A Three-Phase Four-Wire State Estimator Algorithm For Low Voltage Networks Management

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A THREE-PHASE FOUR-WIRE STATE ESTIMATOR ALGORITHM FOR LOW VOLTAGE NETWORKS MANAGEMENT

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ABSTRACT

The low voltage networks management plays even more a key role for distribution system operators due to the fast development of the distributed energy resources. European low voltage networks have particularities (four-wire, unbalance, small network at electric grid extremities) which should be considered when a state estimator is created. This paper presents a three-phase four-wire state estimator algorithm. To analyse the performance of this algorithm, we consider a LV test network.

INTRODUCTION

In the context of Smart Grids and fast development of the distributed energy resources (DER), distribution system operators (DSO) wants to better know the state of their networks. In Europe, DSO should respect the EN 50160 standard on their whole networks. Table 1 describes the constraints for low voltage (LV) networks.

Characteristics	Admissible limit values
Voltage limit	$V_{\varphi n} \pm 10\%$, 95% of the time
Voltage unbalance	$V_2 \leq 2\% \ V_1$

Table 1 EN 50160 standard requirements

In Europe, the nominal line-to-neutral voltage is 230 Volts (1 pu in what follows) for LV networks.

To determine the line-to-neutral voltages of all network nodes and meet EN 50160 standard, DSO use two main techniques: load flow (LF) and state estimation (SE) [1]. LF techniques require a good precision of all the parts of the network. As features of each distribution network are often different from each other, this technique presents some restrictions. SE techniques become critical for distribution systems (DS).
Single phase SE algorithms are already used in high voltage (HV) networks by the transmission system operator. DSO begin to use 3-phases SE algorithms in medium voltage (MV) networks. To work on LV networks, SE algorithm have to be adapted to the particularities of LV networks (3-phases and 4-wire grids).

European LV networks have three major differences compared to HV and MV networks:

- They are three-phase four-wires (Phases A, B, C and neutral N) networks.
- They are very often unbalanced and not always observable.
- LV networks are small networks at electric grid extremities. In that way, SE algorithm should require low amounts of computing resources to run on a concentrator, located in the secondary substation.

This paper presents a three-phase four-wire SE algorithm tested on a LV benchmark test case. In section II we introduce the LVSE particularities. Section III discusses the adaptations done on network model. Section IV shows the SE method used. Section V gives the test cases. The two last sections give results and conclusion.

LOW VOLTAGE STATE ESTIMATION

The SE problem is defined as solving the equation for the measurement model (1).

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_M \end{bmatrix} = \underbrace{\begin{bmatrix} h_1(x_1, x_2, \cdots, x_N) \\ h_2(x_1, x_2, \cdots, x_N) \\ \vdots \\ h_M(x_1, x_2, \cdots, x_N) \end{bmatrix}}_{h(x)} + \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_M \end{bmatrix}}_{e} \qquad (1)$$

Where z is the measurement vector, x the state vector which size is N (n letter is kept for neutral), h the measurement function vector relating x to z and e the vector of measurement errors. [2]

Measurement vector

In LV networks, DSO uses smart meters (SM) to control load consumption and voltage limits. As EN 50160 standard are related to line-to-neutral voltages, SM measure line-to-neutral voltages and feeders active and reactive powers [3]. Figure 1 represent the measurements recorded by the SM at user node.

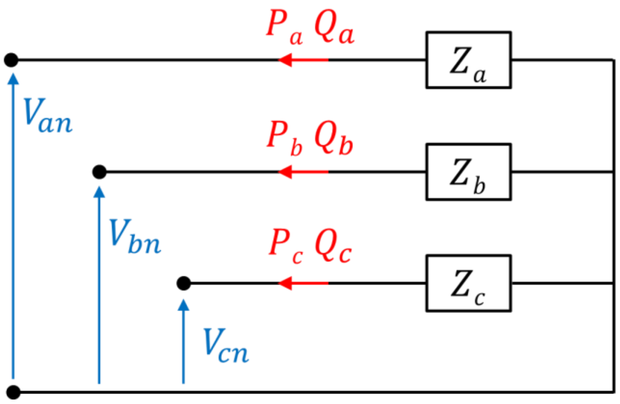

Figure 1 Electric measurements by smart meters at three-phase four-wire user node

In secondary substation, the smart meter measures in addition the neutral-to-ground voltage. All these measurement are set in the z vector.

We make some assumptions of the measurement errors:

- $E[e_i] = 0$ for $i = 1, 2, \cdots, m$.
- $Cov[e] = R_c = diag\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_m^2\}$.

We define the weighting matrix W which is the inverse of the covariance matrix of measurement error.

State vector

As described in [4], the state vector x is the set of all voltage magnitudes and voltage angles. In HV and MV networks, the line-to-ground voltages are always considered for the state vector. In LVSE algorithm [2], [5] and [6], the neutral wire is not explicitly considered and neutral-to-ground voltage is assumed to be equal to zero. Thus, Kron reduction is used to eliminate the neutral wire and line-to-ground voltages are considered as in HV and MV SE. However, single-phase and unbalanced three-phase users are connected to LV networks. [6] recommends to consider the neutral wire. Two ways enable to add the neutral wire to the state vector: either voltages are line-to-ground or neutral-to-ground voltages or voltages are line-to-neutral or neutral-to-ground voltages. The two equations (2) and (3) are the state vector at the node i of the network. Equation (2) is more intuitive because it looks like the state vector of HV and MV SE. Nevertheless, equation (3) includes terms which are directly in measurement vector z. Hence, we choose equation (3). This choice has an impact on network model.

$$x_i = \begin{bmatrix} V_{ag}^i & V_{bg}^i & V_{cg}^i & V_{ng}^i & \delta_{ag}^i & \delta_{bg}^i & \delta_{cg}^i & \delta_{ng}^i \end{bmatrix}^T \quad (2)$$

or

$$x_i = \begin{bmatrix} V_{an}^i & V_{bn}^i & V_{cn}^i & V_{ng}^i & \delta_{an}^i & \delta_{bn}^i & \delta_{cn}^i & \delta_{ng}^i \end{bmatrix}^T. \quad (3)$$

NETWORK MODEL

As neutral wire is modelled, impedance matrices and power flow equations should be adapted.

Admittance matrices

Admittance matrices are separated in series and parallel admittance matrices. In LV network, parallel admittance matrix concerns grounding resistance (capacitive currents are negligible) and series admittance matrix concerns self and mutual admittance.

In SE algorithm, admittance matrices are preferred to impedance matrices. However, impedance matrix is more often used to model LV cable.

Impedance matrix is usually the application of Kirchhoff's law when the line-to-ground voltages are used in the state vector. The impedance matrix is the link between the complex state vector and the complex injection currents. The complex state vector is:

$$\underline{V} = \begin{bmatrix} \underline{V_{an}^1} & \underline{V_{bn}^1} & \underline{V_{cn}^1} & \underline{V_{ng}^1} & \underline{V_{an}^2} & \cdots & \underline{V_{ng}^N} \end{bmatrix}^T$$

As define in [7], a section between nodes k and l of a LV cable is shown in Figure 2. The line current between nodes k and l is $\underline{I_i^{kl}}$ for $i = a, b, c, n$. The injection current at node k is $\underline{I_i^k}$ for $i = a, b, c, n$.

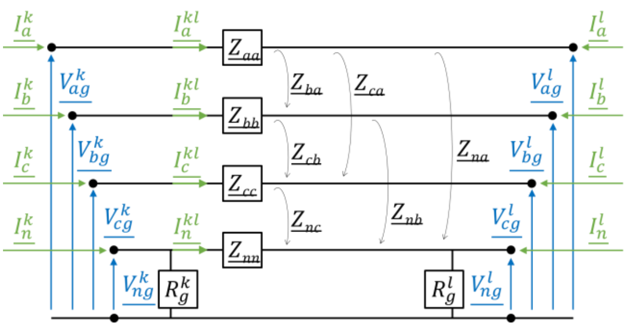

Figure 2 LV cable model

The voltage reference is usually an ideal ground voltage which is modelled by the bottom line. As all terms of state vector are not line-to-ground voltages, the series impedance matrix have to be adapted with the following relation:

$$\begin{bmatrix} \underline{V_{an}^k} - \underline{V_{an}^l} \\ \underline{V_{bn}^k} - \underline{V_{bn}^l} \\ \underline{V_{cn}^k} - \underline{V_{cn}^l} \\ \underline{V_{ng}^k} - \underline{V_{ng}^l} \end{bmatrix} = \begin{bmatrix} \underline{Z_{aa}} - \underline{Z_{an}} & \underline{Z_{ab}} - \underline{Z_{bn}} & \underline{Z_{ac}} - \underline{Z_{cn}} & \underline{Z_{an}} - \underline{Z_{nn}} \\ \underline{Z_{ba}} - \underline{Z_{an}} & \underline{Z_{bb}} - \underline{Z_{bn}} & \underline{Z_{bb}} - \underline{Z_{cn}} & \underline{Z_{bn}} - \underline{Z_{nn}} \\ \underline{Z_{ca}} - \underline{Z_{an}} & \underline{Z_{cb}} - \underline{Z_{bn}} & \underline{Z_{cc}} - \underline{Z_{cn}} & \underline{Z_{cn}} - \underline{Z_{nn}} \\ \underline{Z_{na}} & \underline{Z_{nb}} & \underline{Z_{nc}} & \underline{Z_{nn}} \end{bmatrix} * \begin{bmatrix} \underline{I_a^{kl}} \\ \underline{I_b^{kl}} \\ \underline{I_c^{kl}} \\ \underline{I_n^{kl}} \end{bmatrix}$$

Then we obtain the series impedance matrix between the complex state vector and the line currents. Using the parallel admittance matrices with the series admittance matrices, we obtain the admittance matrices.

Power flow equations

The power flow equations are used to define the measurement function h and especially to calculate the Jacobian matrix H [2]. In LV network, the power measurements (e.g. for phase a at node k) are defined

$$P_{an}^k = Re\left(\underline{S_{an}^k}\right) = Re\left(\underline{V_a^k} * \underline{I_{an}^k}^*\right)$$

$$Q_{an}^k = Im\left(\underline{S_{an}^k}\right) = Im\left(\underline{V_a^k} * \underline{I_{an}^k}^*\right)$$

As we adapted the admittance matrix in previous subsection to link line-to-neutral voltage and line currents, H is calculated in the same way as usual.

STATE ESTIMATION METHOD

To determine the most likely state of the system, we use weight least squares (WLS). This method or its variations are the most common used in DSSE [1]. To avoid instabilities in numerical resolution, we add explicit constrains.

Explicit constraints

The explicit constraints are:

- Some nodes of LV network are not injection nodes. At these nodes, active and reactive powers are equal to zero.
- At all nodes of the LV network even the non-injection nodes, the sum of currents is equal to zero.

Method chosen

To take into account the explicit constraints, we use the augmented approach WLS state estimation [2]. In this approach, the WLS problem can be formulated as follows:

$$\hat{x} = \arg\min_{x} J(x) = [z - h(x)]^T W [z - h(x)] \qquad (4)$$

subject to

$$c(x) = 0$$
$$r - z + h(x) = 0$$

TEST CASE

Network model

The LV network considered is part of the distribution dataset used in the CGMES 2.4.1 interoperability tests performed in 2016 [8]. The main features of this network are presented in Table 2.

Characteristics	Values
Number of nodes	13
Number of users	10
Global consumption	25 kW
Global length	850 m

Table 2 Main features of LV network

No producers are connected to this network. The consumption of users is unbalanced at some nodes.

Features for LVSE

This LV network has 13 nodes. At node 1, the MV/LV transformer is connected. At nodes 3 and 12, no loads are connected. At the other nodes, one load is connected.

Considering the state vector, there are 8 terms at each node. Hence, the state vector has 104 terms. Nevertheless, at secondary substation, we make the assumptions that voltage angles are known. Then, the state vector has 100 terms.
Considering the measurement vector, there are 10 terms at secondary substation and 9 at user node. Hence the measurement vector has 100 terms.

SIMULATION

SE and LF software

To evaluate the performance of this SE, we compare SE results with LF results.
The LF is computed by MATLAB/Simulink via the Simscape Power Systems toolbox and the SE is computed on MATLAB.

Estimation without measurements errors

Voltage results
Figure 3 and Figure 4 show the voltage magnitude at each node. The results of simulation are presented with circles and the results of estimation are presented with crosses.
In Figure 3, we compare the magnitude of line-to-neutral voltage in pu. The algorithm estimate perfectly the line-to-neutral voltage.

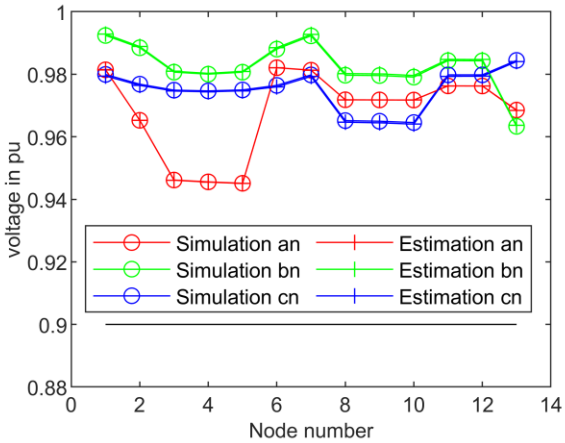

Figure 3 Magnitude of line-to-neutral voltage without measurements errors

In Figure 4, we compare the magnitude of neutral-to-ground voltage in Volts. The estimation has the same shape as the simulation. The error between estimation and simulation is always below 0.5 Volts.

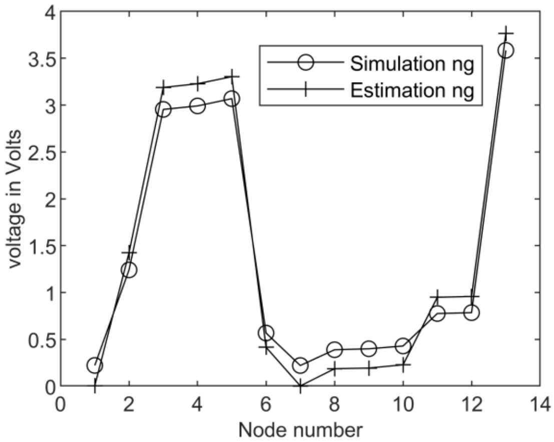

Figure 4 Magnitude of neutral-to-ground voltage without measurements errors

The magnitude of neutral-to-ground voltage is almost hundred times lower than the magnitude of line-to-ground voltage. In SE, it is always difficult to estimate a state when all state vector terms are not in the same order of magnitude. The estimation of neutral-to-ground voltage is satisfying.

Current results

Figure 5 shows the current magnitude at each node. As we add a constraint on currents, current estimations are really near from current simulations.

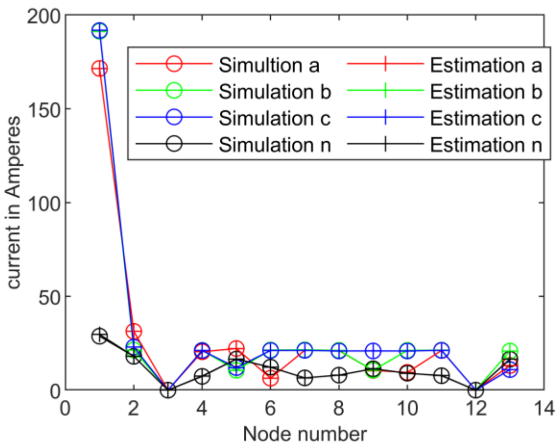

Figure 5 Magnitude of line current without measurements errors

This LVSE algorithm could better estimate the state of the network but it would require more computing resources. Moreover, it is better for DSO if the algorithm provides not very precious results than no results.

Estimation with measurements errors

Measurements precisions

All measurements done by SM do not have the same precisions. The error of measurements is presented in Table 3.

Measurements	Error (%)
Line-to-neutral and neutral-to-ground voltages at secondary substation	0.1
Line-to-neutral voltages at user node	0.5
Active power at secondary substation	0.1
Active power at user node	0.5
Reactive power at secondary substation	0.1
Reactive power at user node	5

Table 3 Measurements errors of the network

We then considered the precisions given in Table 4, which are the covariance of the measurement errors used in matrix R_c.

Measurements	Covariance
Line-to-neutral and neutral-to-ground voltages at secondary substation	1
Line-to-neutral voltages at user node	0.25
Active power at secondary substation	1
Active power at user node	0.01
Reactive power at secondary substation	1
Reactive power at user node	0.0025

Table 4 Covariance of measurements

Voltage results

In Figure 6 and Figure 7, the mean value and the standard deviation of the voltage estimation is shown.

Measurements errors modifies the magnitude of line-to-neutral as described in Figure 6. The maximum standard deviation is about 1 Volts (0.004 pu).

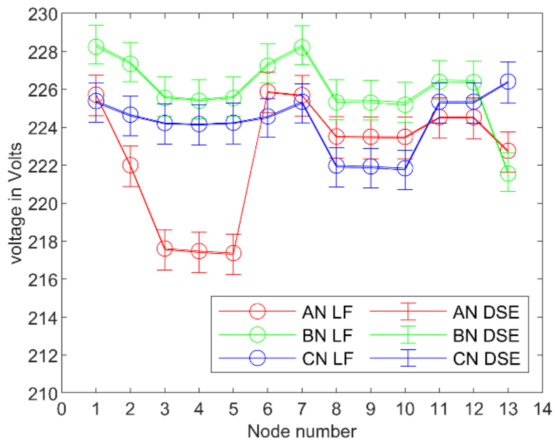

Figure 6 Magnitude of line-to-neutral voltage with measurements errors

In Figure 7, measurements errors have a low influence on neutral-to-ground voltage magnitude. The maximum standard deviation is about 0.5 Volts.

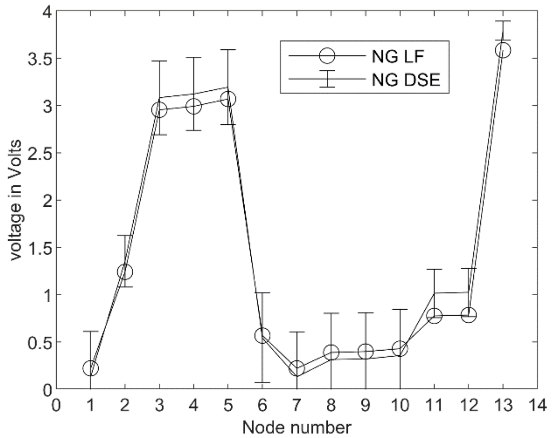

Figure 7 Magnitude of neutral-to-ground voltage with measurements errors

Currents results

In Figure 8, the mean value and the standard deviation of the line current estimation is shown. Measurements errors have a low influence on line currents magnitude. The maximum standard deviation is about 5.5 Amperes.

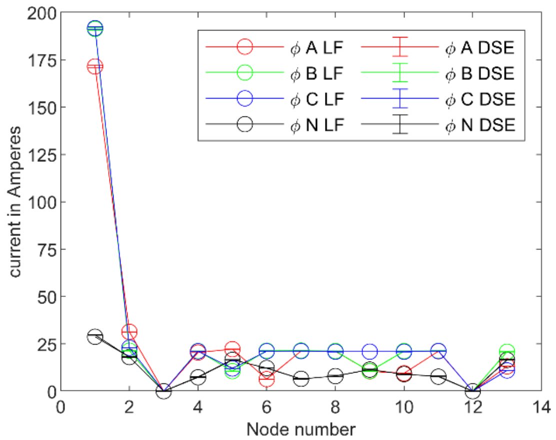

Figure 8 Magnitude of line current with measurements errors

CONCLUSION

The LV networks have characteristics which are different from the HV and MV networks. In that way, SE algorithm for LV networks have to be deeply modified from HV and MV SE algorithm.

In this paper, we propose an LVSE based on the augmented approach WLS. This algorithm estimates the voltages and the currents of the test case with good precision expect for the voltage magnitudes which are far from the average value. Moreover, it is not dependant of measurements errors.

To conclude, this algorithm is adapted for LV networks.

ACKNOWLEDGMENTS

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