects responses from the students with the aim of rectifying the fault.	Stimulates at key moments; maintains interest.	Directs, channelling the positive and negative contributions.
	20 <i>how it is concluded</i>	Proposes his resolution as the correct one.	Proposes his resolution process as the most correct one.	Contributes his conclusions to the group resolution.	Organises the discussion and the final synthesis.

<i>Problems in the evaluation</i>	21	Sanctionable element; emphasis on the result.	Sanctionable element; the steps and attempts are considered within a conventional framework.	Formative instrument which allows the redirection of the process.	Formative instrument which allows for the process to be redirected and the evolution to be valued.
	22 <i>what is evaluated</i>	Weighed analysis of all the parts.	Weighed analysis of all the parts.	Valuation of effort, the implication of the student and the dynamics of the group.	Valuation of personal and disciplinary variables with explanation of routes to improvement
	23 <i>how it is evaluated</i>	Correct or incorrect adjustment of the scheme set out by the teacher.	Adequate or inadequate processes adjusted to the scheme set out by the teacher.	Discussion of the quality of the processes.	Discussion of the quality of the processes and the improvement of those.
	24	Memory of formulas and other facts.	Identification of ideas to be applied.	Implication of the students.	Acquisition of conceptual meanings and heuristics.
	25 <i>what is evaluated</i>	Mechanical application of the concepts taught.	Identification and application of adequate algorithms.	Valuation of the ideas constructed.	Relevancy of notions built up.
	26	No valuation of styles or personal strategies.	No valuation of styles and personal strategies.	Valuation of personal strategies.	Valuation of personal strategies; analysis of alternatives.
	27 <i>reactivation</i>	Training in type exercises, reinforcement.	Training in type exercises, reinforcement.	Change of activity.	Simplification of the problem maintaining the underlying mathematical structure.
	28 <i>concern about the theory</i>	Problems along with the theory; in fact they only serve to measure the theory.	The problems are valued in that they highlight the applicability of the theory.	The eventual conceptual achievements are not of vital concern.	Reflection and analysis of the eventual conceptual achievements.
	29 <i>role of error</i>	Eradication of error; sanction.	Correction of the error for a constructive end.	Warning about the existence of an error.	Constructive use of error.

2.3

AFFECTIVE COMPONENTS OF TEACHERS' COMPUTER BELIEFS: ROLE SPECIFIC ASPECTS

Peter Berger

University of Duisburg (Germany)

Background

In recent years, great efforts have been made to introduce computers and new media into schools. It is widely thought that the effective teaching of and by computers is predominantly influenced by the teachers' cognitive skills, with emphasis on up-to-date technical knowledge. However, as earlier studies by the author (see Berger 1997, 1998b, 1998c, 1998d, 1998e) have revealed, the individual 'computer world view' of a teacher influences the teaching process considerably. It gains didactic relevance, not least in that it may play the part of a 'hidden curriculum', having a selective and directive impact on the teacher's performance.

The present paper reports some outcomes of a larger research project which was aimed at analysing the computer world views of German mathematics and computer science teachers. The project is connected to the international research program of the MAVI (mathematical views) group which has been initiated by G. Törner (Duisburg) and E. Pehkonen (Helsinki)⁸. MAVI is undertaking research

⁸ The author wants to thank the DAAD (German Academic Exchange Program) and the Finnish Academy for their support during several research periods at the University of Helsinki in 1996 and 1997.

into the mathematical and related world views of teachers, pupils, and similar groups (see e.g. Pehkonen & Törner 1998).

The ‘computer world view’ can be loosely described as an individual’s view or philosophy of the ‘world of computers’. More precisely, such a ‘world view’ can be understood as a complex system (‘belief system’) of individual images, convictions, opinions, and attitudes towards a certain topic or context with a strong and, in part, subconscious influence on the manner in which the individual deals with this topic – on his thoughts, emotions, and behaviour, for example as a pupil or as a teacher (see Thompson 1992; Pehkonen & Törner 1996a, 1996b; Törner & Pehkonen 1996).

The study is methodologically based on instruments of qualitative social research (see Lincoln & Guba 1985; Tesch 1990), which are, apart from participant observation, mainly characterised by detailed analyses of exemplary single cases with the help of intensive in-depth interviews. The analysis of the interviews followed the principles of modern hermeneutics (see e.g. Beck & Maier 1994; Berger 1998a). The investigation proceeded in two phases: the first being an exploratory preliminary study (9 respondents) with a questionnaire and open and non-standardised interviews, which was followed by the main study (21 respondents) – designed on the basis of the outcomes of the preliminary study – with a questionnaire and open and standardised interviews. The empirical material consists of the video-taped interviews (1–2 hours each) as well as the transcriptions of these interviews (250,000 words). The respondents work in grammar schools and comprehensive schools in Northrhine-Westfalia, one of the 16 Federal States of Germany, teaching both mathematics and computer science; 50% have degrees in computer science, another 39% have undergone a two-year intensive in-service teacher education program in computer science.

Three social roles – three fields of experience

An investigation of the computer world views of mathematics and computer science teachers should not regard teachers only as teachers. The individual acts in specifically different social roles: in the role of a *teacher*, in the role of an *expert* (on mathematics and computer science), and last but not least in the role of a *private person*, i.e. an active member of society. Corresponding to these social roles, there

are three experiential domains forming a teacher's views of the computer, i.e. *school*, *science*, and *society*. Based on experiences in these domains, each social role separately may have shaped the computer world view of the individual, resulting in specific facets and in overlapping and sometimes even inconsistent partial views.

Corresponding to these domains of experience, the interviews and the questionnaire included three thematical domains, in which the partial views, analogous to the attitudinal components, were manifested – and thus could be analysed – as opinions (judgements on the truth or probability of statements about reality), as affects (feelings of attraction or rejection), and as dispositions of behaviour (predispositions for actions).

Experiential domain 'school' – the view of the teacher

In the preliminary study, three aspects of computer science have been named as the most central ones at school: namely 'computer', 'programming language', and 'algorithm'. Figure 1 shows how the teachers assess the centrality of these aspects in the questionnaire and how this assessment changed during their years of teaching experience. The answers (distribution of 100 points to the three items) are represented by barycentric co-ordinates, where the pin-heads show the present and the pin-points the former positions.

The diagram shows a distinct orientation towards *algorithm*. A thorough analysis of the interviews reveals that we may understand this orientation as a general tendency *from phenomena towards essentials*, i.e. as a didactic concentration on the fundamental aspects. This turning away from the engineer's fascination with hardware and software aspects, which had been accentuated in prior times, and the turning towards the concept of algorithm, which is regarded as fundamental nowadays, is especially prominent among those of the interviewed teachers with an academic background in computer science.

While the period of introducing computers into schools was shaped by a 'pioneering spirit' which quite uncritically accepted the challenge of the new medium, most of the interviewed teachers today show a more sceptical attitude. The acceleration of hardware and software innovations is experienced as an inflationary process, rapidly rendering technical knowledge outdated. As an answer to this 'race

for innovation', the interviewees tend to favour fundamental aspects as the basis of a long-term computer literacy. Following an increasingly intensive didactic reflection on the actual situation of computer science as a school subject and highlighting its contribution to the aims of general education, the basis for a confident and competent handling of the medium 'computer' is increasingly seen to lie in the *emancipation of the human from the machine*. The teaching of local tactics (e.g. detailed knowledge of a certain programming language) recedes in favour of the teaching of global strategies (problem-solving, thinking in complex structures of processes and systems).

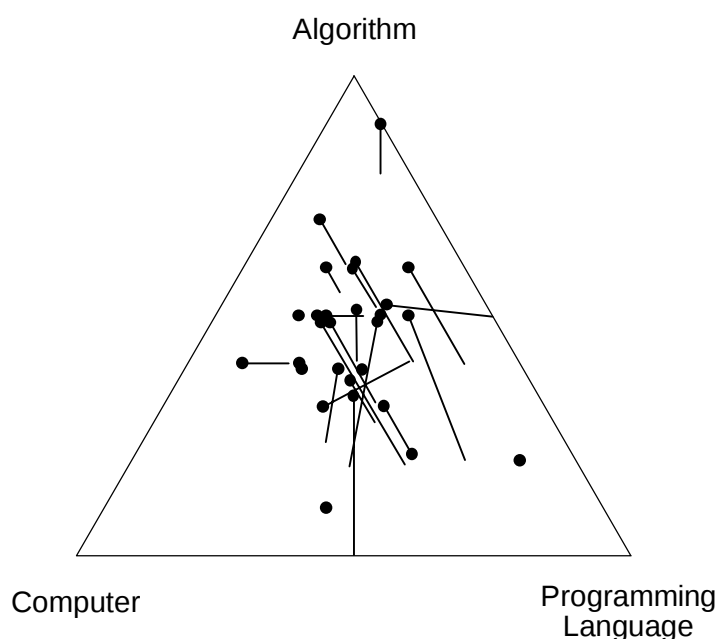


Fig. 1: The view of central concepts (N=28).

Even the academically trained (mathematics and) computer science teachers emphasise that the gap between computer users and developers of hardware and software is today nearly unbridgeable. While during the pioneering period the computer science teachers, for the most part, saw themselves as specialists with an up-to-date technical knowledge who wanted their students to acquire the same expertise, nowadays the teachers aim at making themselves and their students confident and competent computer users provided with a sound background knowledge, at least in the ideal case. In present-day computer science classes, the

computer is no longer seen as a 'magic machine' placed at the centre of teaching and learning activities. It is, rather, from the perspective of the fundamental aspect of algorithms, seen at a relativized position.

Experiential domain 'computer science' – the view of the expert

As one result of the study, it turned out that the interviewed teachers actually do not see computer science predominantly as a 'science of the computer'. The interview statements regarding computer science as a scientific discipline are, rather, thoroughly characterised by a distinct *computer-distant* point of view. This is also confirmed by a quantitative observation, i.e. in the statements describing the essentials of computer science as a scientific discipline, the frequency of the terms 'computer' and 'machine' is significantly low. More than 60% of the respondents do not use the terms at all, or only once. If it is used, it is often done with the intention of restriction and distancing.

- "In computer science everything has somehow got to do with computers. Nevertheless, computers are not the point, but all those theoretical foundations."
- "In my view, the computer is not really a characteristic feature of computer science."
- "The technical know how, the machine, is one aspect. But that is something I would rather regard as engineering, not as computer science."
- Some of the interviewees have rather subjective conceptions of computer science, coming up with a broad spectrum of individual characterisations.
- "Basically, computer science deals with the outside world in our heads. That's actually the same as philosophy does."
- "Computer science is: Given a problem, how to find a solution?"
- "Computer science is to handle complexity. It is itself complicated, and it must be, just because the world is complicated."
- "Computer science has become a rival to mathematics. Mathematics now is annexing subjects of computer science."

- “I think, the crux of the matter is, that we do not exactly know what we are actually doing and what sort of science that might really be, which is copying a little bit here and there.”

The spontaneous individual characterisations of computer science given in the interviews of the main study are, in a standardised form, shown in Table I. For the most part, the teachers see computer science as a dominantly formal science, dealing with abstract topics such as ‘information’, ‘structures’, ‘algorithms’, ‘formal languages’, and ‘complex systems’. It is seen as a science more akin to mathematics and even to philosophy, rather than to engineering. It is regarded as devoted to the machine *computer*, however, focusing mainly on the abstract aspects of those machines. Even some interviewees who consider computer science as ‘science of the computer’ emphasise its foundational aspects.

<i>Computer science essentially is ...</i>	No. of respondents (N=21)
Information science	5
Structural science, like mathematics	3
Computer science, oriented towards applications	3
Theory of algorithms	3
Computer science, oriented towards foundations	2
Theory of formal languages and machines	1
System analysis	1
Formal philosophy	1
Science of complexity	1
A ‘hotchpotch’ of other sciences	1

Tab. I: Teachers’ characterisations of computer science.

Experiential domain ‘society’ – the view of the private person

Whereas the interview statements concerning the experiential domain ‘computer science as a scientific discipline’ are characterised by a distinct *computer-distant*

perspective, the role of the computer in the domain 'school' is mostly seen in a reduced, but still central position, as mentioned above. However, the statements concerning computers and society in many ways form a reasonable contrast to this. Most of the teachers attach great importance to the computer in this domain and see its role here as central. Significantly, frequent individual assessments are made and the comments are often emotionally charged.

The assessments are highly individual, forming a wide range from euphoric agreement to vehement disapproval, from confidence to extreme worry. As an illustration, we quote from different interviews:

- “The computer is the central medium – it secures our standard of living.”
- “I think we could not survive without the computer.”
- “At the moment we are living in a time where the computer is being overestimated, simply because it is a time of radical change. In 50 years, it would be a dead-normal thing, like a kitchen appliance today.”
- “The technological instrument computer has infiltrated us.”
- “A radical change is going on. How things will develop – there are so many tendencies – it’s a horror. One can barely describe it in words – it will crash.”
- “It depends on what man makes from it, it’s another kind of atomic bomb coming our way.”

There is a widely held opinion that having been familiar with computers for a long time, a computer science teacher’s attitude towards computers might not be characterised by fear nor fascination. Some of the interviews and the quotations above, however, reveal a totally different view. On the one hand, some teachers describe their attitudes toward computers from a subliminal perspective of fear:

- “Earlier I feared that people could blame me for things that I couldn’t do. ... But that I don’t fear now. ... However, yesterday I was in the computer centre, and I acted a little bit stupid. I only needed some information, and I immediately said that I was totally stupid, because the person there always behaves somehow arrogantly. So I was not keen on letting him know that I had

studied computer science. I fear I might be blamed for not being able to work with MS-DOS or Unix commands ... ”

On the other hand, teachers with many years of computer experience are revealing their high-flying expectancy of the skills of a next generation of computers, exposing contrary positions of both euphoric agreement and extreme worry within a narrow context:

- “The internet ... a crazy possibility to communicate with a gigantic public. Global village – a fascinating thing ... When it breaks free, it would become a gigantic danger of totally loosing yourself ... that is a gigantic danger – a gigantic danger ... you are separated. We would no longer communicate with all those non-verbal signals, but only by computers. That’s a gigantic danger ... the computer would become an instrument damaging the whole society. On the other hand, I can fully understand that naively euphoric usage. ... It is a totally new experience, you can discover some totally new aspects of your own personality. Thus, it is a widening of one’s own self – definitely. ... I find it a fascinating thing. It is fun. ... This central communicating-machine, I think, will come ... yes, that will be fun.”

Computers and affectivity

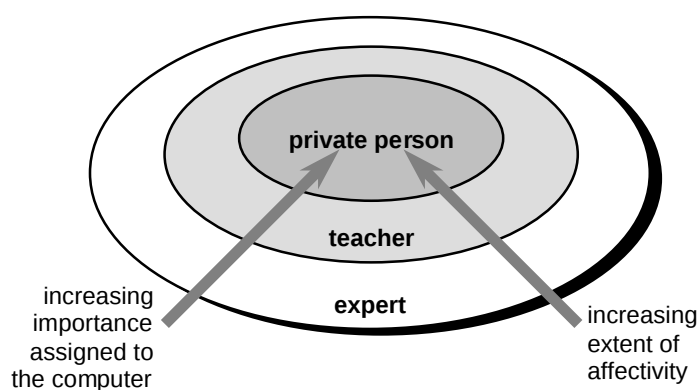
If analysis of the interviews is done by applying the parameters *contents* (what does the interviewee say about the computers’ role in the specific field?), *assessments* (how, and how often, does she or he assess this role?), and *affectivity* (what is the extent of emotion in the presentation of this role?), and if we distinguish only three items of interpretative characterisation (*low*, *medium*, *high* or similar), it turns out that there is essentially one global ‘interview profile’ which characterises all interviews (see Table II).

There is a significant qualitative correlation between the importance assigned to the computer, the frequency of assessments, and the affectivity colouring the corresponding statements. From *science* via *school* to *society*, the computer views of each interviewee take a more and more emotionally charged perspective, while the role of the computer is increasingly considered as central and relevant.

	<i>experiential domains</i>		
	<i>science</i>	<i>school</i>	<i>society</i>
<i>role of the computer</i>	peripheral	central, but qualified	central
<i>importance assigned to the computer</i>	little	medium	high
<i>frequency of assessments</i>	low	medium	high
<i>'affectivity' of the statements</i>	none to low	medium	medium to very high
	<i>expert</i>	<i>teacher</i>	<i>private individual</i>
	<i>social roles</i>		

Tab. II: A global interview profile.

According to the hypothesis that higher affectivity refers to deeper layers of personality, we may depict the situation in a 'shell model' as shown in Figure 2. The more the theoretically specialised character of the field declines and social aspects and everyday experiences become determining, the greater the importance (approving or disapproving) attached to the role of computers becomes. The more the human being is involved, the more the phenomenon computer is seen as 'explosive'. The view of the *expert* is not, as one would expect, computer-centred, but the view of the *private person* is.

**Fig. 2:** Shell model.

Formalism versus creativity

Having certain educational conceptions, teachers want to reach certain pedagogical aims. Logically enough, they can achieve those aims only in co-operation with their students. Teachers, of course, know that as reflections about aims and students belong to their every day professional life. So it could reasonably be assumed that there should be a correlation between a teacher's pedagogical aims and his or her conceptions regarding (a good teaching of) computer science and mathematics on the one hand, and, on the other hand, his or her conceptions of students – particularly of good students. However, as the study revealed, it is not the teachers' *educational concepts* which make them prefer a certain type of student. Rather, those preferences originate in a teacher's personal *likes or dislikes* of persons.

In the open interviews of the preliminary study, the teachers came up on their own account with various descriptions of their views of a good computer science student. Some respondents simply gave descriptions, some showed their personal likes or dislikes of certain types of students. Essentially, there were two different types of students which were contrasted in those descriptions. To gain a deeper insight into the teachers' views of those types of students, the individual characterisations given in the pre-study had been condensed to a standardised question of the main study:

“Teachers sometimes describe two extreme types of computer science students:

- the ‘creative type’ likes problem solving, finds unexpected solutions, but may sometimes work in a somewhat lax manner and may dislike teamwork and explaining her or his ideas to others;
- the ‘formalist type’ likes accuracy, works in a more disciplined way, gives exact explanations, is co-operative, but may sometimes fail to have good ideas.

What are your experiences? Do you know similar types? Which type is more close to you?”

The research questions involved with this topic had been the following: Do the teachers accept those two types of students, named by the respondents of the first series? (Do they modify the descriptions? Will they give descriptions of other types?)

Do teachers really have certain preferences for types of students? And if so, will they admit that they have these preferences? (Doesn't it belong to the role of a teacher not to be influenced by personal likes and dislikes?) How do those preferences – if there are any – correlate with the interviewees' educational concepts and computer science beliefs?

On a merely phenomenological level, the outcome can be described by the following simple observations: All interviewees give detailed answers, based on their own experiences; only one interviewee questions – implicitly or explicitly – the existence of both types. The interviewees give more – and more detailed and vivid – characterisations of creativity than of formalism. 17 interviewees (80%) have a clear preference for one of the types, however, none of both types is significantly preferred more often than the other one (see Table III).

	<i>respondents</i> (N=21)	
strong preference of a formalist student	3	38%
preference of a formalist student	5	
no preference	3	14%
preference of a creative student	5	43%
strong preference of a creative student	4	
(statements too vague)	1	5%

Tab. III: Teachers' preferences for student types.

The respondents made very few contributions to the description of the formalist student type. Their statements mostly stick to the characterisation of the formalist type given in the question. In contrast to this, the comments on the creative student type are altogether vivid, detailed, and frequently coloured by individual assessments. These assessments are often emotionally charged, covering a wide range from euphoric agreement to vehement disapproval. As an illustration, we quote from different interviews:

Teachers preferring a formalist student

- “The problem is, that I have to get along with this chaotic creative student who can hardly be persuaded to co-operate. I wish the good students would stop being obstinate and follow the conventions we made.”
- “Well, why should I change a formally correct student? Because he has no good ideas? But perhaps he is not able to have any. The creative student may be enthusiastic about his marvellous ideas; however, we have to put it into his head that he just causes troubles to himself this way.”
- “I don’t like those ‘single combatants’ in the class room who call themselves ‘cracks’. I think, the student who asks ‘which tools are available to solve the problem?’ is more successful than the creative student who is creative, but unproductive.”
- “If it works in an egocentric way, creativity will simply be useless, especially if you are to produce something in a team.”
- “Creativity is in a way only a preliminary stage; it must be revised later on in a clean and formalistic way.”
- “I definitely prefer the formalist type. I think, the reason is that I myself fall into that category. I see difficulties in integrating the creative type into the class and to motivate him to co-operate. However, something formally correct appeals more to me.”
- “Well, I am against those solitary, reclusive [creative] students who are not able to use their resources economically.”
- “The creative type is much more problematic and requires a terrific lot of teacher’s care.”

Teachers with no preference

- “I do really like this kind of sound creativity, creativity based on learned stuff. But not, however, this chaotic creativity, which is chaos pure, a relapse into anarchy.”
- “Creative phases are oozing with ideas.”

- “It is useless to develop great ideas if you are unable to write the correct program, because you find it too silly to pay attention to details.”

Teachers preferring a creative student

- “Creative students are constantly electrifying the others with good ideas. Certainly creativity is a quality which is somehow encoded in the genes. You just have to offer it the right topics.”
- “Well, to a student who is creatively running away I have to make clear that he has to slow down now and then on his way to ecstasy. Of course I prefer a student who must be bridled and curbed. He makes my job much easier.”
- “The creative, let’s say do-it-yourself man is the more interesting one – I like such students.”
- “I think it’s great if someone is able to be creative. [Commenting on a certain creative student:] In the sixties he would have become a hippie, however, today times have changed and we now have the computer, and so he is realising his ‘flower power ideas’ this way.”
- “A student who likes problem solving – a creative puzzle type – is a very good basis. Whereas a formalist ... You don’t know if he will be able to manage the other things ... if he is able to be creative. The creative type has in any case proved to be talented – he will be able to learn the rest somehow.”
- “Certainly, the creative type is nearer to my heart. In computer science classes, those creative people dominate over the others who are waiting to be supplied with ideas. Nevertheless, I must say it is fun. They are nearer to my heart.”
- “I have a liking for the creative type. Naturally, they are more difficult to treat. [Which type has a better success prognosis?] Well, I might almost think: the formalist – it’s a pity.”
- “Well, this sloppy and spontaneous creative type is closer to me. I don’t know how far creativity can be learned. In any case, I think that it is more difficult to learn creativity than to learn discipline. Actually, discipline means to cut down on one’s innate behaviour. Strictly speaking, that’s nothing positive.”

- “Naturally, we all like that creative student more than mediocre people. It’s great if there is such a student in the class.”

On the basis of a thorough analysis of the interview statements and the outcomes of the questionnaire, we can draw the following conclusions (for a detailed analysis see Berger 1998d):

1. The question of the preferred type of a good student in computer science polarises the group of teachers. The teachers preferring a creative student form a more homogeneous group than the teachers preferring a formalist student – both with regard to the teachers’ self-concept as computer users (the majority sees the computer as creative and motivating) and with regard to the teachers’ educational level in computer science (they dominantly have a university degree). The formalist teacher’s view of a creative student is characterised by keywords such as ‘chaotic’, ‘anarchic’, ‘obstinate’, ‘unproductive’, ‘egoistic’. He or she sees a creative student as a ‘single combatant’ which ‘causes troubles to himself’. In contrast to that, a creative teacher depicts a creative student as ‘sloppy’, ‘spontaneous’, ‘hippie’, ‘electrifying’, ‘oozing with ideas’, ‘on the way to ecstasy’, ‘more difficult to treat’, ‘above mediocrity’.
2. There is a significant difference between the ways teachers describe, substantiate, or justify their preferences for the creative and the formalist student type. The teachers preferring the creative type refer to the positive aspects of a creative student, whereas the majority of the teachers who prefer the formalist type do not refer to the positive aspects of a formalist student, but rather to the negative aspects of a creative student. The teachers are likely to have a central attitude towards the creative type. This attitude conforms to (is induced by?) the teacher’s self-concept. It is this central attitude towards the creative type which determines the ‘satellite’ attitude towards the formalist type.
3. It is not the teacher’s educational concepts of computer science which makes him or her prefer a creative student or a formalist student. Those preferences rather originate in the teacher’s personal likes or dislikes of persons, i.e. in the teacher’s attitudes towards people. With other words, the preference is socially

determined, not conceptionally. Teachers prefer a student type fitting their own type of personality, not fitting their pedagogical conceptions.

Summary

As the study revealed, a teacher's individual computer world view or computer concept, i.e. her or his attitudes towards computers and the context in which they appear, represent a decisive factor in the teaching (and learning) process. The cognitive components constitute only one part of these computer concepts, the other parts being the affective and the operational components. The study showed that it is first and foremost the affective component which has a selective and directive impact on the teachers' thinking about computers, computer science, and teaching in the realm of computers. Even young teachers who, on the surface, seem to be well-equipped for their jobs as mathematics and computer science teachers did not altogether prove to be without an apprehension of computers, if subconsciously. Such observations gain even more importance, as the outcomes of other studies of the author (see e.g. Berger 1996) indicate that a teacher's computer world view may constitute a 'hidden curriculum'.

Moreover, a mathematics teacher's computer world view has a *network of tacit links* to certain of his or her basic beliefs concerning the nature of human thinking and learning, the nature of mathematics, and the nature of teaching and learning of mathematics. In order to contribute to an adequate training of mathematics and computer science teachers, further studies – and especially so intercultural studies – could allow a deeper insight into the affective aspects of teaching and learning processes in the realm of mathematics and the new media.

References

- Beck, C. & Maier, H. (1994). Zu Methoden der Textinterpretation in der empirischen mathematikdidaktischen Forschung. In H. Maier & J. Voigt (Eds.) *Verstehen und Verständigung. Arbeiten zur interpretativen Unterrichtsforschung*. (IDM 19) Köln: Aulis, pp. 43–76.
- Berger, P. (1996). Mathematics Versus Computer Science. Teachers' Views on Teacher Roles and the Relations of both Subjects. In E. Pehkonen (Ed.) *Current State of Research on*

- Mathematical Beliefs III. Proceedings of the MAVI-3 Workshop, University of Helsinki, August 23–26, 1996.* Research Report 170, Helsinki: University Press, pp. 11–18.
- Berger, P. (1997). 'Computer Concepts' of Mathematics and Computer Science Teachers. In E. Pehkonen (Ed.) *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lahti, Finland: University of Helsinki, Vol. 1, p. 218.
- Berger, P. (1998a). Exploring Mathematical Beliefs: The Naturalistic Approach. In M. Ahtee & E. Pehkonen (Eds.) *Research Methods in Mathematics and Science Education*. Helsinki: University Press, pp. 25–40.
- Berger, P. (1998b). Computer World Views of Teachers. Professional Beliefs, Attitudes, and Conceptions of German Mathematics and Computer Science Teachers. In T.W. Chan, A. Collins & J. Lin (Eds.) *Global Education on the Net. Proceedings of the 6th International Conference on Computers in Education (ICCE '98), Beijing, China*. Berlin: Springer, Vol. 1, pp. 670–679.
- Berger, P. (1998c). Teaching in the Realm of Computers. In E. Pehkonen & G. Törner (Eds.) *The State-of-Art in Mathematics-Related Belief Research. Results of the MAVI activities*. Research Report 195, Helsinki: University Press, pp. 97–126.
- Berger, P. (1998d). How Computers Affect Affectivity. Role Specific Computer Beliefs of German Teachers. In T. Breiteig & G. Brekke (Eds.) *Theory into Practice in Mathematics Education. Proceedings from Norma 98*. Kristiansand: Agder College Research Series, pp. 87–94.
- Berger, P. (1998e). *Informatische Weltbilder von Lehrern. [The Computer World Views of Teachers]*. Frankfurt: Lang (to appear).
- Lincoln, Y.S. & Guba, E.G. (1985). *Naturalistic Inquiry*. Beverly Hills, CA.: Sage.
- Pehkonen, E. & Törner, G. (1996a). Introduction to the Theme: Mathematical Beliefs. *International Reviews on Mathematical Education (ZDM)* 28 (4), pp. 99–100.
- Pehkonen, E. & Törner, G. (1996b). Mathematical Beliefs and Different Aspects of their Meaning. *International Reviews on Mathematical Education (ZDM)* 28 (4), pp. 101–108.
- Pehkonen, E. & Törner, G. (Eds.) (1998). *The State-of-Art in Mathematics-Related Belief Research. Results of the MAVI activities*. Research Report 195, Helsinki: University Press.
- Tesch, R. (1990). *Qualitative research*. London: The Falmer Press.
- Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.) *Handbook of research in mathematics teaching and learning*. New York: Macmillan, pp. 127–146.

2.4

RESEARCH ON TEACHERS' BELIEFS: EMPIRICAL WORK AND METHODOLOGICAL CHALLENGES

João Pedro da Ponte
University of Lisbon (Portugal)

Peter Berger
University of Duisburg (Germany)

Lucilla Cannizzaro
University of La Sapienza, Rome (Italy)

Luis Carlos Contreras
University of Huelva (Spain)

Ildar Safuanov
Pedagogical University of Naberezhnye Chelny (Russia)

This subchapter provides an overview of the empirical research regarding teachers' beliefs towards mathematics and mathematics teaching and learning, with a special emphasis on teachers' beliefs regarding selected mathematical topics and problem solving. It also discusses issues regarding the change of beliefs and teacher education. Drawing from issues derived from research regarding teachers' beliefs about the computer, it concludes with a brief analysis of the methodological questions implied in this field of research.

Mathematics beliefs

Beliefs and conceptions may concern mathematics in general, i.e., the nature of mathematics; they may refer to single mathematical concepts or to systems of concepts; and they may concern meta-concepts, meta-aptitude or ability to cope in activity involving proof, recursion, and problem solving, mathematical-epistemological roots, mathematical-cognitive roots, mathematical-social aspects, and computer world views that are interrelated with many of the previous fields⁹.

Most empirical work carried out by researchers strives to define or improve systems for making categories of answers to underline general features and specific differences. Pehkonen (1994) provides an extensive bibliography on the topic. Thompson (1992) presents an extensive overview of empirical research carried out into beliefs and conceptions. Because of these works we concentrate on more recent research reports, pointing out previous research just in cases of specific interest concerning contents or methodology. We select items to point out the variety of specific foci as well as the range of research instruments, given CERME's aims of collaboration and future co-operation.

We start with teachers' and students' *beliefs regarding mathematics*. These were addressed by Middleton (1995), who carried out an investigation focused on teachers' and students' personal constructs regarding intrinsic motivation in this discipline. Ponte et al. (1994) considered the way teachers and students reacted to an innovative curriculum, finding some contrasts and commonalities in their views regarding mathematics and mathematics teaching.

For pre-service elementary school teachers, Franke (1990) used Kogelman & Warren's (1978) framework of mathematics myths. She found that they share many of the mathematical beliefs held by math-anxious people. Lindgren (1995) analysed pre-service teachers' beliefs and conceptions about mathematics and mathematics teaching, referring to the three views on the nature of mathematics proposed by

⁹ Meta-concepts include, for example, proof, conjecture, axiom, counter-example, and representation. Mathematical-epistemological roots include rigor, consistency, mathematical model, and language. Mathematical-cognitive roots refer to understanding concepts or different objects, and reflecting. Mathematical-social aspects concern pedagogy of mathematics, mathematics for every day life, and gender differentiation.

Ernest (1989). She also used Thompson's (1991) hierarchical structure about the evolution of teachers' conceptions about teaching mathematics to generate a new model of three partly overlapping levels¹⁰.

Concerning in-service teachers, an important landmark is Thompson's (1982, 1984) research. She studied the relationship between the conceptions and instructional practices of three junior high school teachers using case studies and a variety of techniques, including observations, audio-recorded lessons, interviews, and Confrey's (1978) six bipolar dimension system for describing mathematics. Quantitative studies have also been carried through. For example, Austin (1992) used a Math Belief Survey Instrument (MBSI) to investigate the effects of mathematics beliefs on mathematics anxiety and mathematics self-concept of in-service teachers.

In-service teachers' beliefs are investigated, almost always, in comparison to students' beliefs or against epistemological frameworks. Pre-service teachers' beliefs are investigated, mainly, referring to commonplace notions about mathematics and in contrast with people not much educated in mathematics. The notion that a teacher can maintain different positions with regard to different contents and the understanding that teachers' practical knowledge is situation-oriented and context-bound, prompted researchers to consider "local" beliefs, in different fields of the school mathematics curriculum, in particular, in the role of problem solving.

Beliefs in specific mathematical topics

Empirical work on beliefs regarding specific mathematical topics is quite heterogeneous. Numeracy as the ability to process, communicate and interpret numerical information in a variety of contexts was observed by Askew, Brown et al. (1997) in a study examining the links between (i) teachers' practices, beliefs, and knowledge, and (ii) pupils' learning outcomes. This work included case studies of 18 primary teachers, selected from a pool of schools, who were considered as highly effective, exploring their beliefs about what it means to be numerate, how pupils

¹⁰ Also, a component factor analysis was done looking, on a selection of items, for significant correlations (see Törner 1995).

became numerate, and about their professional role. Three main groups were identified and labelled as connectionist, discovery, and transmission.

Teachers' conceptions of rational numbers and idiosyncratic beliefs about real numbers were investigated by Pinto & Tall (1996). Undergraduates in the third year of a mathematics education degree, preparing primary and secondary mathematics teachers in a British university, were interviewed with respect to searching roots of misconceptions. A fine-grained analysis was done on the role of definition (framed as formal or distorted) in relationship with number concept image, revealing a rich diversity of imagery these student teachers have about rational numbers.

Beliefs on probability and stochastics are rarely investigated. Troun's research (1997) on the subject deserves special attention because of the specific lens used to observe pre-service teachers' beliefs. The researcher obtained information on confusions and misconceptions analysing students' written assignments.

For the conception of function, Norman (1992) and in general all of chapter 3 of the concept of function in the 25th MAA notes (Harel 1994), provides a review of empirical work within a theoretical landscape to interpreting data. Some chapters in the book edited by Tall (1991) on advanced mathematical thinking are a basic resource to consider teachers' beliefs on different topics in the field of analysis and calculus. A further asset in this area is Artigue's (1992) work on differentiation, comparing physical and mathematical conceptions of differentiation. Teachers' beliefs were also investigated in connection to computers and graphic calculators. Valero & Gomez (1996) studied how a curricular innovation centred on graphic calculators effected a teacher's belief system. They developed a conceptual framework articulated in five different areas and used three different research techniques for observing teacher's behaviour.

Shifting to beliefs on relations between mathematics and real world two researches stand out as prototypes of different senses in which such relations can be interpreted. Kyeleve (1996) measured teachers' attitudes towards mathematical modelling. He reports on the development and validation of a scale built on five factors designed to measure teachers' beliefs about the importance of modelling. Verschaffel (1996) looked at pre-service elementary teachers' conceptions and beliefs about the role of real-world knowledge in arithmetical word problem solving. He

concluded that there is a strong positive correlation of the pupils' tendency to exclude real-world knowledge to analogous student teachers exclusion as well as their exclusion of appreciation of the pupils' answer.

Beliefs regarding mathematics teaching and learning

Teachers' beliefs about mathematics influence his or her choices regarding the contents to be mobilised in the classroom, the aims pursued, and the selection of learning activities. In the same way, the role that students and teachers should play in these activities, the means and methods to expound them and the evaluation schemes concerning the teaching-learning process can emphasise the importance of the teacher's beliefs about mathematics teaching and learning.

The role played by these beliefs seems to be unequal depending on the teachers, the specific aim, and the context, amongst other factors. As Thompson (1984) points out, epistemological differences concerning the nature of mathematics correspond to different options in its planning. However, beliefs about students' learning seem to belong to wider personal schemes, formed by one's own personal professional experience (Clark 1988) and during one's own learning period as a student (Ball 1988) – rather than owing to influences from educational psychology theories.

In spite of the personal nature of beliefs, dominant instructional models have been identified amongst mathematics teachers. For example, the work of Kuhs & Ball (1986) based on a revision of literature in mathematics education and on their own empirical studies, emphasised the existence of different instructional tendencies. These models are useful to describe different points of view about the teaching and learning of mathematics, although owing to the fact that they are “eclectic aggregations of beliefs, values, propositions, and principles” (Thompson 1992, p. 137).

Different studies (Brown 1985; Cooney 1983, 1985; Shaw 1989; Thompson 1984) have shown the existence of inconsistencies between *espoused* beliefs and practices. These inconsistencies suggest that there is no simple cause-effect relationship between beliefs and practices, and makes us aware of the existence of other factors that influence both the professional practice and the institutional context

(Brown 1985; Cooney 1985; Furinghetti 1997; Hoyles 1992). Such inconsistencies are fewer when teachers have the opportunity to reflect on their practice (Thompson 1984). Based on a study carried out on three groups of countries (basically from Europe, Asia, and the Middle East), Philippou & Christou (1997) showed that there were substantial differences in teachers' beliefs that can be attributed to the social, ethical, or philosophical values of each country.

Beliefs about problem solving

Problem solving is, at times, understood as a motivation element or as a way of introducing a topic (Cooney 1985). In other times it is regarded as an element of a repetitive correction sequence of exercise-explanation-exercise (Franco & Teixeira 1987; Marcelo 1987). Both cases involve traditional beliefs about teaching and learning mathematics.

Grouws, Good & Dougherty (1990), in a study of 25 secondary school teachers, found four important types of conceptions: problem solving is word problems, solving problems, solving practical problems, or solving thinking problems. The first three focused on the nature of the problems and their computational aspects, whereas the fourth concentrated on the processes used to reach the solution. This position is less common than the first three, as it becomes evident from these paradigmatic examples. Block, Dávila & Martinez (1991) point out in a study of 48 primary school teachers that standardisation is more characteristic in the classrooms: each problem corresponds to a certain operation or the use of a certain algorithm. They also show that giving definite clues, pointing out mistakes to be corrected, or finishing the process by showing the "correct solution" are other characteristics of traditional beliefs about mathematics teaching and learning.

Ernest (1992) proposed a model considering three types of teachers according to their interpretations of problem solving. The first could be identified by the term "apply" (at the end of the sessions of content transmission), the second by "experiment" (to clarify mathematical strategies and discover structures), and the third by "negotiate" (the meanings to reinforce social construction of knowledge). Chapman (1997) used metaphors to describe the beliefs which apply to the different meanings the teachers give to problem solving. One of the teachers in her study was

labelled as “community” to emphasise the humanist feeling with emphasis in group dynamics and interest in the everyday problems which characterised his practice; another was labelled as “adventure”, linked to curiosity, effort and taking risks; while the third was “play”, related to fun, reward, and challenge. Chapman’s work leads us to think about the elusive nature of the teachers’ beliefs and their implicit nature (Schön 1983). The use of metaphors for their description suggests that beliefs can be inferred from the teachers’ principles and practical rules, more easily observable (Elbaz 1983).

Change of beliefs

Change is a term that refers to dynamic processes. Change in human behaviour is related to the process of learning. As distinctly pointed out by Ernest (1989), research on mathematics learning needs to be complemented by research on mathematics teaching. However, it is rather easy to obtain permission from children to come out of the classroom to be observed, to obtain parents’ agreement to have children observed, to obtain teachers’ collaboration and co-operation to carry out that research. It is much harder to obtain teachers’ time to be questioned regarding characteristics of their teaching. It is even more difficult to take teachers education courses as objects to be observed and reflected upon because of the lack of consideration of school as a place where teachers and teacher educators learn and evolve together with the children.

The two sides of our coin – the theoretical contributions and the concrete indications derived from empirical studies – are closely interconnected. One author, Lerman (1997), addresses the issue of a theory for the psychology of mathematics teachers’ learning. The author provides an overview of theories of teachers’ learning and makes explicit his option for an activity theory approach¹¹. He presents a conceptual framework for understanding the psychology of mathematics teaching, distinguishing between teachers’ thought processes and thought schemes including teachers’ knowledge, attitudes, and beliefs.

¹¹ Essential bases for this discussion may be found in Ernest (1989).

A constructivist theoretical framework is adopted by other authors. One, Pehkonen (1995), considers fifteen possible change factors that may affect teachers' beliefs. Another, Mayers (1994), strives to determine changes in primary student teachers' beliefs about and attitudes towards mathematics and mathematics teaching through their participation in a mathematics education course designed in a constructivist perspective. And a framework based on cognitive psychology (Cognitively Guided Instruction) was used by Fennema (1996) for a 4-year period in which 21 primary teachers were involved in a program to help them understand the development of children's mathematical thinking.

The results of empirical studies may lead to recommendations for teacher education courses and professional development activities. Lerman (1997) reports two studies carried out with pre-service elementary teachers and with experienced teachers engaged in a masters degree in mathematics education. They were asked to identify areas of study that they felt they knew very little about and wished to learn about. The aim was to engage them in their zone of proximal development and to draw on their personal goals and needs. The author indicates that, according to the participants' written accounts, learning indeed took place and they were pleased that they had been able to learn something difficult on their own.

Mayers (1994), in the study mentioned above, used two belief scales and three attitude scales to measure changes in beliefs and attitudes. He registered significant shifts towards a constructivist perspective and a reduction in mathematics anxiety. Several constructivist teaching practices were identified through interviews as having been significant in contributing to changes. Pehkonen (1995) interviewed 13 experienced German middle school teachers and concluded that two main strategies may be highly effective to promote change of teachers' beliefs: change of role (the teacher was forced to identify herself with a student) and change of viewpoint (the teacher conducted a thorough interview of a pupil and saw mathematics from the pupil's point of view). Similar results are reported by Krainer & Posch (1996), indicating that interview activities by teachers (aiming at carefully listening to students' ideas and later reflecting upon their understanding of concepts, views of mathematics etc. as well as on the interview process) are a valuable starting point for challenging teachers' beliefs and practices. Krainer (1998) reports about a mathematics teacher who changed his beliefs as a result of an investigation into his

own teaching: starting with the belief that noise in the classroom must be interpreted as a hindrance of good teaching he changed – on the basis of students' feedback and his own observations – his view towards the belief that noise may emerge through monotonous modes of instruction which do not promote content-related communication between students. As a consequence, the teacher reduced frontal instruction to a minimum and increasingly designed lessons in another way.

Within the outlined landscape some questions seem to receive answers, but others seem to need continuing inquiry. For example: What are the effects of context, if there is any, on changes in beliefs? Which factors influence the change of teachers' conceptions? Might it be a single issue? Or the grade level in where pupils are? Or the kind of academic diploma that the teachers possess? Are there possible hierarchical organisations among such kind of factors? Are there some ways which promise to be more effective for helping teachers to become conscious of their own beliefs and become open to changes? Is establishing a mentor relationship between an experienced teacher and a student teacher an effective method for inducting changes in both?

It seems reasonable that if a teacher clearly changes his or her beliefs, this may lead to a change in his or her practice. It is less evident how that change can be carried out in a short-term; and even less how to promote it through education courses. Ponte et al. (1994) showed that teachers may struggle with the contradiction of being in favour of some curricular approaches but not carry them into practice because they feel insecure about how to do it. It is necessary, as Tillema (1994) suggests, to use other intervention techniques, allowing the integration of beliefs and professional knowledge. In the words of Hohoff (1997), we need to “provide [teachers] the opportunity to construct new understandings about teaching and learning, the roles they assume and the nature of the change in a dialogue community ...” (p. 283).

Research questions and research methodology

Beliefs are phenomena that escape direct observation. They can only be unfolded by the researcher in a complex and thorough process of inference and interpretation. Belief research is mainly interested in deep phenomena and cannot be carried through

in the style of an opinion poll, which would be too superficial an approach as it points into the wrong direction.

Regarding the degree of nearness to a person's self, Rokeach (1979) distinguishes between *central* and *peripheral* beliefs. Inherent in central beliefs is a considerable inertia against change, and that is what makes them relevant to educational research. Improving education always implies a process of change, which finally also means a change of beliefs. Due to their inertia, central beliefs form one important type of beliefs, another being *subconscious* beliefs. Subconscious beliefs both rule a person's actions in a particular strong way and render the change process difficult, as changing cannot be done without knowing what to change. Although certain central beliefs may be subconscious, and vice versa, both form categories which are not too closely connected.

Nevertheless, there are conscious and subconscious levels not only in belief systems, but also – and in the same fundamental sense – in knowledge systems. The most prominent contribution to this question is the concept of *tacit knowledge* as evolved by Michael Polanyi (1958, 1965, 1966). He showed that the processes involved in creative imagination cannot simply be described on the basis of deduction but, rather, need to be understood as emergent processes. The anti-positivist position of Polanyi appeals to modern qualitative social research and therefore it is, if only for that reason, of interest for belief research. In addition to this methodological aspect it gains relevance as subconscious beliefs may have effects similar to those of subconscious knowledge, particularly when we move from one world of experience to another (Ponte 1994). According to an observation of Giddens (1993), tacit knowledge may play a role in the framing of a person's perception of reality like “the discussion of Gödel's theorem in the framing of theories” (p. 158).

In empirical work, theories, methods and outcomes are strongly interrelated. Questionnaires, interviews, and observations are major methodological tools for empirical research on teachers' beliefs. In the last few years, other techniques such as diaries, narratives (see also chapter 3), and repertory grids have also been used in this field. A special reference must also be made to biographical research as described by Kelchtermans (1993, 1995). This method seems to be particularly designed for putting into action changes because of the reflecting phase which they will certainly

provoke in teachers; it seems to be an interesting method, even more from a perspective that aims at actions inducing changes than from a research perspective¹².

	Interviews and observations	Questionnaires	Diaries, Narratives, Repertory grids
Beliefs in general			
Structure of belief systems			
General beliefs versus particular			
Beliefs versus knowledge and practice			
Changing beliefs			

Tab. I: Issues and tools in research in teachers' beliefs.

Different tools may be useful to address the various aspects of research on beliefs, including beliefs in general, beliefs regarding specific topics (e.g. in problem solving), structure of belief systems (central vs. peripheral; conscious vs. unconscious), beliefs versus knowledge and practice, and changing beliefs¹³. One may arrange tools and research issues in a table (see Table I; see also Pehkonen 1994). Of course, different instruments may be used in combination. The obvious question is if researchers choose their methodological tools because they are more “fashionable” or because they are more appropriate to the research question.

¹² Useful contributions on the pertinent use of the method may be seen in Fernandes (1995).

¹³ Each of these tools requires its own ways of elaborating empirical data. Statistical methods (parametric and non-parametric) may be used for processing results of questionnaires. Factor analysis may be used to process repertory grids (see Lehrer & Franke 1992). Some less popular methods (among belief researchers) could be used such as cluster analysis, multidimensional scaling, and fuzzy logic.

Questionnaires are quite convenient in order to deal with large samples. It is possible to find a useful source of ideas and reflections for elaborating questionnaires devoted to identify teachers' beliefs in Cuadra (1997). However, we need to be aware of the elusive nature of beliefs and conceptions, which, as we have seen, often operate at a subconscious or implicit level.

Much research on teachers' beliefs and conceptions uses case studies, which sometimes draw upon quantitative and qualitative data or just on qualitative data. Most of these studies involve short periods of contact between the researcher and the teacher. This suggests the need of longitudinal studies. Researchers on teachers' (and students') understandings or conceptions should keep in mind Mason's (1997) admonition: the absence of evidence of behaviour does not mean convincing evidence of absence of conception.

Looking ahead

As teachers become more aware of the new curriculum orientations, their discourses tend to be quite in favour of such approaches. However, research has shown that change in practice, more than just sympathy for new ideas, requires personal commitment and also the necessary know-how to carry out such new practice (Ponte 1994). To get new insights we need to make creative use of methodological tools and of the new technologies. In order to gain confidence in the results and findings, we need to refine our theoretical constructs and models and improve our methodological tools. Research on teachers beliefs is useful if it is related to theories about teachers' knowledge, attitudes, practice, and processes of change. It is sound if it draws upon methods and procedures that we feel confident about.

Research on teachers' beliefs provided a wealth of information about the way teachers see mathematics, mathematics teaching and learning, both in general and in specific ways. This research provides mathematics educators with important ideas regarding the structure of teachers' knowledge and its relation to the social and institutional context. We got good descriptions about teachers' beliefs but we still know very little about the structure of beliefs and the processes by which they evolve and change.

Let us consider computers as an example. When people describe the functions of computers, they come up with a broad spectrum of individual aspects, yet mostly focusing on the instrumental aspects. To the common view, computers are machines, tools, and media which automate human brainwork. This depicts the function those machines actually are designed and destined for. However, it is not their only function – it is, rather, only the desired or explicit function of computers. When mathematics education researchers focus on computers, for example when investigating the way computers influence, change or even improve mathematics teaching and learning, they normally have in mind this explicit function. From this perspective, however, computers are of no special interest for belief researchers.

Nevertheless, computers also have a hidden or implicit function, which makes them promising and revealing for belief research. The computer serves as an objective for human projections, as a multifarious metaphor – a metaphor for human thinking, for the human brain, and even for a human being itself. The computer is both an instrumental medium and a projective medium, *a tool and a metaphor*.

New technologies not only change the world, but also man's thinking about this world, mostly resulting in a new and more abstract view on it. Clocks influenced man's understanding of time, steam engines changed man's view of labour, telescopes, railways, and telegraphs changed the thinking about distance. Technology always changes both human actions and human thinking. The new technology of computers, however, has an additional quality, as the new machine – in contrast to clocks, steam engines, railways, telegraphs etc. – is a 'thinking' one. It is both a copy of the human brain and a model for it. It is the first machine which changes our thinking about 'thinking'. A person's (a mathematics teacher's) beliefs concerning computers, about their relation to him- or herself or to other human beings reveal a lot of his or her views of the nature of (mathematical) thinking and learning.¹⁴

¹⁴ Summing up her social research in the field of computers, Turkle (1984, 1995) characterises computers as man's 'second self', as 'metaphysical' and 'psychological machines' which are a challenging cause for a metaphysical self-reflection of their users, having considerable psychological impact – on their self-concepts and on their view of human thinking. "People are able to see themselves in the computer. The machine can seem a second self ... In Freud's work, dreams and slips of the tongue carried the theory ... Today, life on the computer screen carries theory." (Turkle 1995, p. 30 and 49).

At present, teaching, and especially mathematics teaching, increasingly is ‘teaching in the realm of computers’. As a study by Berger (see e.g. 1998a and 1998b, see also subchapter 2.3) has shown, a mathematics teacher’s computer world view has a network of tacit links to some of his or her basic beliefs, concerning the nature of human thinking and learning, the nature of mathematics, and the nature of teaching and learning. As the deeper layers of a person’s belief system are not directly accessible, those links provide what we may call a ‘bypass approach’, where the narrative about one topic is readable in another context – although largely unnoticed by the narrator. Thus the ‘oral computer history’ of a computer-experienced mathematics teacher is, within the framework of qualitative social research, a promising means among others to gain access to deep and more or less subconscious mathematical and related beliefs.

To focus on such deep beliefs may, after about two decades of belief research, launch the shift from a period of a mainly descriptive belief-sampling towards a period of developing a grounded phenomenological theory of beliefs.

References

- Askew, M., Brown, M., Rodhes, V., William, D. & Johnson, D. (1997). Effective teachers of numeracy in UK primary school: Teachers’ beliefs, practices, and pupils’ learning. In E. Pehkonen (Ed.) *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lahti, Finland: University of Helsinki, Vol. 2, pp. 25–32.
- Artigue, M. & Viennot, L. (1986). La notion de differentielle en mathematiques et en phisque dans l’enseignement superieur. In *Actes de la IVme Ecole d’Eté de Didactique de Mathematiques*. Paris: Irem Paris 7, pp. 271–294.
- Austin, S. (1992). Effect of beliefs about mathematics on math anxiety and math self-concept in elementary teachers. *Education*, 112 (3), pp. 390–396.
- Ball, D.L. (1988). I haven’t done these since high school: Prospective teacher’s understanding of mathematics. In *Proceedings of the 10th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA)*. Illinois, IL, USA, pp. 268–274.
- Berger, P. (1998a). Computer World Views of Teachers. Professional Beliefs, Attitudes, and Conceptions of German Mathematics and Computer Science Teachers. In T.W. Chan, A. Collins & J. Lin (Eds.) *Global Education on the Net. Proceedings of the 6th International*

- Conference on Computers in Education (ICCE '98), Beijing, China*. Berlin: Springer, Vol. 1, pp. 670–679.
- Berger, P. (1998b). *Informatische Weltbilder von Lehrern [The computer world views of teachers]*. Frankfurt: Lang (to appear).
- Block, D., Dávila, M. & Martínez, F. (1991). Los algoritmos en la resolución de problemas: Concepciones de los maestros. *Epsilon*, 21, pp. 129–138.
- Brown, C.A. (1985). *A study of the socialization to teaching of a beginning mathematics teacher* (unpublished doctoral dissertation, University of Georgia, Athens, GA., USA).
- Brown, M., Askew, M., Rodhes, V., William, D. & Johnson, D. (1997). Effective teachers of numeracy in UK primary school: Teachers' content knowledge and pupils' learning. In E. Pehkonen (Ed.) *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lahti, Finland: University of Helsinki, Vol. 2, pp. 121–128.
- Chapman, O. (1997). Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics*, 32, pp. 210–228.
- Clark, C. (1988). Asking the right questions about teacher preparation: Contributions of research on teacher thinking. *Educational Researcher*, 17 (2), pp. 5–12.
- Confrey, J. (1978). *Conception of mathematics survey*. Unpublished manuscript, Mount Holy Oak College, South Hadley, MA., USA.
- Cooney, T.J. (1983). Espoused beliefs and beliefs in practice: The cases of Fred and Janice. In *Proceedings of the 5th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA)*. Montreal, Canada, pp. 162–169.
- Cooney, T.J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education*, 16 (5), pp. 324–336.
- Cuadra, F.G. (1997). The elaboration of questionnaires in order to determine teachers' beliefs. *Uno, Revista de Didactica de la Matematica*, 4(11), pp. 43–54.
- Elbaz, F. (1983). *Teacher thinking. A study of practical knowledge*. London: Crom Helm.
- Ernest, P. (1989a). The knowledge, beliefs and attitudes of mathematics teachers: A model. *Journal of Education for Teaching*, 15, pp. 13–34.
- Ernest, P. (1989b). The psychology of the teaching mathematics: A conceptual framework. *Mathematics Education for Teachers*, 6, pp. 21–55.
- Ernest, P. (1992). Problem solving: Its assimilation to the teacher's perspective. In J.P. Ponte, J.F. Matos, J.M. Matos & D. Fernandes (Eds.) *Mathematical problem solving and new information technologies*. Berlin: Springer, pp. 287–300.

- Fennema, E. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27 (4), pp. 403–434.
- Fernandes, D. (1995). Analysing four pre-service teachers' knowledge and thought through their biographical histories. In L. Meira & D. Carraher (Eds.) *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Recife, Brazil: University of Pernambuco, Vol. 2, pp. 162–169.
- Franco, A. & Teixeira, A. (1987). *Atitudes dos professores face a resolução de problemas*. Lisbon: APM.
- Franke, M.L. (1990). What myths about mathematics are held and conveyed by teachers? *Arithmetic Teacher*, 37 (5), pp. 10–12.
- Furinghetti, F. (1997). On teachers' conceptions: From a theoretical framework to school practice. In *Proceedings of the First Mediterranean Conference 'Mathematics-Education and Applications'*, Nicosia, Cyprus, pp. 277–287.
- Giddens, A. (1993). *New rules of sociological method: A positive critique of interpretative sociologies*. Cambridge: Polity Press.
- Grouws, D.A., Good, T.A. & Dougherty, B. (1990). Teacher conceptions about problem solving and problem solving instruction. In G. Booker, P. Cobb & T.N. Mendicutti (Eds.) *Proceedings of the 14th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Oaxtepec, Mexico: University of Mexico, Vol. 1, pp. 135–142.
- Harel, G. & Dubinsky, E. (Eds.) (1992). *The concept of function: Aspects of epistemology and pedagogy* (MAA notes 25). Washington, DC.: MAA.
- Hohoff, S. (1997). Mathematics teachers' pedagogical content knowledge and their professional development. In E. Pehkonen (Ed.) *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lahti, Finland: University of Helsinki, Vol. 1, p. 283.
- Hoyle, C. (1992). Illuminations and reflections: Teachers, methodologies and mathematics. In W. Geeslin & K. Graham (Eds.) *Proceedings of the 16th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Durham, NH., USA: University of New Hampshire, Vol. 3, pp. 263–286.
- Kelchtermans, G. (1993). Getting the story, understanding the lives: From career stories to teachers' professional development. *Teaching and Teacher Education*, 9(5/6), pp. 443–446.
- Kelchtermans, G. (1995). A utilização de biografias na formação de professores. *Aprender*, 18, pp. 5–20.
- Kogelman, S. & Warren, J. (1978). *Mind over math*. New York, NY.: McGraw Hill.

- Krainer, K. (1998). Dimensions of teachers' professional practice: Action, reflection, autonomy and networking. In T. Breiteig & G. Brekke (Eds.) *Theory into Practice in Mathematics Education. Proceedings from Norma 98*. Kristiansand: Agder College Research Series, pp. 36–44.
- Krainer, K. & Posch, P. (Eds.) (1996). *Lehrerfortbildung zwischen Prozessen und Produkten*. Bad Heilbrunn: Klinkhardt. [See review in *Journal of Mathematics Teacher Education* 1 (1), pp. 113–116, 1998]
- Kuhs, T.M. & Ball, D.L. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills and dispositions*. East Lansing, MI.: Michigan State University, Center on Teacher Education.
- Kyeleve, J.I. (1996). Measures of teachers' attitudes towards mathematical modelling. In L. Puig & A. Gutiérrez (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Valencia, Spain: University of Valencia, Vol. 3, pp. 209–216.
- Lerman, S. (1997). The psychology of mathematics teachers' learning: In search of theory. In E. Pehkonen (Ed.) *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lahti, Finland: University of Helsinki, Vol. 3, pp. 200–201.
- Lehrer, R. & Franke, M.L. (1992). Applying personal constructs psychology to the study of teachers' knowledge of fractions. *Journal for Research in Mathematical Education*, 23 (3), pp. 223–241.
- Lindgren, L. (1995). *Pre-service teachers' beliefs and conception about mathematics and teaching mathematics* (technical report). Tampere: Tampere University, Department of Teacher Education.
- Lindgren, L. (1996). Thompson's level and views about mathematics. *ZDM*, 28 (4), pp. 113–117.
- Marcelo, C. (1987). A study of the implicit theories and beliefs about teaching in elementary school teachers. *Meeting of the American Educational Research Association*, Washington, DC.
- Mason, J. (1997). Describing the elephant: Seeking structure in mathematical thinking. *Journal for Research in Mathematical Education*, 28 (4), pp. 377–382.
- Mayers, C. (1994). Mathematics and mathematics teaching: Changes in pre-service student-teachers' beliefs and attitudes. In G. Bell, B. Write, N. Leeson & J. Geake (Eds.) *Proceedings of MERGA 17*, Lismore, Australia. Ascot Vale, Vict., Australia: Mathematics Education Research Group of Australasia, Vol. 2, pp. 5–8.
- Middleton, J.A. (1995). A study of intrinsic motivation in the mathematical classroom: A personal constructs approach. *Journal for Research in Mathematical Education*, 26 (3), pp. 254–279.

- Norman, A. (1992). Teachers mathematical knowledge of the concept of function. In G. Harel & E. Dubinsky (Eds.) *The concept of function: Aspects of epistemology and pedagogy*, MAA notes 25, pp. 215–232.
- Pehkonen, E. (1994). On teachers' beliefs and changing mathematics teaching. *Journal für Mathematik-Didaktik*, 15 (3/4), pp. 177–209.
- Pehkonen, E. (1995). What are the key factors for mathematics teachers to change? In L. Meira & D. Carraher (Eds.) *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Recife, Brazil: University of Pernambuco, Vol. 2, pp. 178–185.
- Pehkonen, E. & Törner, G. (1994). *Development of teachers' conceptions about mathematics teaching: What are the key experience for the change?* (Schriftenreihe des Fachbereichs Mathematik, Preprint 270). Duisburg, Germany: University of Duisburg.
- Philippou, G.N. & Christou, C. (1997). A study of teachers' conceptions about mathematics. In E. Pehkonen (Ed.) *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lahti, Finland: University of Helsinki, Vol. 4, pp. 9–16.
- Pinto, M. & Tall, D. (1996). Student teachers' conceptions of the rational numbers. In L. Puig & A. Gutiérrez (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Valencia, Spain: University of Valencia, Vol. 4, 139–146.
- Polanyi, M. (1958). *Personal knowledge. Towards a post-critical philosophy*. Chicago, IL.: University of Chicago Press.
- Polanyi, M. (1965). The structure of consciousness. *Brain*, 88 (4), pp. 799–810.
- Polanyi, M. (1966). *The tacit dimension*. Garden City, NY.: Doubleday.
- Ponte, J.P. (1994). Mathematics teachers' professional knowledge. In J.P. da Ponte & J.F. Matos (Eds.) *Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Lisbon, Portugal: University of Lisbon, Vol. 1, pp. 195–210.
- Ponte, J.P., Matos, J.F., Guimarães, H., Canavarro, P. & Leal, L.C. (1994). Teacher's and student's views and attitudes towards a new mathematics curriculum: A case study. *Educational Studies in Mathematics*, 26 (4), pp. 347–365.
- Rokeach, M. (1970). *Beliefs, attitudes and values: A theory of organization and change*. San Francisco, CA.: Jossey-Bass.
- Schön, D.A. (1983). *The reflective practitioner: How professionals think in action*. New York, NY.: Basic Books.

- Shaw, K. (1989). *Contrasts of teacher ideal and actual beliefs about mathematics understanding: Three case studies* (unpublished doctoral dissertation, University of Georgia, Athens, GA., USA).
- Tall, D. (1991). *Advanced mathematical thinking*. Dordrecht: Kluwer.
- Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15, pp. 105–127.
- Thompson, A.G. (1991). The development of teachers' conceptions of mathematics teaching. In *Proceedings of the 13th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA)*. Blacksburg, VA, USA, Vol. 2, pp. 8–14.
- Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.) *Handbook of research in mathematics teaching and learning*. New York: Macmillan, pp. 127–146.
- Tillema, H.H. (1994). Training and professional expertise: Bridging the gap between new information and pre-existing beliefs of teachers. *Teaching and Teacher Education*, 10 (6), pp. 601–615.
- Törner, G. (Ed.) (1995). Current state of research on mathematical beliefs. *Proceedings of the MAVI Workshop*. Duisburg, Germany: University of Duisburg.
- Truran, K. (1997). Beliefs about teaching stochastics held by primary pre-service teaching students. In F. Biddulph & K. Carr (Eds.) *Proceedings of MERGA 20*, Rotorua, New Zealand. Ascot Vale, Vict., Australia: Mathematics Education Research Group of Australasia, pp. 538–545.
- Turkle, S. (1984). *The second self: Computers and the human spirit*. New York, NY.: Simon & Schuster.
- Turkle, S. (1995). *Life on the screen: Identity in the age of the Internet*. New York, NY.: Simon & Schuster.
- Valero, P. & Gomez, C. (1996). Precalculus and graphic calculators: The influence on teachers' beliefs. In L. Puig & A. Gutiérrez (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Valencia, Spain: University of Valencia, Vol. 4, pp. 363–370.
- Verschaffel, L. (1996). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in arithmetic word problem solving. In L. Puig & A. Gutiérrez (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Valencia, Spain: University of Valencia, Vol. 4, pp. 387–394.

CHAPTER 3

TEACHER EDUCATION AND INVESTIGATIONS INTO TEACHERS' KNOWLEDGE

José Carrillo

University of Huelva (Spain)

Moisés Coriat

University of Granada (Spain)

Hélia Oliveira

University of Lisbon (Portugal)

3.1

BACKGROUND

ON TEACHERS AND ON KNOWLEDGE

The situation and some challenges

Mathematics raises, among others, two kinds of challenges in the different European educational systems (leaving aside other social and structural aspects): challenges related to the role that curriculum designs grant to school mathematics, and challenges related to the pre- and in-service mathematics teachers' education.

Most curricula refer to a mathematical knowledge in continuous evolution, admitting the change of meaning of mathematical concepts through time. This epistemological caution impedes a monolithic, closed approach, and should have a special relevance in our didactic area. Tall (1995) establishes explicitly a distinction between school mathematics knowledge and advanced mathematical knowledge, both regarding the importance granted to spontaneous induction as a starting point for school mathematics learning, and referring to the inadequacy of an automatic translation from formal knowledge to any school context.

Tradition portrays a vision of mathematics as a science of the certainty, either yes or no, true or false. The dual consideration as a science of estimation and approximation (proposed in several European curriculum designs) constitutes a fundamental contribution, mainly for the special incidence of the current impressive applications of mathematics.

Finally, school methodological development in mathematics, generated from the problem solving perspective, overrides the implicit or explicit excess in the selection and application of algorithms. Carrying out inferences, exploring and identifying relationships, looking for likeness and differences, help in enlarging the

development and the acquisition of abilities not exclusively mathematical, which contributes to the individual's education.

These considerations, however, have an unequal implication in teachers' education. Differences between the implementation of the reforms can be explained on the basis of the existence of different groups of teachers, but it is not our concern. On the contrary, we would mention what teachers have in common (and also people responsible for in-service education): an attitude towards their own professional work (concrete knowledge, personal meanings, beliefs on Mathematics, and its teaching and learning, mental images, preferences and so on). We would place all that within Thompson's (1992) expression: teacher's conceptions.

In several countries teachers have been under great criticism due to what some people consider to be their inability to implement current curriculum reforms in which so many hopes rest. In spite of the recognition of many deficiencies in educational systems, teachers are those to whom people often look demanding an explanation since, traditionally, they are viewed as responsible for the knowledge transmission. Paradoxically, it is quite common to hear that, besides subject matter knowledge, teachers don't have any professional knowledge, just experience. So, for the public in general, it might look like as if "anyone can teach".

Clandinin (1986, pp. 8, 9) points out possible causes for this vision on teachers: "They are commonly acknowledged as having had experience but they are credited with little knowledge gained from that experience. The omission is due in part to the fact that we have not had ways of thinking about this practical knowledge and in part because we fail to recognise more practical oriented knowledge."

However, an increasing number of researchers are claiming that teachers do have a professional knowledge that is closely linked to their practice. This knowledge must be interpreted also in terms of the purposes teachers try to achieve and not just of what researchers or the teachers themselves consider they know (Olson 1992, p. 42).

Research is gathering evidence that teacher's knowledge is transformed and grows through classroom interaction, but this process is quite complex. Fennema & Franke (1992, p. 163) refer that there is not much research available "that explains the relationship between the components of knowledge as new knowledge develops

in teaching, nor is information available regarding the parameters of knowledge being transformed through teacher implementation.”

The importance of the studies on this subject strongly relates to the development of a positive personal image on behalf of the teacher. Personal reflection upon professional knowledge is essential to a teacher's self-confidence and development (Guimarães 1996). Therefore, the process of in-service education can benefit from a reflection by the teacher on these issues. It is also essential to identify the features of this knowledge in designing teacher education programs. Pre-service teachers can be helped to perceive what is expected from them as teachers and how their competencies might grow in practice through the analysis of and reflection on real cases.

However, teacher educators should have in mind that their role regards mainly assisting teachers in gaining consciousness of their knowledge and not just identifying it, also creating environments in which they are stimulated to construct personal practical knowledge. Fenstermacher (1994, p. 51) stresses that “The challenge for teacher knowledge research is not simply one of showing us that teachers think, believe, or have opinions but that they know. And, even more important that they know that they know.”

Although briefly, it seems suitable also to mention other questions, related to researchers' and educators' prior knowledge, whose study would bridge different research or issues:

- The connection theory-practice, involving several fields:
 - The progressive specialisation of mathematics curricula, from 6 until 18 years;
 - The excess of mathematical knowledge among mathematics teachers versus an equilibrated conception of mathematical knowledge;
 - The school practices for prospective mathematics teachers (institutional aspects versus relevant aspects according to the community of educators and researchers);
 - The classroom integration of the information technologies;

- The attention to curriculum transverse matters (mathematics and environment; mathematics and sport; and so on);
- The attention to diversity.
- The comparative analysis of the careers of mathematics teachers of primary and secondary levels, including administrative requirements, in the different European countries.
- The lessons we must learn, from the matters relative to the mathematics teachers' education, for the perennial discussion on the curriculum of mathematics in primary and secondary.
- The theoretical frames and research methodologies that different European countries are actually generating on the mathematics teachers' education; compared studies; defining quality of research and quality of teachers' education curricula.

On knowledge

According to Vergnaud (1990), the main question of epistemology is “What is knowledge?” but immediately he guides the reader's attention towards another more specialised question, as far as it appeals to the theories of learning: “How is knowledge acquired?” Vergnaud (1990, pp. 17–23) describes briefly three lines of answers, those of Piaget, Vygotsky, and Fischbein, and maybe a fourth should be added, due to Ausubel (see e.g. Novak 1985). All these references are easily accessible and we ask the reader to refer to the abundant bibliography.

Philosophically there is no agreement about the meaning of the term knowledge; any issue, from the Socratic belief of reminiscences¹⁵ to the post-modern approaches¹⁶, is strongly discussed.

We find a similar situation both concerning mathematics and mathematics education. Different meanings attributed to mathematical knowledge (Platonism,

¹⁵ Knowledge is within us and the teacher's work is as that of a midwife that should make it leave to the light (see Plato's *Meno*).

¹⁶ Both to eliminate the ideal (positivistic) subject and to „save“ knowledge, they don't have another issue than to accept the last as a „product“ of an interaction among individuals.

formalism, etc.) (Kline 1985) cohabit in the mathematicians' practices and, of course, in those of the mathematics teachers. In mathematics, Rorty (1989, p. 40) has expressed very well the main exigency imposed by a Platonic or Neo-Platonic approach: one should try to explain the Theorem of Pythagoras on the basis of the relationship of the reason with the "triangularity".

The main approach used in mathematics education relates to the term evaluation: we consider ourselves able to recognise that somebody knows something, either for his/her behaviour or for the statements that she/he utters. These social and conventional aspects of mathematical knowledge (and of its construction) constitute a line of recent investigation (see Ernest 1994).

Of course, among researchers in mathematics education, there are also different sensibilities in the way of understanding knowledge and mathematical knowledge. These differences are increased by linguistic reasons, as occurs during verbal exchanges among researchers coming from the Anglo-Saxon and the Roman Languages worlds. During one of the *Thematic Group 3* sessions, B. Jaworski (UK) asked for the difference between the terms *savoir* and *connaissance* in French (we generalise to various Roman Languages (as Spanish, French, Portuguese or Italian)). The question led to different exchanges:

- According to Conne¹⁷ (1998), in translating French educational research reports to English, we should use the term *connaissance* as knowing, the individual action addressed to construct personal meaning. On the other hand, one should use the term *savoir* as knowledge, the social product of individual *knowings*, stated and established by a collective sense.
- In Spanish, Moliner (1979a, pp. 728–729; 1979b, pp. 1074–1076) explains that, from a logical point of view, the verb *conocer* has a wider meaning than the verb *saber*. From the point of view of its usage, this doesn't happen due to the functions of the auxiliary verb that Spanish also assigns to *saber* (but not to *conocer*).

Attempting to answer Jaworski's translation question, we hypothesise that *saber* and *insight* are complementary parts of a process; each language attempts to

¹⁷ We thank F. Conne for his contribution in a mail sent after the CERME 1.

highlight only one of those terms. *Saber* and *insight* have a social character: either they are recognised by others or are narrated by the person that confronts the problem-situation (or both).

When speaking about knowledge it is very difficult (if not impossible) to isolate conditions that make it possible. (In mathematics we have mentioned reason as a condition; in politics and in the school cultures force could also be included; the teacher receives it from the institution to impose it, on occasions).

Types and components of knowledge

Educational research reports and essays generally never isolate the word *knowledge*. Let be some examples: Whitehead (1953) claims to reject *inert* knowledge, so diffused in many schools; Section 3.2 will describe, among others, *intended* knowledge, *propositional* knowledge, *logical* knowledge, and so on. As meaningful research constructs, those composed expressions call for some epistemological questions:

- Is there any hierarchy among them?
- Are those meanings of *knowledge* “independent” and, if so, in which sense: disjoint (set theory metaphor)?, dimensional (vector space metaphor)?

What we understand by “types of knowledge” is illustrated in Figure 1. A, B, and C stand for subspecies of Z. A has something in common with B, but not with C. We use the word “types” to tolerate a non-disjunction. If we could speak of *non intersecting knowledges*, we would use the term “class”, even if $A \cup B \cup C$ didn’t equal Z. According to this set theory metaphor, set inclusion describes any hierarchy among *knowledges*. If Z stands for knowledge, obviously all types of knowledge depend hierarchically of Z. Remark that $A \cup B$ and C could appear as two classes of knowledge.

Let us see some examples from the context of school mathematics. Cockroft’s Report (1985) generalised in mathematics education a distinction among conceptual and procedural knowledge. To construct a concept (conceptual knowledge) one needs facts (conceptual knowledge) and skills (procedural knowledge); similarly, to construct a procedure one needs concepts; we can conclude that conceptual

knowledge and procedural knowledge are non-disjoint, thus justifying looking at them as types. Of course, knowing facts appears as less general than knowing concepts; knowing concepts appears as less general than knowing conceptual structures; therefore, conceptual structures “include” concepts, which “include” facts; similarly, procedures “include” skills.

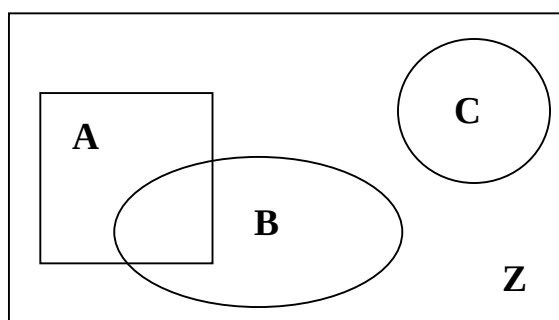


Fig. 1

Sometimes, as for teachers' professional knowledge, it is useful giving some structure to the types of knowledge. Two metaphors are available: vector space (dimensions) and meaningful grouping (components). For instance, Apple's hidden curriculum may be approached either as a curriculum dimension or as a component of any school curriculum. In Section 3.3 we will introduce four components to describe expert teachers' knowledge. Our research approach imposes to locate these four components in the same level of analysis, as far as both their descriptions and their strong relationships should together help deepen teachers' professional knowledge.

Scope of the chapter

In this chapter we have tried, on the one hand, to show some ideas expressed during the meetings of *Thematic Group 3*. On the other hand, we have expressed some ideas, which are not aimed to end relevant discussions; instead, we present these ideas as a basis for reflection and further discussion. A reference in this way is Boero, Dapueto & Parenti (1996), who deal with the problem of the relationships between research on mathematics education and the professional knowledge of mathematics teachers.

They make explicit some challenges and difficulties in this field and call for further studies.

Some words, concepts and challenges arise when we deal with teachers' knowledge and teacher education. "What is knowledge?" is perhaps one of the first questions that meet at that point. But this question calls for a deep study: types and components come to play and contribute the second question, "What knowledge?". Then we have linked them to the difference between expert and prospective teachers' knowledge. We are in favour of distinguishing expert and prospective features, and at the same time we are in favour of considering teachers' education as a process that should begin in the pre-service period. In this sense, we avoid conceptualising this period as an ended phase.

We go on with the construction of teachers' knowledge. Our aim does go beyond a characterisation of knowledge towards a concern about its construction process. In this process we present a discussion around Schön's contribution and we propose collaborative action research as a framework to place research on teachers' knowledge as well as to educate teachers. The focus of situated learning and the use of narratives help close our proposal concerning teachers' growth.

Some consequences for what we consider an open curriculum, some open questions and some looks at the future are given at the end of this chapter. This last section is devoted to challenge future co-operation, communication and collaboration in order to improve our knowledge, which should be related to teachers' and students' knowledge.

3.2

DIFFERENT APPROACHES TO A CHARACTERISATION OF TEACHERS' KNOWLEDGE

Overview

In a first approach to the mathematics teachers' knowledge we establish some limitations of the terms pre- and in-service and we make a quick travelling through different meanings that it is necessary to attribute to teachers' knowledge, both from the education and the educational research perspectives. Even if we want to make a mere description of the mathematics teacher's knowledge (novice or expert), we hope to show the need for some kind of knowledge's structure. Two big ways have been pursued to attempt that structure; we label them as *types* and *components*.

Section "Types of teachers' knowledge" presents a brief retrospective literature on the mathematics teacher's types of knowledge; whenever it makes sense to speak on types of knowledge valuable doubts and disagreements arise among different research groups. A second way, enabling also taking into account professional experience, comes closer to the mathematics teacher's knowledge looking for "components"; these don't banish types from the research discourse, but try some hierarchies in them by considering the mathematics teacher as a professional. The use of components constitutes a challenge for research, as far as it demands to consider more complex data structures. Anyway, we should not forget that types and components are researchers' constructs (although some types or some components are also expressed in the daily language and used by the teachers themselves). For that reason the challenge of the sum of those entities and their evolution along a

professional career arises; this led us to devote a section to the integration of knowledge.

Naturally, and fortunately, all attendants of *Thematic Group 3* proposed, although not always within a closed theoretical frame, ideas that are constant leitmotifs of our respective research: the relationship theory and practice, the social roles of mathematics teachers, the behaviours of these, sometimes guided to palliate their own lagoons with different knowledge, among others. In the last section we have gathered our notes in this respect.

Expert and prospective teachers' knowledge – first approach

The distinction between pre- and in-service education is usually useful to make it explicit both the context in which research is carried out and the constraints the subjects have got. However, we propose a different way forward to include pre-service education as a step before in-service education (it would imply, for example, a higher weight of teaching practices in pre-service education). In addition, in-service education should avoid considering pre-service education as a finished phase from a theoretical point of view, but only from an institutional perspective (it would imply that in-service education should deal with some pre-service contents).

In that way, teacher education would have its own features and also its own aims (not depending on whether education is pre- or in-service). One global aim should be the “prospective to expert transition” (this is neither a question of time nor a matter of instruction). Whatever the knowledge to be dealt with (e.g. practical knowledge, or specific mathematical knowledge), concerning teachers' knowledge (from the point of view of the researcher), we propose to distinguish: intended knowledge, in action knowledge, simulated knowledge, and real knowledge.

In fact, we cannot pinpoint real knowledge; we can only approach it. (Nobody can be completely aware of his or her real knowledge.) Like in the case of beliefs, people have got some barriers that prevent the application of a particular knowledge. We, as researchers, must be conscious of its existence, but our influence has to be directed to the other three types of knowledge. We propose some ideas in order to characterise the intended knowledge (or planned knowledge, stated in the curriculum to be acquired by student teachers), including specific topics. And we can cope with

the knowledge that teachers put into practice (in action knowledge). We call simulated knowledge the knowledge that is made explicit by teachers when they answer our research questions (questionnaires, interviews). This can be more limited than the in action knowledge (above all in relation to its theoretical presentation), but it could be also more apparent. From a different perspective, the intended knowledge is usually determined by others and well established and static. Therefore, teachers do not influence it directly. On the contrary, real, in action, and simulated knowledge play together, linked to each other by somehow a diffuse being which includes beliefs, attitudes, and values.

We all are aware of the gap between intended knowledge and in action knowledge. What we consider desirable knowledge for teachers and students teachers to be acquired is often too different from the knowledge that teachers put into practice. The sentence “the better the intended knowledge is defined, the more similar this and the in action knowledge will be” is quite ingenious. We are conscious of the complexity of this field, but sometimes we are blind to teachers' needs. In our research we must find out a balance between intended knowledge and what pre- and in-service teachers really need.

As far as in-service teachers are concerned we propose action research (Elliott 1991, Zuber-Skerrit 1996) as a framework where the features of the above-mentioned gap could be dealt with. Action research gives room for bridging the gap between research and practice. On the one hand teachers must know research issues in mathematics education, and on the other hand researchers must know the challenges that teachers have got in their practice, and the solutions of these challenges should be one of the main issues to study. Within action research, Feldman's (1993) collaborative research perspective, seems to be a very convenient way to undertake the relationship between research and practice. In collaborative research, researcher and teachers' roles are different from each other, but they all share the same objective: the understanding and improvement of teaching practices. This difference related to the roles implies at the same time a difference concerning traditional researcher's authority.

As Robinson (1998, p. 17) says: “A much neglected reason for the limited contribution of research to the understanding and improvement of educational practice is the mismatch between educational research methodologies and the generic

features of practice. Increasing the match requires an account of practice that clarifies its methodological implications. I propose a problem-based methodology in which practices are treated as solutions to practical challenges and explained by inquiry into the problem-solving processes that gave rise to them.” In addition we have to be aware that the main goal of the improvement of teachers’ knowledge is the improvement of students’ knowledge and behaviour.

The relationship between teachers’ knowledge and students’ knowledge may be one of the major variables provoking the unavoidable role that beliefs, attitudes, and values play in the former, because some students-teachers exchanges call for *improvisational performance* (see below). The influence of beliefs, attitudes, and values may help also to set up differences between the before mentioned types of knowledge. We think that only the intended knowledge is not influenced directly by teachers’ beliefs, attitudes, and values, although it is influenced by researchers’ beliefs, attitudes, and values, and hence, at last, it might be influenced by teachers’ ones.

Teachers’ types of knowledge

Many authors use “types” in the sense given in 3.1, although others avoid the term explicitly. In our research approach we assume that teachers should attempt to reach a type of knowledge of the highest level, which not only includes the knowledge of why and wherefore, but also conditional knowledge¹⁸, including a professional education which supports decision making strategically related to the teaching and learning processes.

From the context of general teacher education, Shulman (1986) considers three types of knowledge: propositional knowledge, case knowledge, and strategic knowledge. We include Borko & Livingston’s (1989) findings in the interplay between types and components of knowledge. They characterise expert teaching as a conjunction of four cognitive skills (pedagogical reasoning, pedagogical content knowledge, schema, and improvisational performance).

¹⁸ Paris, Lipson and Wixson (1983, p. 303) introduce *conditional knowledge*, “to capture this dimension of learning to be strategic.”

- Pedagogical reasoning is defined, with the help of Shulman, “the process of transforming subject matter knowledge into forms that are pedagogically powerful and yet adaptive to the variations in ability and background, presented by the students. This skill should be specific to teaching and “it is relatively undeveloped in novice teachers”.
- Pedagogical content knowledge, “also specific to the teaching profession”, consists, in Shulman’s words, of “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues, represented and adapted to the diverse interests and abilities of learners for instruction”.

It is very difficult to detect these skills in any concrete teacher. One should observe his/her courses, and cross-reference the information obtained from pre- and post-active conversations; however, in this respect, research has not (yet) produced clear conclusions.

- “A schema is an abstract knowledge structure that summarises information about many particular cases and the relationship between them ... Three types of schemata [...] seem to characterise teachers’ knowledge systems: scripts, scenes, and propositional structures.” In our opinion, using schemata makes it less difficult observing teachers in and out their classrooms. These research constructions may find some usefulness in the design of pre- and in-service curricula.
- Improvisational performance is a metaphor. “An improvisational actor enters the stage with a definition of the general situation and a set of guidelines for performing his or her role, rather than working from a detailed written script. Such a performer draws upon an extensive repertoire of routines or patterns of action while playing out a scene, incorporating them into a performance that is continually responsive to the audience and to new situations or events.”

Any teacher acts as an improvisational performer, because any teaching experience faces, unavoidably, unexpected situations or approaches. Improvisation is always an inescapable complement of any planned action, of any schema. “Preparation for such improvisation entails the creation of general guidelines for

lessons that are designed to be responsive to the unpredictability of classroom events.”

“Successful improvisational performance requires an extensive network of interconnected, easily accessible cognitive schemata”. This should explain why improvisational performance skills develop from a teacher’s experience. We faced a basic obstacle in organising all these research patterns. They seem to be based on a belief: processes (even paradigms) lead from a novice “state” towards an expert “state”. In fact, time and expertise, as variables, are related, but one can distinguish easily one from another (see Figure 2).

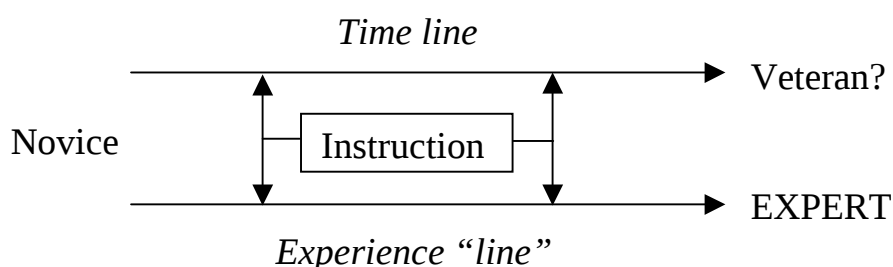


Fig. 2

- Farnham-Diggory (1994) categorises three “Core Instructional Paradigms” or “Models”:
- “In the behavior model, novices and experts are on the same scale(s), and transformation is accomplished through the mechanism of *incrementation*”. The author refers to Thorndike and Joncich, principally.
- The development model distinguishes novices and experts “on the basis of their personal theories and explanations, sometimes called *qualitative models*, of events or experiences.” Piaget is acknowledged as a “founding father” of this model.
- In the apprenticeship model, the key word is *acculturation*. Instructional practices both reduce differences between a novice and an expert and validate cultural practices in their context. The author refers to situated cognition (see subchapter 3.3 below).

Farnham-Diggory's categorisation is intentional. This means, "the three core models are defined as mutually exclusive. Definitional criteria were deliberately selected so that a form of instruction fitting one model would not fit the others". For the author, each model allows for acquiring "exactly five types of knowledge": "declarative, procedural, conceptual, analogical, and logical". Each model accomplishes the novice to expert transition in a different way.

Our own practice in pre- and in-service teacher education leads us to the following remarks which need to be confirmed through research issues:

- It is possible to recognise behaviour and analysis-of-situations patterns that are very different, on the one hand, among prospective teachers and teachers with less than three years of experience, and, on the other hand, among prospective teachers and some teachers with more than five years of experience. School cultures seem to generate an inflexion (not always toward the expertise) between the 4th and the 5th years, and we think that it would be necessary to distinguish among the "veteran" (more than five years of experience) and the expert teachers.
- To characterise *expert teacher*, it is not enough to detect the four cognitive skills mentioned by Borko and Livingston. If the new curricula recognise each teacher as a responsible professional, the teaching community should rationally discuss challenges deriving from school changes (innovations and reforms) and to use (for example) the intellectual anger, the intellectual honesty and the circumspection to modify the patterns of their pedagogic experience. So, added to the aforementioned four cognitive skills, an expert teacher should exhibit an attitudinal pattern that makes him/her be attentive to the new proposals on teaching and learning and to accept or reject them rationally, with the resulting modifications in his/her own performance.

Elstgeest, Goffree & Harlen (1993, pp. 37–39) distinguish five notions linked to knowledge (they emphasise that these are not types neither levels): Social knowledge (how-is-it-called-knowledge), physical knowledge (refers to direct experience and makes prediction possible), logical knowledge (relationships between concepts), technical knowledge (precedes skills and abilities, and motivates to do the

necessary exercise to acquire these), and professional knowledge (the most advanced level; it allows the tackling new situations and grants autonomy).

For Simon (1994), teacher education should include knowledge-of and knowledge-on mathematics. Previously, Ball (1988) had defined knowledge of mathematics as conceptual and procedural knowledge of the matter, and knowledge on mathematics as any understanding about the nature of the discipline (where it comes from, how it changes and how the truth is established). Knowledge on mathematics also includes what it means to know and to make mathematics. Notice that the term “truth” is used in the sense of proof, not in allusion to the existence of a unique way to raise concepts and demonstrations.

Concluding remarks: towards components

The teacher, sometimes possessing exclusively a social knowledge, should try to acquire a professional knowledge, reaching (as well as their pupils), a higher level of reflection and enriching their thinking processes. This consequence can emerge as one of the major aspects of teachers’ knowledge, and should appear in *our* intended knowledge. One reason for that is that teachers’ knowledge consists of a set of capacities, many of which are open, in the sense that they can be managed only increasingly, but never managed totally.

We would like to stress a different perspective when we analyse these types of knowledge. Regardless of terms, not innocuous at all, nowadays one is in favour of differentiating two relevant components in professional knowledge: one of them, static, has a theoretical character, while the other, dynamic, matches to practices. In this dynamic component one could place the term pedagogical reasoning (Wilson et al. 1987). This distinction is particularly relevant when dealing with trials based on a hypothetical transfer from the teacher’s learning to the students’ learning.

The mentioned terms (knowledge; relationship between *saber* and *insight*, knowledge construction, types of knowledge) need more reflection and study; research groups should continue their efforts to integrate these questions in different theoretical approaches.

Components: expert and prospective teachers' knowledge – second approach

In simple terms, the NCTM (1991) proposes four aspects, related to decision making, which teachers need to know in order to be capable of carrying them out: to choose valuable mathematical tasks, organise the presentation in the classroom, create a learning atmosphere, and analyse the teaching and learning. These aspects (maybe, components) refer to educational practices. We are interested in developing a frame that allows a wider theoretical analysis.

Wilson, Shulman & Richter (1987) provide a theoretical model in which they establish the following components:

- Subject matter knowledge (what and the why of the content, giving meaning to the subject).
- Pedagogical content knowledge. It includes those elements that characterise the teachers' style, like the knowledge of the teaching subject (learning features, specific instructional methods and epistemological teachers' beliefs on the teaching subject); general pedagogical knowledge; knowledge about goals and educational objectives.
- Curriculum knowledge (knowledge about alternative materials for a specific notion, curriculum knowledge about other subjects linked to mathematics, and curriculum knowledge about mathematics in the previous courses).

Bromme (1988), on Shulman's (1986) proposal, establishes the following elements to characterise the pedagogical content knowledge:

- Mathematical knowledge (it comes from the academic degree).
- Curriculum knowledge (study plans, mathematical contents of other subjects).
- Knowledge about the classroom (it is essential in order to make decisions with respect to the application of the official program).
- Knowledge about what the students learn (personal strategies, conceptual mistakes and epistemological obstacles).
- Meta-knowledge (mathematics and mathematics teaching and learning beliefs).

- Knowledge about mathematics education (practical and methodological knowledge).
- Pedagogical knowledge (of general character as well as related to school organisation).

Many researchers highlight the importance of considering several components when one speaks of teachers' knowledge. This is not a question of believing that these components perform isolated from each other. We are trying to make four components explicit through the analysis of the following meta-question.

A meta-question

The meta-question is very simple: How does a human group acquire (or produce) a new knowledge in the context of a school? Let us see what this implies¹⁹. This implies, first of all, a certainty on the part of the teacher: I know something. The same fact of being a teacher implies also the certainty of being recognised by some institution. This addresses us to the school culture, but we will not enter in this subject. This implies, secondly, an intention. The teacher wants (as well as the institution for which he/she works wants) a human group to learn what he/she know. This implies, also, a structured interaction among people. The interaction is structured, because teacher and students' roles are predetermined by the institution itself and they cannot be exchanged. The teacher is always a teacher, even when he/she repeats aloud the question of a student; the student is always a student although the teacher invites him/her to speak to the whole group (teacher included).

Again, this implies a variety of decisions about and during the class. For example: (1) what approaches/solutions should the teacher emphasise? (2) How will he/she confront prospective errors? (3) How will she/he co-ordinate different working/solving strategies? (4) How will she/he assist either questions or surprises from the students? (5) How will the materials be used? (6) Which moment of the

¹⁹ It is best to do it with the help of an example. In the Workshop of the *Thematic Group 3* Coriat and Carrillo proposed an example adapted from Gardiner (1996). Any topic of the curriculum as well as any methodological approach are good enough in order to exhibit the relevancy of the use of components. However, for the sake of generality, we will not mention any particular case.

curriculum development is good to implement a concrete topic? (7) Will students work in small groups or as individuals? (8) Classroom or homework task?

This implies, finally, an attitude towards the interaction, both from teacher and students. Mathematics teachers, even experts, do not love mathematics in the same way or with the same intensity. Many teachers can be labelled as authoritarian, others as cordial, many others as demanding with their students. Many students often enjoy a mathematics class, others do not enjoy it, but they work hard to obtain good scores. Many students prefer to work alone, others feel they learn better working in a group. By arranging all those implications, we attribute to the expert teachers' knowledge several components:

4. Discipline component (Mathematics).
5. Human component (To take account of the human group).
6. Curriculum component (Like an "intersection" between pedagogy and mathematics).
7. Attitude component (Appreciation of mathematics, values transmitted by mathematics).

It is true that there is no unanimity in the terms used to refer to the components, their environment or their relationships. First, a curriculum component would include mathematical questions (contents), pedagogical (methodology, objectives) and knowledge about students (evaluation models which include the students progress and attention to diversity). Second, Porlán, Rivero & Martin (1997) place Shulman's pedagogical content knowledge in the interplay between teachers' professional habits (which are related to the conduct) and the implicit theories (which support the conceptions). Research on pedagogical content knowledge usually considers in an inseparable way components (2), human group, and (3), mathematics curriculum. Third, following Berger²⁰, components (2) and (4) need for further discussions in order to clearly distinguish the group feature from the individual one. We think that each classroom group has its own "personality" (we should say "tribality" or "groupness"); therefore, research on the group (as a whole) demands a different

²⁰ Oral communication, *Thematic Group 3*; see also Berger (1998).

approach from that given to the individuals that compose it (although, of course, there will be strong interactions).

The knowledge that the teacher brings into play is more professional than scientific; and it is directed by the impulse of the student's learning. For this reason, the teacher's mathematical knowledge is subordinated by his/her ability or skill in communicating, sharing, and promoting contexts and motivation in such a way that their students learn mathematics educating themselves²¹.

Many meta-answers

If the meta-question is easy to state the same doesn't happen with the meta-answers, the biggest difficulty (and the main wealth) coming from the plurality or multiplicity. One can easily understand that the four components interact and evolve during each teacher's professional life.

The impressive diversity of mathematics expert teachers that we will find in the schools is a starting point for any research on the professional knowledge. The study of this rich variety embraced by the four components needs analysis instruments so that any concrete teacher can be studied, even in the period of his/her education.

Simplest ideas are also powerful. For that reason, our starting point is trivial: as in other professions, the mathematics teacher's experience is related to his/her professional knowledge (see Figure 3).

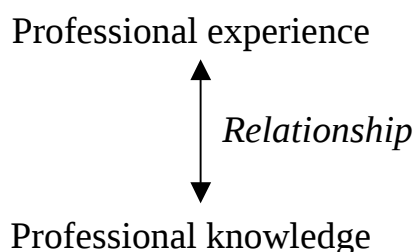


Fig. 3

²¹ Many researchers, such as Shulman (1986, 1987, and 1993) and Llinares (1991) have indicated the complexity of this professional knowledge.

Do we know something about that relationship? We have seen that the meta-question is intentional; it cannot exclude criticism of its content; roles are pre-determined; each answer to it is made explicit through interactions (supported by contexts and situations); and it calls for a variety of decisions.

In order to study the relationship between professional knowledge and professional experience, one can use a looped data structure or a tree data structure. To compare both structures, it seems suitable to bear in mind the strong interactions among the components of the professional knowledge (some of which have been mentioned), which induce the use of a strongly connected structure: Figure 3 spreads out as in Figure 4. Notice the privileged (central) position of the item “decisions” and the impossibility to disconnect that graph due to the unavoidable presence of many decisions during the educational interaction.

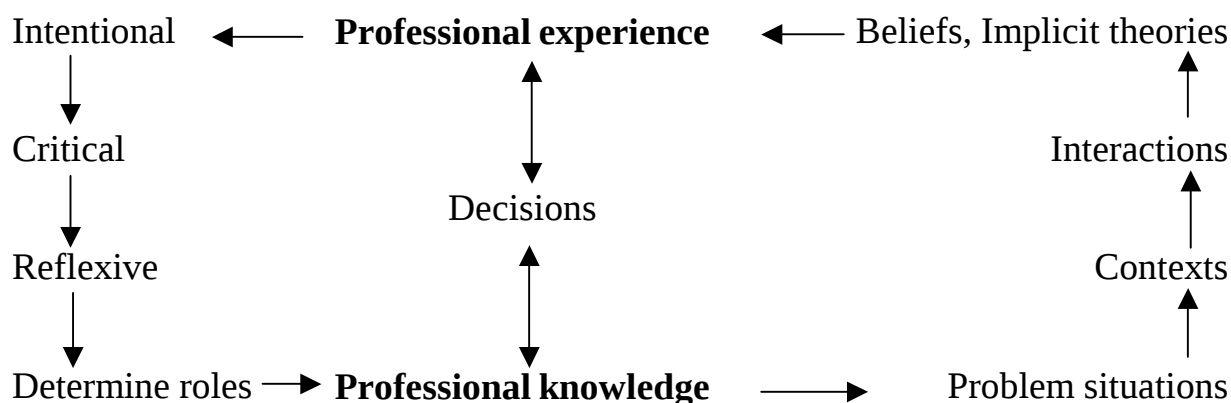


Fig. 4

In a certain sense, the decisions constitute the first manifestation (sometimes, a decision is not translated in behaviour) of our knowledge. This way, in our prior knowledge of the abstract mathematics teacher, we conjecture that the decisions he/she makes (while preparing lessons, interacting with colleagues, during lectures, and so on) influence any study of the relationship between the professional knowledge and the professional experience. Such decisions may be described (for example, it is possible to take a census of them) and compared *a posteriori* with those taken by a concrete mathematics teacher.

By observing a teacher moving forward and backwards from professional experience to professional knowledge, the researcher may face different challenges:

- To consider a teacher's experience as a space for action and, eventually, to help / convince / assist him or her be aware that self-evaluating that space, and enriching it, show ways to improve and enlarge his or her professional knowledge. Intentions, Criticisms, Reflections, and Roles stand for significant "marks" allowing both teacher and researcher to "move down" together systematically.
- To consider a teacher's knowledge as the main structured tool available to face up to his or her professional experience and, eventually, to help / convince / assist him or her be aware that self-evaluating such a tool, and enriching it, show ways to design new interactions with the students. Problem situations, contexts, interactions, and belief and implicit theories stand for significant "marks" allowing both the researcher and the teacher to "move up" together systematically.

Discussion in CERME 1

The following ideas come from discussion within the schedule frame as well as from spontaneous talks of participants between *Thematic Group 3* meetings and further mail communications. We will highlight the contributions related to knowledge components, which were the core of the discussion. Besides those participants who contributed specific terms, others offered some features or descriptions. We think that all these proved to be valuable for everybody and we hope that they help to increase our knowledge.

We have classified the contributions according to several components mentioned above. *Subject matter knowledge* appears as a component and as mathematical competence. *Pedagogical content knowledge* appears as a component and was referred to in statements like:

- "The teacher must formulate good questions to assist the students to overcome the points of blockage."

- “The teacher should create an environment that gives children perspectives to go on, make relationships with comparable problems.”

Curriculum knowledge is present when we think that one teacher's desirable ability is to organise the classroom tasks, to decide which tasks are appropriate and which materials should be used. One deals with *knowledge about students* when one speaks about psychology of learning or about how students learn and react. Other related sentences are:

- “The teacher should manage students' interaction to promote important mathematical ideas.”
- “The teacher has to create an environment that challenges children to think about the problem and to co-operate and collaborate.”

Other component was the *self* (teachers should have got knowledge about themselves): about their own abilities to do mathematics, their confidence in their own abilities (etc.). As we have said, some features appeared together with these components.

- “Knowledge takes place only when interacting with others.”
- “When you have got knowledge about more things, decisions on which kind of knowledge should be used get more complex.”
- “Teachers should be flexible: they should be aware that there are different strategies and possibilities.”
- “Teachers should have their own learning experiences.”
- “Teachers should be aware that there are different didactic aims.”

The components as well as these last sentences might lead us to a deep reflection on knowledge. This is not the point of this subchapter, but it offers a field on which to further reflect in a co-operative environment.

3.3

WAYS OF PROMOTING MATHEMATICS TEACHERS' KNOWLEDGE

Overview

The approaches devised to describe the construction of mathematical knowledge (either individually or socially) have the main advantage of helping to make explicit determined suppositions in teaching that were considered characteristic of the teacher's privacy until the sixties, and that, little by little, have contributed to enrich the discussions on the teaching and learning of mathematics. The different theories in competition, all in construction, help researchers to improve their interactions with teachers, and teachers to identify their intuitions with more elaborated discourses (therefore, establishing some practical consequences).

We are in favour of a balance between theory and practice. We need much caution, because our philosophical beliefs and our research methods either generate bias or prevent us coming to an agreement on the results of our observations. The question gets more complicated if we remember that, for many of us (in *Thematic Group 3*), research on teachers' construction of knowledge is not but another means of school improvement.

Our concern now is mathematics teachers' knowledge from the point of view of its construction. We have already dealt with types and components of teachers' knowledge. Now, the question refers to the methods teachers and researchers might choose in order to facilitate knowledge construction and also the features of that process. First of all, we propose to think about and consider some interrelated issues, and secondly we approach knowledge integration.

Construction of teachers' knowledge

Limits of the Technical Rationality Model

Following Habermas (1992, p. 122), “when choosing a certain sociological concept of action we commit with certain ontological presuppositions”, which leads him to distinguish among *teleological action* (and its variant *strategic action*), *action ruled by norms*, *dramatic action*, and, finally, *communicative action* (pp. 122–124).

The traditional *modus operandi* of teleological-instrumental action (Habermas 1994, p. 385) has been subjected to strong criticism. Schön (1983) appears as a recent champion of this criticism; his book had a seminal influence in many American university milieus; their interest in pre- and in-service teacher education, led them to using Schön's reflection-action model in their research.

Schön starts observing the essential character acquired by the professions in society. He profiles the professional as the person to whom we go to define and solve our problems and by whose intervention we try to progress socially. He explains how we have passed through a time in which the professions understood each other as the true base of society [in North America], to another time in which professionals are criticised in very diverse ways, both from inside and outside the professions. He also diagnoses: “We are bound to an epistemology of practice which leaves us at a loss to explain, or even to describe, the competences to which we now give overriding importance”, such as “making sense of uncertainty, performing artistically, setting problems, and choosing among competing professional paradigms” (Schön 1983, p. 20).

In our opinion, the main reason of the “crisis of confidence in the professions, and perhaps also the decline in professional self-image” [in the United States] (Schön 1983, p. 13) comes from that bond to an epistemology of practice. “According to the model of Technical Rationality ... professional activity consists in instrumental problem solving made rigorous by the application of scientific theory and technique.” (Schön 1983, p. 21). According to Schön, the model of technical rationality “is embedded in the institutional context of professional life. It is implicit in the institutionalised relations of theory and practice, and in the normative curriculum of professional education. Even when practitioners, educators, and researchers question

the model of technical rationality, they are party to institutions that perpetuate it.” (Schön 1983, p. 26).

The idea of hierarchical solution of problems is insufficient. In Schön's opinion, professionals don't only solve problems, they also outline them; to state a problem is a process which means taking decisions, and establishing the ends that we want to reach and the means that we will choose for that. We ask problems starting from problem-situations that, being unique and uncertain, carry out questions of value. Both scientific and technical rigor used in the resolution should depend on these three notions (uniqueness, uncertainty, and value).

In accordance with McFee (1993), “at the centre of Schön's strategy are three related ideas”. The first idea is a rejection of positivism, which generates technical rationality through an epistemology of practice. The second idea proposes “a regard for practice as embodying both knowledge ... and the solution to problems”, intends to go beyond the reflection on means and goals, and takes into account this practice “as a basis for theory”. The third idea is to introduce reflection-action as a new epistemology of practice. The teacher is for Schön a *reflective practitioner*, a professional who considers the practice as the source of his/her problems, challenges, as well as of the solutions to those: the practice is the permanent reference. All these ideas raise many discussions and researches. We will only mention a strong discussion on positivism.

Criticisms to positivism preceded Schön's first book. Ten years before, Wilson (1973) noticed an approach of educational research based on “certain assumptions, often unconscious; ... they are usually too vague and subterranean to be easily pinned down. Hence I call them ‘myths’ ”:

- The questionnaire or ‘hard facts’ myth. This corresponds to believing that different people answering the same questionnaire will assign the same meaning to the same words.
- The ‘replicability’ myth, “borrowed from the physical sciences” [sic]. This corresponds to believing that if “X behaved in way Y because P and Q.... *other things being equal*, P and Q will on any occasion make X behave in way Y”.
- The ‘value-free’ myth. “The myth is that one can do serious research without oneself tackling the question, just by restricting oneself to ‘the facts’ ”.

Wilson concludes that to expel those myths we need “courage as well as clarity. I should guess that the apparent unreality of much educational research, as seen by teachers and other practical workers, stems from these defects”.

More recently, Eisner (1992) wrote: “Positivism ... is a philosophy of science that has an attitude towards metaphysics, that separates value from fact, that embraces methodological monism, that rests upon a foundationalist view of knowledge, that possesses a particular conception of meaning, that regards ethical claims as meaningless utterances, that believe science to be the sole source of knowledge, that seeks to explain ‘reality’ through an appeal to universal laws, and that regards measurement as the quintessential means through which reality, whatever it may be, can be represented”. After having explained his departure from positivism, Eisner concludes: “Educational research does not yield prescriptions teachers get from an educational pharmacy which they then implement in their classrooms. What they can get are ideas, suggestions, possibilities. These important contributions are cues, not prescriptions. Nor are they hypothesis to be validated like those used in drug trials. In the classroom, nothing can replace the teacher’s judgement, including the decision to alter the aims of the lesson. Looked at this way, educational researchers are not engaged in discovering mechanistic universal truths sought by positivists or tidy prescriptions about ‘what works’ ... Educational researchers, rather, provide guidelines and interpretive material intended to liberate the teacher’s intelligence ...”.

Action research

Action research is traditionally seen as a paradigm immersed in in-service teachers’ education, but there are voices (see Krainer 1996) arguing that action research projects and other kinds of involvement of teacher students in investigations can be integrated in pre-service teacher education.

García (1993) presents a model of practice for prospective teachers from which some theoretical aspects are worth being emphasised. “1. The singular and unique character of educational processes. 2. Teaching as a reflective activity in a dialectic relationship: practice – reflection – hypothesis of action practice. 3. Teaching as a form of action research, following the paradigm of research of action (Stenhouse 1984; Elliott et al. 1985; Carr & Kemmis 1986). 4. Reflection, deliberation, and

discussion of all the subjects who take part in the educational process and who will help its improvement (Schön 1983; Zeichner 1987). 5. Teacher's professional growth comes from his/her emancipatory self-evaluation (Elliott 1987). 6. The resolution and development of conflictive situations and challenges as a teacher's art related to the application of his/her theories to specific contexts and pupils (Stenhouse 1985; Contreras 1985; Gimeno 1983). 7. The prospective teacher's practice as a socialisation process as well as a process characterised by a permanent reflection on action (Elliott 1985; Zeichner 1987). 8. The use of the ecological model in teaching research, instead of the means-ends and mediation models (López de Ceballos 1987)" (p. 339).

This conceptualisation of practice helps to draw a line from pre- to in-service education. Moreover, it offers a framework in which prospective teachers perform as researchers in and on their own practice, laying the base for their desirable future role as teaching professionals. They can reflect on their own practice, on their colleagues' and on in-service teachers'. This reflection emerges as a way to learn from practice²².

Zuber-Skerrit (1994) mentions three types of action research: technical (goal: efficiency of educational practice), practical (goals: the former and transformation of conscience) and emancipator (goals: the previous and the transformation of organisation and educational system). Zuber-Skerrit (1996, p. 3) states that every group that tries to bring about the third type must be immersed in a cyclical process defined by "1. Strategic planning. 2. Action, that is to say, planning application. 3. Observation, evaluation, and self-evaluation. 4. Critical reflection and self-criticism of the results concerning the points 1 – 3 and decisions for the next cycle of action research."

One can argue that it is too difficult to find teachers willing to undergo a long education process. Sometimes this is the argument to propose short courses and very local and partial training, which is, in general, somewhat superficial, inconsistent, and irrelevant after a short period of time. On the contrary, in order to improve their professional knowledge, teachers have to feel and be the "stars" of the process. They should understand this process as something which is never-ending. We have said

²² See also MILE project in chapter 4.

that we do not consider pre-service education as a finished phase. Therefore, this is also a feature that could be linked to action research (see chapter 5).

Our proposal of training / research considers teachers as in the sense illustrated by Figure 4. We demand teachers to be able to apply research strategies and results to their professional experience, teachers who demand criticism on their professional knowledge. We think that this attitude creates an intellectual activity that leads to professional development. In this realm, we are in favour of collaborative research (already mentioned): teachers have different roles from researchers, but they share many goals.

Situated Learning

Following the perspective of situated learning (Brown, Collins & Duguid 1989), learning and cognition are basically situational: “The activity in which knowledge develops and is used (...) is not separable from or auxiliary to learning and cognition.” (p. 32).

As Lave & Wenger (1991) say, it does not only mean thoughts and actions locations in space and time, nor the fact that it implies other people, nor that they depend on a social meaning. Situated learning perspective implies in addition a new dimension: the notion of peripheral participation in practice communities. Situated learning activity conceived in such a way by Lave and Wenger, moves “in a centripetal direction, motivated by its location in a field of mature practice. It is motivated by the growing use value of participation, and by newcomers’ desires to become full practitioners. Communities of practice have histories and developmental cycles, and reproduce themselves in such a way that the transformation of newcomers into old-timers becomes unremarkably integral to the practice.” (Lave & Wenger 1991, p. 122).

This notion implies a change of perspective. Learning is not simply situated in practice, but it is considered as an integral part of every activity. “There is a significant contrast between a theory of learning in which practice (in a narrow, replicative sense) is subsumed within processes of learning and one in which learning is taken to be an integral aspect of practice (in a historical, generative sense).” (Lave & Wenger 1991, pp. 34–35).

Situated learning and collaborative research emerge as two poles around which one organises teachers' growth. The challenge is to engage teachers and researchers in common commitments. Teachers must be makers of their own learning. Their challenges, troubles, and strengths, if they are issues for the teachers themselves, must be the issues for researchers.

In the way forward, situated learning and collaborative research offer a shift with respect to teachers' and researchers' roles. The teachers should become real stars of new curricula, as well as their students. These two poles mentioned before concern not only research and/or teachers' education perspectives, but meta-issues inside the classroom. Indeed, the teachers who learn in a situated way and use to reflect on their professional challenges within the framework of collaborative research would be very keen to implement teaching methods in accordance with it. Therefore, they would promote situated learning for their students and enhance discussion between them and between them and him/her.

This aspect deals with the transfer of research issues, perspectives or features to the teaching. Such a transfer is a desirable characteristic of every research project, which intends to enhance students' learning through teachers' education and growth. Moreover, this link is unavoidable in action research.

Narratives

Narratives have been presented as a particularly adequate way of knowing and thinking to study teachers' knowledge (Carter 1993). As teacher's knowledge has specific features, Mattingly (1991, p. 236) considers that narrative may facilitate the reflection upon the tacit knowledge that orients practice, bringing out "deep beliefs and assumptions that people often cannot tell in propositional or denotative form". Narratives also present themselves as a rather suitable way of representing knowledge proceeding from action, and, according to that, they have stimulated a growing interest in the educational researchers' community, mainly in studies related to teachers and teacher education. As Elbaz (1991, p. 3) suggests: "Story is the very stuff of teaching, the landscape within which we live as teachers and researchers, and within which the work of teachers can be seen as making sense ... teachers' knowledge in its own terms is ordered by story and can be best understood in this way."

Narratives facilitate the approach to beliefs, ideas, experience, and practice of teachers, from their own perceptions, or, using Cortazzi's (1993) words, "from inside". Moreover, as narratives are a common way of communicating, the authenticity of teachers' expressions is prone to be greater and, consequently, the final product will be more genuine.

Teachers' practical knowledge has a strong component of case knowledge. We just have to sit for some minutes in the staff room of an ordinary school to hear endless stories about teaching, full of ingredients. Gudmundsdottir (1991 p. 211) refers that "These stories tend to be about cases, a case being a difficult child, a good class, mathematics materials, reading books, a topic, or group work. Case knowledge is characteristic of practitioners who work with people."

Teachers' personal practical knowledge is strongly linked to the context, because it expresses itself in the physical surroundings in which the teacher's action takes place, and is internalised in himself, generally not needing him to make it explicit (Cortazzi 1993). That is why teachers' descriptions of their teaching sounds more like stories than theories because they are full of information which comes from their own experience.

In a study with novice teachers, Carter (1993, p. 7) tried to understand "what they know, how their knowledge is organised and how it changes with additional experience in observing classes and in teaching", through the analysis of the events they recorded as being relevant. The remembered incidents presented themselves as little stories resulting from their experience. According to the author, professional development results from the acquisition of structured knowledge in harmony with situations experienced. Therefore, this experiential knowledge assumes a narrative form.

Gudmundsdottir (1991) similarly refers to the structural role of narratives in knowledge. This knowledge of practical nature (teachers' particular knowledge of a content to teach) is organised in narrative structures. In a research with two experienced teachers, she describes the stories (presents a narrative) they told about the content which was to be taught. She concludes, "their excellence as teachers is mostly due to the storied nature of their pedagogical content knowledge and the interesting stories they tell their students" (p. 207).

The deepening of this area might be stimulated, as we have seen, by the adoption of a narrative approach in investigation. The fact that teachers' knowledge is strongly linked to the context and structured according to situations experienced seems to support this kind of approaches.

The value of the stories that we create every day is illustrated by Iris Murdoch (quoted in Mattingly 1991): "When we return home and "tell our day", we are artfully shaping material into story form. (...) We are constantly employing language to make interesting forms out of experience which perhaps originally seemed dull or incoherent" (p. 237).

The construction of narratives is used to give order, coherence, and significance to lived experiences. Therefore, these might contribute, through systematic reflection, to making teacher's practical knowledge, which is tacit by nature, explicit. Narratives might have an important role in promoting reflection, in the context of teachers' education or, in a broader way, in teachers' professional development. In fact, stimulating narrative use by the teacher, simultaneously his reflection capacities upon practice are being stimulated.

Cortazzi (1993) presents a literature review concerning the issue, giving a summary of some empirical data obtained in studies on teachers' personal knowledge and events related to the classroom. For instance, he refers that Connelly and Clandinin use stories to help teachers to reflect about personal practical knowledge. Their strategy consists in asking teachers to write three detailed stories about themselves in the classroom. Then, each teacher shares his/her stories with another teacher he/she trusts, trying to see how those stories express a view of learners, subject content, teaching, classroom relationships and the educational context. In the end they search patterns examining larger collections of stories. The author considers that positive stories, that teachers share, "provide direction, courage, and hope in their work" (p. 139). But negative stories might also have an important role as they "may be a social lubricant, reducing friction in schools and allowing them to function more smoothly" (p. 139). So this process of story telling can contribute to teachers' self-confidence.

The question of integration of knowledge

Theory and practice, different knowledge components, pre-service and in-service education, social expectations ... The main research question is integration! But integration is not an easy issue. It demands for teachers to forget comfort, for researchers to cope with more complex research objects. Within integration we should distinguish at least two features. The first one is the content of integration; this has to do with what should be integrated. The second feature refers to the dynamic character of integration and concerns the process of integration. Both, content and dynamic character, although they can be distinguished in order to make the analysis easier, relate to each other.

We have already stated the need for the integration of knowledge components. In general, one gives the responsibility for that to teachers. These should organise their lessons in such a way that the different components play together. They should try to do their best in order to look at themselves as professionals, not only as mathematicians or pedagogues; they need a broad set of elements to face their teaching (and professional) tasks.

However, researchers have also a role at integration. Pieces of research show features of the human being, but this can be divided in order to highlight some aspects. One can study beliefs on mathematics teaching and learning, or problem solving modes, or whatever, and one can try to sketch conceptual maps which reflect a part of the teacher's thinking, but, in fact, every elemental part has played its role. Researchers on mathematics education should deal with research issues within a complex framework. We should too be aware that integration of knowledge, as a goal, is very difficult to achieve during the research process. Here we can appreciate the dynamic character of integration. The researchers can promote integration and assess its acquisition process; but any success belongs to the teachers themselves.

Moreover, people have got an integrated knowledge. This may seem contradictory with the previous paragraphs, however it is not the case. It refers to a different kind of integration. Previously, we have dealt with *inter*-integration (integration between several components). Now we are speaking of *integration-in* (integration with external parts): a teacher's knowledge is integrated in his/her values,

professional context, school culture, and family's constraints and, in general, in a way of life. This kind of variables build the mixture in which *inter*-integration evolves.

The integration between theory and practice is a hot issue in pre-service education. The subjects are usually theoretically biased and one expects that the prospective teachers shall make the integration in the future, when they become teachers. And this perspective goes on in the in-service education.

This point of view consists of proposing, to teachers, the different structured combinations and specialised knowledge and allows stimulate the teacher to generate integration, for example by creating rich learning environments. This is very costly in terms of time in academic training and does not allow for the possibility that the in this way educated teacher will organise this knowledge hierarchically and in a definitive way. Lappan & Theule-Lubienski (1992, p. 253), quoting Feiman-Nemser (1983), point out a limitation in the initial education: the theorisation of the material, which gives the "professional life" the responsibility for the integration between theory and practice, between traditional academic knowledge and professional knowledge.

As follows from Figure 4, we propose the integration of all the components distinguished before to occur in a situational way. The trainee teacher has the possibility of coming into contact with a critical reflection on what his/her teaching challenges will be and to understand different conceptions of mathematics, which are connected to cultural characteristics of the group with which he/she is working.

The situational approach mentioned above does not reject different knowledge; it allows for the possibility that it avails of the theoretical frameworks that aid the teacher to make sense of his/her educational experience. In the words of Russell (1994, p. 205): "Learning from experience is neither simple nor straightforward and certainly not automatic." Moreover, as Edwards & Mercer (1987) point out, on many occasions experience provokes a mechanical knowledge, instead of the desired knowledge of the basic principles. There are research projects, for example the project by Fernandes & Vale (1994), that highlights the manner whereby the educational context conditions the conceptions in the teachers' practice. Indeed, the two novice teachers analysed, both having a conception belonging to the investigative tendency, carry out their teaching in different ways. One of them, immersed in the traditional context, does not use the problems as a methodology, fundamentally basing his classes on the textbook. The

other, in a completely different context, develops a teaching practice more in line with his stated conceptions. The context inevitably conditions, but the teacher must learn how to analyse it in a critical way, detecting ‘cracks’ and making decisions with the aim of intervening in the group in an effective way which does not contradict his convictions. Studying practice carefully, reflectively, and collaboratively is a means to this learning.

Looking at another point, one can see that the integration of components or the integration theory-practice goes together with the integration of knowledge elements or features. In this way, a particular type of knowledge (let’s say mechanical) might be improved by the influence of such elements and, thus, a teacher’s knowledge can grow from a qualitative point of view. Teachers, researchers, and teachers’ educators should be aware of the complexity of educational challenges, as well as of the need for models to implement the above mentioned ideas.

One of us (Carrillo) proposes a model in which teachers are considered as problem solvers in several directions. As members of a school, they are faced with challenges related to organisation and relationships amongst colleagues. As teachers *inside the classroom*, they must tackle educational challenges: adequacy of program to students, quality of explanations, classroom climate, and so on, particularly they have to state some problems and to cope with students’ troubles to approach them. Therefore, as mathematicians who try to enhance students’ learning and to improve their understanding of students’ thinking processes, they need also to pose and to try to solve mathematical problems. In relation to that, we propose a research & in-service education model in which teachers present their common challenges (whatever) and they deal with mathematical problems. Dealing with mathematical problems, ways of putting them into practice, and/or approaching their solution following a resolution schema like Schoenfeld’s, paying special attention to the whole process (not only the solution) and to the revision stage, seems to be a good way to challenge beliefs on mathematics and on mathematics teaching and learning. This challenge should accompany knowledge’s growth.

3.4

SYNTHESIS AND FUTURE WORK

Open questions

Qualitative research is not aimed at generalising, as with quantitative research, but it is not infrequent that even *qualitative* researchers try to expand their piece of research to some extent. This consideration is also applicable to research on teacher's knowledge. Whenever a meeting is held on teacher education or research on teacher's knowledge, teachers and researchers share their perspectives, beliefs, ideologies, frameworks, methodologies, results, and beyond the usual agreeable atmosphere one can feel that the question of applicability is not solved yet.

We do not expect any global and general solutions when dealing with this question. We are not in favour of dealing with the question in order to achieve a general solution. On the contrary, we think that this question might serve as a challenge in order to enhance communication, co-operation, and collaboration amongst teachers, amongst researchers, and amongst teachers and researchers.

The question of applicability does not refer exclusively to the feasibility of using some research results or methodology in different contexts and cultures. It concerns also the relationships between teachers and researchers: one of the major questions (and doubts) expressed by teachers is about the role that they and their students play in research. In too many occasions it seems to teachers that researchers have not considered their concerns and those of their students.

This feeling should become an essential question. Researchers should ask themselves about their goal when they are dealing with research on teachers. This question evolves in two further ones. On the one hand, teachers and researchers must sit together in order to define their roles. One approach to this issue comes from

collaborative research (Feldman 1993), but one needs to go deeper. On the other hand, researchers have got their own epistemological goals, and these must be compatible with teachers' goals, and these with students' goals. Moreover, all of these epistemological goals have the need for being shared and for supporting each other.

In addition, some (not new) questions arose from the work at the CERME 1, which may be formulated as follows:

- What knowledge does the in-service teacher have and how does it grow? Types and components give us some clues. In relation to the second one, many variables play together and, in fact, one does not have final and general answers, only some ways to make that growth easier and consistent.
- How do student teachers construct their personal professional knowledge? What knowledge? What is the role of discourse and collaboration?

At this point, the different status of pre-service and in-service teachers' knowledge arises. Furthermore, the way to acquire knowledge and to grow (in a professional sense) – for example, discourse and collaboration – seems to have its own features.

Some other questions were:

- How can we design teacher education (pre- and in-service) in order to demonstrate to teachers the full complexity of mathematics learning?
- Connections between students' activities and the acquisition of mathematical knowledge. How do teachers manage?
- These questions show the deep relationships between students' and teachers' learning. And
- What is the interplay between knowledge, confidence, and beliefs?

Problem solving and mathematical problem solving was proposed as a tool to know mathematics teaching beliefs and to improve mathematical knowledge, as well as mathematics teaching. Indeed, problem solving might be an attractive realm in which teachers acquire mathematical and pedagogical knowledge, reflect on their own beliefs and search for coherency.

Finally, we would like to make it explicit that we believe that situated learning provides a basis to design meeting points for prospective and expert teachers. However, its complex nature makes it difficult to obtain good research designs. The challenges are on air.

Consequences for research on the open curriculum for teacher education

The following premises allow us to suggest the idea of an open teacher education curriculum:

- Primary or secondary mathematics curricula change from time to time. If we suppose a professional life of 30 years, each teacher should work at least in the context of two curricula.
- School cultures evolve, following with certain slowness the changes of the society (under non-revolutionary or catastrophic hypotheses).
- Mathematics curricula change slowly at the university level.
- In recent years, people have gone into teaching due to two main reasons: vocation and unemployment.
- Each person is unique (under non-clonic hypotheses). So, students interact with different teaching conceptions.

All this leads to conceiving teacher education as an open process; for that reason we need to speak of open curriculum. Open, because:

- The diversity of teaching styles constitutes a value.
- Teachers should be able to adapt themselves to different changes (curriculum, social and of school culture).
- Each teacher should teach (at most, every three years) to new groups of people.
- Not everything can be learned through pre- and in-service education courses.
- There are always unpredictable questions that experience sets down.

Paraphrasing Canguilhem (1971, p. 11), we can consider the teaching profession as a technique or art placed at the crossroads of many sciences, rather than the practice of a sole science.

Throughout pre-service teacher education courses, situations arise which can trouble future teachers. Theoretical courses advocate an innovative attitude in mathematics teaching, whereas practical courses refer to traditional processes. Coordinating both theoretical and practical preparations is yet a pending task.

An open curriculum to embrace teachers' education has consequences for research in mathematics education (within pluralistic societies, in the sense of Gellner 1994). We state some of these, expressing them either as needs or as questions):

- It is necessary to carry out comparative studies of the actual mathematics teachers' education plans and to recognise the current closure elements (in order to propose modifications).
- It is necessary to study the concrete social roles allotted to mathematics teachers in the different European countries.
- It is necessary to agree on criteria related to the quality of research. This is a very difficult task, because, on one side, no philosophical option can prevail as the result of an axiom, and, on the other, exporting-importing, among countries, either theoretical options or practices constitute, at least, a research question and, certainly, it is not a matter of evidence.
- If the economic and labour situations of mathematics teachers are comparable in two or more European countries, are they also comparable in their respective beliefs, school performances, professional knowledge, and professional experiences?
- If it is a goal giving meanings to the expression "European mathematics teaching profiles", it is necessary to establish the main characteristics of each one of those supposed profiles as well as of their evolution.

Looking ahead: communication, co-operation, collaboration

Our group agreed with the philosophy of ERME. Our main aim consists of fostering communication, co-operation, and collaboration amongst researchers. Therefore, our main goal is to create a real research community. Instead of going on with isolated researchers, trying to do their best by themselves, the new challenge is to find ways

and fields of research that could be tackled by many researchers, sharing programs and results.

We think that our aim is very ambitious and that encourages us, because at the same time we are aware that progress in education comes usually step by step. In order for that aim to be achieved, it is necessary that we concentrate all our efforts in its pursuit. With respect either to the whole group or to the subgroup, we propose:

- To share doctoral programs and dissertations, and doctoral tutoring,
- To take part together in European projects,
- To write monographic numbers in some specialised journals,
- To write some books and papers,
- To present some papers at conferences,
- To organise specialised conferences, meetings or seminars in different countries.

From all the previous proposals we would emphasise the combination of the communication, co-operation, and collaboration amongst Ph. doctors and the aim to improve future Ph. doctors' qualifications. They should already work within this philosophy.

References

- Ball, D.L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*, 8 (1), pp. 40–48.
- Berger, P. (1998). Teaching in the Realm of Computers. In E. Pehkonen & G. Törner (Eds.) *The State-of-Art in Mathematics-Related Belief Research. Results of the MAVI activities*. Research Report 195, Helsinki: University Press, pp. 97–126.
- Boero, P., Dapuerto, C. & Parenti, L. (1996). Didactics of Mathematics and the Professional Knowledge of Teachers. In A.J. Bishop et al. (Eds.) *International Handbook of Mathematics Education*. Dordrecht, The Netherlands: Kluwer.
- Borko, H. & Livingston, C. (1989). Cognition and Improvisation: Differences in Mathematics Instruction by Expert and Novice Teachers. *American Educational Research Journal*, 26 (4), pp. 473–498.

- Bromme, R. (1988). Conocimientos profesionales de los profesores. *Enseñanza de las Ciencias*, 6 (1), pp. 19–29.
- Brown, J.S., Collins, A. & Duguid, P. (1989). Situated Cognition and the Culture of Learning. *Educational Researcher*, 18 (1), pp. 32–42.
- Canguilhem, G. (1971). *Lo normal y lo patológico*. Buenos Aires: Siglo XXI.
- Carr, W. & Kemmis, S. (1986). *Becoming Critical: Education, Knowledge, and Action Research*. London: The Falmer Press.
- Carter, K. (1993). The Place of Story in the Study of Teaching and Teacher Education. *Educational Researcher*, 22 (1), pp. 5–12, 18.
- Clandinin, D.J. (1986). *Classroom Practice: Teacher images in Action*. London: The Falmer Press.
- Cockroft, W.H. (1985). *Las matemáticas sí cuentan*. Madrid: Servicio de Publicaciones del Ministerio de Educación y Ciencia.
- Contreras, J. (1985). ¿El pensamiento o el conocimiento del profesor? Una crítica a los postulados de la investigación sobre el pensamiento del profesor y sus implicaciones para la formación del profesorado. *Revista de Educación*, 277, pp. 5–28.
- Cortazzi, M. (1993). *Narrative Analysis*. London: The Falmer Press.
- Edwards, D. & Mercer, N. (1987). *Common Knowledge: the development of understanding in the classroom*. London: Routledge.
- Eisner, E.W. (1992). Are All Casual Claims Positivistic? A Reply to Francis Schrag. *The Educational Researcher*, 21 (5), pp. 8–9.
- Elbaz, F. (1991). Research on Teachers Knowledge: The Evolution of a Discourse. *Journal of Curriculum Studies*, 23, pp. 1–19.
- Elliott, J. et al. (1985). *Investigación-acción en el aula*. Valencia: Generalitat Valenciana.
- Elliott, J. (1987). Autoevaluación, desarrollo profesional y responsabilidad. In M. Galton. & B. Moon (Eds.) *Cambiar la escuela, cambiar el curriculum*. Barcelona: Martínez Roca.
- Elliott, J. (1991). *Action research for educational change*. Milton Keynes: Open University Press.
- Elstgeest, J., Goffree, F. & Harlen, W. (1993). Education for Teaching Science and Mathematics in the Primary School. In W. Harlen (Ed.) *Science and Technology Education*. Document Series, 46. Paris: UNESCO.
- Ernest, P. (Ed.) (1994). *Mathematics, Education, and Philosophy: An international Perspective*. London: The Falmer Press. (Refer, in particular, to Chapters 2, 4, and 15).
- Farnham-Diggory, S. (1994). Paradigms of Knowledge and Instruction. *Review of Educational Research*, 64 (3), pp. 463–477.

- Feiman-Nemser, S. (1983). Learning to teach. In L. Shulman & G. Sykes (Eds.) *Handbook of teaching and policy*. New York: Longman.
- Feldman, A. (1993). Promoting equitable collaboration between university researchers and school teachers. *Qualitative Studies in Education*, 6 (4), pp. 341–357.
- Fennema, E. & Franke, M.L. (1992). Teachers' Knowledge and Its Impact. In D.A. Grouws (Ed.) *Handbook of Research on Mathematics Teaching and Learning*. New York: MacMillan, pp. 147–164.
- Fenstermacher, G. (1994). The Knower and the Known: The Nature of Knowledge in Research on Teaching. *Review of Research in Education*, 20, pp. 3–56.
- Fernandes, D. & Vale, I. (1994). Two young teachers' conceptions and practices about problem solving. In J.P. da Ponte & J.F. Matos (Eds.) *Proceedings of the 18th International Conference for the Psychology of Mathematics Education*, Vol. II. Lisbon, Portugal: University of Lisbon, pp. 328–335.
- García, J.N. (1993). Un modelo de prácticas de enseñanza para futuros maestros. *Revista de Educación*, 301, pp. 337–358.
- Gardiner, A. (1996). Problem-solving Or problem solving? *Math. Gazz.* 80 (487); pp. 143–148.
- Gellner, E. (1994). *El arado, la espada y el libro*. Barcelona: Ediciones Península.
- Gimeno, J. (1983). El profesor como investigador en el aula. Un paradigma de formación de profesores. *Educación y Sociedad*, 2, pp. 51–73.
- Gudmundsdottir, S. (1991). Story-Maker, Story-Teller: Narrative Structures in Curriculum. *Journal of Curriculum Studies*, 23 (3), pp. 207–218.
- Guimarães, F. (1996). *O Conhecimento Profissional do Professor de Matemática: Dois Estudos de Caso* (tese de mestrado). Lisboa: DEFCUL.
- Habermas, J. (1992). *Teoría de la acción comunicativa, I. Racionalidad de la acción y racionalidad social*. Madrid: Taurus.
- Habermas, J. (1994). *Teoría de la acción comunicativa: Complementos y estudios previos*. 2^a ed. Madrid: Cátedra.
- Kline, M.S. (1985). *Matemáticas. La pérdida de la certidumbre*. Madrid: Siglo XXI Editores.
- Krainer, K. (1996). Action Research als Chance zur Verbindung von Forschung und (Aus- und) Fortbildung im Bereich der Mathematikdidaktik? In J. Juna & P. Kral (Eds.) *Schule verändern durch Aktionsforschung*. Innsbruck-Wien: StudienVerlag, pp. 121–144.
- Lappan, G. & Theule-Lubienski, S. (1992). Training teachers or educating professionals? What are the issues and how are they being resolved? In D.F. Robitaille, D.H. Wheeler & C. Kieran (Eds.) *Selected Lectures from the 7th International Congress on Mathematical Education*. Québec: Les Presses de l'Université Laval.

- Lave, J. & Wenger, E. (1991). *Situated Learning. Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.
- Llinares, S. (1991). *La formación de profesores de matemáticas*. Sevilla: GID.
- López de Ceballos, P. (1987). *Un método para la investigación-acción participativa*. Madrid: Editorial Popular.
- Mattingly, C. (1991). Narrative Reflections on Practical Actions: Two Learning Experiments in Reflective Storytelling. In D. Schön (Ed.) *The Reflective Turn: Case Studies In and On Educational Practice* (235–257). New York: Teachers College Press.
- McFee, G. (1993). Reflections on the Nature of Action Research. *Cambridge Journal of Education*, 23 (2), pp. 39–45.
- Moliner, M. (1979a). *Diccionario de uso del español*. Tomo I (A-G). Madrid: Gredos.
- Moliner, M. (1979b). *Diccionario de uso del español*. Tomo II (H-Z). Madrid: Gredos.
- NCTM (1991). *Professional Standards for Teaching Mathematics*. Reston, Virginia: NCTM.
- Novak, J.D. (1985). *Teoría y práctica de la educación*. Madrid: Cátedra.
- Olson, J. (1992). *Understanding Teaching: Beyond Expertise*. Milton Keynes: Open University Press.
- Paris, S.G., Lipson, M. & Wixson, K. (1983). Becoming a strategy reader. *Contemporary Educational Psychology*, 8, pp. 293–316.
- Porlán, R., Rivero, A. & Martín, R. (1997). Conocimiento profesional y epistemológico de los profesores. I: Teoría, método e instrumentos. *Enseñanza de las Ciencias*, 15(2), pp. 155–171.
- Robinson, V.M.J. (1998). Methodology and the Research-Practice Gap. *Educational Researcher*, 27(1), pp. 17–26.
- Rorty, R. (1989). *La filosofía y el espejo de la naturaleza*. Madrid: Cátedra.
- Russell, T. (1994). Teachers' Professional Knowledge and the Future of Teacher Education. *International Analysis of Teacher Education*, 19 (4 & 5).
- Schön, D.A. (1983). *The Reflective Practitioner*. New York: Basic Books.
- Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), pp. 4–14.
- Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), pp. 1–22.
- Shulman, L.S. (1993). Renewing the Pedagogy of Teacher Education: The Impact of Subject-Specific Conceptions of Teaching. In L. Montero & J.M. Vez (Eds.) *Las didácticas específicas en la formación del profesorado (I)*. Santiago de Compostela: Tórculo Edicións.

- Simon, M.A. (1994). Learning mathematics and learning to teach: learning cycles in mathematics teacher education. *Educational Studies in Mathematics*, 26 (1), pp. 71–94.
- Stenhouse, L. (1984). *Investigación y desarrollo del curriculum*. Madrid: Morata.
- Stenhouse, L. (1985). El profesor como tema de investigación y desarrollo. *Revista de Educación*, 277.
- Tall, D. (1995). Cognitive Growth in Elementary and Advanced Mathematical Thinking. In L. Meira & D. Carraher (Eds.) *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education (PME)*, Recife, Brazil.
- Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.) *Handbook of research in mathematics teaching and learning*. New York: Macmillan, pp. 127–146.
- Vergnaud, G. (1990). Epistemology and Psychology of Mathematics Education. In P. Nesher & J. Kilpatrick (Eds.) *Mathematics and Cognition*. Cambridge: Cambridge University Press.
- Whitehead, A.N. (1953). *Science and the Modern World*. Cambridge: University Press.
- Wilson, J. (1973). Three Myths in Educational Research. *Educational Research*, 16 (1), pp. 17–19.
- Wilson, S. et al. (1987). '150 Different Ways' of Knowing: Representations of Knowledge in Teaching. In J. Calderhead (Ed.) *Exploring Teachers' Thinking*. London: Cassel Education.
- Zeichner, K. (1987). Preparing reflective teachers. *International Journal of Educational Research*, 11 (5), pp. 565–575.
- Zuber-Skerrit, O. (1994). Learning and action research. In P. Nightingale & M. O'Neil (Eds.) *Achieving Quality Learning in Higher Education*. London: Kogan Page.
- Zuber-Skerrit, O. (1996). Introduction: New Directions in Action Research. In O. Zuber-Skerrit (Ed.) *New Directions in Action Research*. London: Falmer Press.

CHAPTER 4

TEACHER EDUCATION AND INVESTIGATIONS INTO TEACHERS' PRACTICE(S)

Preview

This chapter has been built on two *papers* (see subchapters 4.2 and 4.3) and the co-operation of four teacher educators and researchers (see subchapter 4.1).

The *first paper* (Perrin-Glorian, subchapter 4.2) shows an investigation of secondary mathematics teaching practice in a French ‘lycée’, focusing on specific subject matter (absolute value) and taking special theoretical points of view (Brousseau and Chevallard).

In the *second paper* (Goffree and Oonk, 4.3) the reader will see how information and communication technology can be functionally used in order to design a learning environment for primary mathematics student teachers in which ‘teaching practice’ has got a central position. Two Dutch student teachers, as learners in this environment, have been followed (4.3).

The first subchapter (4.1) is the outcome of a co-operative group (Goffree, Oliveira, Serrazina & Szendrei). The leading idea originated during the process of bringing together two different contributions and some ideas which already arose in the first discussions. What position should teaching practice get in a learning environment of future teachers? Looking for an answer new questions announced themselves:

- What is good practice in the context of teacher education?
- What do you register in real practice to get a good representation?
- What can research do to create a good representation of good practice?
- How do you present good practice to student teachers?
- How do students learn from practice?

Some of these questions are discussed in subchapter 4.1.

4.1

GOOD PRACTICE

Fred Goffree

Freudenthal Institute (The Netherlands)

Hélia Oliveira

University of Lisbon (Portugal)

Maria de Lurdes Serrazina

Escola Superior de Educação in Lisbon (Portugal)

Julianna Szendrei

Budapest Teacher Training College (Hungary)

Introduction

In this subchapter, ‘research in practice’ – investigating daily life in mathematics classrooms – is studied from the perspective of teacher education. These efforts have been made at different distances, so to speak, from teacher education, but always with the aim of supporting the *creation* or *improvement of learning environments* for future teachers. ‘Practice’ takes up a central position in these environments. It constitutes everything that goes on in mathematics classrooms, but mostly learning, teaching, subject matter, interactions, social and emotional phenomena, and institutional influences.

Sometimes *real* classroom practice forms part of the learning environment of prospective teachers who, in that case, are like apprentices learning their future job on the shop floor. But in many other situations real practice is, instead, *represented* by

means of case studies, teachers' stories or narratives, critical situations, and so on. By making the right selection of good practice representations, it is possible to bring 'daily life in classrooms' into the colleges of education.

It might be supposed that '*real*' practice, acquired in schools, would represent the best approximation to '*good*' practice. But teacher educators, after years of experience, have reasonable ground for doubting this. They will argue that there is no one single 'practice in schools' as such, and will point out that daily life in classrooms comes in many shapes and forms. For instance, there are classrooms that really do provide rich learning environments for both pupils and student teachers. But there are also classrooms where little or nothing happens that is likely to be of interest for student teachers, or, worse, things happen that put them on the wrong track.

Moreover, simply being a guest in a classroom does not automatically mean that anything is being learnt about teaching. Student teachers visiting classrooms often find a working place rather than a learning place in the classroom (Cohen 1998). Their main motivation is to do the teaching job properly, often to meet the goals of the educator, or to satisfy the classroom teacher.

If they are to learn from practice in schools, student teachers – particularly first-year student teachers – will need an *inspiring coach* and a *reflective expert* in the environment. The coach shows (by modelling or explanation) how to act and gives hints for finding the right track. The expert, as a reflective practitioner, accompanies the student on the track and is always there when needed. He or she points out relevant events and interesting phenomena, asks questions and discusses the answers, gives explanations and makes theoretical reflections that show how to use 'theory-in-action' (Schön 1983).

The first section of this subchapter describes a *curriculum experiment in Hungary*. It is a teacher education experiment in which school practice takes a central place. Using an elementary school classroom in Budapest and the different areas of expertise of (1) the classroom teacher, (2) the mathematics educator and (3) the general educationalist of the College of Education, a *learning environment* was created. The student teachers are here primarily learners, actively involved in the study of daily life in a mathematics class. Their main perspective on the experiment will therefore be the aim of 'learning to teach mathematics to elementary school children'.

Learning from practice can also take place within the walls of a School of Education (Teacher's College). To make such an approach to teacher education possible, a representation of 'good practice' should be a prominent part of the learning environment. Student teachers should have frequent opportunities to encounter situations from daily life in classrooms in order to observe relevant events, ask personal and professional questions, discuss interpretations and explanations and evaluate learning and teaching using mathematical knowledge and didactical theory.

Good practice in the context of teacher education focuses on the learners as well as the teacher. In 'good practice' learning processes and the act of teaching are closely related, and must also be so in a *representation* of good practice.

Good practice can be represented by stories, narratives, cases, classroom episodes, teachers' log books or research diaries, critical situations, clinical interviews with pupils, tests and test scores, metaphors, events, anecdotes, etc. Both old media and new information and communication technologies (ICT) can be used to create up-to-date presentations of such representations. Using ICT, it is even possible to create a digital representation of everything that is going on.

The *story of teacher Rosa* (section 2 of this subchapter) is an example of good practice in teacher education. It is based on an observation in the first-grade classroom of an experienced and committed teacher in Portugal. She likes to teach mathematics as an activity to young children and knows how to use manipulatives in order to make children construct their own knowledge. She is creative and shows how to design a learning environment where children feel free to participate. In Rosa's story the pupils are also the actors. A story like Rosa's, with so many reasons to learn from practice, is what we, following Hans Freudenthal (1991, p. 76), would call 'a paradigm of good practice'. Keeping in mind the perspective of teacher education, what makes Rosa's story a paradigm? Thinking forwards to the story itself, we can mention some criteria:

- It is a transparent story of good mathematics teaching.
- It is about the use of manipulatives – a relevant topic in the didactics of mathematics for young children.
- It shows the children as active learners.

- It also shows the teacher as a designer of a learning environment.
- There are links with ‘the’ theory.
- It shows elements of the (pedagogical) classroom climate.
- It may stimulate student teachers to improve their didactical repertoire.

As always, it is not difficult to point to things that are missing:

- There are no after-lesson reflections by Rosa.
- You are not told what happened before and what will happen after this lesson.
- We do not know much about the individual children. (Etc.)

Rosa’s story is told by a teacher educator, but you can also ask teachers to tell the stories themselves, reflectively, from a professional and a personal point of view. (McEwan & Egan 1995). These ‘narratives’ can be completed by observations in the classroom and reflections supported by stimulated recall. The outcome of research like this can be used to create items to represent ‘good practice in teacher education’.

The *stories of two secondary school mathematics teachers*, Theresa and Isabel (third section of 4.1) were analysed ‘narratively’ and supplemented with interviews, reflections, and classroom observations.

Although this narrative research was not primarily intended as a way of improving learning environments for teachers, the data collected, the research process itself, the innovative context, the mathematical content, and the personal and professional backgrounds of the both teachers nevertheless do have this effect.

A curriculum experiment in Hungary

Following a German tradition, the teacher training institutes in Hungary are associated with primary schools with the aim of making it possible for the undergraduate students to teach and gradually improve their practical teaching abilities in a realistic situation. In fact, practical teaching has always been an important part of the training of primary teachers and, as in many other European countries, it still is.

As a consequence, 'school practice' is an essential part of the teacher education curriculum in Hungary and, in the curriculum experiment described here, school practice plays a major role. This experiment was set up as a piece of preliminary research in teacher education. Working in co-operation with primary school teachers, other educators and student teachers, teacher educators designed a learning environment for student teachers in which many relevant aspects of teacher education were realised. It has been put in chapter 4 because it demonstrates the importance of studying school practice in teacher education.

All teacher education activities at the Budapest Teacher Training College seek to answer the question "How can you help prospective primary mathematics teachers to better understand the thought processes of their students?" as well as closely related questions such as "How can we use theory in practice?"

The experiment was carried out at the Department of Mathematics of the Budapest Teacher Training College. Altogether, ten college tutors and mentor teachers of the practice school were involved.

Learning environment for student teachers

The experiment aimed at creating an environment for the student teacher in which he or she is part of a group of peers and two senior educators. They work and observe together, in an interactive way, in the classroom. By careful, non-aggressive interpretation of the educators, and continuous communication with peers, the student comes to understand the overt and hidden processes as they occur in the class. This progress hopefully leads the student to grow into a teacher able to choose between alternative ways of teaching, as appropriate, and not become a traditional 'schoolmaster' only able to teach according to an inflexible schema.

In the experiment, the student is one of a group of 14–15 members who try to explain and understand their experiences together; but in his or her future practice, the student will have to predict and analyse the concrete and mental processes in the classroom on their own.

Activities in the learning environment

- *First session*

The process starts with the class teacher giving a lesson in his or her own classroom. After the lesson, the teacher presents his way of planning the class activity. This includes *a priori* analysis. The teacher also makes an *a posteriori* analysis of the classroom activity and listens to the reflections of the student teachers. The teacher gives some additional information about the class and about the individual pupils of the student teachers.

- *Week 1*

The primary school teacher gives the goals and tasks of a mathematics lesson, which will be led by one of the student teachers in two weeks' time. All student teachers then individually plan a whole-class activity at home.

- *Week 2*

The group of students, the mentor teacher, and the college educators discuss their preparatory work and make an *a priori* analysis of the next class activity and choose two students, one of whom will teach the lesson in the following week while the other is designated as a deputy teacher who will teach only if the first is absent.

The primary school teacher gives the aims, goals, objectives, and tasks of the lesson two weeks ahead to allow for preparation.

- *Week 3*

The student teacher teaches the class. The group of students, the primary school teacher, and the mathematics educator take notes. The deputy student teacher may make some observations for his or her colleagues.

On the same day, the student teacher, led by the deputy student teacher, makes a self-evaluation, and the group, the primary teacher, and the maths educator make reflections on the lesson. (This is an *a posteriori* analysis which results from the work of the whole group.) The main aim of this collective effort is to be able to understand the classroom events in a way that makes optimum use of the fact that 15 different people are observing and reflecting on the same situations.

The primary school teacher gives the aims, goals, objectives, and tasks of the lesson two weeks in advance to allow time for preparation. This process is repeated six times in the first semester and three times each in the other three semesters. The students remain with the class and the class teacher during the semester to practice school subjects other than mathematics.

Actors in the learning environment

The actors in this collaborative learning environment are the student teachers, the teacher, the educator (a specialist in mathematics education) and the educator, whose specialism is educational psychology and pedagogy. They all participate in the design, analysis, and evaluation of mathematics teaching.

The teacher (mentor) provides the content and goals of the next lesson, which is to be taught by a student teacher. The educator of mathematical didactics (pedagogical content knowledge) prepares the lesson in the presence of the teacher of the class and the group of student teachers. He or she gives special support to the student teacher who is going to teach the lesson and the main characters – the student teachers – study the curriculum of the class and prepare for the lesson in accordance with the content and aims as specified by the teacher. They discuss the drafts they have individually prepared for the lesson and brainstorm about them. One student is chosen to teach the lesson and another is selected to take their place if, for some reason, this is not possible.

Good practice for teacher education

It goes without saying that this collaborative learning environment must also be a promising tool for staff development. However, this is not the point we wish to emphasise here. What happens in this environment, in which school practice has been given such a central position, may offer guidance for researchers and educators. Both, after all, need a clear concept (image) of 'good practice in the context of teacher education'. Scientific researchers attempt to contribute to the knowledge base of what happens in schools and try to enrich these environments. Teacher educators, and particularly teachers-to-be, like to recognise their own images and hear the teachers' voices in the results of research. The educators base their creative work on their own images and the results of research representations of 'good practice'. These

representations give good reasons for investigating, studying, and discussing in order to learn about teaching.

In the following sections, in which researchers and educators present their contributions to teacher education, some of the foregoing characteristics of ‘*good practice in the context of teacher education*’ may be recognised:

Maria de Lurdes Serrazina presents a paradigm of good practice – an observation of primary school teacher Rosa – and continues with a reflective note about research methods in the context of teacher education. Hélia Oliveira uses a narrative research approach to study the teaching practice of two secondary school mathematics teachers. The teacher’s voice can be heard.

Paradigms of good practice

What is meant by practice? For longer than anyone cares to remember, ‘practice’ has been described by the components of the so-called ‘didactical triangle’ where ‘the student’, ‘the teacher’, and ‘the subject matter’ are the vertices and where the sides represent the interactions between the aforementioned actors. However, in observing good practice these days we see more than the six elements of a triangle. In describing these practices we must also pay attention to such things as the learning environment and the media used.

So when presenting examples of good practice we should pay attention to the components of a sort of ‘extended’ didactical triangle. Moreover, in the context of teacher education, good practice should really be considered as a rich source for student teachers in which can be found many good reasons to learn the essentials of the profession.

What should be understood by a paradigm? Hans Freudenthal, quoted earlier in this chapter, gives a clear (paradigmatic) explanation:

“(...) I will try to make clear what I mean by observation of learning processes by means of an example: I always spoke with Bastiaan (born on 27 April 1970) in adult language, sometimes purposely introducing words that I supposed were new to him. Whenever he did not understand, he would say: ‘What did you say Grandfather?’ and then I would repeat what I had said previously but in different words. On 15 February 1975 he did not say ‘What did you say

Grandpa?', but he said 'What does 'elsewhere' mean?' This proved to be a watershed, for never again did he react to an unknown word or construction with the words: 'What did you say Grandpa?' but he would always ask about its meaning. (...) I was not helping him, I could not say whether this was on purpose or not. But at any rate I enabled him to reach a discontinuity in his learning process, a learning jump. I regret however that I did not draw his attention to this fact at the time: he had replaced the global question about the meaning of a linguistic utterance by a much more direct question about a word. (...)”

Later, he added in the same article:

“One learns more from one paradigmatic case than from a hundred that do not apply. This is how you should interpret my observation of 15 February 1975. The situation described by me is a unique one, in particular when I tell the previous history as well. It is a windfall when one can observe a caesura as clear as this one, but one should make the most of the opportunity too ...” (Freudenthal quoted in Goffree 1993, p. 33).

A paradigm might be called a peak experience in qualitative research. It is an observation that places a given problem in a novel perspective and represents a local theory, by which the new understanding can be embedded into existing knowledge or maybe even improve it. The observation becomes a strong and transparent example that represents the theory as situated cognition or contextualised knowledge. Finding a paradigm means creating new understanding and opening new perspectives.

Paradigms of good practice in the context of teacher education are examples, cases or just observations with a high degree of pedagogical and didactical, practical, and theoretical wisdom. Analysing and discussing paradigms are essential elements of discourse in teacher education.

Paradigms can be presented as stories, either as live oral accounts or recorded on audio or videotape. They can also be discovered by student teachers themselves when investigating good practice. Discoveries like these happen in MILE (see subchapter 4.3).

Rosa and the use of manipulatives for constructing knowledge

Manipulatives

Since 1991, the Portuguese primary school mathematics curriculum has emphasised the problem-solving approach and the understanding of concepts. It calls for the use of manipulative materials for constructing knowledge and it begins by saying that the ‘teacher’s main task is to instil in every pupil an enjoyment of mathematics’ (DGEBS 1990, p. 125). The use of manipulatives is encouraged in the curriculum and curriculum authors justify it in the following way: “If the manipulation of materials can allow the construction of certain concepts, it can also be suitable for the representation of abstract models, thus allowing a better structure of those concepts” (DGEBS 1990, p. 129). In the following we follow Maria de Lurdes Serrazina’s description of teacher Rosa.

Rosa

Rosa is a primary school teacher of 17 years experience. She likes mathematics and usually works with manipulatives. I was observing her first-grade class of 14 pupils when she suggested working on subtraction with her pupils. She started by inviting them to sit around a large table. All the children could see the whole table. She sat on a small chair in order to be at the same level as the pupils.

Rosa had a box of coloured cubes, with which the pupils were already familiar. She started by spreading out the cubes on the table and each pupil picked up some of them until they were all taken.

Rosa took advantage of the situation by asking how many cubes each one had. This led to individual counting. While they finished the counting, some children kept spontaneously making towers, squares or other groupings on the table.

Afterwards, the teacher asked the children to make bars of ten cubes of the same colour and put them on the table. As some children did not have enough cubes, the teacher asked each one how many cubes they needed to have ten. She asked them to complete their bars by asking other children for cubes. In the end, there were many bars of ten cubes on the table, but some pupils did not yet have ten. They counted the bars that did not have ten orally, thought about how many were missing and

completed them with different coloured cubes. They recorded what they had made using coloured plastic numbers.

The teacher took some cubes from one of the bars and asked a pupil to record what had happened. Then she made a game: Rosa ordered one pupil to close her eyes, took out some cubes from one bar and when she opened her eyes she had to say how many cubes had been taken out just by looking at the configuration. Afterwards, the conjecture would be confirmed by the child by putting back the number of cubes that she had said were missing. The situation was again recorded using coloured numbers. Usually they recorded the last operation. That is, they did the addition to verify if it was right.

After the pupils had returned to their places, the last problems solved with cubes were written on the blackboard by one of them while the others recorded it on their worksheets.

As the class continued, Rosa presented new problematic situations for the children to solve with manipulatives and record symbolically on their worksheets. For each situation, one pupil went to the blackboard, solved the situation, and recorded it. For instance, the teacher took out some objects, the number of which the children knew, from a bag and asked them to work out how many were left inside. She also asked someone to take out, from a known whole, a certain quantity while the pupil at the blackboard had to work out the number of objects which were left; they were also asked how many extra objects would be needed to get a given quantity. ... In this way, the teacher was able to exploit different subtraction situations.

At the end of the class, the children seemed to have understood the mathematical ideas implicit in what they had just done. This appears to be a good way to introduce the concept of subtraction for small children using manipulatives. The way the class was conducted seemed to favour pupil-pupil and pupils-teacher interaction. Rosa seemed to have understood that 'using manipulatives for teaching mathematics is always a means to an end and never an end in itself' (Pimm 1995, p. 13).

Recommendations of methodology

If we consider the study of practice in schools to be essential in order to improve existing, theory-oriented teacher education programmes, we need to decide what methodology will be most appropriate. The question is ‘how to study practice in order to acquire knowledge?’. Knowledge in this case means knowledge about learning and teaching mathematics which is useful for teachers and which is also transferable in teacher education programmes.

Teachers also can investigate their own practice in order to improve it and become a better teacher. The approach, which will be recommended in a next chapter, is that of action research. If we want to know teachers’ instructional practices, we will need to systematically observe a variety of mathematics classes and discuss certain aspects of teachers’ practices, as there are meanings and norms implicit in practices of individuals, practices which cannot be conceived as a set of individual actions but which are essentially modes of social relation, of mutual action.

Qualitative approaches emphasise the importance of getting close to the people and situations being studied in order to understand personally the realities (Patton 1987). If we want to have a deep understanding of practice, we should carry out interpretivist research while bearing in mind that ‘the purpose of doing interpretivist research is to provide information that will allow the investigator to ‘make sense’ of the world from the perspectives of participants; the researcher must be involved in the activity as an insider and able to reflect upon it as an outsider’ (Eisenhardt 1988, p. 103).

The methodology should allow the researchers to interact with teachers in their natural setting, that is, their classrooms and schools, and should be sufficiently flexible to embrace a variety of research techniques, as and when they became relevant. A qualitative approach should mainly involve observation (supported by field notes) and interviews (taped and transcribed). The role of the researcher should be that of a participant observer: ‘Participant observation combines participation in the lives of the people under study with maintenance of a professional distance that allows adequate observation and recording of data’ (Fetterman 1989, p. 5).

The research should take the form of case studies. Data collection should be done by a variety of methods: interviews, class observations, and the study of written documents (lesson plans, worksheets given to the pupils, etc.).

A narrative completion – the stories of Teresa and Isabel

The idea that research should take the form of case studies if it is to be useful for teacher education brings us closer to the concept of personal practical knowledge (Elbaz 1983). Practice, from the standpoint of teacher education, can thus be considered as a rich source of practical knowledge. Moreover, stories constitute an interesting and fruitful way of gaining access to teachers' thinking and knowledge (...).

A learning environment like MILE (see 4.3) has been designed to offer each student teacher the opportunity to create his or her own case study and to personally reconstruct the practical knowledge contained in the case. The personal practical knowledge of teachers, following Carter (1993) and Gudmundsdottir (1995), is a form of narrative knowing. It means that teachers construct, organise (Bruner 1991), memorise, and communicate their knowledge about practice by telling each other stories.

Stories help us to put order and coherence into our experience and to give meaning to incidents and events in our lives. Or, as Elbaz (1991) put it: 'story is the very stuff of teaching, the landscape within which we live as teachers and researchers, and within which the work of teachers can be seen as making sense. Teachers' knowledge in its own terms is ordered by story and can be best understood in this way' (Elbaz 1991).

Next, an example will be given of creating, structuring, and analysing narratives. Two mathematics teachers have been followed in their teaching of mathematical investigations. So the context of this research is more 'innovation' than 'teacher education'. But process as well as product of the research may contribute to the creation of good practice for use in teacher education.

Narratives for Studying Teachers' Practice.

Data have been collected by classroom observation with audio and video recordings, along with audio-recorded interviews and reflections with the teachers about their classes using stimulated recall. The data was interpreted in terms of teacher's narratives, so that Labov's evaluation model could be used for a narrative analysis (Cortazzi 1993). This is a socio-linguistic approach that relates the formal structural properties of stories with their social functions (see Table I).

<i>Structure</i>	<i>Question</i>
Abstract	What was this about?
Orientation	Who? When? What?
Complication	Where?
Evaluation	Then what happened?
Result	So what?
Coda	What finally happened?

Tab. I (after Cortazzi 1993).

In the classroom

The researcher assigned some tasks, individually, to two teachers, Teresa and Isabel, which involved investigative work (see Figure 1). The teachers suggested a slightly different formulation and presented it to one of their 8th-grade classes. The mathematical investigations were accomplished during two or three 50-minute lessons. Both classes had about 30 pupils and teachers decided to form groups of four or five pupils. In general, the teachers made a short introduction to the task, then the pupils worked in groups and, finally, the teacher proposed a whole group discussion.

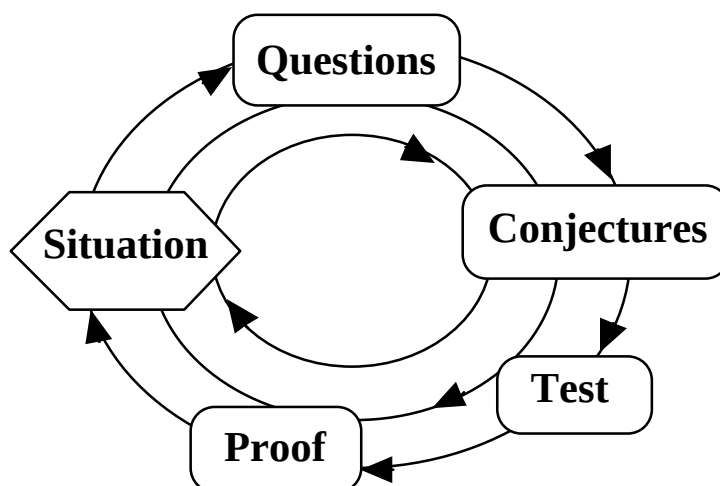
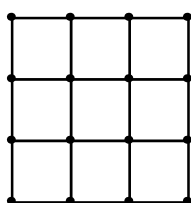


Fig. 1: Mathematical investigations.

These teachers told several stories about classes with mathematical investigations and these stories were analysed according to the evaluation model. In the first two, we see some aspects of the teacher's role while in the other ones we see the pupils' role. Stories 1 of Teresa and Isabel concern a lesson with the task 'squares with matches':



- How many matches were used to make this square?
- Find out how many matches you need to make any square of this type.

Story 1 (Teresa)

Teresa tried to help the pupils to generalise the numbers of matches for every square by suggesting that they count the matches in a systematic way because their focus was only in the numbers they obtained after counting. But then she expressed some doubts about the legitimacy of her support because she thought the pupils could do it in a different way (complication). Since two groups got the expression in a different

manner, she felt that maybe it is better not to make suggestions but rather to wait to see what happens (evaluation):

“There was a moment when I felt that they wouldn’t be able to get there (to the general expression) because they hold very much to this, to the expression of the numbers. So I said to some groups, ‘Look at the way you count the matches. Find a systematic approach to counting’ and so on.” [Abstract]

“As it was the way I got there, I thought that it would be the better way of doing that.” [Orientation]

“But afterwards I thought, ‘How can I say that pupils won’t get there by another process?’ ” [Complication]

“So I stopped saying that. I said to myself ‘I am not going to say anything. Absolutely anything, not even if they don’t find anything at all.’ And I got the feeling that they were not going to find anything during the entire lesson, that they were not going to find the general expression.” [Result]

“But finally the two other groups got there using other processes. So I don’t know if it would be better not to tell them anything and let them disengage, and then we will see.” [Evaluation/Coda]

Story 1 (Isabel)

Isabel also found it difficult helping pupils to write down the expression for the number of matches in any square. Two groups were able to explain how to obtain the numbers in the sequence recursively but the teacher wanted more than that. However, she didn’t know what to tell them that could be helpful without saying too much (complication). She solved the problem when she realised that her expectations were greater than the ones in the notes for the teacher (result). These helped her to feel satisfied with pupils’ work.

“In this lesson the most complicated thing was the part dealing with generalisation, because they saw there the result of the sequence – they saw the numbers, compared. But they were only able to explain what happens now by finding the expression and that is much more complicated.” [Abstract]

“I gave them a little hint at the beginning. I saw that they needed one of those expressions and they were trying and presented $4n$, but it was not that.” [Orientation]

“And there was my greatest difficulty, ‘How am I going to help them to generalise?’ Because they had all the calculations and their thinking was correct. But I could not find the way out, ‘This is not the expression, it’s another one, but how am I going to explain this?’ And so I just called their attention to that sequence formed by the integers, and for the other numbers that were even.”

“But I became a bit frustrated in this lesson. And that frustration grew from the fact that they were understanding, everything they did was correct, but it was that bridge to generalisation. It was that bridge to generalisation that I was not able to pass over, ‘How am I going to explain them and still be non-directive?’ For me the only solution there was being directive.” [Complication]

“But after that I read in your sheets [teacher’s suggestions] that, for this level, it’s acceptable that they simply explain their thinking ... And I thought, ‘Good, this is exactly for these two groups.’ ” [Result]

“But the question is that they can explain their thinking and show their calculations ...” [Evaluation]

Story 2 (Teresa)

Teresa’s second story regards a different lesson not included in this research. Nevertheless, she told it to show the pupils’ role in the lesson. The time constraints forced the teacher to act in a way that was not consistent with what the pupils were expected to do. It is interesting to note that they are the ones who, in this episode, remind the teacher of their role in the investigation.

“They usually discuss everything. They don’t jump to the following question without understanding the previous one, they can’t do it. And it is funny ... look, Carolina told me something that I was ...” [Abstract]

“(...) There was a part in which I asked for an angle and they had to find it out, but it was rather difficult. I moved from group to group during the lesson but just arrived at that group in the end. And I don’t know exactly how it was but they were late and I wanted them to hurry up. What I really wanted in that lesson was to advance. I saw that they have had a lot of discussion, they drew many lines, and many angles, many triangles ... They discussed everything deeply but they didn’t arrive at the end.” [Orientation]

“So I erased their lines and said, ‘Look, this figure is too complicated. Don’t draw any more lines and let’s try to interpret this situation as it is.’ In the meantime, the bell was ringing and I started explaining: ‘Are you seeing this angle ...’ and so on. And Carolina tells me, ‘Teacher, didn’t we have to discover this by ourselves?’ But she was really angry! And I answered, ‘Yes, you did, but you have been so slow. The bell is ringing and you still haven’t reached the end.’ ” [Complication]

“And Carolina says, ‘So, teacher, let us find out by ourselves. Let us find out by ourselves’ as if to say: ‘Go away!’ ” [Result]

“It was so funny. They didn’t ask anything and I went there and erased their work, which spoiled their thinking, and I start thinking in a different way, which they didn’t ask for. So, clearly, I was meddling in matters that didn’t concern me.” [Evaluation/Coda]

Story 2 (Isabel)

Isabel’s second story comes from a lesson with a task where the pupils were asked to write the following numbers in the sequence as powers of three: 81, 27, 9, 3, 1, $1/3$, $1/9$, $1/27$... This was the first time they would be using negative exponents. Through this story, Isabel shows how important it is for her that pupils explain and argue for their ideas.

“It happened [students explaining and convincing their colleagues] with this task.” [Abstract]

“I think that happened in Beatriz and Fábio’s group.” [Orientation]

“Because in that question, which had to do with representing the numbers on the sequence on the form of powers of three, Beatriz had $1/3$, after, $(1/3)^2$, $(1/3)^3$... And I had already told Alexandre to pay attention to the exponent sequence because he had 31, 30. And he said, ‘Ah! So it is $3-1$, $3-2$.’ But Beatriz didn’t have that.” [Complication]

“So I said, ‘Oh Beatriz, now speak with Alexandre. Alexandre explains how and why, and we’ll see if he convinces his friends.’ ” [Result]

“That subject of convincing others also came up in Miguel’s group.” [Orientation]

“Miguel had noticed that the exponent sequence was decreasing, but Ana expressed more strongly her opinion that it should be $(1/3)^1$, $(1/3)^2$, $(1/3)^3$, and so on.” [Complication]

“As it was Miguel who had the [right] sequence, I told them, ‘Miguel, try to convince Ana.’ But meanwhile she says, ‘Ah, but I have already understood.’ She understood after Miguel explained for the first time.” [Result]

“So I think that it is very fruitful, in this kind of work, when there is someone who reaches the conclusion quicker than the others, that they convince each other.” [Evaluation/Coda]

Teacher's voice

An analysis of the narratives synthesises teachers' experiences in this innovative context. Using the findings may contribute to the creation of good practice in teacher education:

About the role of the teacher:

- Creating conditions for student activity.
- Creating a favourable disposition towards mathematical tasks.
- Promoting investigative processes.
- Sustaining students' activity.
- Promoting communication and thinking and stimulating the development of concepts and procedures.

About the students:

- Taking initiative and risks.
- Assuming more responsibility for arguing and convincing their colleagues.
- Developing more and more intellectual autonomy.

About teachers' dilemmas:

- Giving support without providing the answers.
- Making suggestions without leading students off the paths they have chosen.
- Asking for justifications up to a reasonable point.

- Promoting whole class discussion while balancing available time and individual intervention.

Reflection

Creating good practice for student teachers in order to learn about mathematics teaching requires careful and specific research into teachers' practices. Teacher educators will use the results of this research to design learning environments for future teachers. This use is not limited to the last chapter in research reports, in which the results and recommendations are simply listed. Design, instruments, data, analysis and possibly anecdotes and reflective notations linked to the learning process of the researcher (Streefland 1993) are helpful ingredients for someone designing learning environments. We could say that putting research into teachers' practices into the perspective of teacher education will give a new dimension to the research enterprise.

References

- Bruner, J. (1991). The Narrative Construction of Reality. *Critical Inquiry*, 18, pp. 1–21.
- Carter, K. (1993). The Place of Story in the Study of Teaching and Teacher Education. *Educational Researcher*, Vol. 22, No. 1, pp. 5–12.
- Cortazzi, M. (1993). *Narrative Analysis*. London: The Falmer Press.
- DGEBS (1990). *Programas do 1º ciclo do ensino básico*. Lisbon, Portugal: Direcção Geral do Ensino Básico e Secundário, Ministério da Educação.
- Eisenhart, M. (1988). The ethnographic research tradition and mathematics education research. *Journal for Research on Mathematics Education*, 19(2), pp. 99–114.
- Elbaz, F. (1983). *Teacher thinking: A study of practical knowledge*. London: Croom Helm.
- Elbaz, F. (1991). Research on Teachers' Knowledge: The Evolution of a Discourse. *Journal of Curriculum Studies*, 23, pp. 1–19.
- Fernandes, D. (1985). *A mathematics needs assessment study of elementary school teachers of Viana do Castelo*. Unpublished masters thesis. Boston, MA, US: Boston University.
- Fetterman, D.M. (1989). *Ethnography step by step*. Sage Publications.
- Freudenthal, H. (1991). *Revisiting Mathematics Education. China lectures*. Dordrecht: Kluwer Academic Publishers.

- Goffree, F. (1993). HF: Working on mathematics education. In L. Streefland (Ed.) *The Legacy of Hans Freudenthal*. Dordrecht: Kluwer Academic Publishers, pp. 21–50.
- Gudmundsdottir, S. (1995). The narrative nature of pedagogical content knowledge. In H. McEwan & K. Egan (Eds.) *Narrative in teaching, learning, and research*. New York: Teachers College, Columbia University, pp. 24–38.
- McEwan, H. & Egan, K. (Eds.) (1995). *Narrative in teaching, learning, and research*. New York: Teachers College, Columbia University.
- Patton, M.C. (1987). *How to use qualitative methods in evaluation*. Sage Publications.
- Pimm, D. (1995). *Symbols and meanings in school mathematics*. London: Routledge.
- Pires, M. C. (1995). *Os conceitos de per'metro e área em alunos do 6º ano: concepções e processos de resolução de problemas*. Lisbon, Portugal: APM.
- Reys, R.E. (1971). Consideration for teachers using manipulative materials. *Arithmetic Teacher*, December 1971.
- Ribeiro, A. (1995). *Concepções de professores do 1º ciclo a Matemática, o seu ensino e os materiais didácticos*. Tese de Mestrado, Universidade de Lisboa. Lisbon, Portugal: University of Lisbon.
- Sowell, E.J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education*, 20, pp. 498–505.
- Streefland, L. (1993). The design of a mathematics course. A theoretical reflection. In L. Streefland (Ed.) *The Legacy of Hans Freudenthal*. Dordrecht: Kluwer Academic Publishers, pp. 109–135.

4.2

A STUDY OF TEACHERS' PRACTICES: ORGANISATION OF CONTENTS AND OF STUDENTS' WORK.

Marie-Jeanne Perrin-Glorian

Université Paris 7 et Université d'Artois (France)

Introduction

This paper presents a research which takes place in a larger project about the study of teachers' practices in relation with the knowledge involved and with the students' practices inside class and outside. A result of my previous research (Perrin-Glorian 1990, 1993) was the identification of a phenomenon which affects '*devolution*' and '*institutionalisation*' (Brousseau 1987), specially in *low achievers classes*: constraints felt as conflicting by teachers lead them to renounce conditions of learning and to give to students means to succeed nevertheless expected tasks by negotiation (negociation '*à la baisse*' in French). Then '*vicious circles*' may happen ending at '*no-learning*' of some students. Institutionalisation seems to be particularly of interest because it is the moment when the teacher makes a connection between the mathematical knowledge involved by students in problems and the notions to learn.

To study the institutionalisation process, I had to question *knowledge*, *teachers*, and *students*. Concerning *knowledge*, my questions are: What is there to know about this knowledge at this level? Are there different possible interpretations of syllabuses? Which kind of tasks are students expected to succeed? Which techniques are expected for that? Which explanations? Which theories?

Concerning *teachers*, my questions are: What knowledge is actually institutionalised? How is it related to knowledge used by students in problem solving? How is new knowledge connected to previous knowledge or to future knowledge? Which other means (besides knowledge) does he offer to students to succeed exercises?

Concerning *students*, I wish to find, about a precise notion and for some students what knowledge is used to solve problems, spontaneously or with help of the teacher, how it is modified during interactions between students or between teacher and students, or by the lesson of the teacher, or during assessment tests. I also wish to know the working methods of students in class or outside (library, home) and their relations with long term and short-term success.

To study those questions I used different methods: class observations, interviews with teachers, with some students, analyses of students' productions. To be able to connect the declarations of teachers and students with data collected from observations, it seemed necessary to me to gather all that was done or said in class by teachers and students about one precise notion. I chose 10th grade (15–16 years old) which is in France the first year of 'lycée' and the last one before the choice between literary, scientific, economics or technical studies. One year before, there is a first orientation to professional schools so that this grade does not gather all students of the same age but a large part of them. In addition, this grade presents two other interests: in the schedule, forty-five minutes are provided for 'modules' that teachers are free to organise, dividing their students into two groups as they like. Moreover, a national assessment is carried out in every class of that grade at the beginning of the scholar year so that we have a way to compare different classes. I chose to observe the teaching of absolute value and approximations. It is a new notion in this grade since 1991 (Perrin-Glorian 1995, 1997).

I hypothesise that beyond differences in the organisation of teaching, there are conditions which favour learning but that these conditions may be performed in many various ways. In order to try to identify some of them, I consider two ways: on the one hand a precise study of what is taught and on the other hand the evolution of students' knowledge. In this paper, we are only interested in the practices of teachers independently of their effects on students. In other publications (Perrin-Glorian 1995,

1997) I brought some issues about the connection between contents actually taught and performances of students.

Framework, problem, and methodology

Theoretical references

My main references are those usually used in the French research in mathematics education, in particular the 'theory of situations' mainly elaborated by Guy Brousseau (e.g. 1996) and the anthropologic approach of 'didactic transposition' mainly worked out by Yves Chevallard (e.g. 1991).

According to these references, my purpose is not to try to look for a model of teacher but rather for a model of teaching. I mean that it is not the teacher himself with his internal references (his beliefs, his knowledge, his thinking ...) who is the main object of my research. It is rather the teacher as a function that I try to study. I am mainly interested in *external references* as institutional constraints and in the actions of the teacher: *relations between constraints and actions* and *between actions and their effect on the students' learning*.

Nevertheless, as one can see through the hypotheses and the results of this research, I approach somewhat the issue of teachers' knowledge, dividing it in *four categories*:

- mathematical knowledge, revised through the filter of his experience (classes he taught before or now) and the filter of syllabuses, especially those of ulterior classes,
- general knowledge about students and their difficulties to learn a specific concept (mainly coming from his experience) and specific knowledge about his students as particular students,
- practical knowledge of the effect of some devices ...
- habitus (Bourdieu 1972) coming from experience, specially concerning class management.

What are we calling practices in this paper? What practices are we talking about?

I have now to precise my *research question* (problematique) for the present research. The teachers accomplish many tasks inside class and outside in order to practise his profession. To me, the organisation of some of these tasks is a practice. Some of those practices may be called mathematical practices and others rather concern class management and organisation of students' work. The mathematical practices develop inside devices more or less constraining, some of them being put in place by the scholar institution, others by the teacher himself. Those devices leave more or less latitude to teacher and students. Hence the teacher makes choices, consciously or not. We gather them into two large domains: choices concerning the mathematical practices and choices concerning the organisation of students' work, class management.

Organisation of practices – choices for the study

Therefore, I distinguish *two objects* and *two levels* for my analysis of teachers' practices. The *two objects* are the following. On the one hand the contents actually taught, that is to say the institutional relationship to knowledge expected in the class (Chevallard 1992). On the other hand the way in which the teacher's course is related to the students' own work, which includes the devices used and the way the teacher manages them. The *two levels* are the 'global level' (on a scale of one year or some weeks) and the 'local level' (on a scale of one class session). For these two objects and levels, we are trying to characterise the choices of teachers and the variables that control those choices.

Concerning the contents at the global level, I try to characterise the choices concerning *didactic transposition* and *didactic time* (Chevallard 1991, 1992), that is: the organisation of contents and their distribution during the year, the parts that are more or less developed, the choices for the presentation of each notion, the objectives for long or short term, the dialectics between previous and new knowledge. I try to connect those choices with an epistemological analysis of contents and an analysis of the 'institutional relationship' (Chevallard 1991) to contents as they appear through syllabuses and handbooks. The main institutions I consider are the scholar institution, the curriculum in 10th grade institution, the curriculum in class A (or B, C, D, E) institution. For each institution, I regard the following questions: What is the

institutional relationship to absolute value and approximations made of? What methods, technics (technical skills) are taught? What justifications of these methods are expected from students?

Concerning the relation between the teacher's course and the students' work, at the *global level*, I try to identify first the devices used by the teacher to organise the work of students inside classroom and also outside and pedagogic means provided to students, second the variables leading the teacher to choice one device or another. The devices may be: working groups, the distribution of different tasks among different organisations provided by the institution (whole class, half class, 'modules'), organisation of taking notes and homework, frequency of tests, way of correcting, documentation provided, use of calculators, computers ...

At the *local level*, I use the *theory of situations* (Brousseau 1987) to try to reconstruct from observations some didactic situations, as defined by Brousseau. Then I make an a priori analysis of these situations and I analyse the different 'didactic contracts' (Brousseau 1996) used by the teacher. How can the student know if he is right or wrong when he solves a problem? Is there an 'adidactic milieu'? What is the didactic contract at hand? What are the regulations of the teacher? Are they in the domain of orthodoxy (he goes on the same situation) or in the domain of pathology (the teacher changes the situation, for instance changing the milieu)? I also make an analysis of the teacher's discourse to identify how the relationships between lessons and exercises as well as between previous knowledge and new one are managed, what place is left to students' own work, if there is some differentiation depending on students ...

Moreover, I think that not only the mathematical content, but also the 'didactic contract' concerning one notion in a class or in a grade, requires the use of some standardised procedures, symbols that will be essential guides for the activity of students and marks to interpret the teacher's expectations: What to do, what to say, what to write to succeed? For a more precise study of the relation between construction of sense and development of technical skills for students, I am particularly interested in the use of various symbols and representations, referring for their study to Duval (1995) and Chevallard (1996).

An analysis of mathematical contents and syllabuses allows us to determine the possible types of problems, methods of solving, types of writings. The local analysis of practices allows us to determine the effective choices of teachers and students.

What conducts the teacher's decisions?

I hypothesise that to conduct his choices and to adapt them to his students the teacher has two systems of reference, related to his 'institutional subjections':

- a system that I call '*internal*' which contains the teacher's own knowledge of concepts to teach, of their relations with other concepts, his conception of mathematics, but also his own experience of learning, his own working methods. For a large part, this system is implicit or even unconscious.
- a more explicit system that I call '*external*', concerning for instance his knowledge of scholar institution, syllabuses of ulterior grades, or students' projects.

I hypothesise that, in order to lead his class serenely, the teacher needs to be in agreement with his *internal references*. Moreover, wishing to be more efficient, the teacher tries to fit his teaching to his students, privileging at the same time what seems to him very essential and useful for the projects of most of his students. To do that, he uses his experience of ulterior grades or his reading of syllabuses. Of course, we can only observe the results of the interactions between *internal and external references*. We can observe the action of the teacher, his discourses, his choices, the devices he manages for his students. Most of the devices used by a teacher are tightly connected with his global choices, concerning mathematical concepts as well as the relation he does between his lessons and the students' own work. These devices are chosen or fitted by the teacher in order to balance as well as possible internal and external constraints. Yet these devices leave some latitude for the local choices of the teacher, conducted by 'habitus' (Bourdieu 1972).

The subject of the present paper

I present here the results of the analysis of interviews with five teachers, completed by class observations concerning four of them. The whole progressions of these teachers on absolute value are described in Perrin-Glorian (1997). I give here a

synthesis of the analyses, trying to derive some *regularities* and *differences* concerning *choices of teachers* and concerning variables which affect those choices. From interviews, we can only derive global choices, so I will also present two examples of local analysis of very short episodes extracted from observed sequences, in order to show the interaction between local and global analysis.

Results of the analysis of the interviews

Choice of teachers and gathering of data

Interviews with five teachers, A, B, C, D, E, took place in November 1994. All of them had been teachers for ten years at least, often twenty. Three of them (A, B, C) led in-service teacher education, and four of them (A, B, C, D) received trainee teachers in their classes. Therefore, the sample is not representative of ordinary teachers but I think that it is rather favourable for my purpose. We may think that teacher educators can have a better view on their practices and can explain and justify their choices more easily than other teachers, making explicit the variables they take into account. Interviews showed that it seemed difficult indeed for teacher E, who had not the same experience, to speak of her practice. Moreover, for other reasons, no observation took place in her class so that I have less information about her practices. Other classes were observed between November 1994 and February 1995. Some classes of teacher A were also observed during the two previous years. The school of teachers A, D, E was situated in rather far suburbs, with many low-class students. The classes of teachers A and E had bad results in the national assessment. The one of teacher D was low middle. The schools of teachers B and C were in suburbs bordering Paris, with children from a higher social environment and in competition with Parisian 'lycées'. Their classes were rather good.

Interviews were flexible: I asked teachers to describe the progression they planned for the year and the difficulties they were expecting from students. The interview was then directed according to teachers' answers and reactions. I asked them to precise their plans concerning functions and absolute value, topics which I wished to observe. I announced as an aim of my research to study how students integrate new notions and use them in their own work.

Method of analysis

I transcribed on paper the whole of the recording tapes and then I tried to reconstruct some clues through the teachers' talk. I cut those transcriptions according to the criteria drawn above, concerning the two objects of my study, but I tried also to identify some issues emphasised by each teacher, which seemed to come along as a leitmotif. Thus, I divided teachers' declarations into three large items:

- didactic transposition choices and variables directing them: organisation of different topics along the year, more precise choices about functions and absolute value.
- devices allowing the articulation between the course and the students' own work: 'modules'-management, relations made by the teacher between lessons and exercises, organisation of whole class or half class sessions, organisation of working groups or not, form of the lessons, organisation and checking of students' notebooks, homework, evaluation (organisation, marking, correcting)
- main cares of each teacher.

I listed a priori some variables the effect of which I tested by requesting, if they did not give by themselves, teachers' justifications about the devices they used. Those *variables* were:

- institutional constraints about contents: syllabuses, handbooks ...
- institutional constraints about students' work: schedule, general or specific to each school devices as organisation of half class sessions, 'modules', size of classes ...
- level of students' and teachers' beliefs about that level
- teachers' own relation with contents
- teachers' beliefs about learning, importance of their own work, of interactions with parents.

These variables were mainly those we met through results but it appears that not only most of them intervene in both domains but also that didactic transposition choices may interfere with organisation devices. While analysing the interviews, I tried to put in evidence the mutual influence of contents organisation and of students'

work organisation. Then I looked for some regularities and differences in teachers' choices and in the influence of the different variables. I present here some *institutional constraints*, the *regularities* and *differences* found from analysis.

Some institutional constraints

In tenth grade, the regular week schedule for mathematics is divided in two hours and a half in whole class, one hour in half class for 'directed works' and forty five minutes for 'modules' in a half class, often organised in one hour and a half a week every other time, that is to say four hours and a quarter for students and six hours for teachers.

Because of some school projects, there may be different distributions: for instance in the school of teachers A, D, E, 'directed works' and 'modules' were confused so that students got four hours of maths (two hours in whole class and two in half class) while the teacher got six hours anyway. The reduced time for students was supposed to be balanced by the fact that there was much time in reduced size groups. This organisation, as well as the week timetable has an influence on the choices of teachers. The succession of half class and whole class sessions has also an effect as well as the fact that half sessions alternate or not with another subject (like French or History).

Some regularities

Quite early in the year, every teacher plans to meet the notions that he or she regards as very important, and to keep for the end of the year those which seem to him easier or less useful. Teacher B specified that perhaps the notions she deals with at the end of the year seem to her easier because she has no time enough to see students' difficulties about them. But the topics regarded as important change according to teachers and classes. Functions are always regarded as a very important topic and studied at the very beginning or in the middle of the year, according to the importance that the teacher gives to problems eventually encountered by students to adapt themselves to a new educational establishment. Vectors and equations are also studied quite early in the year both because they are tools needed all year along in many other topics and because they allow a connection with middle school. Trigonometry is always taught at the end of the year, it's no wonder since it needs

functions and vector angles. Statistics too, are usually approached during the last term: they are regarded as easy and with no repercussion on other topics, but their exact place depends on the teacher and on the class. For example, teacher D said that, this once, she would not leave statistics for the very end of year because most of students should orientate towards the economics section so it will be important for them.

A variable taken into account by all the teachers is the level of the class and the orientation they anticipate for most of the students. It has an affect as well on contents organisation, time devoted to each topic, choices for exercises, as on the use of some devices as working groups. Teachers take in grand account all that will be useful for following grades until ‘baccalauréat’ but not later. The way in which contents are developed depends on the class but it is mainly teachers with classes that have bad results in the national assessment who talk about that. Some topics are less deepened in such classes and even perhaps not met at all: for instance, teacher E thinks that she will do nearly nothing about absolute value, perhaps nothing at all. Sometimes, a notion regarded as no essential according to the anticipated orientation of students is yet studied, but for other reasons. As an example, teacher A treats of space geometry with ‘bad’ classes because it is a topic where students do not feel unsuccessful.

All teachers observe syllabuses and propose ‘introducing activities’ or ‘practical works’²³ for new topics before their lesson and training exercises after it. Nevertheless, the change from one type of activity to another (practical works, lesson, exercises, problems) is more or less indicated by the teacher and more or less easy to identify by students.

Some differences

The five teachers do not agree with what to study at the beginning of the school year. Some of them wish to begin with something quite important and new (as functions) to emphasise the breaking from middle school while others prefer to let an adaptation time to students. We saw that differences concerning order of topics or degree of

²³ That is to say a problem where the notion is useful, but usually, it is not an adidactic situation but a problem that allows the teacher to show the new concept or new theorem.

deepening of contents are often related to the supposed orientation of students. Nevertheless, some differences appear concerning space geometry which teachers A and C seem to regard as more important than the other teachers. Concerning what is actually taught, the differences are not easy to derive from interviews. Differences appear from the precise observation of what is really treated of a subject and what is asked to students through evaluation. Then, we can see the close relationship between what is asked in tests and the way that the notions are institutionalised (Perrin-Glorian 1997).

Considering what teachers regard as very important and what they actually ask to students in tests, we can identify a variable seemingly of large effect on contents really taught: the previous professional experience of teachers. Thus, a teacher who had a very long experience in middle school before teaching in 'lycée', like teacher A, will be concerned about relation with middle school and will take care of repeating some notions taught in middle school but which he knows to be hard to master in middle school. A teacher who has a large experience of teaching in the last class (12th grade) of scientific section, like teacher B, will be more attentive to difficulties of students of that grade and will take care of emphasising contents which are not yet acquired two years later. For instance, to study the variations of a function in 10th grade, teacher B prefers to use inequalities and composite functions even if it does not fit in any case (see $x^2 - x$ on $[1; +\infty[$ for instance), instead of the study of the sign of $f(a) - f(b)$ privileged on the contrary by teacher C who emphasises algebraic calculations and inequations solving. Moreover, there is a great *variety of working forms* used by the five teachers:

- Only one (teacher A) uses working groups in class. This point seems in relation with private preferences linked to internal references.
- From the writing of the whole lesson on the blackboard (teacher D) to notes taken by students with very few indications and completed with short duplicated notes provided by the teacher (teacher A), there is a great diversity in written traces left to students.
- It is the same concerning distinction between lesson and exercises: marked in different ways (like 2 or 3 different notebooks) with some teachers, it is nearly invisible with others.

- ‘modules’ integrated to the progress of the course (teacher C) or used for a differentiation fitting to students (teacher B). This different choice may be related to different heterogeneity of classes.
- differentiation between whole class sessions and half class sessions is also variable: strongly distinguished for some teachers (A, C), the two types of sessions are quite similar for others (D). For example, teacher C prepares in advance during ‘modules’ sessions some important subjects before treating them in whole class sessions, in the same way, space geometry is prepared by the use of a software during ‘modules’ sessions, thus she often deals with two subjects at the same time. On the contrary, teacher D makes no difference at all between whole class and half class sessions.

There are also differences between what seems primordial to each teacher, what he or she spontaneously mention several times:

- to keep master of her class and to fit her teaching to the supposed orientation of her students, for teacher A
- not to discourage students and to get acquired the bases sometimes lacking two years later for students in last year of scientific section, for teacher B
- to balance previous and new knowledge, numerics and geometry, reinforce middle school knowledge, for teacher C
- to treat of the whole syllabus, to fit the work schedule to the anticipated orientation of most of students, for teacher D
- to help students encountering difficulties for teacher E, but she does not speak of difficulties of contents in themselves.

In conclusion, the expressed differences seem related to differences between *classes* (level of students and future orientation of most of them), but also to differences in the *professional experience* of the teachers (kind of classes in which he is used to teach, in which he teaches at the same time, in which he taught during previous years ...). Moreover, there are of course different individual characteristics, and differences in the relation to mathematics, the last being often related to

professional experience. So, we can see the effect of different obligations ('assujettissements' in Chevallard's theory) of teachers.

Results of the analysis of two short episodes

We showed previously (Perrin-Glorian 1996) that teacher A, in two different classes during the same year, one of low level and one of high level, made very few differences in global choices. But we could identify light but important differences in the local analysis. First, a precise observation allowed us to see that, in fact, students did not do the same work during 'directed works' sessions because the teacher did not bring further information in the same moment. In 'bad' classes, most of the time is devoted to the problem 'devolution', and to work on prerequisites needed for entering into situation: some knowledge needed for the 'devolution' and which should have been in the 'milieu', is not in fact, so that the students' work turns on it. In 'good' classes, the 'devolution' phase is shorter and the teacher presents sooner the lacking knowledge so that students actually work on the new knowledge at stakes in teaching. As the lesson of this teacher is held as a dialogue with students from the 'directed works', that lesson relies on an actual reflection of students in 'good' classes but not in others. Moreover, a detailed comparative analysis of a lesson given in the two classes and made of comments on the same duplicated sheet, shows deep differences even if they are not visible at first sight: some explanations appear only in the 'good' class; in the 'bad' class, explanations are more contextualised, nearer to the precise tasks the students will have to do, formulations are less mathematical, the discourse is focused on how to do, often with gestures.

We will now compare the *correction of an exercise in two 'good' classes with two different teachers*, in two different schools (teacher A, February 1993, teacher B, November 1993). The subject is the translation between different definitions of intervals, namely with absolute value or set notations.

First, look at the exercises chosen for the session as a whole. We can see that teacher B treats of one kind of conversion at a time, taking difficulties one by one while teacher A gives mixed exercises, with all kinds of conversions and various difficulties: students have to fill the 4 columns of a table corresponding to the 4 ways to describe an interval, one of them being given. This kind of task is used for

evaluation by both teachers A and B. Teacher A uses the same during teaching, but teacher B does not.

Second, we compare the way they treat of the same difficulty: substitution of ‘−’ by ‘+’ in two cases: the student makes an error or not.

First case: The students had to translate in absolute value terms $x \in [-7; -2]$ in class A, $x \in [-6; -2]$ in class B. Students succeeded both. Teacher B only repeats what the student did, emphasising the essential point ‘ $x + 4$ because $-(-4) = +4$ ’, and she questions the students who did not understand before. Teacher A gives a lot of explanations and recalls the lesson to the students.

Second case: Solve $|x + 5| = 3$ in class B, $|x + 3/2| < 2$ in class A. Both students failed.

Once again, teacher B intervenes in a moderate and very short way: she thanks the student for his error ‘because it was something I wanted to emphasise’. She gives an explanation by herself, then questions the students who did not understand another exercise with the same difficulty. Teacher A first gives an help ‘to decode this, you need a minus sign’ but, just after, she asks ‘what is the notion lying behind this coding’ in order to help the student to find the reason of that minus sign, referring to distances. The student cannot answer so the teacher reminds a large part of the lesson and some students became agitated.

It is possible to compare those ‘local choices’ to ‘global choices’ of these teachers, namely the way to give the lesson and to relate it to exercises. Remember that teacher A gives a lesson not very distinguished from exercises and emphasises what is to retain from correcting exercises: she needs to identify parts of the lesson through exercises solving, asking for reasons of students. Teacher B dictates her lesson, which is thus clearly identified by students, so during exercises correcting, she deals only with specific difficulties of students. To work again on some difficult point, or to answer some question, she gives an explanation only on the specific point concerned without coming back to definitions but she also proposes another exercise focused on that specific point. So does she in order to answer a question of one student on the difference to solve $|x - a| < r$ and $|x - a| > r$. Moreover, we saw that teacher B takes difficulties one by one. She lets a very restricted domain to students

but, on this restricted domain, she watches over the engagement of each student. Teacher A gives more complex exercises.

Those local differences may perhaps be related to different relationships of these teachers to discipline: Teacher A is afraid of possible dissipation of her students, she cares much of discipline because she needs to keep it during long periods when students have to listen her explanations. In class B, moments when students have to listen without writing are very short, which may explain why teacher B seems not to care about discipline.

As a conclusion, I think that the frame I defined for this analysis allows the identification of discriminating variables for teachers' practices. This frame must be tested with other experienced teachers. It remains also to evaluate the effects of observed variations on students' learning before being able to make proposals towards teachers training.

References

- Bourdieu, P. (1972). *Esquisse d'une théorie de la pratique*. Genève: Droz.
- Brousseau, G. (1987). Fondements et méthodes de la didactique des mathématiques. *Recherches en didactique des Mathématiques*, Vol. 7/2. Grenoble: La Pensée Sauvage, pp. 33–115.
- Brousseau, G. (1996). L'enseignant dans la théorie des situations didactiques. *Actes de la 8ème Ecole d'été de Didactique des Mathématiques*. Clermont Ferrand: IREM, pp. 3–46.
- Brousseau, G. (1997). *The theory of didactic situations*. Dordrecht: Kluwer.
- Chevallard, Y. (1991). *La transposition didactique. Du savoir savant au savoir enseigné*. 2ème édition. Grenoble: La Pensée Sauvage.
- Chevallard, Y. (1992). Concepts fondamentaux de la didactique : perspectives apportées par une approche anthropologique. *Recherches en didactique des Mathématiques*, Vol. 12/1. Grenoble: La Pensée Sauvage, pp. 73–111.
- Chevallard, Y. (1992). Fundamental concepts in didactics perspectives provided by an anthropologic approach. In *Research in Didactique of Mathematics. Selected papers*. Grenoble: La Pensée Sauvage. (This text is an English version of the precedent one.)
- Chevallard, Y. (1996). Les outils sémiotiques du travail mathématique. *Petit x*, n° 42. Grenoble: IREM, pp. 33–57.
- Duval, R. (1995). *Semiosis et pensée humaine*. Bern: Lang.

- Perrin-Glorian, M.J. (1990). Reflexions sur le rôle du maître dans les situations didactiques à partir du cas de l'enseignement à des élèves en difficulté. In G. Booker, P. Cobb & T.N. Mendicutti (Eds.) *Proceedings of the 14th Conference of the International Group for the Psychology of Mathematics Education (PME)*. Oaxtepec, Mexico: University of Mexico, Vol. 2, pp. 209–216. (English or Spanish version available by the author.)
- Perrin-Glorian, M.J. (1993). Questions didactiques soulevées à partir de l'enseignement des mathématiques dans les classes 'faibles'. *Recherches en didactique des Mathématiques*, Vol. 13/1.2. Grenoble: La Pensée Sauvage, pp. 5–118.
- Perrin-Glorian, M.J. (1995). The absolute value in secondary school. A case study of 'institutionalisation' process. In L. Meira & D. Carraher (Eds.) *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education (PME)*, Recife, Brazil, Vol. 2, pp. 74–81.
- Perrin-Glorian, M.J. (1996). Gestion didactique des moments didactiques. Enseignement de la valeur absolue en classes de seconde. *Actes de la 8^{ème} Ecole d'été de Didactique des Mathématiques*. Clermont Ferrand: IREM, pp. 69–82.
- Perrin-Glorian, M.J. (1997). Institutionnalisation en classe de seconde : valeur absolue, intervalles, encadrements, approximations. Choix globaux des enseignants et résultats des élèves. *Cahier DIDIREM n° 29*. Paris: IREM, Université Paris 7.

4.3

A DIGITAL REPRESENTATION OF 'FULL PRACTICE' IN TEACHER EDUCATION: THE MILE PROJECT

Fred Goffree and Wil Oonk

Freudenthal Institute (The Netherlands)

Context and theoretical framework

Standards for primary mathematics teacher education in the Netherlands

Since the Wiskobas project (Treffers 1987 etc.), Dutch mathematics educators have been working on a suitable programme for teacher education (Goffree 1977). In addition to the learning processes of children and teaching mathematics, the learning processes of students learning to teach mathematics should also be taken into consideration (Goffree 1979). The influential publications dealing with 'standards for Mathematics Evaluation and Teaching' (NCTM 1989, 1992) inspired the Dutch Association of Primary Mathematics Educators (NVORWO) to submit a request to the National Institute of Curriculum Development to draft a similar publication specifically for teacher education. In 1990, a group comprising approximately 10 mathematics educators started developing national standards. In 1995, the group presented the results to colleague mathematics educators (Goffree & Dolk 1995). The results can be regarded as a handbook for teacher educators in this subject area. It consists of three segments: 1) Standards for primary mathematics teacher education. 2) On the shop floor; the educator and his student teachers in practice. 3) Qualifications of beginner teachers of primary mathematics.

In this handbook the Dutch mathematics educators can read among others that primary mathematics teacher education is based on three pillars: *constructive*, *reflective*, and *narrative*. In other words, that students are taught to create their own (practical) knowledge primarily through *reflection of practical situations*, and that their acquired knowledge usually has a narrative character.

And also that knowledge construction in the area of mathematics education can especially be stimulated in a rich learning environment, in which student teachers are encouraged to work together. Recent research data in the field of collaborative learning environments, in which the students and tutor/experts are the actors, offer a promising perspective (Bereiter & Scardamalia 1997).

Principles and paradigms of realistic mathematics education

Realistic mathematics education in the Netherlands is characterised by five pairs of *fundamental learning and teaching principles* (Streefland 1991). These express that the development and implementation of realistic mathematics education is based on insight into children's learning processes. The following pairs are relevant to this paper: 'levels versus models' (levels can be recognised in the learning processes of children, and increases in level can be effected by working with 'conceptual models'), and 'social context versus interaction' (learning is not merely a solo activity, but something that occurs in a society and is directed and stimulated by its own socio-cultural context). The consequence of this is that mathematics education should by nature be interactive, in order that pupils can exchange ideas, persuade each other using logic and reason, discuss different meanings, etc. (Treffers 1991). The literature in this area contains many practical examples of realistic mathematics education (Gravemeijer 1994, van den Heuvel 1996, Streefland 1991, Van den Brink 1989). Some of these examples clearly verify a particular theoretical insight and make further observations of this point unnecessary. Such cases are referred to by Freudenthal as 'paradigms' (Freudenthal 1980), they particularly are relevant for use in teacher education.

What a primary mathematics teacher should do

The development of a curriculum for teacher education has to be based on the requirements inherent to the teaching profession. In order to draw up a list of relevant

qualifications one need to characterise the teacher's task (Nieuwenhuis 1993). In the Netherlands, the initial requirements for beginning teachers were drawn up with this model (SLO/VSLPC 1997). The aforementioned principles (level versus models and social versus interaction) are expressed in these requirements by the following:

- “(...) The teacher's task also has a more traditional element. He introduces and works with contexts and problem situations. He teaches the pupils to use conceptual models, gives them the opportunity to work at and increase their own level as well as providing them with suitable material to practice skills and memorise facts (...).
- (...) In the mental arithmetic lessons, the teacher stimulates the use of strategies based on order and structure. The personal inventions and the different approaches of the pupils can be effectively demonstrated in this respect, if the lessons have an interactive character. *Interaction* is considered extremely important in modern mathematics curricula and it is closely related to the principles of *construction* and *reflection*. Interactive teaching that deals with different approaches and levels can only be successful if there is a good pedagogical climate and working atmosphere (...).” (SLO/VSLPC 1997, p. 45)

Practical knowledge and knowing practice

One of the main questions that teacher educators have to answer now is what teachers have to know to be able to carry out the aforementioned tasks. The micro-level development of a curriculum, i.e. compiling material and lessons for student teachers, requires concrete data on the necessary knowledge (and insights, skills, beliefs, and attitude).

Since the publication by Elbaz (1983), educators have increasingly thought in terms of ‘practical knowledge’ (Verloop 1992). This includes the information a teacher knows and can apply in practical teaching and learning as well as the knowledge that gives direction to his actions in real life situations. For teachers, this knowledge often remains implicit and are usually even ‘tacitly’ assumed. This means that personal elements can be woven in, for example, the teacher's own experiences as a pupil during education, his (sub)conscious vision on learning and teaching and his image of the ideal teacher. Practical knowledge also includes insight into the daily

situation – yesterday’s experiences with the group or observing a breakthrough in the understanding of a “weak” pupil. Practical knowledge is context-dependent and consists of situated cognition. The literature contains global interpretations of this, constructed on general educational themes. However, virtually no research has been carried out on the subject specific elements. Shulman (1986) does adopt a subject specific line of approach and differentiates ‘content knowledge’ (the teacher must know his subject matter), ‘pedagogical content knowledge’ (the teacher must know the best way of presenting the subject matter to specific pupils) and ‘curricular knowledge’ (the teacher must know the objectives, the available resources, and the programme he must align his teaching to). There is, however, more: Shulman also refers to ‘case knowledge’ (prototypes, precedents, and parables) and ‘strategic knowledge’, which both give more direction to the teacher-pupil interaction. Strategic knowledge is knowledge that teachers will build up when other knowledge sources prove inadequate. Lampert touches on this subject when she characterises teachers as a dilemma-managers (1985).

Subject oriented practical knowledge makes it possible to understand, to anticipate, and to prepare realistic mathematics in the classroom as well as giving the teacher the manoeuvrability to function as a discussion partner for pupils, a coach and an expert. Practical knowledge also helps making links to the ‘big ideas’ in the theory and makes theory-on-action possible (Schön 1983). Practical knowledge in the area of mathematics education means, for example, that teachers know that money is a good context for mental arithmetic and that it is best to let pupils construct number lines (as models) themselves. Practical knowledge also contains good ideas for stimulating and maintaining the teacher-pupil interaction and the experience that interaction stimulates reflection.

The narrative character

Sigrun Gudmundsdottir (1995) is one of many researchers who take the point of view that teachers mainly communicate, arrange, and memorise their practical knowledge in the form of ‘narratives’ (stories). Although many teachers’ stories are in circulation, the number with a subject-specific element is also in this case exceedingly limited (e.g. Jalongo & Isenberg 1997).

Acquiring practical knowledge

In general, it is assumed that practical knowledge and '*knowing about practice*' can be acquired in real life situations. However, teacher educators and their student teachers often know better. Fieldwork (practical training) is often more a case of trial-and-error and survival than learning about how to teach. Learning *about* teaching, made possible by a digital representation of real life situations in classrooms, provides greater opportunities to learn the practicalities of the profession constructively and reflectively (Cohen 1998, Lampert & Ball 1998).

To be able to learn something *from* practical situations, student teachers must adopt an *investigative attitude*. This requires a suitable environment in which:

- specific events elicit specific associations (recognition, admiration, and knowledge);
- students can acquire information about preceding events;
- students can find out the motivation behind the teacher's actions;
- students can look up information in the teachers' guide;
- students can consult an expert within a short space of time;
- students can study the written work that pupils do during the lesson;
- the lesson can be halted at 'interesting moments';
- interesting moments can be reviewed;
- students can make reflective notes during the lesson;
- students can exchange ideas about particular observations;
- students can refer to relevant literature (theory).

A suitable environment is the MILE (Multimedial Interactive Learning Environment). This project and its use in teacher education is explained below.

The MILE project – student teachers investigate experienced teachers’ practice

MILE is being developed using a (rapid) prototyping method. The first prototype to become available, Petit-MILE, consists mainly of narratives – real time registration of mathematics teaching in grade 3 (during 5 weeks) and grade 5 (one week) – arranged in video fragments with an average length of 1 to 2 minutes, including transcripts. Each fragment is considered a separate narrative and has its own title summarising the essence of the ‘story’. The fragments can be accessed with the search engine (PUMA, full text retrieval).

Student teachers as pioneers in MILE

Students use MILE for investigating mathematics teaching and learning in classrooms. The hypothesis is that they will construct their own (subject oriented) practical knowledge. In the perspective of systematic research on knowledge construction in a Computer Supported Collaborative Learning environment, two advanced students has been followed during their ILP (Investigation for Learning Project). The ILP in MILE carried out by the two student teachers from Amsterdam is described below.

Pre-knowledge and perspective

From 16 May to 27 June 1997, Dieneke and Hayet, students taking the accelerated two-year education course for primary school teachers, carried out an ILP with MILE. Their ‘assignment’ was aimed at enriching, expanding, and deepening their knowledge of mathematics education in elementary school. Their pre-knowledge had largely been acquired in lectures and workshops during their ‘Basic Skills’ course, the main study material in this course is ‘*Het Fundament*’ (The Foundation) (Goffree 1994). They acquired their ‘knowing about practice’ as students and interns working in Amsterdam primary schools.

Investigation for learning

The ILP was similar in nature to that of Josie Cekola of the teacher education course of the University of Michigan (Goffree 1997). Because of the limited amount of

supervision, the education received an open character: students could formulate, edit, and revise their own hypotheses. The idea behind this was that the research of personally formulated didactic hypotheses and the reflective activities that this especially involved would significantly benefit the students' acquisition of practical knowledge.

Observations in MILE

The tableau below (Figure 1), taken from the second meeting of the students' ILP, illustrates how they worked with MILE. The continuing discourse of the meeting resulted in reflection and study of contexts and models.

In addition to Dienneke and Hayet, Fabienne was also present at some meetings; she graduated a short while ago, now has her own class and attended the meetings as an interested participant.



Fig. 1: Blackboard sketch of Dwaen's answer.

The students look at a video fragment of a lesson for grade 2. In the video, they see how the teacher (Minke) discusses the answer given by Dwaen, one of the pupils, with the whole group.

Before this, Dwaen and the other children had been asked to make as many 100 Dutch Guilder combinations as possible with toy money (5, 10, and 25 guilder notes). The context was a story about a child that had lost her purse.

The video shows the teacher drawing the bank notes that Dwaen used on the board (Figure 1). The class then ‘checks’ the answer by reading aloud the row of numbers cumulatively (in this case: 25, 50, 60, 70, 80, 90, 100).

Starting criticising the teacher

Dieneke asks if the video can be stopped. Fabienne reacts immediately: “It was stupid of the teacher to do this. She starts counting from right to left without giving any explanation. She did it the other way round just now.” Fabienne supports this comment by pointing out the importance of consistently reading and writing from left to right. With number lines, children should also work from left to right, from smaller to larger numbers.

Starting a discussion

The tutor wonders aloud what might have motivated Minke, the teacher, to act in this way. Dieneke feels challenged by this and defends Minke: “Perhaps she knows from experience that the children find it difficult to jump from 50 to 75 and 100 and easier to fill in the line 0, 25, 50.” The tutor eagerly expands on Dieneke’s remark and points out that the idea of the ‘story’ – the money context and its pictographic representation on the board – places the actions of Minke a different light. Until now, the emphasis of the discussion has been mainly about ordering numbers using the number line and left-right orientation as models. The context, however, invites to do mental calculations, using a systematic approach and finding creative solutions.

The tutor acting as a coach

The tutor asks the students to think constructively and especially about what they have seen and to illustrate and support the comments and 'statements' in their reports with arguments.

Theoretical orientation

In her report of this meeting, Dienneke goes into the discussion in detail. She still wonders whether Minke's approach is in line with the opinion that children should work as much as possible from left to right and whether she should adhere to the principle of sequence on the number line. After all, this group and this method regularly resort to this type of line. Dienneke decides to search the literature for information about 'money arithmetic' (Goffree 1997). The information she finds makes her think and reflect on the use of money as a tool for learning the place value of numbers. She feels that the number line does not support the cumulative counting of monetary units. She concludes that she likes Minke's approach, but has reservations about whether it may have been better for Minke to explain to the children explicitly why she deliberately started on the right in this case instead of the left. You will probably be able to deduce from the children's remarks that the representation of the reality is more important than the use of a model (the number line). Because the context is strong in this case it does not require a model. At this point in her report, the tutor makes a note (annotation via e-mail) about using money as a conceptual model, in order to support thinking.

Reflections

At the end of her reflection, Dienneke indicates (explicitly) that she has learnt a great deal from the discussions – especially to supplement the material covered in the lessons at the PABO (*Teacher Education College for Primary Education*). She refers to key questions that never received in-depth attention – not even during her school practice periods, such as: In which situations does context suffice? When should you use a model? Do the children become confused? What is the most natural approach?

In her report of this meeting, Hayet reflects mainly in a metacognitive sense. She refers to the contents of the discussion caused by that 'minor but interesting

incident' as the most informative part of the whole meeting. A great deal suddenly occurred that interested her, and made her think about such things as the role of the reading and writing direction, the number line, reversals, the importance of sequence, could all the children follow it, etc. For her, the tutor's intervention was a revelation. It made her aware of the one-sidedness of thinking on the basis of models. After all, you can also use the context as a point of departure and, for example, choose to stick (toy) money criss-cross on a flannelboard. The theoretical reflection taught her that you must be able to deal with the subject matter with (more) flexibility.

Hayet calls the discussion a springboard for forming an opinion and for making people aware of personal habits or blind spots. She will make additional remarks substantiating this statement later in the ILP.

Searching further: deepening, enrichment, and theorising

After this meeting, Dienneke and Hayet work alone with MILE searching for other interesting fragments about the number line. Amongst others, they find a situation in which the teacher places numbers on a classical line together with the children. By deliberately reversing a number card, she manages to increase all the children's participation and interaction. In another video fragment, she makes the children imagine a section of the number line with their eyes closed. Dienneke and Hayet watched a video about the introduction of the 5-times table several times. The video shows a teacher, Willie, giving an explanation. On the edge of the board, there are ten bags with real tangerines. A stripe has been drawn on the board that also contains ten bags and at the bottom, there is a number line.

The fragment makes Dienneke and Hayet think about how the children will interpret this. They wonder if the transition from context to model is too large. They discuss their own practical experience and formulate a statement about the switchover from context to the use of models. The tutor makes extensive notes on the use of representations and models in teaching multiplication in their reports.

Hayet uses the discussions about the number line as the foundation of theoretical research. In the chapter of her report entitled 'Theory discussed in a row', she reflects on models, phasing, and levels.

Tentative conclusions

The above can be considered representative of activities that take place during Dienneke and Hayet's ILP. It shows a number of the concepts that characterise MILE and express the objectives with which it was set up:

- The ILP shows *how narratives* taken from 'full practice' *are subjects for* observation, analysis, and discussion (Gudmundsdottir 1995).
- The *discourse* motivates and encourages the students, especially stimulating them to undertake further research. Students working together leads to interaction and the need for reflection (Bereiter & Scardamalia 1992).
- The tutor functions as *coach and expert*. In the case of Dienneke and Hayet, he sometimes thinks aloud with them or forces them to view the situation from a different perspective (Lampert & Ball 1998). He provides them with theoretical considerations, which makes them reflect on the fundamentals of realistic mathematics education.
- The ILP shows signs of *knowledge construction*. This applies especially to the fragment about the 5-times table. The students are seen to construct new knowledge: the discussion induces them to contribute their own knowledge from practical experience and to adapt newly acquired theoretical knowledge. The *practical knowledge*, acquired in this case mainly through reflection (Verloop 1991), can be considered as a form of *narrative knowing* (Gudmundsdottir 1995). The way in which students construct knowledge in MILE throws a new light on how the professional knowledge of teachers is ordered (Shulman 1986), how ideas are formulated about subject oriented practical knowledge (Goffree 1997) and the initial requirements of beginning teachers (Goffree & Dolk 1995, SLO 1997).
- It seems that the students naturally acquire *metacognitive knowledge* about how to assess acquired knowledge and about their personal *learning styles* (Vermunt 1992).

Further research will investigate the significance of working together in MILE to the way in which students construct knowledge.

References

- Ball, D.L. (1993). With an Eye on the Mathematical Horizon. Dilemma's of teaching Elementary School Mathematics. *The Elementary School Journal*, Vol. 93, No. 4.
- Bereiter, C. & Scardamalia, M. (1992). An architecture for collaborative Knowledge Building. In E. De Corte, M.C. Linn, H. Mandl & L. Verschaffel (Eds.) *Computer based Learning Environments and Problem Solving*. Berlin: Springer.
- Bereiter, C. & Scardamalia, M. (1997). *Knowledge Forum*. Toronto.
- Cohen, D. (1998). Experience and Education: Learning to Teach. Afterword. In M. Lampert & D.L. Ball (Eds.) *Teaching, Multimedia, and Mathematics. Investigations of real practice*. New York: Teachers College Press.
- Elbaz, F. (1983). *Teacher Thinking. A study of practical knowledge*. London: Croom Helm.
- Elbaz, F. (1991). Research on teacher's knowledge: the evolution of a discourse. *Journal of Curriculum Studies*, Vol. 23, No 1, pp. 1–19.
- Goffree, F. & Dolk, M. (Eds.) (1995). Standards for primary mathematics teacher education. Enschede, Utrecht: SLO/NVORWO.
- Gravemeijer, K.P.E. (1994). *Developing realistic mathematics education*. (Ph. D. thesis) Utrecht: CDß.
- Gudmundsdottir, S. (1995). The narrative nature of pedagogical content knowledge. In H. McEwan & K. Egan (Eds.) *Narrative in teaching, learning, and research*. New York: Teachers College, Columbia University, pp. 24–38.
- Jalongo, M.R. & Isenberg, J.P. (1995). *Teacher's Stories. From Personal Narrative to Professional Insight*. San Francisco: Jossey-Bass Publishers.
- Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice. In *Harvard Educational Review*, Vol. 55, No. 2.
- Lampert, M. & Ball, D. (1998). *Investigating Teaching. New Pedagogies and New Technologies for Teacher education*. New York: Teachers College.
- Oonk, W. (1998). *Pioneers in MILE* (MILE-brochure No. 9), Utrecht, The Netherlands: Freudenthal Institute [in press].
- Schön, D.A. (1983). *The Reflective Practitioner*. New York: Basic Books.
- Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), pp. 4–14.
- Streefland, L. (1991). *Fractions in Realistic Mathematics Education. A Paradigm of Developmental Research*. Mathematics Education Library. Dordrecht: Kluwer.

- Streefland, L. (Ed). (1991). *Realistic Mathematics Education in Primary School*. Lecture on the occasion of the opening of the Freudenthal Institute. Utrecht, The Netherlands: Freudenthal Institute, CDß.
- Treffers, A. (1987). *Three Dimensions*. Mathematics Education Library. Dordrecht: Kluwer.
- Treffers, A. (1991). Didactical background of a mathematics program for primary education. In L. Streefland (Ed.) *Realistic Mathematics Education in Primary School. On the occasion of the opening of the Freudenthal Institute*. Utrecht, The Netherlands: Freudenthal Institute CDß, pp. 21–65.
- van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. (Ph. D. thesis) Utrecht, The Netherlands: Freudenthal Institute CDß.
- Vermunt, J. (1992). *Leerstijlen en sturen van leerprocessen in het hoger onderwijs (Learning styles and directing learning processes in Higher Education)*, hoofdstuk 2. Lisse: Swets & Zeitlinger b.v.

CHAPTER 5

TEACHER EDUCATION THROUGH TEACHERS' INVESTIGATION INTO THEIR OWN PRACTICE

Barbara Jaworski

University of Oxford (United Kingdom)

Background

One session of the Thematic Group “From a Study of Teaching Practices to Issues in Teacher Education” (TG 3) at the First Conference of the Society for Research in Mathematics Education (CERME 1) was devoted to considerations of the developments which occur in teaching resulting from mathematics teachers engaging in research or inquiry into aspects of their own teaching practice. A number of members of the group have experience and expertise in this area of study, but no papers had been submitted formally to the group. Thus, rather than a discussion of papers, this session took the form of an introduction followed by questions to experienced members of the group. There was then a whole group discussion of issues and a raising of further questions for continuing study. This chapter provides an account of the session, including responses from a number of participants, and indicates directions for further work in the area of mathematics teaching development related to teachers’ action research.

Introduction to the session

“Imagine a classroom in which there is a mathematics lesson: a teacher and students are working together on some aspects of mathematics – for example, on decimals. Try to think of a real classroom with people you know, in a specific context with which you are familiar. Now, please keep this in mind as we continue the session.”

These instructions were given to the group by Barbara Jaworski at the beginning of the session. It was intended that discussion should be firmly rooted in our varying experiences of classrooms, mathematics, teaching and learning.

Initially focus was directed toward the teacher in the situation described and visualised. The complexity of the teaching process was emphasised. The teacher has many objectives for the lesson, including those related to the student group, particular students, the curriculum, personal philosophies of learning and teaching, and so on. Some of these objectives are overt, others are tacit in the operation of the teacher. In “acting and doing” in the classroom the teacher draws on his or her personal knowledge and beliefs, which may themselves be overt or tacit. Central to the teaching activity are first of all the students and their learning, and secondly the subject matter of the lesson, the mathematics. Partly, the complexity for teaching lies in organising the learning experiences of students so that they can gain understanding of the mathematics which is the focus of the lesson.

However, this classroom does not exist in a vacuum; it is part of a complex social process with influences from society, culture, politics, economics, curriculum, and assessment, the particular school or institution, and so on. These influences will vary for the different participants (teacher and students) in the teaching-learning interface in the classroom lesson.

Research has shown that teachers operate at varying levels of explicitness in their objectives for teaching, drawing on personal knowledge and beliefs. For some teachers, a high level of expertise and quality of teaching can be achieved with only tacit recognition of the knowledge underpinning this expertise (see e.g. Calderhead 1987; Polanyi 1957; Schön 1987; Othman 1995). When the relationship between knowledge and expertise is largely tacit, it is hard to trace the elements of

development of teaching, or indeed to recognise how teaching develops. For many teachers, this development is implicit in their teaching practice.

There would seem to be value in encouraging activity on the part of the teacher directed towards making more explicit the elements of teaching development; perhaps in order to be more aware of, and therefore in more control of the directions of this development. Findings from research conducted in classrooms with teachers shows that the teachers themselves become more questioning of their theories and beliefs in motivating their practice (see e.g. Jaworski 1994). This provides opportunity for recognition of issues and concomitant changes in practice. If the teachers themselves are the ones conducting the research, the process seems likely to be more directly focused on their particular interests and needs, and therefore more fruitful for their developing practice. Recent research has shown this to be the case (see e.g. Krainer & Posch 1996; Jaworski 1998).

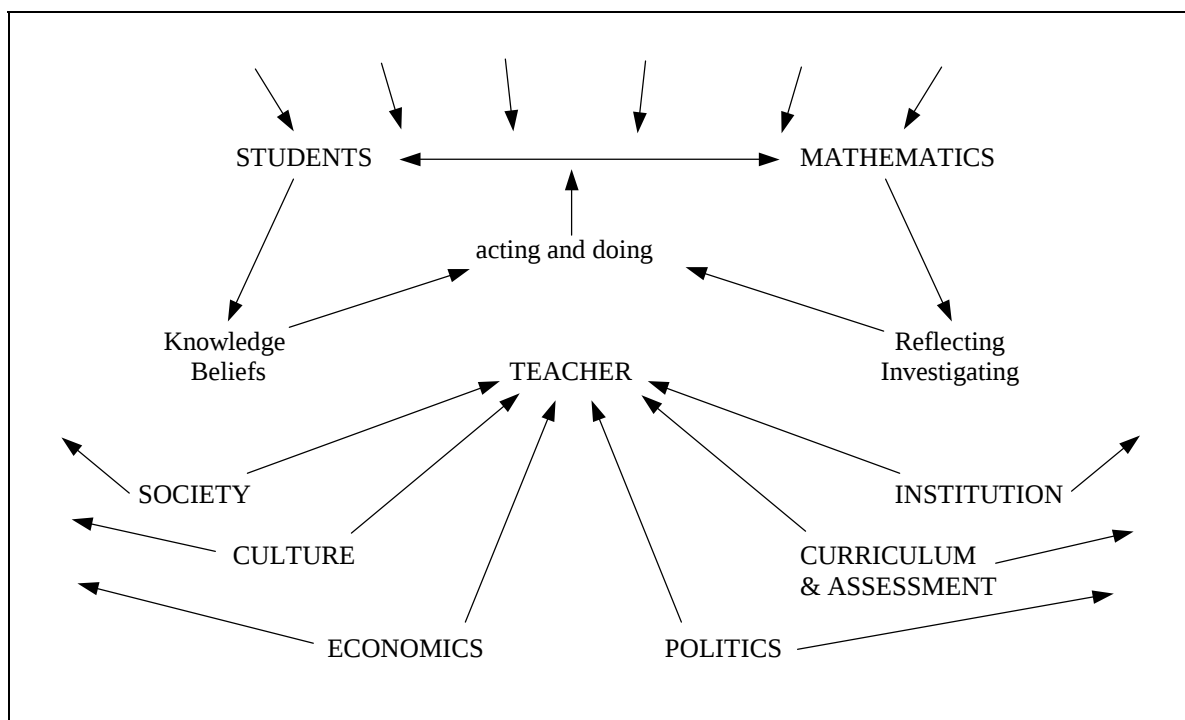


Fig. 1

Figure 1 is designed to capture some of the above factors in sketching the position of the teacher operating in a classroom situation in which the main focus is students learning of mathematics.

The teacher research element is shown in the terms *reflecting* and *investigating*, which are indicative of the teacher's explicit interaction with *knowledge* and *beliefs* in constructing teaching-learning processes. Such teacher research is often known as "action research". Briefly, this involves teachers taking *action* in the classroom related to their reflections, and evaluating this action for further development and learning (for an exposition of action research, see e.g. Elliott 1991). Action research has become a much-used paradigm in different parts of the world in mathematics education as well as in other curriculum areas (e.g. Crawford & Adler 1996).

The following sections will expand on this brief introduction to raise and elaborate issues in the processes of teachers' action research and its influences for the development of mathematics teaching. The next section will consist of the questions that were addressed to some members of the group and their responses to these questions.

Questions to and responses from members of the group experienced in action research

These members were Konrad Krainer (Austria), Elisabeth Thoma (Austria), Razia Fakir-Mohammed (Pakistan), and Barbara Jaworski (UK). Questions were asked by Barbara Jaworski.

"How do teachers get to a point where they can engage in classroom inquiry?"

Response by Konrad Krainer:

There is a manifold of starting points for teachers to get engaged in classroom inquiry. I confine myself to sketch two examples of such starting points which led to teachers' action research processes within the framework of the PFL-Programme in Austria (see e.g. Krainer & Posch 1996). Participants of these two year teacher in-service education courses (e.g. mathematics, science, German, English) are regarded as professionals who systematically try to develop their competence in and towards the different dimensions of professional practice, on the average writing two case

studies during the course. For example, one female mathematics teacher's motivation to start an inquiry was her feeling that most of the time she has to deal with students' errors. Motivated by other teachers and staff members of the course, she strove to learn more about her students' misconceptions and errors in learning algebra as a way to improve her teaching. She studied articles about error analyses, investigated students' misconceptions and errors from her own classes and finally identified about fifty types of 'error frames'. However, one major result of her study was the insight that 'the appreciation of an achievement [of a student] ... and the teacher's interest in investigating the thinking behind the work ... are important factors for student motivation'. Another teacher's action research process within the PFL-programme did not start with a content-related issue, but with a pedagogical one. He was concerned about the noise level in one of his classes. As typical for discussions about possible starting points of investigations into his own teaching within PFL, he was motivated to have a closer look at the situation and to gather relevant data concerning the assumed high noise level in this class. Primarily, based on regular observations and notes in a research diary over a period of seven months, he found out, to his surprise, that it was primarily he himself who judged his instruction as too loud, and that a considerable part of this 'noise' is caused by content-related communication between students. This discovery was the starting point of his second case study in which he reflected on the tremendous changes in his approach to teaching caused by the 'noise study'. These examples – and many others – show that the first starting point of a teacher's investigation into his/her own teaching is not so important. In many cases the starting point changes and very often the processes go very directly and deeply to the core challenges of these teachers: it is more the process with all the needed courage and assumed added value of critically reflecting on one's own practice that defines the direction of teachers' professional development.

“Please give an example of classroom inquiry in which you have engaged”

Response by Elisabeth Thoma:

Being not very satisfied (internal factor) with my double role as a facilitator for students' mathematics learning and as an assessor for the students' progressing in their career, I tried to change the common assessing methods I used during the last ten years and created a new model. At the same time I attended the four semester

course PFL-Mathematics (Pedagogy and Subject-specific Methodology for Mathematics Teachers) for experienced teachers at the university and became familiar with action research and its methods to improve a teacher's situation. Combining both the new idea and the methods of action research I developed, step by step, a new assessing model by implementing pieces of the model and reflecting and negotiating the pupils' reactions to them. After one school year when all pieces were experienced I revised the model and wrote a case study concerning the whole process. This study was on the one hand the required paper for a successful participation in the course and on the other hand could be and is spread among teachers and parents being interested on this topic. Thus one teacher's research revealed that knowledge based in experience can be shared with others in the sense of 'teachers are learning from teachers'.

“What is your theoretical background for engaging in teacher research or action research?”

Response by Konrad Krainer:

We should first speak about the terms 'teacher research' and 'action research'. For me, teacher research mainly indicates that teachers do research, nothing is said, for example, about the methodology, the goal and content of this research or how it relates to teachers' practice. For example, one teacher might be a part of a university research team, investigating the international results of the Third International Mathematics and Science Study (TIMSS), whereas another teacher might investigate his/her own teaching within the framework of action research. Action research, for example, defined as the systematic reflection of practitioners on action in order to improve it (see e.g. Elliott 1991 or Altrichter, Posch & Somekh 1993) mainly indicates a specific methodology, there is no restriction that only teachers do that kind of investigation. Action research is used in a variety of practice fields (e.g. ecology, economics); in particular, it also seems to be a valuable approach for our practice field, namely teacher education. Here, reflecting on the impact of our interventions into the educational process, we are hopefully the systematically reflecting practitioners who aim at improving our actions within pre-service and in-service teacher education courses or professional development projects. Most of the 'stories' of successful projects in the teacher education literature are reports based on

more or less systematic reflection of practitioners (teacher educators) on action, enriched by additional data from different involved people and perspectives (which is a standard in action research). For this reason, I like the word 'action research' because it reminds us very clearly that reflection on one's practice is not only a challenge for teachers but also for ourselves. In many of our projects – where teachers do action research – we investigate our facilitation process and both are part of joint discussions. John Elliott introduced for these two investigation processes (practised by teachers and facilitators) the terms 'first order' and 'second order' action research.

My engagement in promoting teachers (and also principals, teacher educators, ...) doing action research is based on the theoretical assumption that complex practical situations and problems (e.g. the improvement of learning and teaching) cannot be resolved outside the practice through general propositions to be transferred (e.g. through in-service education) as 'ready knowledge' to practitioners who then would only have to apply this knowledge in practice. In contrast to this model of 'technical rationality', my thinking is grounded on a model of 'reflective rationality' which assumes that complex practical situations and problems need particular solutions that only can be developed in the specific context of their appearance (see e.g. Schön 1991 or Altrichter, Posch & Somekh 1993). Therefore action research (defined as the systematic reflection of practitioners on action in order to improve it, see e.g. Elliott 1991 or Altrichter, Posch & Somekh 1993) is the central feature of many development and research activities which we initiate in Austria, like the above mentioned PFL-programme.

More and more it has become apparent through organisation theory, system theory or reflections on our own projects that the promotion of professional development of individual teachers is only one approach to foster innovations in schools. Another approach is to work with the whole teaching staff of schools, the school community or a group of representatives of it in order to facilitate the whole school's further development towards a learning organisation (see e.g. Brunner et al. 1997). The challenge is to find a bridge between both demands, the promotion of teaching development and the promotion of school development. Otherwise, on the one hand there might be a wonderful professional development of some individual teachers but the positive effect is confined to their classrooms and partially there is

(hidden or open) resistance against their innovations because they intersect with traditional rules and habits of many other colleagues; on the other hand there might be a wonderful plan for introducing new structural elements and working groups at a school but there are no or only few impacts on teaching in classrooms. One possibility is to collaborate, for example, with the whole group of mathematics teachers of one school. A pilot project in Austria shows some first encouraging results (see e.g. Krainer 1999).

“In what way(s) have you been involved in teachers’ action research?”

Response by Razia Fakir-Mohammed:

The notion of teachers’ research on their practice is very new in the context of Pakistani schools. A one year project of action research was initiated by the Institute for Educational Development at the Aga Khan University in 1997. A group of university based researchers worked with teachers in their school, having a dual role of mentor/researcher (M-R), to facilitate the improvement of mathematics teaching and research the processes involved. There were four M-Rs who each worked with two teachers in two school sectors: government and private. In this project teachers inquired critically into their teaching in collaboration with the M-Rs who played different roles and used different strategies according to the situation.

This was my first experience being involved in research of this kind, and I found it an interactive and democratic way to analyse my own understanding of teaching-learning processes and my role as a M-R. I see this as an active, social, and reflective experience. The teachers too started to question their personal practice and philosophy: What are the reasons for my problems? How do I solve them? How do children learn? What factors are involved in their learning, inside the classroom and outside the classroom? How could I help my children to learn in an effective manner?

I developed my own meanings for action research from my own experiences as well as from literature. As one of my teacher researchers said, “It is a source of learning, through which one could interact and exchange thoughts”, while the other said

“In the daily routine, teachers explore a lot of questions, but ignore them because of their involvement in different activities. Isolation does not allow

one to concentrate on these questions. This process of research focuses one's thoughts towards particular problems. Conversation and observation with the facilitator helps me to explore problems and see multiple ways of solution."

Both teachers perceived their activity as self inquiry by themselves in a social situation to improve their own practice. I acted as a facilitator in this process, trying to help in synthesising issues arising from their reflective processes, but not imposing on the teachers. This is a way of professional development that enables participants to construct knowledge from their own stories of the classroom and to devise their own ways to solve problems. It involved a deliberate and conscious effort by teachers alongside other routine school work and the realities of time constraints.

For me, this was a dynamic and cyclic process in which teachers focused on a problem, reflected on it in the real classroom situation, constructed knowledge, and planned and acted accordingly. Both teachers developed reflective and critical attitudes. Reflection and dialogue on the classroom events helped them to see problems in multiple ways and to identify factors in children's learning both inside and outside the classroom situation. For one of the teachers the process was of a diagnostic nature. She explored different ways of solving problems, in terms of improving her classroom management skills and instructional strategies. However, she did not have the skills identified as necessary, so a need for training was realised by the mentor and suggested by the teacher, an example of mutual agreement and democracy. The other teacher was trained and had sufficient skills but was not using them in an appropriate way. So she developed a better understanding of her role in mathematics classroom.

As well as the teachers' learning, facilitators (the M-Rs) were also in a process of learning. In the beginning I was looking on one aspect of it, the issue of acceptance of my role as a facilitator rather than provider. As I engaged in the process, I realised that it was a very challenging task for my self as well as for the teachers. It required interpersonal and personal skills and an open attitude. At various times I was a stimulus (motivating teachers to reflect), a listener (encouraging identification of issues), a clarifier (encouraging focusing of thinking), a task keeper (planning different strategies and diagnosing areas of improvement) and an overall guide (without imposing anything).

I was confused many times: for example, in encouraging teachers to reflect – sometimes they just wanted answers from me (the easy solution); deciding when to intervene or not to intervene; sharing my analysis about issues of learning; waiting for the time researchers reach that point of self-analysis. I needed to be very patient and careful. This issue leads to a question of whether teachers would be able to do action research collaboratively themselves – do they have the relevant skills and attitudes? If we assume they will learn from their own experiences of involvement, as I learn myself, then what would be their basic level of understanding the research? Teachers were very devoted in terms of spending time having dialogue, and reflecting on the classroom events despite time constraints. However, on some occasions, when I could not meet them, they did not meet themselves. One of them said “your absence makes this process very slow”. Although from the beginning they recognised the value of their learning, it seemed that all the time they required a social ear.

At the end I would like to say that despite all the issues and the demanding role of researchers and facilitators this one year project had promising outcomes. I believe that it encouraged teachers to engage in the process of critical self reflection which never ends. For me its influence is like a chain reaction; change in teachers directly affects their students and colleagues, and the circle of influence grows bigger when these affected people interact with others. If this process could be continued in an organised and systematic way, it could play a vital role to improve mathematics learning.

“In what ways does action research depend upon an institutional context in higher education?”

Response by Barbara Jaworski:

In the research discussed by my colleagues above, there has been an institutional element in the teachers’ engagement in action research. In one case, this involved the teacher being enrolled in a course at the institution, such as an in-service course (K. Krainer, E. Thoma). In another case it involved teachers being part of a research & development initiative involving mentors from the university (R. Fakir-Mohammed).

To some extent the involvement in the course or programme directed or influenced the teachers in undertaking their research.

“Is it possible for teachers to engage in research without the structure provided by a university course or in-service education programme?”

Response by Barbara Jaworski:

Some years ago, I engaged in research with teachers in which my questions as a researcher proved to be stimulation for teachers' thinking and corresponding classroom activity. It seemed that if only teachers could ask and explore such questions themselves, this could be a most fruitful form of teaching development. I subsequently initiated a research project to explore the processes in teachers asking their *own* questions without being a part of a formal programme at the university. Explicit in this research was the nature and directions of teachers' own questioning, and its relationship to development in teachers' thinking and teaching.

Volunteer teachers were sought who would explore self-chosen aspects of their teaching without any direct input from an official course or programme. Their teaching and thinking about teaching would be studied by researchers from the university. A major consideration in the study was to what extent the teachers' research was self-motivated, and to what extent it was influenced by external factors, such as interviews with researchers, or meetings with other teachers participating in the research. Conclusions indicated that these factors were both strongly influential in the teachers' growth within their research; for example, anticipation of interviews with researchers triggered research action, and meetings with other teacher researchers helped sustain the inquiry process. However, teachers were able to generate and explore their own research questions through classroom observation and action. Reflections on the outcomes of this exploration led to revised questions and new directions of inquiry. The teachers' research was described as *evolutionary*, as their knowledge and confidence in the research process developed (see e.g. Jaworski 1998). It was nevertheless unlikely that any of these teachers would have engaged in research without the initial stimulation of the project. It thus needs still to be questioned to what extent action research as a tool for teaching development can exist independently of institutions with research expertise.

Responses and feedback from members of the wider group

After the questions and responses reported above, a period of 45 minutes was given to small group discussions within the wider group. A set of questions was provided from which small groups could choose a focus. These questions were as follows:

1. How does/can action research contribute to
 - a. Growth of knowledge of the mathematics teacher?
 - b. Development of mathematics teaching?
2. What counts as knowledge in mathematics teaching, and how does it grow?
3. What is teachers' action research? What do we regard as research in a teaching context?
4. Is teacher research the same as action research?
5. How does research by teachers fit with norms of established educational research – particularly in terms of validity and rigour?
6. What are the outcomes of action research and for whom are they significant?
7. Is action research most significant in terms of its contribution to the development of teaching, or does it also enhance knowledge in the wider community?
8. In what ways does/can teaching be seen to develop relative to action research?
9. How does action research relate to critical reflective practice?
10. In what ways might the cycle of teacher planning, classroom activity, and feedback be regarded as a research process?
11. What counts as theory in action research?
12. In what ways is action research in mathematics teaching specifically related to mathematics as a discipline?

The following includes a representative selection of comments and questions received in written feedback from the small groups:

- *The problems of theory*: Formally constituted research always has a well defined theoretical basis. Where/what are the theoretical constructs in action research, and how are they conceptualised?
- *Research methodology*: In what ways do teachers become familiar with research methods, and learn to judge their appropriateness for exploring particular research questions?
- *Communication in action research*: Perhaps Associations of Mathematics Teachers can provide opportunity for dissemination of action research, and also for collaborative work and support between teacher researchers.
- *Action research and problem solving*: In what ways is action research different from problem solving? One possibility is that action research encompasses problem solving, and, in its cyclic process, has the potential to refine problems and enable more knowledgeable problem solving.
- *Action research and public knowledge*: Is action research more likely to result in practical innovation than in knowledge enhancement? In what ways can the outcomes of action research contribute to knowledge?
- *Action research and the teacher educator*: Perhaps action research allows teacher educators clearer visions of teaching practice and the developing thinking of teachers.
- *Collaborative work between teacher researchers, teacher educators, and academic researchers*: Perhaps such collaboration can be extremely fruitful in the enhancement of mathematics teaching and development of related knowledge in the public domain.

There were clear indications of a strong interest in this area as well as the necessity for further research into the issues raised. However, it is also important to recognise that in the time available in the meeting it was impossible to do justice to the research that already exists. As a result of this research, many of the above issues are already being addressed (see e.g. Jaworski 1994 and 1998; Krainer 1994 and 1999; Crawford & Adler 1996).

Some of these issues are addressed in more lengthy responses to several of the above questions, for example, concerning questions 2, 5, and 7.

Question 2: What counts as knowledge in mathematics teaching, and how does it grow?

Response by Konrad Krainer:

Let us come back again to the example of the teacher in the PFL-programme who investigated in his first case study the noise level in one of his classes, and let us regard this story through the lens of four dimensions of teachers' professional practice, namely action, reflection, autonomy, and networking (see e.g. Krainer 1998). For this teacher, "noise" was a problem which influenced his classroom activities enormously. Collecting data and reflecting on this data brought him new information and new insights he would not have got by simply referring to his original "practical theory" of the situation, which mainly said that noise has to be seen as a factor that hinders teaching. The process of reflecting led to a more complex and deeper understanding of the situation, for example, realising that noise may be an expression of students' need for content-related communication and that "noise may emerge through monotonous modes of instruction"; but this also had consequences on his actions, from an alternative way of dealing with "noise" to starting "occasionally to design lessons in another way". This process shows the impact that the close interplay of the teacher's actions and reflections had on his beliefs and his practice.

Regarding the teacher's further progress we now place our emphasis on the interplay between autonomy and networking. For his second study, entitled "Mathematics instruction for one's different" (Kliment 1996), the teacher read literature on teaching and learning objectives, formulated his own objectives and found consequences for his teaching, for example, stating:

"It is clear that – with regard to the objectives formulated in the preceding section – frontal instruction now plays only a small role. But what are the alternatives? One of the most effective incentives in changing my teaching was a study by colleagues."

Here he refers to a case study written by participants and one staff member of a former course. This shows one advantage of writing down teachers' investigations and of making the writing accessible to others: teachers' local knowledge can be linked with the experiences of others through being published in studies available for a larger community. This means that teachers' autonomous work can be networked and therefore used as one contribution to increasing professional communication among teachers. The teacher increasingly turned to a child-centred, application-oriented, and computer-supported form of instruction and realised his ideas in a teaching experiment which lasted for a period of about ten weeks. The evaluation was predominantly very positive.

To sum up, more reflection on action may improve practitioners' activities which in turn may lead to new questions and reflections. But this interplay between action and reflection is not only confined to the learning of individual teachers, it can also be used as a starting point for professional exchange among teachers: the learning process is directed towards autonomy as well as towards networking – it is the interplay of both which leads to progress.

Question 5: How does research by teachers fit with norms of established educational research – particularly in terms of validity and rigour?

Response by Konrad Krainer:

Altrichter (1991), comparing traditional-empirical research (concerned with quantification) and so-called alternative research (e.g. including qualitative, ethnographic and action research) especially with regard to validity, stresses that “to validate is to investigate” (and not to demonstrate the worth of something) and concludes (p. 84):

“This interpretation of validation, however, seems to be compatible with the views and practices of most of the alternative researchers and with the procedures they use, as, for example, communicative validation, saturation, etc. ... Thus, there is no need for different methodologies for alternative and traditional-empirical validation.”

Are there major differences between academic research and a teacher's self-critical inquiry on his/her own practice? A profound discussion on whether teachers'

“action research” can be seen as research is given in Altrichter (1990) who comes to a positive conclusion and outlines the theoretical and methodological foundations.

Burton (1991, pp. 120–121) states:

“I wish to conclude by reaffirming my purpose in doing research through which is to make clearer the processes through which mathematics is learnt and taught. With this aim I cannot find myself occupying a distinctly different role as researcher than that which I occupy as a teacher although it is clear that role demands and imperatives do vary. If that is valid for me, I cannot do other than recognise its validity for all teachers and researchers.”

The understanding of research underlying this statement has much in common with the idea of Eisner (1993) who views research as an art, as a process of understanding. Of course, the attempts to understand should have attributes like systematic and self-critical, to assure that the research is properly done. Feynman (1987, p. 454), winner of the Nobel prize in physics, stresses that the most fundamental principle for conducting research is that of not cheating oneself or other scientists.

What all kinds of research have in common, is the wish to get a deeper understanding of a situation, a problem, etc. The danger of being kept in the trap of cheating oneself and others is given for all researchers, for the scientist, for example, through aiming at getting proved the hypotheses he/she had in his/her mind from the very beginning, or for the teacher doing action research, for example, through aiming at getting confirmed that he/she is doing good teaching (instead of trying to find out specific strengths and weaknesses and ways to improve). In order to strengthen the self-critical element of scientific work and to minimise self-fulfilling prophecies, it is crucial to make one's approach (methods, data, interpretation, ...) open to public discussion and feedback. Of course, this is quite more intensively done in the case of scientists working at universities, in particular through publishing, reviewing, lecturing at conferences etc. than in the case of teachers investigating their own practice. Nevertheless, this process of making one's results visible and discussible by other professionals is an inevitable part of action research. In this case, so-called ‘critical friends’, this means, for example, colleagues from their own school, from other schools or from the university, play an important role: They are ‘friends’ in a

sense that they aim at promoting someone's further development in a positive and helpful way; at the same time they have the task to be 'critical' in a sense that they help to avoid the traps mentioned above. 'Critical friends' can act at various levels, for example through helping to gather data (e.g. observing teaching or conducting interviews with students), through giving feedback to oral or written interpretations, through giving advice for further methods of data collection or for the overall structure of the investigations, or through helping to present the findings at professional meetings (e.g. at their own school) or to publish it (as practised e.g. in the PFL-programme where case studies of teachers run through a certain feedback cycle before they get part of the PFL case study series).

In action research, similar to other kinds of so-called alternative research, it is not something 'foreign' that is being investigated but situations in which the acting researcher is playing a crucial part. There is a clear tendency to try to understand the uniqueness and particularity of the case. There is not much interest to look for the generalizability to other (or even all) cases.

We do not need special methodologies when co-operating with teachers or when speaking about teachers' or teacher educators' investigation into their own practice. What we need first of all is an extended understanding of methodology, an understanding that includes an attitude of mind, namely self-critically reflecting and trying to understand something to which we want to make a significant contribution.

Question 7: Is action research most significant in terms of its contribution to the development of teaching, or does it also enhance knowledge in the wider community?

Response by Konrad Krainer:

The teachers' systematic reflections on their own practice can not only improve their teaching but can also have consequences for the further development of teacher education (see e.g. Krainer & Posch 1996), for mathematics education (see e.g. Fischer & Malle 1985) or for the personal and professional development of team members (see e.g. a mathematics educator's reflective paper on his activities within the course; Peschek 1996).

The fact that writing case studies causes some problems for both the teachers, and the teacher educators supporting them, should not be withheld. In writing case

studies, teachers have to do at least three things which are rather unusual in their normal practice:

- They have to gather data and to reflect on them systematically (and not only take action),
- they have to write down their findings (and not just communicate them orally), and
- they have to formulate these results for other people (and not just practice something within their own classrooms).

That this is more difficult for teachers than for us – living in a “culture of publishing” – should be taken into consideration. Nevertheless, it seems to be worth promoting teachers’ investigations for at least four reasons: Systematic reflection on their own work creates new knowledge which in turn positively influences their teaching; writing down is an additional opportunity to learn; writing a study (to be read by others) increases the opportunities for communicating and co-operating with interested people (teachers, theoreticians, administrators); and finally, it gives us an additional opportunity to learn from them.

Conclusions and future directions

From research conducted in the area of mathematics teacher education, it can be said

- that action research is possible, and is, moreover, a most effective instrument for the development of mathematics teaching;
- that supportive mechanisms such as the attention of external researchers, and teacher researcher support groups are beneficial to effective research and development;
- that communicative networks of teacher researchers sharing experiences either directly or through their writing are particularly valuable to the development of action research;
- that, as yet, action research is under-used in European countries as a tool for the development of mathematics teaching.

Recommendations include the following:

- that better communication of the processes, practice, and potential of action research for mathematics teaching development be sought between those who seek to develop mathematics teaching;
- that co-operative cross-national groups be initiated through which research findings and issues can be shared and debated;
- that those interested in this area of research and development might work more closely with researchers from other disciplinary areas, to learn from insights in those areas. One example is CARN, the Collaborative Action Research Network.

A postscript – concerning mathematics

In preparing this chapter on the conference sessions and editing the contributed items, it became clear to me that there is little talk of mathematics in this chapter. From my own research in this area, it is evident that teachers take their mathematics largely for granted and focus often on the methodological and pedagogical issues when talking about doing research. That their research is about mathematics learning and teaching often seems to be hidden in these discussions. However, there are examples of mathematics teachers' case studies intensively dealing with mathematical issues of learning and teaching (see e.g. Mayer 1994 and Kliment 1996). It seems important, when reporting on action research in mathematics education, to bring mathematics closer to the centre of the stage to see more clearly in what ways action research relates to the teaching and learning of mathematics, and leads to better mathematical learning environments for students.

References

- Altrichter, H. (1990). *Ist das noch Wissenschaft? Darstellung und wissenschaftstheoretische Diskussion einer von Lehrern betriebenen Aktionsforschung*. München: Profil.
- Altrichter, H. (1991). Do we Need an Alternative Methodology for Doing Alternative Research? In O. Zuber-Skerrit (Ed.) *Action Research for Change and Development*. Aldershot-Brookfield-Hong Kong-Singapore-Sydney: Avebury, pp. 79–92.
- Altrichter, H., Posch, P. & Somekh, B. (1993). *Teachers Investigate their Work. An Introduction to the Methods of Action Research*. London: Routledge.

- Brunner, I., Hruska, E., Krainer, K., Piber, C., Posch, P. & Racher, K. (1997). School Renewal through Action Research: a school-university partnership in Austria. *European Journal of Intercultural Studies*, Vol. 8, No. 1, pp. 75–96.
- Burton, L. (1991). Models of Systematic Co-operation between Theory and Practice. In *Zentralblatt für Didaktik der Mathematik* 23, 4, pp. 112–117.
- Calderhead, J. (Ed.) (1987). *Exploring teachers' thinking*. London: Cassell.
- Crawford, K. & Adler, J. (1996). Teachers as Researchers in Mathematics Education. In A.J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.) *International Handbook of Mathematics Education*. Dordrecht: Kluwer, pp. 1187–1205.
- Eisner, E.W. (1993). Forms of Understanding and the Future of Educational Research. *Educational Researcher*, Vol. 22, No. 7, pp. 5–11.
- Elliott, J. (1991). *Action Research for Educational Change*. Milton Keynes, UK: Open University Press.
- Feynman, R. (1987). "Sie belieben wohl zu scherzen, Mr. Feynman!" München: Piper.
- Fischer, R. & Malle, G. (1985). *Mensch und Mathematik. Eine Einführung in didaktisches Denken und Handeln*. Mannheim-Wien-Zürich: Bibliographisches Institut.
- Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: Falmer Press.
- Jaworski, B. (1998). Mathematics Teacher Research: Process, Practice, and the Development of Teaching. *Journal of Mathematics Teacher Education*, Vol. 1, No. 1, pp. 3–31.
- Jaworski, B., Wood, T. & Dawson, S. (Eds.) (1999 expected). *Mathematics Teacher Education: Critical International Perspectives*. London: Falmer Press.
- Kliment, F. (1996). Mathematikunterricht einmal anders. In K. Krainer & P. Posch (Eds.). *Lehrerfortbildung zwischen Prozessen und Produkten*. Bad Heilbrunn: Klinkhardt, pp. 235–248.
- Krainer, K. (1994). Integrating research and teacher in-service education as a means of mediating theory and practice in mathematics education. In L. Bazzini (Ed.) *Theory and Practice in Mathematics Education*. Pavia: ISDAF, pp. 121–132.
- Krainer, K. (1998). Dimensions of teacher's professional practice: Action, reflection, autonomy, and networking. In T. Breiteig & G. Brekke (Eds.) *Theory into practice in mathematics education. Proceedings from Norma 98*. Kristiansand: Agder College Research Series, pp. 36–44.
- Krainer, K. (1999 expected). PFL-Mathematics: Improving Professional Practice in Mathematics Teaching. In B. Jaworski, T. Wood & S. Dawson (Eds.) *Mathematics Teacher Education: Critical International Perspectives*. London: Falmer Press.

- Krainer, K. & Posch, P. (Eds.) (1996). *Lehrerfortbildung zwischen Prozessen und Produkten*. Bad Heilbrunn: Klinkhardt. [See review in *Journal of Mathematics Teacher Education* 1 (1), pp. 113–116, 1998]
- Mayer, C. (1992). *Fehler*. Unpublished case study of PFL-Mathematics 1991–93. Klagenfurt: University of Klagenfurt, IFF.
- Othman, M.Y. (1995). *A pilot study into the teacher craft of three mathematics teachers and its relation to learning in the classroom: methodological issues and indications for future research*. Unpublished M.Sc. Thesis, University of Oxford.
- Peschek, W. (1996). PFL – eine wissenschaftliche wie auch persönliche Herausforderung. In K. Krainer & P. Posch (Eds.) *Lehrerfortbildung zwischen Prozessen und Produkten*. Bad Heilbrunn: Klinkhardt, pp. 167–180.
- Polanyi, M. (1958). *Personal Knowledge*. London: Routledge and Kegan Paul.
- Schön, D.A. (1987). *Educating the reflective practitioner*. Oxford: Jossey-Bass.
- Schön, D.A. (1991). *The reflective practitioner*. 2nd edition. London: Temple Smith.

CHAPTER 6

INVESTIGATIONS INTO TEACHER EDUCATION: TRENDS, FUTURE RESEARCH, AND COLLABORATION

Konrad Krainer

University of Klagenfurt / IFF (Austria)

Fred Goffree

Freudenthal Institute (The Netherlands)

Trends of investigations into teacher education

In meeting 1 of *Thematic Group 3*, the participants were invited – as one part of their personal introduction to the group – to name one or two *research questions* in the field of teacher education that are of special interest for them or that they propose to investigate more intensively in order to get a better understanding of important issues in teacher education.

In the following we look at how these research questions are distributed concerning the different kinds of *research in teacher education* we sketched in the preface:

Research in the perspective of teacher education (the investigators do not in the first place think about use in teacher education and do not focus on interaction processes within teacher education, however, its results can be used as a basis for designing learning environments in teacher education courses):

- To what extent mathematics teachers' general beliefs relate to local beliefs (as e.g. to specific mathematical topics as teaching algebra)?

- Are there “computer world views” of computer science and mathematics teachers? What are they like and how do they influence the performances of teachers (seen in a holistic way)?
- Is it possible to understand mathematics teaching practice as a “game” where the teacher is against the pupil?
- What are conditions and constraints that influence teaching practice?
- How do teachers manage the connection between students’ activities and the acquisition of mathematical knowledge?
- What is the interplay between mathematical knowledge and ability, self-confidence, personal history and conceptions of primary mathematics teachers?
- How do internal factors interplay with external factors concerning the professional development of teachers?
- How can problem solving be used as a tool to find out mathematics teachers’ beliefs in order to improve teachers’ mathematical knowledge and mathematics teaching?

Research in the context of teacher education (the investigators think about use in teacher education from the very beginning, however, they do not focus explicitly on interaction processes within teacher education; the role and influence of teacher educators is not explicitly reflected):

- Considering the professional development of teachers (within pre-service and in-service education): what is the interplay between cognitive processes and cultural, social, affective processes?
- What kinds of knowledge do teachers bring to in-service education and how does it grow?
- Is the gap between what teachers learn at the university (pre-service education) and their practice in schools evident and how could we explore it?
- How do (student) teachers construct (what) knowledge? What is the role of discourse and collaboration? [Reflections on discourse and collaboration with teacher educators could include elements of research on teacher education.]

- Context of a school-university collaboration: Why and how do mathematics teachers from one school (want to) change their teaching practice using alternative learning and teaching methods?
- Can teacher educators change beliefs of mathematics student teachers by the way higher mathematics is taught to them? [Reflections on the influence of the interaction process would include elements of research on teacher education.]
- Pre-service teacher education: Are there paradigms of learning to learn from practice?
- In-service teacher education: What hindering and promoting factors influence teachers' growth? [Reflections on the influence of the interaction process would include elements of research on teacher education.]
- How do student teachers develop their understanding of childrens' ways of thinking during school practice?

Research on teacher education (the investigations focus explicitly on interaction processes within teacher education: teacher education is the object of research):

- In what ways can teacher educators in mathematics influence the ways of working and thinking of teachers in order to promote more effective learning of students? (As I cannot change anybody but myself.)
- How can teacher educators promote mathematics teachers' consciousness that in the teaching process many different language registers are used?

Research as teacher education (teachers investigate their own practice, interacting with and supported by "critical friends"):

- (There was no research question explicitly relating to this.)

Meta-research on teacher education (analysis of research activities or general conditions in the field of teacher education; see e.g. Wideen, Mayer-Smith & Moon 1998):

- Which cultural differences in teacher education systems (e.g. concerning language, economics, ...) are essential and meaningful?

Some of the questions dealt more with the *challenge of designing teacher education* than investigating issues:

- Which skills of the experienced teacher are missing in the pre-service teacher education curriculum but can be included?
- How can we design teacher education in order to demonstrate to teachers the full complexity of mathematics learning? Which concrete examples and particular situations should be arranged in pre- and in-service teacher education?

It is interesting to compare now *the distribution of participants' research questions* concerning the different kinds of research in teacher education by just indicating their home countries:

Research in the perspective of teacher education

Germany, France, Portugal, Spain, Switzerland. Here the emphasis is put very clearly on research on teachers, investigating their beliefs, knowledge, and practice. *Theory* seems to play a crucial role, the challenge seems to be more to find a bridge between the research results and the *practice* of teacher education.

Similarly the papers in the book written by colleagues from these countries also show their main strengths to be in theoretical frameworks for describing teachers' beliefs, knowledge, and practice (see e.g. chapter 2 and subchapter 4.2). However, partly influenced by the discussions that started at the TG 3 meetings, some papers show considerable attempts to focus more clearly on the practice of teacher education (see e.g. subchapter 3.3).

Research in the context of teacher education and research on teacher education

Austria, Hungary, Italy, Portugal, Russia, The Netherlands, United Kingdom. Here the research questions mainly focus on teachers' professional growth in a context of teacher education, sometimes the role of teacher educators is taken into consideration. *Practice* seems to play a crucial role, the challenge seems to be more to find adequate *theoretical* frameworks for describing teachers' practice and professional growth.

Similarly the papers in the book written by colleagues from these countries also show their main strengths to be in designing of and investigating into rich learning environments in teacher education (see e.g. subchapter 4.3). However, partly influenced by the discussions that started at the TG 3 meetings, some papers show considerable attempts to focus on theoretical issues (see e.g. subchapter 4.1 and chapter 5).

As one element of meeting 1 of *Thematic Group 3*, group work was initiated in order to discuss the research questions mentioned above. The concrete task for each group was the same, namely to negotiate the three to five most important research questions and to present the reasons for their choice in the plenary. We confine ourselves to presenting very briefly the outcome of one group's considerations (for more details see Thoma 1998): Using a triangle with the edges "researchers", "teachers" and "students", the group looked for a structure to combine the six chosen questions. Questions

- "In what ways can teacher educators in mathematics influence the ways of working and thinking of teachers in order to promote more effective learning of students?"
and
- "How can we design teacher education in order to demonstrate to teachers the full complexity of mathematics learning? Which concrete examples and particular situations should be arranged in pre- and in-service teacher education?"

were considered as the *aim of research*, and question

- "Is the gap between what teachers learn at the university (pre-service education) and their practice in schools evident and how could we explore it?"

as the *research focus*, while questions

- "Are there paradigms of learning to learn from practice?"
- "What hindering and promoting factors influence teachers' growth?"
and

- “How can problem solving be used as a tool to find out mathematics teachers’ beliefs in order to improve teachers’ mathematical knowledge and mathematics teaching?”

were seen as *possible* ways or means. Finally, the group created a new research question:

- “How can teacher educators and researchers promote teachers’ confidence and professional growth in order to improve their situation in school, including classroom management, teaching etc.?”

Another group indicated the necessity of a meta-methodology in order to be able to connect all different kinds of research in teacher education. In an exemplary way, the results of these groups show the integrative attempts to get a clearer picture of the complexity of research in teacher education.

In the discussions of *Thematic Group 3* it became clear that some of the research questions were preferred to others. Synthesising the arguments for preferring some questions more than others, mainly the following criteria were used:

- clearness of formulation
- clearness of research focus
- relationship to classroom practice
- relationship to learning and professional growth of (student) teachers
- relationship to curriculum development in teacher education
- possibility of transferring research outcomes across cultures

All in all, the discussions in *Thematic Group 3* showed an increased consciousness of the complex task of acting as a researcher and teacher educator and the need to find fruitful links between *theories* on and *practices* of teacher education. There was the feeling that the bridge should be built from both sides. The desire to co-operate and collaborate in the field of (research in) teacher education, was loud and clear, also, during all sessions of *Thematic Group 3*.

This leads us deeper into the issue of *trends* of research in teacher education. First of all, one major trend is the trend of *an increased interest in research in teacher education* itself. Indicators for that movement are, for example, increasing

literature, conferences and – last but not least – the large number of CERME 1 participants interested in this Thematic Group (see also preview). But what about other trends?

In the following, we sketch some major *trends* (of course, there is some overlapping) that can be sifted out of the papers and discussions concerning research in teacher education in Europe:

- Serious attempts to find bridges between theories and practices of teacher education (see above). In particular, the idea of viewing learning environments for (student) teachers at the same time as a meta-learning environment for teacher educators who investigate into (student) teachers' growth and at the same time reflect on their influence within the interaction process).
- A broader understanding of research in teacher education, for example: a) Investigations should not only focus on teachers (e.g. concerning their beliefs, knowledge, and practice) but also on the interaction process between teacher educators and teachers; b) Investigations are not only restricted to academic researchers in order to construct new general knowledge, but also (student) teachers are increasingly engaged in investigations into their own teaching or into the practice of other teachers in order to construct situated and local knowledge (which can be linked with outcomes of research projects).
- Giving systematic reflection and investigation into classroom practice a central position. This holds true for pre-service and in-service teacher education, and is supported by the use of multi media, for example: The MILE environment – used by student teachers to investigate into classroom practice (see e.g. subchapter 4.3).
- Increasing importance of “stories”, used as a general term for narratives, curricula vitae, cases, etc. In two subchapters (3.3 and 4.1), for example, Elbaz (1991, p. 3) is quoted, stressing: “Story is the very stuff of teaching, the landscape within which we live as teachers and researchers, and within which the work of teachers can be seen as making sense ... teachers' knowledge in its own terms is ordered by story and can be best understood in this way.” And it is indicated that narratives are a particularly adequate way of knowing, representing, and studying teachers' knowledge. Also, teachers' curriculum

vitae are used as one source of data describing teachers' growth (see e.g. Krainer 1994). The importance of case studies has several reasons, among which others we select the following: Firstly, teachers' practical knowledge has a strong component of case knowledge; secondly, case studies on teachers not only inform us about their practice and growth but they also extend our theoretical knowledge on teachers; and thirdly, case studies as an outcome of teachers' efforts to investigate into their own teaching and to write their experiences down, for them as an additional circle of reflection, for other colleagues and researchers as an insight into teachers' challenges and change.

- Increased importance of action research as the systematic reflection of practitioners into their own practice (see e.g. subchapter 3.3 and chapter 5). This doesn't hold true only for in-service teacher education but increasingly also for pre-service education. Action research is based on the theoretical assumption that complex practical situations and problems (e.g. the improvement of learning and teaching) cannot be resolved outside the practice through general propositions to be transferred (e.g. through in-service education) as 'ready knowledge' to practitioners who then would only have to apply this knowledge in practice. In contrast to this model of 'technical rationality', the thinking of many people promoting action research is grounded on a model of 'reflective rationality' which assumes that complex practical situations and problems need particular solutions that only can be developed in the specific context of their appearance (see e.g. Schön 1991 or Altrichter, Posch & Somekh 1993).
- More attention to cultural, situated, and organisational aspects of processes in classrooms and teacher education courses. There is an increasing awareness that each learning and teaching process is not only influenced by the beliefs and actions of the individuals involved, but also is dependent, for example, on the genuine culture of a region, on the specific environment or milieu, on how teachers' practice is supported by the culture of a specific school, or on how knowledge is institutionalised (see e.g. subchapter 4.2). All in all, this means that research in teacher education is increasingly challenged to look at the whole system.

- Not separating things so much but looking more for integration and interconnections. This observation goes hand in hand with the analysis of Cooney (1994) who stressed that progress in the field of teacher education has been made in discarding false dichotomies that pervaded teacher education, stating that we are now more and more aware of the necessity of blurring the distinction between theory and practice, content and pedagogy, researchers and teachers. We would like to add to that trend – which might be seen as the “mega-trend of integration” because it includes most of the trends mentioned above – two other pairs of concepts that are more and more seen as strongly interconnected. Firstly, we relate again to the idea of action research and also to Schön’s concept of the “reflective practitioner”. In both cases, the partition between knowing and acting – or more clearly: those who know or construct knowledge, and those who act on the basis of knowledge constructed by others – has become obsolete. Linked with that, we secondly refer again to our pair of concepts “understanding” and “improving”: It is no longer possible to see “understanding” only as essential for researchers in order to construct new knowledge and “improving” for teachers in order to change their old beliefs etc., but to see both concepts as crucial abilities for all people involved in teacher education, researchers, teacher educators, and (student) teachers. It is of course needless to say that clearly different roles have to be taken into consideration. In this context it should be mentioned that the next conference of subnetwork F of TNTEE, The Thematic Network for Teacher Education in Europe, will focus on the topic “Developing the reflective practice of teachers and teacher education through partnerships between researchers and practitioners” (Catholic University of Lisbon, Portugal, May 28–31 1999).

On the complexity of investigations into teacher education

It was mentioned several times throughout this book that investigating into teacher education is a complex task. Here, we will point out some issues concerning that complexity.

Investigations – seen from a systemic point of view – refer to *three domains of learning*, strongly interconnected with one another, namely students' learning, (student) teachers' learning, and researchers' and teacher educators' learning.

Students' learning

Mathematics classrooms are the most important learning environments for primary and secondary mathematics students. Students' rich learning of mathematics is the main goal of mathematics teaching and therefore the background of all our development and research activities in mathematics education. However, the growth of students' mathematical knowledge is, among others, considerably influenced by teachers' beliefs, knowledge, values, attitudes, their way of designing learning and teaching processes, their way of communicating and interacting with students, etc. This again is influenced, for example, by the way *teachers* have been socialised when they were (mathematics) students at school, by the culture of living and working in their country, society, school system, family, etc., by their pre-service teacher education, by in-service education courses and other forms of professional development, by the culture at their school, the kind of leadership, and collaboration among teachers, students, and parents. All in all, this means that *teachers' learning* – as a continuing and ongoing process – is a crucial factor in students' learning. However, there is another factor that has an impact on mathematics teaching, namely the results of research in mathematics education, expressing *researchers' and teacher educators' learning*. This knowledge, in general, doesn't have a direct influence on classrooms but is mediated via teachers who acquire this knowledge through teacher education or through reading of literature. Increasingly, teachers – in addition to their ongoing and natural process of knowledge construction through reflection on their teaching – generate knowledge through more systematic investigations into their own practice that become relevant for their teaching but also other teachers' classrooms.

(Student) teachers' learning

This domain of learning relates to learning processes within pre-service teacher education, in-service education courses and other forms of professional development. There is an increasing awareness that classroom practice – the domain of students'

learning – is a really important learning environment on different levels. We confine ourselves to sketching four examples.

- Firstly, it is the field where pre-service teachers can make their first steps as teachers and observe other student teachers or experienced teachers (e.g. in order to enrich their didactical repertoire), having sometimes the role of teachers and sometimes as learners. This happens quite in contrary to their former role as students (mostly only a few years before) where they learned mathematics in a process designed by their former mathematics teachers.
- Secondly, it is the field where – within the framework of in-service courses or other kinds of professional development – experienced teachers, for example, carry out experiments in their classrooms, being observed by other teachers or teacher educators as “critical friends”, in order to draw individual and joint conclusions from these findings.
- Thirdly, classroom practice can be presented with the help of videos or, more recently, as a part of multi media products (e.g. representing real teaching practice as an environment for student teachers to investigate learning and teaching in a very specific way; see Lampert & Ball, 1998; Goffree & Oonk, subchapter 4.3). This can be used in all forms of teacher education. It has the advantage that the analysis of classroom practices is more or less free of personal dependent relations between the teacher and the observers, it allows enough time to interpret and investigate, and thus gives the opportunity for individual research questions and provides the possibility to share different views on the presented practice even with bigger groups and repeated times with different groups of learners (also allowing to make comparisons and investigations into different perspectives of different people).
- Fourthly, classroom practice can be a “topic” of a (classical) teacher education unit where teacher educators report on research results concerning, for example, students’ learning of algebra or interactions between the teacher and a group of students. They can be used as a starting point for discussions on students’ learning as well as on teachers’ learning, but also as a matter for reflection on possible conclusions of the participants of that teacher education unit for their professional development and practice.

All in all, these cases show that the *roles of teacher educators* and *(student) teachers* can be manifold. (Student) teachers can act as learners on several levels, for example, as students (e.g. getting informed about new research results, reflecting on the learning process within the teacher education course), as teachers (learning to teach or to observe and reflect on teaching), or as researchers (investigating their own teaching, those of (teacher) student colleagues or those of experienced teachers). Teacher educators mainly have the role of teachers promoting (student) teachers' learning through designing rich learning environments for them. However, they also can use (student) teachers' learning as a learning environment for them. This is discussed below.

Researchers' and teacher educators' learning

When we speak in this book about teacher *educators* and *researchers* we often speak about the same people, meaning that – besides administration and management work – they have the task of educating teachers and carrying out investigations into specific fields of interest. In many cases, they (we) try to find a link between the interests and duties they have concerning teacher education and research. Although each kind of research in mathematics education has some relevance for teacher education (and can be used as a topic for a teacher education course, and therefore can be seen as research in perspective of teacher education), research in the context of teacher education or research on teacher education promise a closer connection between research results and their applicability in the practice of teacher education, whereas research as teacher education is the most concrete realisation of finding the bridge between theory and practice. However, this kind of research – yielding the construction of situated and local knowledge of teachers – has to be accompanied by the other kinds of research mentioned above and by meta-research on teacher education as well. It is the task of researchers, teacher educators, teachers, relevant research, and teacher education institutions and the school authority to find fruitful connections between these kinds of research.

For teacher educators and researchers *understanding* and *improving* are crucial processes. They do not relate only to teachers' learning, in the sense that a deeper understanding of classroom practice implies new ideas for improving practice, and that processes of improvement itself often are the starting point for the motivation to

understand processes better etc. Understanding and improving have a similar meaning for teacher educators acting as teachers in a teacher education context. It is valuable to understand the interaction processes within the teacher education courses and to use these processes as learning environments for learning about teacher education (be it in a joint reflection process with the participants or as an individual reflection process, maybe supported by another teacher educator as “critical friend”). This new understanding can lead to an improvement of one’s own teacher education practice, for example, through designing modified learning environments in the future or to draw immediate conclusions through changing the programme of a running course or unit. The task of researchers is mainly to try to achieve a better understanding through investigating into a topic or situation very deeply. However, also having in mind the role of being a teacher educator, it seems valuable to find links between the results of that research and its application in teacher education. In particular, investigations into teachers’ beliefs, knowledge, and practice seem to be very important, however two challenging questions remain: How can we use this new knowledge for designing learning environments in teacher education? What do we know about our beliefs, knowledge, and practice and its influence on the impact on the success of teacher education? A closer look at the interplay between the processes of understanding and improving might bring us more insight into researchers’ and teacher educators’ learning processes, which are then a crucial factor for (student) teachers’ learning, and which in turn have a big influence on students’ learning.

All in all, we can stress that all three *domains of learning* are closely linked with one another and that there is no easy learning transfer mechanism, for example, from researchers’ results and teacher educators’ perfect learning environments for teachers, to teachers’ recipes for designing good teaching, and finally to predicted success of students’ learning of mathematics. Moreover, we are embedded in a series of challenges concerning different kinds of learning, teaching, investigating, and reflection on our own beliefs and values. This shows one element of the complexity of (research in) teacher education. However, there are also other factors contributing to this complexity.

In the following, we confine ourselves to some remarks on the *complexity of research into teachers’ practice*, for example through investigations of academic researchers into teachers’ practice. We do not refer to the different kinds of

approaches, methodologies, the number of teachers whose practice is observed and the kind of involvement of teachers in this research. We are more interested in making clear the complexity from a more general point of view.

Firstly, such research can be mainly focused on an actual inventory of teachers' practice. Their practice, as pointed out several times throughout the book, is deeply influenced by their beliefs, conceptions, and knowledge. Therefore the investigations could take into consideration the influence of teachers' beliefs, conceptions, and knowledge. Another issue to consider is the question in what way the research itself (e.g. through observations and interviews or questionnaires) influences the picture of teachers' practice. The question of how the teaching is embedded in a context or milieu (culture, nation, school system, curriculum, school, class, colleagues, principal, family, etc.) also seems very important. Another decisive factor is the context in which the investigations are carried out (e.g. a funded research project with experienced teachers as volunteers or an obligatory in-service course) or the different contexts in which the different teachers work.

Secondly, such research can be focused on a learning process of teachers, for example, starting with an actual inventory of teachers' practice, investigating into the continuous progress, and finally doing an inventory again in order to be able to compare teachers' practice at the beginning and at the end of the research and to present a theoretical framework for explaining the processes and factors which led or influenced (or didn't lead or influence) teachers' change and growth. This increases the complexity of research enormously.

Thirdly, such research can – in addition to its focus on the change process of teachers – also focus on the learning process of the teacher educators and researchers. This could include a variety of questions, for example: What beliefs are behind our research questions and hypotheses (as a mirror of our goals)? Do our research questions and hypotheses change? If yes, when, how, and why? How does our knowledge grow? What are the decisive factors? How dependent is it on context or milieu? How can we learn from each other (from researchers, teacher educators, and teachers)?

Learning from investigations

Finally, we will briefly discuss different possibilities of learning from investigations in teacher education. What prospective teachers and practising teachers may learn from research is far more than the outcomes; but also far less than some outcomes may suggest to (student) teachers. Outcomes are often abstractions from a concrete process that occurred in the classroom. We like to stress the value of this process for teacher education. Each stage may contribute to the learning process of student teachers, but the whole process of investigation may be of specific interest from the perspective of future action research in our/their own classrooms. Let us look at some stages and link this to several chapters in our book.

Learning from research questions

Professional researchers spend much time formulating the core problem and derived research questions. For example, Jaworski (chapter 5) emphasised the starting points that inspired in-service teachers to carry out action research. Some good and particularly personal reasons were mentioned. The student teachers in MILE (subchapter 4.3) needed much time and discussion to come to leading questions. Sometimes professional researchers find their reasons and questions in the literature, for teacher education the reasons and questions found in practice are worthwhile. For example, the problem of ‘organising subject matter’ in relation to the needs of students and the requirements of the curriculum standards, or quite simple, what questions you may ask according to the didactics of explaining absolute value, as elaborated by Perrin-Glorian (subchapter 4.2). And how to distinguish between roles of teachers in mathematics and computer science in order to find out about their beliefs in the domain, as investigated by Berger (subchapter 2.3).

Learning from research methodology

Obviously qualitative research methods are preferable to quantitative ones. In the preceding chapters are many illustrations which show the usefulness of approaches like observing classroom teaching, interviewing teachers and students, designing appropriate instruments (see e.g. Contreras, Climent, and Carrillo, subchapter 2.2), pre-structuring interviews, carrying out narrative analysis (see e.g. Oliveira,

subchapter 4.1), etc. It is particularly the description of the research activities, which may contribute to the learning of future teachers, because it gives them a new view of what is going on in practice, from a more or less theoretical point of view.

Learning from elaborating the data

Professional researchers collect data from the perspective of their research goals. Novice researchers sometimes collect data (by self made questionnaires or open interviews) without a clear perspective. The real problem then is how to handle the answers or the audio tapes. Collecting data in classrooms, carried out by practitioners, needs a clear perspective from the very beginning which also provides an organiser for analysing and presenting the data. The research by Berger (subchapter 2.3) and that by Oliveira (subchapter 4.1) show good illustrations of what is meant here.

Learning from presenting the research

Research ‘in the context of teacher education’ has to be presented in more detail than it usually has to be done. Moreover the original sources of data, the instruments used, explanations of (statistical) methods etc. must be accessible for users. The presentation of the research of Contreras, Climent, and Carrillo (subchapter 2.2) partly meets these conditions.

It will be not very difficult to formulate conditions for using the outcomes of research and also for reviewing research. Maybe, in continuing the collaboration and discussions in *Thematic Group 3*, a ‘manifesto of research in the context of teacher education’ may be framed. This idea leads us the last section of this book.

Future research and collaboration

The research and development activities discussed during the meetings of *Thematic Group 3* offered a rich domain full of reasons for future research and collaboration.

As far as we see it now, in Fall 1998, supported by a lot of ideas mentioned in former chapters, it will be helpful to form a *research community* of mathematics teacher educators as an essential part of ERME. This community would have to do several pre-starting tasks:

- Taking an inventory of European research related to mathematics teacher education.
- Setting up a research agenda for the next couple of years, organised in a framework of research ‘platforms’.
- Creating a discussion document in order to come to a ‘manifesto’ for ‘research in the field of mathematics teacher education’.
- Organising a network for communication, co-operation, and collaboration between members of the community, supported by electronic mailing, video-conferencing, etc.

Maybe it will be possible to launch the new community during CERME 2 (planned in the year 2000), maybe earlier. In the meantime collaborative research projects in the field of teacher education may be organised from the perspective of the community. Summing up inspiring ideas mentioned in the chapters before, and adding some new ones, we find the following activities to be valuable steps towards joint efforts to promote mathematics teacher education in European countries:

- Carrying out comparative studies on the actual mathematics teachers’ education curricula and plans.
- Studying social roles allotted to mathematics teachers and mathematics teacher educators and researchers.
- Studying the different cultures (beliefs, knowledge, experience, values, etc.) in mathematics teacher education.
- Sharing doctoral programs and dissertations, and doctoral tutoring.
- Documenting and reflecting on research and development projects in mathematics education where (student) teachers were involved actively and the project’s impact on (student) teachers’ growth was investigated.
- Investigating the construction, growth, and change of teachers’ beliefs, conceptions, knowledge, practices, and ways of reflections, and their interconnection.

- Designing digital representations of “classroom practice” in European countries, sharing these representations and evaluating the use in teacher education.
- Studying the interconnection between mathematics classroom development and school development (importance of climate, leadership, collaboration among students, teachers, and parents, integration of new colleagues, ...).
- Organising conferences, meetings, summer schools, and seminars on special themes.
- Initiating and supporting networks of “reflective practitioners” who investigate into their teaching and share their findings with “critical friends” (other teachers, academic researchers, ...).
- Collaborating with existing networks on teacher education, for example CARN, The Collaborative Action Research Network, or TNTEE, The Thematic Network for Teacher Education in Europe.
- Writing a book about paradigms of “good classroom practice” which can be used in pre- and in-service teacher education and in other forms of professional development of mathematics teachers.
- Writing a handbook on action research methodologies which can be used in teacher education or by teachers themselves in order to find starting points for systematic and self-critical reflection on their own practice.

Having started in chapter 1 with two quotations, we will finish the book in the same way. The first one has to do with our understanding of the role we ascribe to research in teacher education. The quotation from Fenstermacher (1994, p. 51) relates to teachers’ knowledge, but it expresses also a general attitude of mind to look for a bridge between research results and the practice of teachers:

“The challenge for teacher knowledge research is not simply one of showing us that teachers think, believe, or have opinions but that they know. And, even more important that they know that they know.”

The second quotation highlights that collaboration, in particular concerning teacher education, should also include more intensive collaboration with (student)

teachers, and should also promote collaboration among (student) teachers and investigate its effect on their professional growth. Here we need more openness than the traditional attitude that Bishop (1992, p. 719) criticises as follows:

“Yet, in reality, it seems as if researchers are not talking directly to the other people who are key aspects of the educational system, but are increasingly talking to each other. Research journals are edited by researchers, with editorial boards of researchers. And they are increasingly read only by researchers.”

It is evident that this book is mainly written in the context of a conference for researchers, and maybe most readers will also be researchers. However, it contains also a clear focus on self-reflection on our activities in the field of teacher education. We think that self-reflection is an important step to broaden our understanding of (research in) teacher education, to see things from another point of view, to look for new approaches, to experiment with our activities, and to initiate joint reflections with or among student teachers, teachers, teachers educators, and researchers. In this way, we hope, that this book (for example, through discussing parts of it in courses etc.) has some impact on our beliefs and our practices and generates an impulse for further reflections on the field of teacher education.

References

- Altrichter, H., Posch, P. & Somekh, B. (1993). *Teachers Investigate their Work. An Introduction to the Methods of Action Research*. London: Routledge.
- Bishop, A. (1992). International Perspectives on Research in Mathematics Education. In D. Grouws (Ed.) *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan, pp. 710–723.
- Cooney, T.J. (1994a). Research and Teacher Education: In Search of Common Ground. *Journal for Research in Mathematics Education*, 25, pp. 608–636.
- Elbaz, F. (1991). Research on Teachers' Knowledge: The Evolution of a Discourse. *Journal of Curriculum Studies*, 23, pp. 1–19.
- Fenstermacher, G. (1994). The Knower and the Known: The Nature of Knowledge in Research on Teaching. *Review of Research in Education*, 20, pp. 3–56.
- Krainer, K. (1994). PFL-Mathematics: A Teacher Inservice Education Course as a Contribution to the Improvement of Professional Practice in Mathematics Instruction. In J.P. da Ponte & J.F. Matos (Eds.) *Proceedings of the 18th Conference of the International Group for the*

Psychology of Mathematics Education (PME). Lisbon, Portugal: University of Lisbon, Vol. III, pp. 104–111.

Lampert, M. & Ball, D.L. (1998). *Teaching, Multimedia, and Mathematics. Investigations of real practice*. New York: Teachers College.

Schön, D.A. (1991). *The reflective practitioner*. 2nd edition. London: Temple Smith.

Thoma, E. (1998). Documentation of CERME 1. Unpublished report. Klagenfurt, Austria: University of Klagenfurt / IFF.

Wideen, M., Mayer-Smith, J. & Moon, B. (1998). A Critical Analysis on Learning to Teach: Making the Case for an Ecological Perspective on Inquiry. *Review of Educational Research*, Vol. 68 (2), pp. 130–178.

Name Index

- Adler, J. 47, 49, 204, 213, 220
 AKU/IED 18, 35
 Altrichter, H. 206, 207, 215, 216, 219, 230, 241
 Anderson, D.S. 58
 Arnal, J. 52, 58
 Artaud, M. 9
 Artigue, M. 82, 92
 Askew, M. 81, 92, 93
 Austin, S. 81, 92
 Ball, D.L. 14, 36, 83, 92, 95, 116, 141, 191, 197, 198, 233, 242
 Bardin, L. 52, 53, 58
 Bazzini, L. 19, 35
 Beck, C. 64, 77
 Bereiter, C. 188, 197, 198
 Berger, P. 9, 34, 48, 63, 64, 76, 77, 78, 79, 92, 93, 119, 141, 237, 238
 Berliner, D. 21, 36
 Biddle, B.J. 52, 58
 Biehler, R. 19, 36
 Bishop, A. 241
 Block, D. 84, 93
 Boero, P. 107, 141
 Borko, H. 47, 48, 112, 115, 141
 Bourdieu, P. 173, 176, 185
 Breen, C. 19, 36
 Breiteig, T. 17, 36
 Brekke, G. 17, 36
 Bromme, R. 21, 36, 117, 142
 Brousseau, G. 148, 171, 173, 175, 185
 Brown, C.A. 44, 45, 46, 47, 48, 83, 93
 Brown, J.S. 130, 142
 Brown, M. 81, 92, 93
 Brown, S. 17, 21, 36
 Bruner, J. 161, 168
 Burton, L. 216, 220
 Calderhead, J. 202, 220
 Campos, T. 27, 35
 Canavarro, P. 96
 Canguilhem, G. 139, 142
 Cannizzaro, L. 9, 34, 79
 Carr, W. 128, 142
 Carrillo, J. 9, 24, 34, 48, 51, 52, 53, 55, 58, 59, 99, 118, 136, 237, 238
 Carter, K. 131, 132, 142, 161, 168
 Chapman, O. 47, 49, 55, 59, 84, 93
 Chevallard, Y. 9, 148, 173, 174, 175, 183, 185
 Christou, C. 84, 96
 Clandinin, D.J. 102, 133, 142
 Clark, C. 53, 59, 83, 93
 Clarridge, P. 36
 Climent, N. 34, 48, 51, 237, 238
 Cockroft, W.H. 106, 142
 Cohen, D. 150, 191, 198
 Cohors-Fresenborg, E. 1
 Collins, A. 130, 142
 Comiti, C. 14
 Confrey, J. 81, 93
 Conne, F. 9, 105
 Contreras, J. 45, 129, 142
 Contreras, L.C. 9, 34, 45, 48, 49, 51, 52, 55, 58, 59, 79, 237, 238

- Cooney, T.J. 14, 15, 16, 17, 21, 36, 43, 44, 45, 46, 47, 48, 49, 83, 84, 93, 231, 241
- Coriat, M. 9, 24, 34, 99
- Cortazzi, M. 132, 133, 142, 162, 168
- Crawford, K. 47, 49, 204, 213, 220
- Cuadra, F.G. 90, 93
- Cushing, K. 36
- D'Ambrosio, B. 27, 35
- D'Ambrosio, U. 19, 35
- Dapueto, C. 107, 141
- Dávila, M. 84, 93
- Dawson, S. 220
- De la Cruz, Y. 15, 36
- Del Rincon, D. 58
- DGEBS 158, 168
- Dolk, M. 19, 37, 187, 197, 198
- Dougherty, B. 55, 59, 84, 94
- Doyle, W. 20, 36
- Dubinsky, E. 94
- Duguid, P. 130, 142
- Duval, R. 175, 185
- Edwards, D. 135, 142
- Egan, K. 152, 169
- Eisenhart, M. 168
- Eisner, E.W. 128, 142, 216, 220
- Elbaz, F. 85, 93, 131, 142, 161, 168, 189, 198, 229, 241
- Elliott, J. 20, 37, 111, 128, 142, 204, 206, 207, 220
- Elstgeest, J. 115, 142
- Ernest, P. 81, 84, 85, 93, 105, 142
- Fakir-Mohammed, R. 9, 35, 204, 208, 210
- Farnham-Diggory, S. 114, 115, 142
- Feiman-Nemser, S. 135, 143
- Feldman, A. 111, 138, 143
- Fennema, E. 86, 94, 102, 143
- Fenstermacher, G. 103, 143, 240, 241
- Fernandes, D. 89, 94, 135, 143, 168
- Fetterman, D.M. 160, 168
- Feynman, R. 216, 220
- Fischer, R. 217, 220
- Franco, A. 84, 94
- Franke, M.L. 80, 89, 94, 95, 102, 143
- Frasier, C. 16, 37
- Freudenthal, H. 151, 156, 157, 168, 188
- Furinghetti, F. 84, 94
- García, J.N. 128, 143
- Gardiner, A. 118, 143
- Gaulin, C. 55, 59
- Gellner, E. 140, 143
- Giddens, A. 88, 94
- Giménez, J. 19, 37
- Gimeno, J. 129, 143
- Goetz, J.P. 52, 59
- Goffree, F. 9, 19, 24, 34, 35, 37, 115, 142, 148, 149, 157, 169, 187, 192, 195, 197, 198, 223, 233
- Gomez, C. 82, 97
- Good, T.A. 55, 59, 84, 94
- Gravemeijer, K.P.E. 188, 198
- Grouws, D.A. 18, 37, 55, 59, 84, 94
- Guba, E.G. 52, 59, 64, 78
- Gudmundsdottir, S. 132, 143, 161, 169, 190, 197, 198
- Guevara, F. 51, 59
- Guimarães, F. 103, 143
- Guimarães, H. 96
- Habermas, J. 126, 143
- Harel, G. 82, 94
- Harlen, W. 115, 142
- Hölzl, R. 17, 37
- Howson, G. 15, 37
- Hoyles, C. 17, 37, 84, 94
- Hruska, E. 220
- Isenberg, J.P. 190, 198
- Jaji, G. 13, 37
- Jalongo, M.R. 190, 198
- Jaworski, B. 9, 24, 35, 105, 201, 202, 203, 204, 210, 211, 213, 220, 237
- Johnson, D. 92, 93
- Jones, D. 17, 36
- Kadunz, G. 17, 37

- Kautschitsch, H. 37
Keitel, C. 16, 37
Kelchtermans, G. 88, 94
Kemmis, S. 128, 142
Kilmann, R. 17, 38
Kilpatrick, J. 13, 14, 37, 55, 59
Kliment, F. 214, 219, 220
Kline, M.S. 105, 143
Kogelman, S. 80, 94
Korcz, M. 9
Krainer, K. 9, 13, 14, 15, 16, 19, 21, 24, 33, 35, 36, 37, 38, 86, 95, 128, 143, 203, 204, 206, 208, 210, 213, 214, 215, 217, 220, 221, 223, 230, 241
Kuhs, T.M. 83, 95
Kyeleve, J.I. 82, 95
Lampert, M. 190, 191, 197, 198, 233, 242
Lappan, G. 135, 143
Latorre, A. 58
Lave, J. 130, 144
Leal, L.C. 96
Lecompte, M.D. 59
Lehrer, R. 89, 95
Lerman, S. 85, 86, 95
Lincoln, Y.S. 52, 59, 64, 78
Lindgren, L. 80, 95
Lipson, M. 112, 144
Livingston, C. 112, 115, 141
Llinares, S. 19, 37, 58, 59, 120, 144
López de Ceballos, P. 129, 144
Maier, H. 64, 77
Malle, G. 217, 220
Marcelo, C. 84, 95
Martín, R. 144
Martínez, F. 93
Mason, J. 90, 95
Matos, J.F. 96
Mattingly, C. 131, 133, 144
Mayer, C. 219, 221
Mayers, C. 86, 95
Mayer-Smith, J. 225, 242
McEwan, H. 152, 169
McFee, G. 127, 144
Mercer, N. 135, 142
Middleton, J.A. 80, 95
Mitroff, I. 17, 38
Moliner, M. 105, 144
Moon, B. 225, 242
Mousely, P. 19, 38
Mousley, J. 19, 38
NCTM 117, 144, 187
Nebres, B. 15, 38
Nespor, J. 44, 49
Neubrand, M. 17, 37
Norman, A. 82, 96
Novak, J.D. 104, 144
Nygara, L. 13, 37
Oliveira, H. 9, 34, 99, 148, 149, 156, 237, 238
Olson, J. 102, 144
Onk, W. 9, 34, 148, 187, 198, 233
Ossimitz, G. 37
Othman, M.Y. 202, 221
Pajares, M.F. 44, 49
Parenti, L. 107, 141
Paris, S.G. 112, 144
Patton, M.C. 160, 169
Pehkonen, E. 9, 63, 64, 78, 80, 86, 89, 96
Perrin-Glorian, M.J. 9, 34, 148, 171, 172, 176, 181, 183, 186, 237
Peschek, W. 217, 221
Peter, A. 18, 38
Peter-Koop, A. 17, 38
Peterson, P. 53, 59
Philippou, G.N. 84, 96
Piber, C. 220
Pimm, D. 159, 169
Pinnegar, S. 36
Pinto, M. 82, 96
Pires, M.C. 169
Polanyi, M. 88, 96, 202, 221

- Ponte, J.P. 9, 15, 24, 34, 38, 43, 44, 45, 49, 79, 80, 87, 88, 90, 96
- Porlán, R. 119, 144
- Posch, P. 19, 38, 86, 95, 203, 204, 206, 207, 217, 219, 220, 221, 230, 241
- Racher, K. 220
- Reys, R.E. 169
- Ribeiro, A. 169
- Rivero, A. 119, 144
- Robinson, V.M.J. 111, 144
- Robson, M. 13, 37
- Rodhes, V. 92, 93
- Rokeach, M. 88, 96
- Rorty, R. 105, 144
- Russell, T. 135, 144
- Sabers, D. 36
- Safuanov, I. 9, 34, 79
- Sánchez, V. 19, 37
- Santos-Wagner, V. 17, 38
- Saraiva, M. 9
- Scardamalia, M. 188, 197, 198
- Schneider, E. 37
- Scholz, R.W. 19, 36
- Schön, D.A. 20, 26, 38, 85, 96, 108, 126, 127, 129, 144, 150, 190, 198, 202, 207, 221, 230, 231, 242
- Schultz, K. 18, 37
- Schwank, I. 1
- Seeger, F. 19, 38
- Semenov, A. 9
- Serrazina, M. 9, 34, 148, 149, 156, 158
- Shaw, K. 83, 97
- Shulman, L.S. 21, 38, 112, 113, 117, 119, 120, 144, 190, 197, 198
- Simon, M.A. 116, 145
- Somekh, B. 206, 207, 219, 230, 241
- Sowell, E.J. 169
- Stein, P. 36
- Steinbring, H. 19, 38
- Stenhouse, L. 20, 38, 128, 145
- Strässer, R. 19, 36
- Streefland, L. 168, 169, 188, 198, 199
- Sullivan, P. 19, 38
- Szendrei, J. 9, 34, 148, 149
- Tall, D. 82, 96, 97, 101, 145
- Teixeira, A. 84, 94
- Tesch, R. 64, 78
- Theule-Lubienski, S. 135, 143
- Thom, R. 43, 49
- Thoma, E. 9, 28, 35, 39, 204, 205, 210, 227, 242
- Thompson, A.G. 15, 43, 44, 45, 46, 49, 64, 78, 80, 81, 83, 97, 102, 145
- Thompson, V. 15, 36
- Tillema, H.H. 87, 97
- Törner, G. 63, 64, 78, 81, 96, 97
- Treffers, A. 187, 188, 199
- Truran, K. 97
- Turkle, S. 91, 97
- Vale, I. 135, 143
- Valero, P. 82, 97
- Vergnaud, G. 104, 145
- Vermunt, J. 197, 199
- Verschaffel, L. 82, 97
- Viennot, L. 92
- Vollrath, H.J. 27, 39
- Warren, J. 80, 94
- Wenger, E. 130, 144
- Whitehead, A.N. 106, 145
- Wideen, M. 225, 242
- William, D. 92, 93
- Wilson, J. 127, 128, 145
- Wilson, S. 116, 117, 145
- Winkelmann, B. 19, 36
- Wittrock, M.C. 52, 59
- Wixson, K. 112, 144
- Wood, T. 220
- Zeichner, K. 129, 145
- Zuber-Skerrit, O. 111, 129, 145