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The Lensing Effect a Proof of Large Scale Entanglement in Holographic CFTs for Non-Black Holes Objects

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Abstract

Many experimental works at small scale (photons, electrons, etc.) try to confirm the entanglement, in this work we will show that the Einstein ring could be a such proof of entanglement but at very large scale (cosmic objects), when, due of a QCD operation (Schwinger effect) inside nucleons is infused in quantum a flux of energy (pulse) that corresponds to stress-energy tensor onto attached nucleons as of n -sheeted Riemann surfaces glued to the boundaries (CFT), there producing a strain of space—the lensing effect. This approach is tested on light lensing around objects (Earth).

1. Introduction

In my recent works, see in the following, I found that a Black Holes (BH) could be assimilated with space-time region created at the epoch of EWPT (more QGP), where the inside is filled with quarks open strings, which rotate in equilibrium with an accretion disk, so which don't eat nothing, it look more like SATURN planet with its rings, in rest is a imagination about being a monster!

“Stefan Mehedinteanu. More On ”What Would Be Inside of M87* to Explain the Event Horizon Telescope (EHT) Results ?”. 2019. hal-02263360”,

“Stefan Mehedinteanu (Issue 11), The Entanglement in Holographic CFTs- an Alternative Explanation of the Binary Black Hole Merger (LIGO) Experiment, International Journal of Advanced Research in Physical Science (IJARPS) Volume 5, Issue 11, 2018, PP 31-42 ISSN No. (Online) 2349-7882”

Now, we know that the light is bended by planets, stars, any structured matter objects etc. that is explained by geodesic of spacetime deformation, since the quarks are confined in thin tubes due of gluonic confinement inside nucleons, they don't radiate bit threads as in case of BH, for that we have also an explanation in entanglement theory, I will present it in this paper.

2.. *Universality of Gravity from Entanglement*

In the following we realize a compilation of some articles which describe the role of entanglement in the explanation of the “distance effect” mainly in the transfer of stress-energy to the boundary of the objects.

Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles are generated or interact in ways such that the quantum state of each particle cannot be described independently of the others, even when the particles are separated by a large distance—instead, a quantum state must be described for the system as a whole.

According to AntiDeSitter/Conformal Field Theory (AdS/CFT) correspondence, gravitational theories on AdS3 space of radius R are dual to (1+1)D CFTs with the central charge (c), see Fig.1 reproduced from ref. [6].

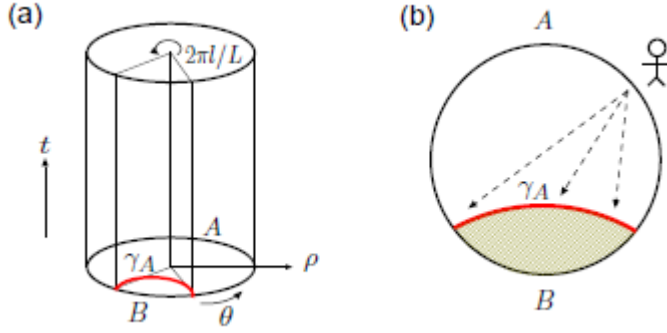


FIG.1 from ref [6] : (a) AdS3 space and CFT2 living on its boundary and (b) a geodesics γ_A as a holographic screen.

With the central charge

$$c = \frac{3\mathfrak{R}}{2G_N^{(3)}}$$

, the entropy becomes
$$S = \frac{c}{3} \log \left(e^{\frac{L}{a}} \sin \left(\frac{\pi d}{L} \right) \right)$$

If the gravitational theory in the bulk is Einstein gravity, then the horizon entropy is given by the usual Bekenstein-Hawking formula, $S = \frac{Ac_{light}^3}{4\eta G}$, and this construction provides a derivation of the Ryu-Takayanagi prescription [6] for a spherical entangling surface.

Emergence of cosmological Friedmann equations from quantum entanglement

A link between the semiclassical Einstein equation and a maximal vacuum entanglement hypothesis is established [5]. The hypothesis asserts that entanglement entropy in small geodesic balls is maximized at fixed volume in a locally maximally symmetric vacuum state of geometry and quantum fields. A qualitative argument suggests that the Einstein equation implies validity of the hypothesis. A more precise argument shows that, for first-order variations of the local vacuum state of conformal quantum fields, the vacuum entanglement is stationary if and only if the Einstein equation holds. For nonconformal fields, the same conclusion follows modulo a conjecture about the variation of entanglement entropy. The generalized second law (GSL) thus points to a deep link between vacuum entanglement and the Einstein equation [5].

First approach

In [4], [5] is studied the deep connections between the concepts of quantum information theory and cosmology. Employing Fermi normal coordinates and conformal Fermi coordinates, we follow these works to construct a relation between Friedmann equations of Friedmann-Lemaitre-Robertson-Walker (FLRW) universe and entanglement. Fermi normal coordinates (FNC) are determined by an orthonormal reference frame $e_{(\alpha)}^\mu$. $e_{(0)}^\mu$ is the timelike unit vector that is

tangent to the central geodesic O. The point $P(x_F^\mu)$ is found by first following O for a proper time τ and following a certain orthogonal geodesic at a proper distance λ .

Friedmann equations are derived with the first law of entanglement under the assumption that entanglement entropy in a geodesic balls is maximized at fixed volume. More brief, we can expand the spacetime elements as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x_F^i)$, so in the Fermi normal coordinates (x_F^i) the metric becomes the following equations

$$g_{00} = -1 - {}^F R_{0i0j}(O) x_F^i x_F^j$$

where ${}^F R_{ikjl}$ denotes the projection of the Riemann tensor.

This leads a new presentation of the FLRW metric in Fermi normal coordinates (for $|x| \ll H^{-1}$)

$$ds^2 = -\left(1 - (H(t_F) + H(t_F)^2) \bar{x}_F^2\right) dt_F^2 + \left[\delta_{ij} - \left(\frac{K}{a^2} + H(t_F)^2\right) \frac{x_F^i x_F^j}{3} \right] dx_F^i dx_F^j$$

The FNC obtained here can also be evaluated via g_{00} . In particular, the spatial components of the Ricci scalar can be easily evaluated as

$${}^F R_{ij}^{ij} = 6 \left(\frac{K}{a^2} + H(t_F)^2 \right), \quad i, j \neq 0$$

The area deficit

Let us first consider the FNC system central at the geodesic O in a spacetime of dimension d.

The geodesics sending out from O orthogonal to u^a forms a (d-1) dimensional spacelike ball Σ . We assume that l is the radius of the ball. Consider a Fermi normal coordinate system based at O, with the timelike coordinate be x_F^0 and spacelike ones $x_F^i = m^i$, where r is the geodesic distance and n^i is a unit vector at O yielding $\delta_{ij} n^i n^j = 1$. Assume the radius of the ball is much smaller than the local curvature length (i.e. $l \ll H^{-1}$). Note that this condition can be relaxed by considering conformal Fermi coordinates.

More briefly, we can expand the spacetime elements as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x_F^i)$ on Σ is given in [4]. The volume element of Σ to the second order of the FNC coordinates is given by

$dV = \sqrt{h} d^{d-1} x$; h the spatial metric h_{ij} on Σ is expressed in terms of R_{ijkl} - the spatial components of the spacetime Riemann tensor evaluated at o.

Then, the area variation at fixed volume satisfies [5]

$$\delta_R A_{V,l} = -\frac{\Omega_{d-2} l^d}{2(d^2 - 1)} \mathfrak{R} \quad (a)$$

$$\mathfrak{R} = \frac{1}{R^2} = 2G_{00}; \quad \mathfrak{R} \text{-scalar Ricci curvature}$$

The area deficit can thus also be expressed as

$$\delta A|_V = -\frac{\Omega_{d-2} l^d}{(d^2 - 1)} G_{00}$$

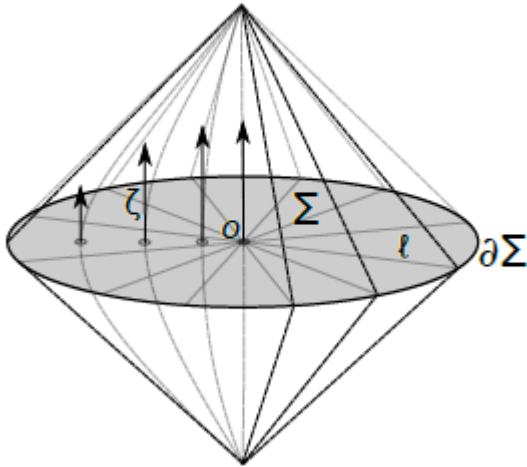
Then, using the Einstein equation $G_{ab} = 8\pi G T_{ab}$, we see that the area deficit is proportional to the energy density,

$$\delta\mathcal{A}|_v = -\frac{8\pi G\Omega_{d-2}l^d}{(d^2-1)}T_{00} \quad (a')$$

For the Friedmann-Robertson-Walker metric, the spatial Ricci scalar is given by $\mathfrak{R} = 6(H^2 + k/a^2)$.

Causal Diamond Structure

The past and future development of the region enclosed by a $(d-2)$ -sphere form a causal diamond $D(\Sigma)$. Each causal diamond is the domain of dependence of a spherical region and the space of all $(d-2)$ -spheres is the same as the space of all causal diamonds.



See FIG. 1 from [5]. Causal diamond, in a maximally symmetric spacetime, for a geodesic ball Σ of radius l with center O and boundary $\partial\Sigma$. The dashed curves are flow lines of ζ , the conformal Killing vector field, whose flow preserves the diamond and which vanishes at the top and bottom vertices and on $\partial\Sigma$. The vectors show ζ at four points of Σ . Also, see the correspondence Figure from [7].

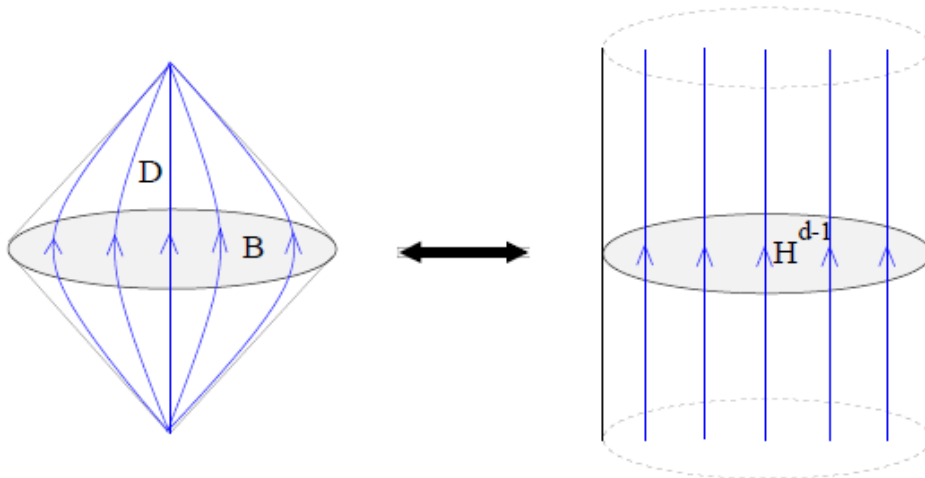


Figure 1. from [7] Causal development D (left) of a ball-shaped region B on a spatial slice of Minkowski space, showing the evolution generated by H_B . A conformal transformation maps D to a hyperbolic cylinder $H^{d-1} \times \text{time}$ (right), taking H_B to the ordinary Hamiltonian for the CFT on H^{d-1} .

Causal development D (left) of a ball-shaped region B on a spatial slice of Minkowski space, showing the evolution generated by H_B . A conformal transformation maps D to a hyperbolic cylinder $H^{d-1} \times \text{time}$ (right), taking H_B to the ordinary Hamiltonian for the CFT on H^{d-1} .

The causal diamond $D(\Sigma)$ and the conformal isometry for the Minkowski line element were discussed in great detail in [5]. It was derived that the unique conformal isometry preserving the diamond with spherical symmetry is generated by the Killing vector [5]

$$\zeta = \frac{1}{2l} \left[(l^2 - r^2 - t^2) \partial_t - 2rt \partial_r \right]$$

For the FLRW geometry with line elements $ds^2 = -dt^2 + a^2(t) d\bar{x}^2$, we can define a conformal time coordinate

$$\eta = \int_{-\infty}^t \frac{dt'}{a(t')}$$

For general FLRW metric ds^2 at arbitrary space curvature, it takes the form

$$ds^2 = -d\xi^+ d\xi^- + \bar{r}^2 d\Omega_2^2$$

with null coordinates

$$d\xi^+ = - \left(dt - \frac{a(t)}{\sqrt{1-kr^2}} dr \right)$$

$$d\xi^- = - \left(dt + \frac{a(t)}{\sqrt{1-kr^2}} dr \right)$$

The conformal Killing vector then takes the form

$$\zeta = \frac{a}{\sqrt{k}} \sin^{-1} \frac{\sqrt{kl}}{a} \left[\left(\cos \frac{\sqrt{kl}}{a} - \cos \frac{\sqrt{k}\xi^+}{a} \right) \partial_{\xi^+} + \left(\cos \frac{\sqrt{kl}}{a} - \cos \frac{\sqrt{k}\xi^-}{a} \right) \partial_{\xi^-} \right]$$

As $k = 0$, the conformal Killing vector field yields a new form

$$\zeta = \frac{1}{2l} \left[(l^2 - \xi^{+2}) \partial_{\xi^+} + (l^2 - \xi^{-2}) \partial_{\xi^-} \right]$$

For non-conformal matter systems, the entanglement entropy in the causal diamond $D(\Sigma)$ is consisted of two contributions, the UV part and the IR part. If we consider the variation of the geometry and the state of the quantum fields, the total variation of the entanglement entropy is contributed from both the area change induced by δg_{ab} (the UV part δS_{UV}) and the IR part δS_{IR} from $\delta|\psi\rangle$. The total entropy variation can be written as

$$\delta\mathcal{S}_{total} = \delta\mathcal{S}_{UV} + \delta\mathcal{S}_{IR} = \eta dA + \delta\mathcal{S}_{IR} \quad (\text{b})$$

If $\eta = 1/4\eta G$, coincides with the variation of Bekenstein's generalized entropy, here interpreted as simply the total entropy in the diamond [5].

Jacobson proposed the maximal vacuum entanglement hypothesis (MVEH) in [5]: when the geometry and quantum fields are simultaneously varied from maximal symmetry, the entanglement entropy in a small geodesic ball is maximal at fixed volume. The MVEH indicates that the above total entropy variation at fixed volume is vanishing at first order.

That is to say

$$\delta\mathcal{S}_{total} = 0$$

They in [4] proceed to derive the Friedmann equations under the MVEH first in the Fermi normal coordinates system and then we turn to conformal normal coordinate

Entanglement and Friedmann equations

In this section, we will make use of the first law of the entanglement $\delta\mathcal{S} = \delta\langle H_\xi \rangle$ and derive the Friedmann equation of the universe.

Now let us assume the quantum states enclosed by the causal diamond are in thermodynamical equilibrium. Above, it is derived the area variation at fixed volume.

This reminds us to take into account the minimal Helmholtz free energy $F = E - TS$ at fixed volume in the ordinary thermodynamic system in equilibrium. The vacuum state of the Quantum Field Theory (QFT) restricted to the diamond can be expressed as a thermal density matrix

$$\rho = Z^{-1} e^{-\frac{K}{T}};$$

where K is related to the modular Hamiltonian and T the temperature. Note that for infinite diamond that overlaps with the Rindler wedge in Minkowski space, T is the Unruh temperature $\eta a/2\pi c k_B$ or Hawking temperature when the acceleration $a = g$, see ref. [19, 20] cited in [4]. The modular free energy is given by $F_K = \delta\langle K \rangle - TS$ denotes quantum expectation value of the modular Hamiltonian and S represents the von Neumann entropy

$S = -k_B \text{Tr} \rho \ln \rho [JK^{-1}]$ in the causal diamond, ρ -density matrix as resulted from Gibbs entropy $S = -k_B \sum p_i \ln p_i$; $p_i = 1/W$ the probability of a microstate; W -the number of microstates, and the Boltzmann eq. $S = k_B \ln(W)$. The variation δF_K must vanish due to the minimal Helmholtz free energy principle. This in turn leads to the minimal Helmholtz free energy principle. This in turn leads to

$$\delta\mathcal{S}_{IR} = \frac{2\pi}{\eta} \delta\langle K \rangle \quad (\text{c})$$

This can be interpreted as the usual Clausius relation for a thermal state. The connections between K and H_ξ were discussed in great detail in [5].

In general K in ρ above, is not a local operator, and does not generate a geometric flow. For a CFT, however, K is equal to H_ζ , the Hamiltonian generating the flow of the conformal boost Killing vector above. (This result is conformally related to the better-known version that holds for any Poincaré invariant QFT restricted to the Rindler wedge.) That is, H_ζ is given by the integral

$$H_\zeta = \int_\Sigma T^{ab} \zeta_b d\Sigma_a$$

If the quantum field state is varied away from the vacuum, with an excitation length scale much longer than the diamond size,

$$l \ll L_{excitation}$$

then $\langle T_{ab} \rangle$ can be treated as constant, and using the Killing field above is find

$$\delta \langle H_\zeta \rangle = \frac{\Omega_{d-2} l^d}{d^2 - 1} \delta \langle T_{00} \rangle \quad (c')$$

If ζ were a Killing vector, δH_ζ would vanish for all fields invariant under the Killing flow. For matter treated as a fluid, it would not vanish, because the fluid potentials do not share the symmetry of the metric. If ζ is only a conformal Killing vector, δH_ζ does not vanish.

In general it receives contributions both from matter and from the gravitational field. If the matter field is not conformal, K is not given by above H_ζ , and we cannot directly use

$$\delta H_\zeta = \delta \oint_{\partial \Sigma} dQ_\zeta \text{ from appendix (D2) of [5], see below. However, suppose that the matter is}$$

described by a QFT with a UV fixed point, so it is asymptotically conformal at short distances, and that, in addition to $l \ll L_{excitation}$, the diamond is much smaller than any length scale in the QFT,

$$l \ll L_{QFT}$$

Then we conjecture—and we shall assume—that $\delta \langle K \rangle$ has the form of that from the Appendix (D2) with an additional term δX that is a spacetime scalar,

$$\delta \langle K \rangle = \frac{\Omega_{d-2} l^d}{d^2 - 1} (\delta \langle T_{00} \rangle + \delta X) \quad (d)$$

where $\delta \langle T_{00} \rangle$ is the change in the energy density in comparison to the vacuum state and δX is a spacetime scalar, and where Ω_{d-2} is the area of the unit $(d-2)$ -sphere, X was first introduced in [5], and see also the comment in [7f] about δX , see below. That it was developed on the base of the next section as from [7d], which could be some scalar operator in the QFT. In what follows we only consider a special case in which $d = 4$.

First law of causal diamond mechanics [5]

Here, if ζ is only a conformal Killing vector, δH_ζ does not vanish. In general it receives contributions both from matter and from the gravitational field. If the theory is general relativity, and the background solution is Minkowski spacetime, then the gravitational field contribution to δH_ζ turns out to be $\delta H_\zeta = -(d-2)\kappa\delta V/8\pi G l$ (see [5], [13]). Since, following [13], the variation of the Lagrangian (L) for vacuum general relativity can be written as

$$\frac{d}{d\lambda} L(g) = E(g) \cdot \frac{d}{d\lambda} g + d\theta \left(g; \frac{d}{d\lambda} g \right) \quad (d')$$

where $E = 0$ are the field equations and θ corresponds to the boundary term that would arise if the variation were performed under an integral sign, namely, see eq.(17) from [13],

$$\theta_{a_1 \dots a_{d-1}} = \frac{1}{16\pi} v^c \varepsilon_{ca_1 \dots a_{d-1}},$$

where $v^a = g^{ac} g^{bd} \left(\nabla_d \frac{d}{d\lambda} g_{bc} - \nabla_c \frac{d}{d\lambda} g_{bd} \right)$

The symplectic current (D - 1)-form ω is defined as

$$\omega \left(g; \frac{\partial}{\partial \lambda_1} g; \frac{\partial}{\partial \lambda_2} g \right) = \frac{\partial}{\partial \lambda_1} \theta \left(g; \frac{\partial}{\partial \lambda_2} g \right) - \frac{\partial}{\partial \lambda_2} \theta \left(g; \frac{\partial}{\partial \lambda_1} g \right)$$

If both perturbations satisfy the linearized equations of motion, then it follows by taking a second, antisymmetrized variation of (d') that ω is closed,

$$d\omega = 0$$

Concretely, $\omega(g; \gamma_1, \gamma_2)$ is given by

$$\omega_{a_1 \dots a_{D-1}} = \frac{1}{16\pi} w^c \varepsilon_{ca_1 \dots a_{D-1}}$$

The symplectic form of the linearized Hamiltonian equations of

motion $W_\Sigma(g; \gamma_1, \gamma_2) = \int_\Sigma \omega(g; \gamma_1, \gamma_2)$ is obtained by integrating ω over a Cauchy surface Σ , or

as in [5]

$$\delta H_\zeta = \frac{1}{16\pi G} \int_\Sigma w^a(g, \gamma_1, \gamma_2) d\Sigma_a,$$

where finally as in [5] $w^t = -2 \frac{d-2}{l} \frac{\delta \sqrt{h}}{h}$, inserting this we have

$$\delta H_\zeta = -(d-2)\kappa \delta V / 8\pi G l; \text{ where, } \kappa \text{ is the surface gravity}$$

$$\kappa = \frac{c^3}{8\pi G M} [s^{-1}],$$

for Planck particle mass results $\kappa = 1.6 \times 10^{42} s^{-1}$.

The surface gravity $\kappa = 1$ is unity for the chosen conformal Killing field, so this yields the result claimed above.

Here, is used the Lie derivative of the Minkowski metric along ζ is

$$\mathfrak{L}_\zeta \eta_{ab} = -(2t/l) \eta_{ab}$$

When combined with the area variation term $-\kappa \delta A / 8\pi G$ this yields $-\kappa / 8\pi G$ times

the area variation $\delta A|_V$ at fixed volume, [Eq. (a), above]. Moreover, the conformal boost energy of the matter integrated over Σ is given by the right hand side of [Eq. (a'), above], when T_{00} is

constant. In this way we see that $\delta H_\zeta = \delta \oint_{\partial \Sigma} dQ_\zeta$ recovers the statement of [Eq. (a'), above], that

addition of matter energy to the diamond decreases the area of the boundary at fixed volume, where, H_ζ is the Hamiltonian generating evolution along the flow of ζ (constructed from the symplectic form evaluated on the Lie derivative of the fields along ζ), Q_ζ is the associated Noether charge (d - 2)-form, and the variation is to any neighboring solution [14].

Now suppose that the quantum fields in the causal diamond is in thermodynamic equilibrium and are dominated by the vacuum state.

The total entanglement entropy variation at fixed volume is zero. That is, from (b), (c), (d) and (a), we obtain

$$\delta\mathcal{S}_{UV} = -\delta\mathcal{S}_{IR} = -\frac{2\pi}{\eta} \delta\langle K \rangle$$

This leads to

$$\frac{\eta}{2} \mathcal{R} = \frac{2\pi}{\eta} (\delta\langle T_{00} \rangle + \delta X)$$

and then

$$3\left(H^2 + \frac{K}{a^2}\right) = \frac{2\pi}{\eta\eta} (\delta\langle T_{00} \rangle + \delta X) \quad (e)$$

We can assume $\delta X = 0$ in what follows, $\eta = 1/4\eta G$. To the first order, we assume the universe is not empty but dominated by some matter and energy. We choose to model the matter and energy in the universe by a perfect fluid. The energy-momentum tensor for a perfect fluid can be written $\delta\langle T_{\mu\nu} \rangle = (\rho + p)U_\mu U_\nu + pg_{\mu\nu}$

where ρ and p are the energy density and pressure (respectively) as measured in the rest Fermi frame, and U_μ is the four-velocity of the fluid. The four-velocity is given by $U^\mu = (1,0,0,0)$.

The energy-momentum tensor can be simply expressed as

$$\delta\langle T_\nu^\mu \rangle = \text{diag}(-\rho, p, p, p)$$

We can then recast (e) into the standard Friedmann equation

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho \quad (f)$$

But, that is *true only for objects like BHs*, and as I will show below it is not the case for *structured matter* of planets, stars etc. as constituted from nucleons when the stress-energy have discrete values in the bulk, see below.

Together with the continuity equation of the perfect fluid

$$\dot{\rho} = -3H(\rho + P)$$

we obtain another Friedmann equation

$$H\dot{K} - \frac{K}{a^2} = -4\pi G(\rho + P) \quad (g)$$

where the dot denotes derivative with respect to t_f . Thus, are obtained the Friedmann equations for the FLRW universe by applying the first law of entanglement. In this sense, the emergences of the Friedmann equations can be considered as a consequence of the change in entanglement $\delta\mathcal{S}_A$ caused by matter $\delta\langle H_A \rangle$. The validity of the derivation is restricted to a small geodesic ball region with radius much smaller than the horizon. To overcome this limitation, in [4] is explored the conformal Fermi coordinates, which is a generalization of FNC. The authors in ref. [17] cited in [7] introduced the conformal Fermi coordinates (CFC) in order to cosmological application. Compared to the FNC, the CFC are valid outside the horizon.

Similar to FNC, the CFC are constructed nearby a timelike central geodesic. In the CFC frame, the lowest order CFC metric is at FLRW spacetime.

Thus are obtained the Friedmann equations (f), (g) in the CFC frame for flat universe, but in case of $K = 0$.

2.3. An approach for structured matter

In the following we use and follow the model described in [6], [8].

For theories with entanglement the entropy is computed by the *Ryu-Takayanagi formula* as

$$S = \frac{\text{Area}_{\text{of}} \gamma_A}{4G^{(d+2)}},$$

where γ_A is the dimensional static minimal surface in AdS_{d+2} whose boundary is given by ∂A , and $G_{(d+2)}^N$ is the $d + 2$ dimensional Newton constant. Intuitively, this suggests that the minimal surface γ_A plays the role of a holographic screen for an observer who is only accessible to the subsystem A. They show explicitly this relation in the lowest dimensional case $d = 1$, where γ_A is given by a geodesic line in AdS3.

The different approach [8]

For the case of a 1+1-dimensional critical system, whose continuum limit is a conformal field theory with central charge c , in [11c] is re-derived the result $S_A \approx (c/3) \log l$ of Holzhey et al. [8] when A is a finite interval of length l in an infinite system, and extend it to many other cases: finite systems, finite temperatures, and when A consists of an arbitrary number of disjoint intervals. For such a system away from its critical point, when the correlation length ξ (inverse mass) is large but finite, is shown that $S_A \approx A(c/6) \log \xi$, where A is the number of boundary points of A [11c]. Thus, in the case of a massive 1+1- dimensional field theory it results $S \approx (c/6) \log \xi/a$ from completely general properties of the stress tensor in the relevant geometry, a is the lattice spacing. This is then verified for a free bosonic massive field.

So, is rederived $S = \frac{c+\bar{c}}{6} \ln \frac{\sum}{\varepsilon}$ from [6], [8]. in a manner which stresses how the entropy arises through coarse graining in real space, without explicit use of mode expansions or modular invariance. The calculation follows ideas of Cardy [13] cited in [8].

Path integral formula for the Entanglement Entropy

Consider a lattice quantum theory in one space and one time dimension [11b]. The lattice spacing is a , and the lattice sites are labelled by a discrete variable x . The domain of x can be finite, i.e. some interval of length L , semi-infinite, or infinite. Time is considered to be continuous. A complete set of local commuting observables will be denoted by $\{\hat{\phi}(x)\}$, and their eigenvalues and corresponding eigenstates by $\{\hat{\phi}(x)\}$ and $\otimes_x |\{\phi(x)\}\rangle$ respectively. For a *bosonic lattice field* theory, these will be the fundamental *bosonic fields* of the theory; for a spin model some particular component of the local spin. The dynamics of the theory is described by a time-evolution operator \hat{H} . The density matrix ρ in a thermal state at inverse temperature β is

$$\rho(\{\phi(x'')\}|\{\phi(x')'\}) = Z(\beta)^{-1} \left\langle \{\phi(x'')''\} \left| e^{-\beta \hat{H}} \right| \{\phi(x')'\} \right\rangle$$

where $Z(\beta) = \text{Tr} e^{-\beta \hat{H}}$ is the partition function

This may be expressed in the standard way as a (euclidean) path integral:

$$\rho = Z^{-1} \int [d\phi(x, \tau)] \prod_x \delta(\phi(x, 0) - \phi(x)') \prod_x \delta(\phi(x, \beta) - \phi(x)'') e^{-S_E} \quad (2.1)$$

where $S_E = \int_0^\beta L_E d\tau$, with L_E the euclidean lagrangian.

The normalization factor of the partition function ensures that $Tr\rho = 1$, and is found by setting $\{\phi(x)''\} = \{\phi(x)'\}$ and integrating over these variables. This has the effect of sewing together the edges along $\tau = 0$ and $\tau = \beta$ to form a cylinder of circumference β .

Let A be a subsystem consisting of the points x in the disjoint intervals $(u_1, v_1), \dots, (u_N, v_N)$. An expression for the reduced density matrix ρ_A may be found by sewing together only those points x which are not in A . This will leave open cuts, one for each interval (u_j, v_j) , along the line $\tau = 0$. See figure 1 from [24] for a pictorial representation of this.

The normalization factor of the partition function ensures that $Tr\rho = 1$, and is found by setting $\{\phi(x)''\} = \{\phi(x)'\}$ and integrating over these variables. This has the effect of sewing together the edges along $\tau = 0$ and $\tau = \beta$ to form a cylinder of circumference β . We may then compute $Tr\rho_A^n$, for any positive integer n , by making n copies of the above, labelled by an integer k with $1 \leq k \leq n$, and sewing them together cyclically along the cuts so that $\phi(x)'_k = \phi(x)''_{k+1}$ and $(\text{and } \phi(x)'_n = \phi(x)''_1)$ for all $x \in A$. Let us denote the path integral on this n -sheeted structure by $Z_n(A)$.

$$Tr\rho_A^n = \frac{Z_n(A)}{Z^n} \quad (2.2)$$

Now, since $Tr\rho_A^n = \sum_\lambda \lambda^n$, where λ are the eigenvalues of ρ_A (which lie in $[0, 1)$), and since

$Tr\rho_A = 1$, it follows that the left hand side is absolutely convergent and therefore analytic for all $Re n > 1$. The derivative wrt n therefore also exists and is analytic in the region. Moreover, if the entropy $S_A = -\sum_\lambda \lambda \log \lambda$ is finite, the limit as $n \rightarrow 1+$ of the first derivative converges to this value.

They conclude that the right hand side of (2.2) above has a unique analytic continuation to $Re n > 1$ and that its first derivative at $n = 1$ gives the required entropy:

$$S_A = \lim_{n \rightarrow 1} \frac{tr_A \rho_A^n - 1}{1 - n} = -\frac{\partial}{\partial n} Tr_A \rho_A^n \Big|_{n=1} = -\frac{\partial}{\partial n} \log tr_A \rho_A^n \Big|_{n=1} = -\lim_{n \rightarrow 1} \frac{\partial}{\partial n} \frac{Z_n(A)}{Z^n}$$

as the eq. from [8] if we taking the trace, respectively,

$$S = -\left(\frac{d}{dn}\right)_{n=1} \frac{Z(n)}{Z(1)^n} = \left(1 - n \frac{d}{dn}\right)_{n=1} \ln Z(n) \quad (2.2')$$

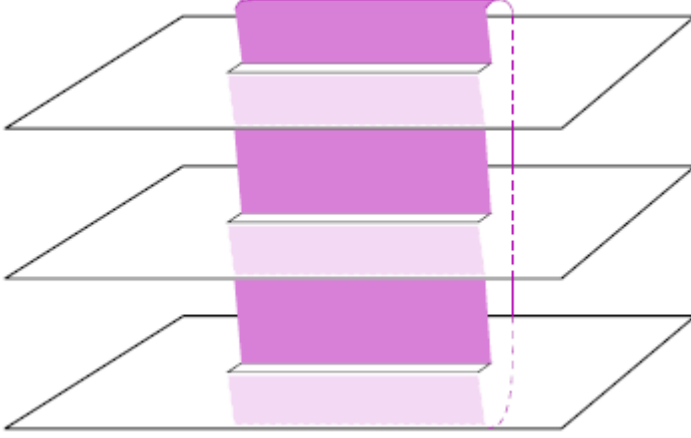


Figure 2 from [24]. A representation of the Riemann surface $\mathfrak{R}_{3,1}$.

Path integral formulation and Riemann surfaces

So far, everything has been for a discrete space domain. We now discuss the continuum limit, in which $a \rightarrow 0$ keeping all other lengths fixed [24]. The points x then assume real values, and the path integral is over fields $\phi(x, \tau)$ on an n -sheeted Riemann surface, with branch points at u_j and v_j . In this limit, SE is supposed to go over into the Euclidean action for a quantum field theory.

We indicate these n -sheeted surfaces with $\mathfrak{R}_{n,N}$ and they are fully defined by the $2N$ branch points u_j and v_j . Whenever the value of n and N is not important, we will simply indicate the surface with \mathfrak{R} .

In the simplest instances it is possible to directly calculate the partition function on a n -sheeted Riemann surface, but in most of the cases this is very difficult. However, the surface we are dealing with has curvature zero everywhere except at a finite number of points (i.e. the boundaries between A and B u_j and v_j above). Since the lagrangian density does not depend explicitly on the Riemann surface \mathfrak{R} as a consequence of its locality, it is expected that the partition function can be expressed as an object calculated from a model on the complex plane \mathbb{C} , where the structure of the Riemann surface is implemented through appropriate boundary conditions around the points with non-zero curvature. Consider for instance the simple Riemann surface $\mathfrak{R}_{n,1}$ needed for the calculation of the entanglement entropy of a single interval $[u_1, v_1]$, made of n sheets sequentially joined to each other on the segment $x \in [u_1, v_1]$, $\tau = 0$.

We expect that the associated partition function in a theory defined on the complex plane $z = x + i\tau$ it can be written in terms of certain “fields” at $z = v_1$ and $z = u_1$.

The partition function (here $L[\phi](x, \tau)$ is the local lagrangian density)

$$Z_{\mathfrak{R}} = \int [d\phi]_{\mathfrak{R}} \exp\left[-\int_{\mathfrak{R}} dx d\tau L[\phi](x, \tau)\right]$$

essentially defines these fields, i.e. it gives their correlation functions, up to a normalization independent of their positions. However in the model on the complex plane, this definition makes them non-local (see for a complete discussion [23]).

Locality is at the basis of most of the results in field theory, so it is important to recover it.

The solution to the problem consists in moving the complicated topology of the

world-sheet \mathfrak{R} (i.e. the space where the coordinates x, τ live) to the target space (i.e. the space where the fields live). Let us consider a model formed by n independent copies of the original model. (Note that n is the number of Riemann sheets necessary to describe the Riemann surface by coordinates on the plane.) The partition function above can be re-written as the path integral on the complex plane

$$Z_{\mathfrak{R}} = \int_{C_{u_1, v_1}} [d\varphi_1 \dots d\varphi_n] \exp \left[- \int_C dx d\tau (L[\varphi_1](x, \tau) + \dots + L[\varphi_n](x, \tau)) \right]$$

where with $\int_{C_{u_1, v_1}}$ we indicated the *restricted* path integral with conditions

$$\varphi_i(x, 0^+) = \varphi_{i+1}(x, 0^-), \quad x \in [u_1, v_1], \quad i = 1, \dots, n$$

where we identify $n + i \equiv i$. The lagrangian density of the multi-copy model is

$$L^{(n)}[\varphi_1, \dots, \varphi_n](x, \tau) = L[\varphi_1](x, \tau) + \dots + L[\varphi_n](x, \tau)$$

so that the energy density is the sum of the energy densities of the n individual copies.

Hence the expression of $Z_{\mathfrak{R}}$ does indeed define local fields at $(u_1, 0)$ and $(v_1, 0)$ in the multi-copy model [23].

Notice that $\tilde{\mathfrak{Z}}_n$ can be identified with \mathfrak{Z}_{-n} (and in fact they were called Φ_n and Φ_{-n} as in [11c], also see below. (see Fig. 2 for the case T).

More precisely, we have as from [25], where $u_1 = a_1, v_1 = a_2$

$$Z(L, M_{n, a_1, a_2}) \propto \left\langle \mathfrak{Z}(a_1, 0) \tilde{\mathfrak{Z}}(a_2, 0) \right\rangle_{L_n, \mathfrak{R}^2}$$

This can be seen by observing that for $x \in [a_1, a_2]$, consecutive copies are connected through $y = 0$ due to the presence of $\mathfrak{Z}(a_1, 0)$, whereas for $x > a_2$, copies are connected to themselves through $y = 0$ because the conditions arising from the definition of $\mathfrak{Z}(a_1, 0)$ and $\tilde{\mathfrak{Z}}(a_2, 0)$ cancel each other.

More generally, the identification holds for correlation functions in the model L on M_{n, a_1, a_2} , this time with an equality sign:

$$\left\langle O(x, y; \text{sheet } _i) \dots \right\rangle = \frac{\left\langle \mathfrak{Z}(a_1, 0) \mathfrak{Z}(a_2, 0) O_i(x, y) \dots \right\rangle_{L_n, \mathfrak{R}^2}}{\left\langle \mathfrak{Z}(a_1, 0) \tilde{\mathfrak{Z}}(a_2, 0) \right\rangle_{L_n, \mathfrak{R}^2}}$$

where O_i is the field in the model $L^{(n)}$ coming from the i^{th} copy of L .

The generalization to Riemann surfaces with more branch points is straightforward, but will not be needed here.

The conformal dimension of branch-point twist fields was calculated in [26]. Consider the model L to be a conformal field theory (CFT). Then also $L^{(n)}$ is a CFT. There are n fields $T_j(z)$ in $L^{(n)}$ that correspond to the stress-energy tensors of the n copies of L , and in particular the sum $T^{(n)}(z) = \sum_{j=1}^n T_j(z)$ is the *stress-energy tensor* of $L^{(n)}$. The central charge of $L^{(n)}$ is nc if c is that of L .

Consider the stress-energy tensor $T(w)$ in L . We can evaluate the one-point function

$\left\langle T(w) \right\rangle_{L, M_{n, a_1, a_2}}$ by making a conformal transformation from z in \mathfrak{R}^2 to w in M_{n, a_1, a_2} (here z and w are complex coordinates, with for instance $z = x + iy$) given by

$z = \left(\frac{w-a_1}{w-a_2} \right)^{\frac{1}{n}}$, see below the continuation.

ENTANGLEMENT ENTROPY IN 2D CONFORMAL FIELD THEORY

Now specialize the discussion of the previous section to the case when the field theory is relativistic and massless, i.e. a conformal field theory (CFT), with central charge c , and initially consider the case of zero temperature [11c].

We show that in this case the ratio of partition functions in (2.2) is the same as the correlation function arising from the insertion of primary scaling operators $\Phi_n(u_j)$ and $\Phi_{-n}(v_j)$, with scaling dimensions $X_n = 2\Delta_n = (c/12)(1-1/n^2)$, into each of the n (disconnected) sheets.

Moreover, this $2N$ -point correlation function is computable from the Ward identities of CFT. *Single interval*

They first consider the case $N = 1$ and no boundaries, that is the case considered by Holzhey et al.[8] of a single interval of length ℓ in an infinitely long 1d quantum system, at zero temperature. The conformal mapping $w \rightarrow \zeta = (w-u)/(w-v)$ maps the branch points to $(0, \infty)$.

This is then uniformised by the mapping $\zeta \rightarrow z = \zeta^{1/n} = ((w-u)/(w-v))^{1/n}$.

This maps the whole of the n -sheeted Riemann surface \mathfrak{R}_n to the z -plane \mathbb{C} . Now consider the holomorphic component of the stress tensor $T(w)$. This is related to the transformed stress tensor $T(z)$ by [22].

$$T(w) = (dz/dw)^2 T(z) + \frac{c}{12} \{z, w\}, \quad (2.3)$$

where $\{z, w\}$ is the Schwarzian derivative $(z'''z' - 3/2z''^2)/z'^2$. In particular, taking the expectation value of this, and using $\langle T(z) \rangle_C = 0$ by translational and rotational invariance, we find

$$\langle T(w) \rangle_{\mathfrak{R}_n} = \frac{c}{12} \{z, w\} = \frac{c(1-(1/n)^2)}{24} \frac{(v-u)^2}{(w-u)^2(w-v)^2} \quad (2.4)$$

This is to be compared with the standard form [22] of the correlator of T with two primary Operators $\Phi_n(u)$ and $\Phi_{-n}(v)$ which have the same complex scaling dimensions

$$\Delta_n = \bar{\Delta}_n = (c/24)(1-(1/n)^2)$$

$$\langle T(w)\Phi_n(u)\Phi_{-n}(v) \rangle_C = \frac{\Delta_n}{(w-u)^2(w-v)^2(v-u)^{2\Delta_n-2}(\bar{v}-\bar{u})^{2\bar{\Delta}_n}} \quad (2.5)$$

where $\Phi_{\pm n}$ are normalized so that $\langle \Phi_n(u)\Phi_{-n}(v) \rangle_C = |v-u|^{2\Delta_n-2\bar{\Delta}_n}$.

Eq. (2.5) is equivalent to the conformal Ward identity [22]

$$\langle T(w)\Phi_n(u)\Phi_{-n}(v) \rangle_C = \left(\frac{\Delta_n}{(w-u)^2} + \frac{\Delta_n}{(w-v)^2} + \frac{1}{w-u} \frac{\partial}{\partial u} + \frac{1}{w-v} \frac{\partial}{\partial v} \right) \langle \Phi_n(u)\Phi_{-n}(v) \rangle_C \quad (2.6)$$

In writing the above, we are assuming that w is a complex coordinate on a single sheet C , which is now decoupled from the others. We have therefore shown that

$$\langle T(w) \rangle_{\mathfrak{R}_n} \equiv \frac{\int [d\phi] T(w) e^{-S_E(\mathfrak{R}_n)}}{\int [d\phi] e^{-S_E(\mathfrak{R}_n)}} = \frac{\langle T(w)\Phi_n(u)\Phi_{-n}(v) \rangle_C}{\langle \Phi_n(u)\Phi_{-n}(v) \rangle_C} \quad (2.7)$$

Now consider the effect of an infinitesimal conformal transformation $w \rightarrow w' = w + \alpha(w)$ of \mathfrak{R}_n which act identically on all the sheets of \mathfrak{R}_n . The effect of this is to insert a factor

$$\frac{1}{2\pi i} \int_C \alpha(w) T(w) dw - \frac{1}{2\pi i} \int_C \bar{\alpha}(w) \bar{T}(\bar{w}) d\bar{w} \quad (2.8)$$

into the path integral, where the contour C encircles the points u and v . The insertion of $T(w)$ is given by (2.5). Since this is to be inserted on each sheet, the right hand side gets multiplied by a factor n .

Since the Ward identity (2.4) determines all the properties under conformal transformations, we conclude that the renormalized $Z_n(A)/Z^n \propto \text{Tr} \rho_A^n$ behaves (apart from a possible overall constant) under scale and conformal transformations identically to the n th power of two-point function of a primary operator Φ_n with $\Delta_n = \bar{\Delta}_n = (c/24)(1 - (1/n)^2)$. In particular, this means that

$$\text{Tr} \rho_A^n = c_n ((v-u)/a)^{-(c/6)(n-1/n)} \quad (2.9)$$

where the exponent is just $4n\Delta_n$. The power of a (corresponding to the renormalization constant Z) has been inserted so as to make the final result dimensionless, as it should be. The constants c_n are not determined by this method. However c_1 must be unity.

Applying the formula (2.2') to (2.9) by differentiating with respect to n and setting $n=1$, we recover the result of Holzhey et al.[8]

$$S = (1-n \frac{d}{dn})_{n=1} \left(\frac{l}{a} \right)^{-(c/6)(n-1/n)} = \left(\frac{l}{a} \right)^0 \left(-n \frac{d}{dn} (c/6)(n-1/n) \left(\frac{l}{a} \right)^{-(c/6)(n-1/n)} \log \frac{l}{a} \right)_{n=1} =$$

$$1 \times (c/6)(2n-0)_{n=1} \times 0 \times \log \frac{l}{a} = \frac{c}{3} \log \frac{l}{a}$$

,where, a is the UV cut off (or lattice spacing) and we set $l = v-u$, and the central charge is

$$c = \frac{3\mathfrak{R}}{2G_N^{(3)}} .$$

In the following we use the w coordinates as in [8], where the system is represented by an annulus restricted to the lower halfplane. The positive real axis is the subsystem where observations are made, and the negative real axis is the rest of the universe. Forming the density matrix of the accessible subsystem we trace over variables on the negative real axis. Then the density matrix is given by a functional integral over fields over the whole annulus, with the indices X and X' of $\rho_{XX'}$, specifying fields on the lower and upper side of the positive real axis. As before we use the *replica trick* as in (2.2')) above

$$S = (1-n \frac{d}{dn})_{n=1} \ln Z(n)$$

where $Z(n) \propto \text{tr} \rho^n$ is the partition function of an annulus covered n times. Extending n analytically to be slightly less than 1, we can interpret $Z(n)$ as the partition function on a cone with $2\pi n$ in angular circumference.

Now we coarse grain the system by taking $\varepsilon \rightarrow (1+\alpha)\varepsilon$. The annulus has outer radius $R_2 = \frac{\Sigma}{\varepsilon}$

which decreases and inner radius $R_1 = \frac{\varepsilon}{\Sigma}$ which increases.

By dimensional analysis $\ln Z$ depends only on the ratio of the two radii, so we can choose to rescale only the outer radius twice as much and keep the inner one fixed.

This is implemented by the rescaling $x^\mu \rightarrow x'^\mu = (1-2\alpha)x^\mu$, in the limit where not only α but also R_1 is treated as small, i.e. we squeeze the inner boundary to a conical singularity.

Using conformal invariance of the path integral measure and the definition of the energy momentum tensor as the generator of coordinate transformations it is easy to show [8]

$$d \ln Z = -\frac{1}{2\pi} \int \langle T_\mu^\nu \rangle \frac{\partial x'^\mu}{\partial x^\nu} d^2 r = \frac{\alpha}{\pi} \int \langle T_\mu^\mu \rangle d^2 r$$

With $\alpha = \frac{\delta\varepsilon}{\varepsilon}$ we find

$$\frac{\partial S}{\partial \ln \varepsilon} = (1-n \frac{d}{dn})_{n=1} \frac{\partial \ln Z(n)}{\partial \ln \varepsilon} = (1-n \frac{d}{dn})_{n=1} \frac{1}{\pi} \int \langle T_\mu^\mu \rangle d^2 r \quad (2.10)$$

We find it illuminating to proceed slightly differently, writing

$$\int \langle T_\mu^\mu \rangle d^2 r = \int \frac{\partial x^\mu}{\partial x^\nu} \langle T_\mu^\nu \rangle d^2 r = \int x^\mu \langle T_{\mu\nu} \rangle dS^\nu = -i \int w \langle T(w) \rangle dw + h.c. \quad (2.11)$$

, where we introduced complex coordinates. Performing the surface integral we are to integrate over the outer surface only. The expectation value of $T(w)$ on a cone with angular circumference $2\pi n$ is easily found by mapping to the cone with $n=1$ which is simply a disc.

We map $w = y^n$ and impose $\langle T(y) \rangle = 0$ on the disc. This is appropriate to our problem, since the primary object of study is the geometric entropy relative to the ground state on the disc. Using the standard transformation formula for a conformal field theory with central charge c , [8], [11(a,b,c)], see eq. (2.12) from [22]

$$T(w) = \left(\frac{\partial y}{\partial w}\right)^2 T(f(y)) + \frac{c}{12} S_y(w); \quad S_y(w) = \frac{y'''y' - \frac{3}{2}(y'')^2}{(y')^2} \quad (2.12)$$

they find

$$\langle T(w) \rangle = \frac{c}{24} \left(1 - \frac{1}{n^2}\right) \frac{1}{w^2}$$

Inserting this in (2.11), we find

$$\int \langle T_\mu^\mu \rangle d^2 r = \frac{c+\bar{c}}{24} \left(1 - \frac{1}{n^2}\right) 2\pi n \quad (2.13)$$

which we insert in (2.10) to find

$$\begin{aligned} \frac{\partial S}{\ln \varepsilon} &= \frac{1}{\pi} \left(1 - n \frac{d}{dn}\right)_{n=1} \left(\frac{c+\bar{c}}{24} \left(2\pi n - \frac{2\pi}{n}\right) \right) = \left(\frac{c+\bar{c}}{12} \left(n - \frac{1}{n}\right) \right)_{n=1} - n \left(\frac{d}{dn} \right)_{n=1} \left(\frac{c+\bar{c}}{12} \left(n - \frac{1}{n}\right) \right) = \\ &= 0 - \frac{c+\bar{c}}{12} \frac{d}{dn} (n^2 - 1)_{n=1} = -\frac{c+\bar{c}}{12} \quad (2) \end{aligned}$$

$$\frac{\partial S}{\ln \varepsilon} = -\frac{c + \bar{c}}{6}$$

By integrating this we recover $S = \frac{c + \bar{c}}{6} \ln \frac{\Sigma}{\varepsilon}$

Notice that in this procedure a finite ε -independent term cannot be excluded. In this derivation the conformal anomaly plays a crucial role. Formally, and classically, the trace of the energy-momentum tensor vanishes as a consequence of conformal invariance; but the necessity of regulating the quantum theory brings in the correction term c .

Another Approach: Entanglement Entropy from Weyl Anomaly

Central charges in CFTs can be defined from the Weyl anomaly (or conformal anomaly) $\langle T_{\mu}^{\mu} \rangle$ [6]. Define the energy-momentum tensor $T^{\mu\nu}$ in terms of the functional derivative of the (quantum corrected) action S with respect to the metric $g_{\mu\nu}$.

$$\text{We can compute } tr_A \rho_A^n = \frac{Z_n}{(Z_1)^n} \quad (\text{k})$$

from the partition function Z_n on the n -sheeted $d + 1$ dimensional manifold M_n as in the 2D case [6], and [11c].

Define the energy-momentum tensor $T^{\mu\nu}$ in terms of the functional derivative of the (quantum corrected) action S with respect to the metric $g_{\mu\nu}$

$$T^{\mu\nu} = \frac{4\pi}{\sqrt{g}} \frac{\delta S}{\delta g_{\mu\nu}}$$

In 2D CFTs, the Weyl anomaly is given by the well-known formula

$$\langle T_{\mu}^{\mu} \rangle = -\frac{c}{12} \mathfrak{R} \quad (\text{l})$$

where \mathfrak{R} is the scalar curvature. We can regard this as a definition of the central charge c in 2D CFTs.

Where l is the length of the subsystem A .

We thus reproduce the known result below.

$$S_A = \frac{c}{3} \log \frac{l}{a} \quad (\text{r})$$

where a is the UV cut off (or lattice spacing) and we set $l = v - u$.

When we are away from a critical point, the logarithmic scaling law Eq. (r) does not persist for $l > \xi$, where ξ is the correlation length (inverse of the mass gap). For large $l (>> \xi)$, the entanglement entropy saturates to a finite value [6], [11c].

$$S_A = A \frac{c}{6} \log \frac{\xi}{a} \quad (\text{s})$$

where A is the number of boundary points (n) that separate A from its complement. Thus, unlike critical (1+1)D systems, the area law holds for the massive case.

To compute the entanglement entropy, we first consider the partition function Z_n on the $d+1$ dimensional n -sheeted manifold M_n . Then we find the trace of ρ^n reduced to the subsystem A is given by the formula (k).

The entanglement entropy can be found by taking the derivative of n with the $n \rightarrow 1$ limit. If we define the length scale of the manifold A by l , then the scaling of l is related to the Weyl scaling [6]. They should be the same at least in the $n \rightarrow 1$ limit. In this way we find similarly with (2.1)

$$l \frac{d}{dl} \log[\text{tr}_A \rho_A^n] = 2 \int d^{d+1} x g_{\mu\nu}(x) \frac{\delta}{\delta_{\mu\nu}(x)} [\log Z_n - n \log Z_1] = -\frac{1}{2\pi} \left\langle \int d^{d+1} x \sqrt{g} T_\mu^\mu(x) \right\rangle_{M_n} + \frac{n}{2\pi} \left\langle \int d^{d+1} x \sqrt{g} T_\mu^\mu(x) \right\rangle_{M_1} \quad (\text{m})$$

When we consider a CFT on $M = \mathfrak{R}^{d+1}$, the second term (i.e. integral on $M = M_1 = \mathfrak{R}^{d+1}$) become obviously vanishing. Below we omit writing the second term explicitly just to make the appearance of expressions simple even if M is a curved manifold. Then the entanglement entropy satisfies

$$l \frac{d}{dl} S_A = -\lim_{n \rightarrow 1} l \frac{d}{dl} \left(\frac{\partial}{\partial n} \log[\text{tr}_A \rho_A^n] \right) = \frac{1}{2\pi} \lim_{n \rightarrow 1} \frac{\partial}{\partial n} \left\langle \int d^{d+1} x \sqrt{g} T_\mu^\mu(x) \right\rangle_{M_n} \quad (\text{n})$$

Eq. (m) can be used to relate the entanglement entropy and central charge in a direct fashion. It is also possible to derive (s) from (n) by noting that

$$m \frac{\partial S_A}{\partial m} = l \frac{\partial S_A}{\partial l} = \frac{1}{2\pi} \lim_{n \rightarrow 1} \frac{\partial}{\partial n} \left\langle \int d^2 x \sqrt{g} T_\mu^\mu \right\rangle \quad (\text{t})$$

When $A = A_{SL}$ (i.e., $A = 1$ in (s)), the integral on the right hand side is evaluated as

$$\int d^2 x \sqrt{g} T_\mu^\mu = -\pi \frac{c}{6} \left(n - \frac{1}{n} \right) \quad (\text{u})$$

by an argument similar to Zamolodchikov's c-theorem [11a,b]. We thus recover (s), with

$$c = \frac{3\mathfrak{R}}{2G_N^{(3)}}$$

We will call eq. (u) as Ryu-Takayanagi formula eq. (4.26) from [[6]

Similarly, but to account for the above ‘‘boundary points’’, we have as follow: we accounts of stress tensors integral by multiplying by a factor $1/2\pi$ corresponding to the conventional normalization of the stress tensor [11c],

$$\frac{2\pi}{\eta} \int d^2 x \sqrt{g} T_\mu^\mu = \frac{1}{2\pi} \pi \frac{c}{6} \left(n - \frac{1}{n} \right) \cdot \frac{1}{\eta}, \text{ or,}$$

$$\frac{2\pi}{\eta} \int d^2 x \sqrt{g} T_\mu^\mu = \frac{3\mathfrak{R}}{2 \cdot 6 \cdot 2G} n \frac{1}{\eta}; \text{ or,}$$

$$\frac{2\pi}{\eta} \int d^2x \sqrt{g} T_\mu^\mu = \frac{2\pi}{\eta} \rho = \frac{3H^2}{2 \cdot 2 \cdot G} n \frac{1}{\eta} \quad (\text{u}')$$

Finally, $\delta T_{00} = \rho$ we have from eq. (u'):

$$\frac{8\pi G \rho|_{bulk}}{3n} = H^2 [s^{-2}] \quad (\text{v})$$

, where $n = n_{RiemannSheet}$, and $\rho|_{bulk} = n \rho|_{nRiemannSheet} = n \frac{T_j}{c^2 \lambda_c^3}$;

$T_j = V = \varepsilon = -nucleon - stress - energy - Tensor$; see below.

Magnetic field generation at the Confinement mechanism

At present, we have no analytic proof of the existence of the condensate of abelian magnetic monopoles in gluodynamics and in chromodynamics.

However, there are two large gaps between QCD and the dual-superconductor picture [12].

1. The dual-superconductor picture is based on the Abelian gauge theory subject to the Maxwell type equations, while QCD is a non-Abelian gauge theory.

2. The dual-superconductor picture requires color-magnetic monopole condensation as the key concept, while QCD does not have such a monopole as the elementary degrees of freedom.

In [28] is found another confinement mechanism. In this note, where is shown that the dual Meissner effect in an Abelian sense works good even when monopoles do not exist, performing Monte-Carlo simulations of quenched $SU(2)$ QCD with Landau gauge fixing. Instead of monopoles, time-dependent Abelian magnetic fields regarded as magnetic displacement currents are squeezing Abelian electric fields. The dual Meissner effect leads us to the dual London equation and the mass generation of the Abelian electric fields which suggests the existence of a dimension 2 gluon condensate. The present numerical results, hence, suggest the Abelian dual Meissner effect is the real universal mechanism of color confinement which has been sought for many years. Moreover the relation of the Abelian dual Meissner effect with the dimension 2 gluon condensate sheds new light on the importance of the gluon condensate, cited [13, 14, 15, 16, 17].

Hence, the Abelian fields satisfy kinematically the simple Abelian Bianchi identity

$$\overset{\vee}{\nabla} \times \overset{\vee}{E}_A^a = \partial_4 \overset{\vee}{B}_A^a \quad \overset{\vee}{\nabla} \cdot \overset{\vee}{B}_A^a = 0 \quad (15)$$

The dual Meissner effect says that the squeezing of the electric flux occurs due to cancellation of the Coulombic electric fields and those from solenoidal magnetic currents.

Now what happens in a smooth gauge like the Landau gauge where monopoles do not exist?

From Eq.(15), only $\partial_4 \overset{\vee}{B}_A^a$ regarded as a magnetic displacement current could play the role of the solenoidal current.

It is very interesting to see Fig.4 from [28], in which this happens actually in Landau gauge.

Note that the solenoidal current has a direction squeezing the Coulombic electric field. Let us see also the detailed distributions shown in Fig.5 from [31]. The other components of the magnetic displacement current $\partial_4 B_{Ar}$ and $\partial_4 B_{Az}$ are not vanishing but they are much suppressed

consistently with Fig.2 from [31]. In comparison, we show the case of MA gauge also in Fig.5 of [28].

Now they have shown that the magnetic displacement currents are important in the dual Meissner effect when there are no monopoles. Then how can we understand the origin of the dual Meissner effect without monopoles?. The Abelian dual Meissner effect indicates the massiveness of the Abelian electric field as an asymptotic field.

DIMENSION 2 GLUON CONDENSATE

Now in [28] is shown that the magnetic displacement currents are important in the dual Meissner effect when there are no monopoles. Then how can we understand the origin of the dual Meissner effect without monopoles? The Abelian dual Meissner effect indicates the massiveness of the Abelian electric field as an asymptotic field:

$$(\partial_\rho^2 - m^2) \vec{E}_A \approx 0 \quad (16)$$

This leads us to a dual London equation which is a key to the dual Meissner effect. Let us evaluate the curl of the magnetic displacement current. Using Eq.(15), we get

$$\vec{\nabla} \times \partial_4 \vec{B}_A = \vec{\nabla}(\nabla \cdot \vec{E}_A) - \vec{\nabla}^2 \vec{E}_A$$

From Eq.(16), we get the dual London equation:

$$\vec{\nabla} \times \partial_4 \vec{B}_A \approx (\partial_4^2 - m^2) \vec{E}_A \quad (17)$$

Neglecting gauge-fixing and Fadeev-Popov terms, we have equations of motion $D_\mu^{ab} F_\mu^b = 0$ and

the (non-Abelian) Bianchi identity $D_\mu^{ab} * F_\mu^{ab} = 0$. Applying D operator to the Bianchi identity

and using the Jacobi identity and the equations of motion, we get

$$(D_\rho^2)^{ab} F_\mu^b = 2g\epsilon^{abc} F_{\mu a}^b F_{\nu a}^c$$

$$\text{Notice } (D_\rho^2)^{ab} = \partial_\rho^2 \delta^{ab} + g\epsilon^{acb} (\partial_\rho A_\rho^c) + g^2 (A_\rho^a A_\rho^b - \delta^{ab} (A_\rho^c)^2)$$

Hence if $\langle A_\mu^a A_\nu^b \rangle \delta^{ab} \delta_{\mu\nu} v^2 \neq 0$ we see asymptotically that the electric fields become massive

$(\partial_\rho^2 - m^2) E_k^a \approx 0$ with $m^2 = 8g^2 v^2$ as in [29]. Now the Abelian electric field is also massive asymptotically $(\partial_\rho^2 - m^2) E_{Ak}^a \approx 0$. Hence the dual London equation (17) is obtained.

Most estimates in the literature refer to the gluon mass, related to the $m_A^2 = \frac{3}{32} g^2 \langle A^2 \rangle$ by the

formula [29],

$$m_A = (625 \pm 33) \text{MeV}. \quad (18)$$

From eq. (u') above we have an expression in terms of glued Riemann sheets n

$$\begin{aligned} \frac{2\pi}{\eta} \int d^2 x \sqrt{g} T_\mu^\mu &= \frac{2\pi}{\eta} \rho = \frac{3H^2}{2 \cdot 2 \cdot G} n \frac{1}{\eta} \\ \frac{2\pi \cdot c^3 \cdot \rho}{\eta} &= \frac{3H^2 \cdot c^3}{4G} \frac{1}{\eta} \quad \text{or} \quad \frac{c^3 \cdot \rho}{3\eta} = \frac{H^2 \cdot c^3}{8\pi G} \frac{1}{\eta} \quad \text{or,} \quad \frac{c^3 \cdot \rho \cdot l_p^2}{3\eta} = H^2 [s^{-2}] \quad , \text{ or,} \\ \frac{c^3 \cdot \rho \cdot l_p^2}{3\eta} \frac{1}{c^2} &= H^2 [m^{-2}] \end{aligned} \quad (20)$$

$$\rho|_{bulk} = n\rho|_{n_{RiemannShear}} = n\rho_{bubble} = n\rho = n \frac{T_j}{c^2 \lambda_C^3}$$

With data for the confinement epoch we have

$$\rho = \frac{M_U}{n_{bubbles} \cdot \lambda_C^3} = 6.2 \times 10^{19} [kg/m^3] \quad ; \quad n = n_{bubbles} = \frac{M_U c^2}{m_A} = 5.2 \times 10^{51} \quad ; \quad l_P = 7.8 \times 10^{-35} \quad , \text{ that results}$$

$$H = 1878[m] \quad ; \quad H = 1740[m]$$

from eq. (20) , which corresponds with the above value of

In the case of a homogeneous potential directed along the z-axis [12] eq. (2.2), the Einstein stress-energy tensor is:

$$T^{00} = T^{11} = T^{22} = -T^{33} = \rho = \frac{\epsilon_0 c^2 B^2}{8\pi} ; T^{0i} = 0, \text{ where } \rho_B [J/m^3] \text{-the magnetic energy density.}$$

$$\epsilon = \frac{V_{vol} \epsilon_0 c^2 B^2}{8\pi} = \rho V_{vol} = V [J],$$

$V_{vol} = 2\pi \lambda_C \lambda_c (4\lambda_c) \cong 8\pi \lambda_C^3$, at Compton length equally with the penetration length $\lambda_C = \lambda$, that results

$$E^2 = \frac{(V)}{\epsilon_0 (\lambda_C^{e*})^3}$$

With $V = \epsilon_{gluons}$ as above is obtained $B = E_{q\bar{q}}/c$.

$$\epsilon = \frac{V_{vol} \epsilon_0 c^2 B^2}{8\pi} = \rho_B V_{vol} = V = m_A = [J],$$

$V_{vol} = 2\pi \lambda_C \lambda_c (4\lambda_c) \cong 8\pi \lambda_C^3$, at Compton length $\lambda_C = \eta/mc$

$$E^2 = \frac{(V)}{\epsilon_0 (\lambda_C^{e*})^3} \quad , \text{ also } E = Bc = \frac{\eta c}{e \lambda_C^2}$$

Here, the Hubble constant is defined as, see eq. (3.20,3.21,3.22) from [12].

$$H^2 = \frac{8\pi G V^4}{3(\eta c)^3 c^4} [m^{-2}], \text{ and by use of the Compton length as:}$$

$$H^2 = \frac{1}{R^2} = \frac{8\pi G V^4}{3(\eta c)^3 c^4} \rightarrow \frac{8\pi G V^4}{3\lambda_C^3 (mc^2)^3 c^4} \rightarrow \frac{8\pi G}{3} \frac{V}{\lambda_C^3 c^4} [m^{-2}] \quad (21)$$

Now, the rate per unit volume of quarks pair creation is given by using the Schwinger effect R inside the nucleon or EW bubbles, when this electric field E is induced by $e^+ - e^-$ quarks pairs which decay in W^\pm that explaining β^- decay, of leading order behavior

$$R = (E/E_{cr})^2 (c/\lambda^4) (8\pi^3)^{-1} * \exp(-\pi E_{cr}/E) \quad (22)$$

or $E/E_{cr} \ll 1$, positron charge e , mass m , Compton wave-length $\lambda_C = \eta/mc$ and so-called “critical” electric field

$$E_{cr} = m^2 c^3 / e\eta$$

the volume is given by:

$$V_{matter} = (\lambda_C)^4 \frac{1}{c} [m^3 s]; \text{ the mass of quarks.}$$

The strain at Earth surface

Strain Model

From [27] we have:

$$\lambda = \kappa^{-1} e^{kv}; \kappa^{-1} = 4M; v = t + r^*; C = 1 - 2M/r; \text{ and } r^* = \int C^{-1} dr; C_{,r} = 2M/r^2$$

We will assume that $T_{vv}(v)$ represents ingoing radiation which changes the black hole's mass by only a small fractional amount, $|\Delta M| \ll M$. We can then take κ to be constant to lowest order. If we change the independent variable from λ to $v = t + r^*$, then

$$\frac{d}{d\lambda} = e^{-kv} \frac{d}{dv}; v = 1/\omega; \kappa = \frac{c^3}{4GM}$$

And

$$T_{\mu\nu} k^\mu k^\nu = e^{-2kv} T_{vv}$$

For spherically symmetric pulses, the shear and vorticity vanish, and the Raychaudhuri equation, Eq. (36) from [36], becomes

$$\frac{d\theta}{dv} = -\frac{1}{2} e^{kv} \theta^2 - 8\pi e^{-kv} T_{vv}(v)$$

The equation for the horizon area can be expressed as

$$\frac{dA}{dv} = e^{kv} A \theta$$

If we take $A_0 = 16M_0^2$ to be the initial area of the black hole in the distant past, where M_0 is its initial mass, then we have

$$\frac{\Delta A}{A_0} \cong \frac{a}{\kappa} e^{kv_+} = 2 \frac{\Delta M}{M_0}$$

In this approximation, the change in the mass of the black hole is

$$\Delta M = \frac{a}{2\kappa} M_0 e^{kv_+} = \frac{aA_0}{8\pi} e^{kv}$$

where we have used $\kappa = 1/(4M) \cong 1/(4M_0)$.

This agrees with the result obtained by calculating the change in mass directly from Eq. (23) as

$$\dot{M} = \frac{dM}{dt} = F = \int T_t^r r^2 d\Omega$$

Here, it was defined an "effective magnetic field", B_{eff} , in terms of the total energy density in the magnetic field of MF Vortex,

$$\varepsilon_0 = \frac{\varepsilon_0 c^2 B_{eff}^2}{8\pi} \quad (23)$$

On the horizon, $T_t^r = T_{vv}$, $r = 2M + \varepsilon$

Since the quarks generated inside nucleons or in EW bubbles are generated by a pulsating process with frequency $\nu = \omega^{-1}$, a such pulse of stress-energy it could be $8\pi T_{\mu\nu} k^\mu k^\nu = a\delta(\lambda)$,

a is a positive constant, and the surface gravity $\kappa = \frac{c^3}{GM} [s^{-1}]$.

We can observe that $\frac{\Delta A}{A} = \frac{8\pi GM}{c^2} \cdot \frac{1}{R} = \frac{r_{Schw}}{R}$, or, we have obtained the classical formula for deformation.

The build of spacetime is obtained by using well-known Inflation models [34a,b], which in our opinion is nothing else than a spreading of entanglement-energy source-horizon end, where the scale leaving the horizon at a given epoch is directly related to the number $N(\varphi)$ of e -folds of slow-roll inflation that occur after the epoch of horizon exit. Indeed, since H -the Hubble

length is slowly varying, we have $d \ln k = d(\ln(aH)) \cong d \ln a = \frac{d \ln a}{a} = H dt$. From the definition

this gives $d \ln k = -dN(\varphi)$, and therefore $\ln(k_{end}/k) = N(\varphi)$, or, $k_{end} = k e^N [m]$, where k_{end} is the scale leaving the horizon at the end of slow-roll inflation, or usually $k^{-1} \ll k_{end}^{-1} [m]$, the correct equation being $k = k_{end} e^N [m^{-1}]$.

During Universe evolution, the horizon leave is when $a_{leave} = k_{leave}/H_{leave} = 1$, and $k_{leave}^{-1} = H_{leave}^{-1} = 10^{-18} [m]$.

From (18) we have at hologram site $\varepsilon_{QCD} \cong 0.7/a_{end} = 27 \times 10^{-2} GeV \rightarrow 4.4 \times 10^{-12} J$; $a_{end} = 25$; $k_{end}^{-1} = 2.5 \times 10^5 [m]$, with eq. (20); $R = 7.1 \times 10^6 m$, and $H_{end}^{-1} = 6.5 \times 10^6 [m]$, $t_{end} = H_{end}^{-1}/c = 0.02 s$, $\lambda_C = 6.7 \times 10^{-15}$; $H_{leave}^{-1} = k_{leave}^{-1} = 10^{-18} [m]$ we found $N = 53.9$

The strain at Earth surface-light bending

We use the above model

With $a = \frac{c}{R} [s^{-1}]$; where $R = 6.5 \times 10^6 [m]$; $a = 45$; with $M = 1 \times 10^{24} [kg]$; $\kappa = 10^{-18} [s^{-1}]$; if we

have the generation eq. (22), $\nu = \omega^{-1} = (R/V \cdot V_{vol})^{-1}$; where

$$R/V \times V_{vol} \cong 2 \times 10^{57} \cdot \lambda_C^3 = 6.3 \times 10^{14} [s^{-1}],$$

$$\nu = 1.5 \times 10^{-15} [s]; e^{\kappa \nu} \cong 1.0.$$

In other words the pulse is transferred from nucleons via electromagnetic field as glued Riemann sheets to CFT viewed as a hologram, that it means the greater contribution to expansion.

So, the deformation is $\frac{\Delta A}{A} = 6.8 \times 10^{-10}$, or the expansion due of EW epoch since continue till

today since $\Delta A = 6.8 \times 10^{-10} \cdot A = 4.4 \times 10^{-3} [m]$, that corresponds with well known Einstein ring, here $A = H^{-1} = R = 6.5 \times 10^6 [m]$.

To note, that the gravitational potential for Earth is $U = \frac{GM_{Earth}^2}{R_{Earth}} = 3.6 \times 10^{32} J$; the stress-energy tensor is: $T_j \cong \varepsilon = U/1.44 \times 10^{-8} \times n_{nucleons} = 4.2 \times 10^{-12} J$;

$$n_{nucleons} = M_{Earth} / 10^{-27} = 6 \times 10^{51}$$

Conclusions

In this work are confirmed the following models:

- author' model of electromagnetic fields in nucleon,
- the origin of the gravitational potential,
- the Schwinger effect to create particles inside nucleons,
- the holographic derivation of entanglement for structured matter objects,
- the origin of the deformation of the horizon for structured matter objects,
- the lensing effect due of magnetic flux passing the hologram-CFT.

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