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# Experimental testing of pre-stressed granular assemblies as a surrogate material for the dynamic analysis of launcher cryogenic tanks

Sinh-Khoa Nguyen\*, Pierre-Louis Chiambaretto, Miguel Charlotte, Guilhem Michon, Joseph Morlier, Yves Gourinat

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## ABSTRACT

This paper focuses on the experimental validation of a new methodology for substitution of Liquid Hydrogen (LH2) on a simplified model of launcher cryogenic tank in vibration analysis by using pre-stressed grains as surrogate materials. The objective of the experiments presented here is to analyze the modal behavior of the (Tank and Granular materials) system during pre-stress procedure. In these experimental tests, a cylindrical tank is fully filled with granular materials and vibrated under swept sine and random excitations. Then, the natural frequency of each vibration mode is measured at different levels of pre-stress. The results obtained from the tests show the evolution of each flexural mode as a function of applied pre-stress. Nonlinear behavior of the analyzed system during vibration was also observed. These experimental values carried out with Tank-Granular material system (TGM) are compared with numerical and analytical results to prove the potential of the new methodology. Finally, based on obtained experimental results, some solutions are proposed in order to improve the new methodology for vibration analysis of cryogenic tank which is an important thematic in aerospace engineering.

## 1. Introduction

The experimental certification of launcher cryogenic tanks during vibration is an important issue in aerospace industry. In this context, some critical problematics are raised with the tanks filled with *Liquid Hydrogen* (LH2), which is largely used in cryogenic engines due to its high efficiency [1]. Indeed, the vibration tests with that fuel are however too dangerous to be performed in laboratory due to the LH2 fugacity and explosiveness [2]. In fact, while for other liquid fuels, surrogate liquids such as water are used to analyze the vibratory behavior of tank, in contrary, it is difficult to apply this method for LH2 because of its lowest mass density.

The previous works [3,4] suggested the possibility that pre-stressed granular medium could be used to overcome difficulties in experimental certification. These preliminary analyses provide good analytical results for vibratory modes in an in-plane 2D hypothesis. Following this idea, a new methodology [5,6] for more engineering applications of cylindrical tanks in 3D geometry in coupling interaction with fluid and granular materials were proposed and shown a good agreement with numerical simulations. In that 3D case, the modal analyses of two families of modes (lobed modes and flexural modes) were realized with the help of Donnel-Mushtari's shell theory and

Timoshenko's beam theory. The proposed methodology for vibration analyses of tank filled with LH2 by using granular materials is developed based on analytical formulations of these theories. The essential idea of the new methodology [5] (which includes 5-steps) is that the modal equivalency between Tank – Fluid system (TFM) and Tank – Grains system (TGM) can be found by applying different pre-stresses on the granular medium with the help of the elastic homogenization model proposed by Emeriault and Cambou [7]. In this new methodology, it is important to compute the evolution of eigen-frequency of each vibratory mode as a function of axial pre-stress during oedometric compression. Then, the required pre-stress can be found as intersection between *frequency-axial stress* curve given by TGM system and the expected value given by TFM system. Some important conclusions were deduced by using this strategy. For lobed modes, it is necessary to apply different values of pre-stress for different modes in order to substitute a same fluid in vibration analysis. In contrary, we only need one level of pre-stress for different flexural modes to substitute LH2 in vibratory test. Numerical simulations were also performed in order to verify the suitability of the proposed method. To summarize; as presented in the referenced paper [5], good agreements between analytical and numerical results for TFM and TGM systems were found and showed the potential of that new methodology.

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It is however important to conduct vibratory tests in laboratory in order to experimentally validate the proposed methodology. Hence, as an extension, the work reported here aims at describing the results in particular for two families of vibration modes: the 1st one is lobed modes in radial direction and the 2nd one is flexural modes in the longitudinal direction. This choice was guided by the fact that the compressive and torsional modes can be neglected because LH2 is a non-compressible and non-viscous fluid [6] in relation with the considered structural modes (viscosity of LH2 at storage state is  $120.10^{-7}$  kg/ms [8]). It is obviously impossible to do experiment on real cryogenic tanks because of its huge dimension [9]. Thus, in this work, we perform the test on a small tank made of polycarbonate. It is also impossible to do tests on tank filled with any liquid because of safety issues. By the virtue of these facts, the experiments in this paper insist on the evolution of natural frequency of each vibratory mode for different compression levels. If this tendency is observed, we have a good evidence to show the potential of the new method. The chronological order of the experiment is, first, a cylindrical tank is fully filled with granular grains. Then, the granular assembly inside the tank is pre-stressed to the maximal value of compression in oedometric conditions that is inferior to elastic yield of embedded granular material. The whole system is vibrated under swept or random excitations. During this step, the modal shapes are captured with help of accelerometers; the natural frequency of this mode is also measured. All these steps are repeated with another value of pre-stress. The pre-stress in the granular medium is decreased by applying a vibratory motion at the frequency band near the lowest frequency of the whole system. By recording the values of the natural frequency with respect to axial pre-stress, we are able to study the evolution of *frequency pre-stress* and compare it with analytical results obtained by the proposed methodology. The experiment scheme is characterized by the flow chart Fig. 1.

The second purpose of this experiment is to study some physical behaviors (eg non-linearity, energy dissipation...) of the complex vibrating system which are not taken into account into analytical formulations in [5,6].

This paper contains three main sections. First, the equipment setup and the data acquisition is detailed. The second section presents the results of our experimental tests for empty tank and tanks fully filled

with pre-stressed grains. These results are compared with analytical and numerical ones obtained with the modeling of the present paper (carried out on Tank-Granular material system) in order to validate the potential of the proposed method. It is important to take into account that both the numerical simulation and the analytical modeling presented in this paper are carried out with the actually measured geometrical and mechanical properties of the tank and granular material. The last section will discuss about the complexity of the dynamic system. In this section, we will find some explanation for some existing problematics encountered during the experimental practice. We will propose some possible solutions to circumvent them and to improve the methodology.

## 2. Experiment setup

In this work, the ‘composite’-structure which is the object of vibratory tests contains two main parts: the 1st part is the cylindrical tank while the 2nd part is the granular medium that fills the tank. However, that structure is also fixed on a 3rd part, which is an aluminum clamping support. On this last one, there is a system of screws which are used to assemble the tube to vibrating table. A general view on this setup is illustrated in Fig. 2.

The tank containing pre – stressed granular material is fixed and vibrated in the x direction with the help of a vibrating table. Modal shapes of vibratory modes are captured with a system of accelerometers. Natural frequencies are determined with *frequency response function* (FRF). Details on mechanical and geometrical properties of tank and granular material, accelerometers, vibrating table, pre-compression process and vibratory experiments are carefully presented in the next subsections.

### 2.1. Properties and dimension of testing structure

The experimental tests were performed on a cylindrical tank made of polycarbonate. This material was chosen because of its transparency allowing the observation of granular packing inside the tank. The geometry and mechanical properties of tank are described in Table 1.

The tank is fully filled with spherical grains provided by ATECA.

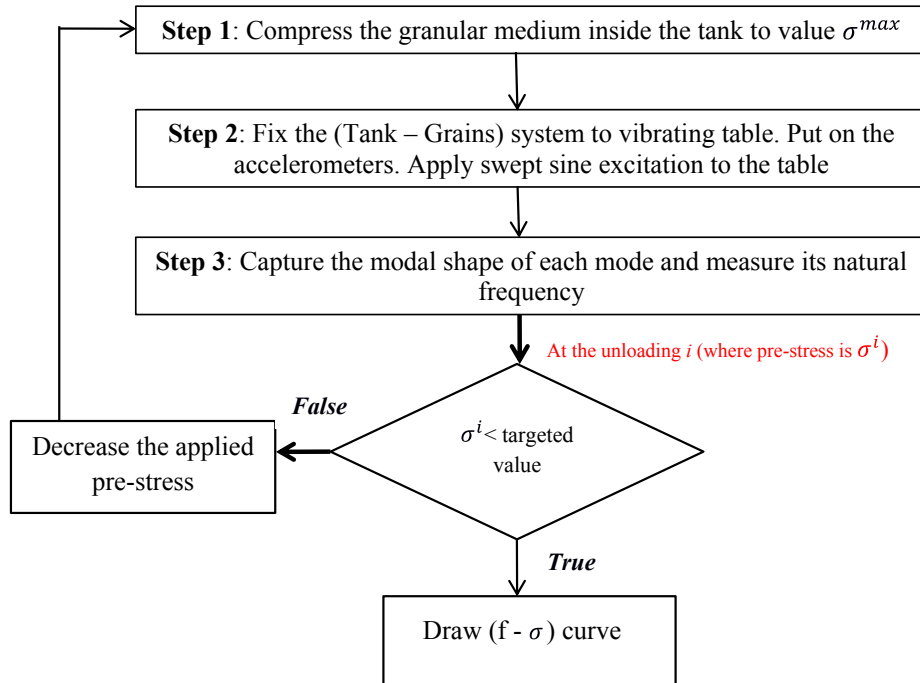


Fig. 1. Experimental scheme.



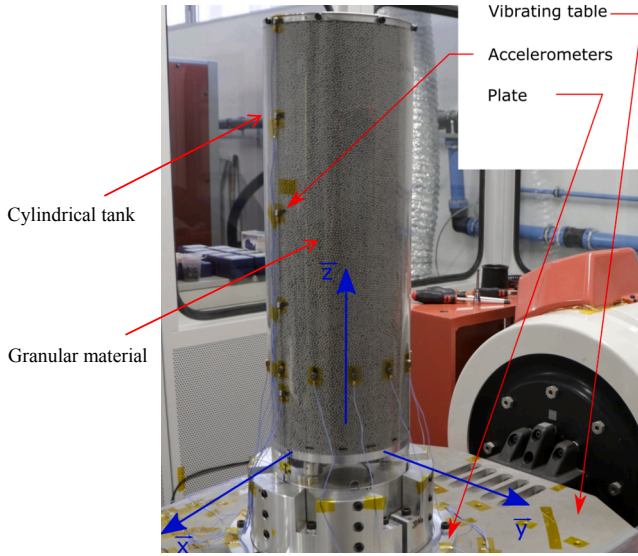


Fig. 2. Experiment setup for vibration test.

**Table 1**  
Properties of the tank under test.

Length	0.602 m
External radius	0.1 m
Internal radius	0.097 m
Thickness	0.003 m
Young's modulus	2.3 Gpa
Poisson's ratio	0.45
Mass density	1189 kg m <sup>-3</sup>



Fig. 3. Grains used in the experiment.

Individually, each grain is made of polystyrene and coated outside with epoxy resin. The nominal external diameter and thickness of one grain are respectively 3.33 mm and 0.2 mm. A picture of these grains are shown in Fig. 3

These properties are determined in the previous works [3,4,10] and shown in Table 2.

The upper bound of tank is covered with an aluminum disk with thickness of 0.01 m, radius of 0.1 m in order to apply the pre-stress in granular system. The total weight of this disk is 0.8482 kg.

**Table 2**  
Properties of the grains.

Radius	1.665 mm
Specific weight	603 kg m <sup>-3</sup>
Young's modulus $E_m$	600 MPa
Poisson $\vartheta_m$	0.015

## 2.2. Experiment setup

There are three main phases in our experimental setup process.

- **Step 1:** Filling of granular grains in cylindrical tank.
- **Step 2:** Compression of granular grains contained in the tank.
- **Step 3:** Vibratory test.

Details of these steps are mentioned in following sections.

### 2.2.1. Filling of granular materials in cylindrical tank

Filling of granular grains is an important issue because of its heterogeneous and discrete nature. Many studies in literature show that the mechanical behavior of granular materials depends on internal organization of contacts in granular system [11,12,23,26,27]. More importantly, the filling process should be done carefully and slowly in order to avoid the localized distribution of grains in the system. Accordingly, in this work, the grains are progressively filled in tank layer by layer. One can observe that the level of granular medium inside the tank is decreased after vibration at the lowest frequency of the system (which is the 1st flexural mode) due to the reorganization of the particles in granular media. As consequence, a small space between the last layer of grains and the cover disk is opened. Hence, all pre – stress created in compression phase is lost. This phenomenon is shown in Fig. 4.

To avoid this phenomenon, the filling process is realized in two phases: the 1st phase is manual filling mentioned above; the 2nd phase is vibratory packing. In detail, we open the cover to add more grains, then, the system is vibrated with the frequency of the 1st flexural mode during 1 min.

### 2.2.2. Compression of granular grains contained in cylindrical tank

In this step, the granular assembly is compressed by using the machine BOSE ElectroForce 3330. A general view on the setup for the compression phase is shown in Fig. 5.

Once the tank is completely filled, the granular assembly contained inside is pre-stressed by applying a compression force on top of the cover disk. A small space between the tank wall and the lower aluminum support (which is fixed to machine BOSE) allows the tank to slide along downward direction and to produce a pressure inside the granular medium. The granular assembly is compressed with a displacement rate equal to 0.5 mm/min (strain rate equivalents to  $8.10^{-4} \text{ s}^{-1}$ ) which is slow enough to avoid dynamic behavior of granular medium [25]. When this pre-stress reaches the expected value, a system of 18 screws on the clamping support is fastened. By doing this action, the polycarbonate tank is absolutely fixed to the aluminum support and the pre-stress obtained during compression is prevented for the sake of vibratory test. Three force sensors HBM-U9B-2kN are put under the plate which is in contact with grains in order to measure the pre-stress before, during and after the vibration test. This method is the better one to obtain the convenient representation of the filled tank.

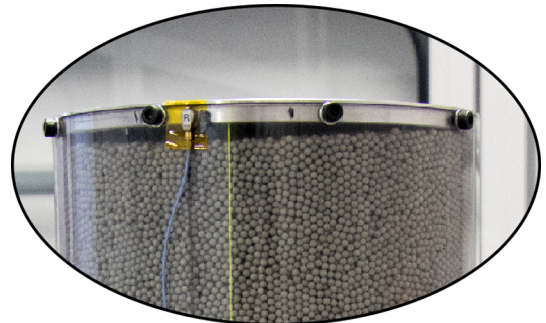


Fig. 4. Space between the last layer of grains and the cover disk.

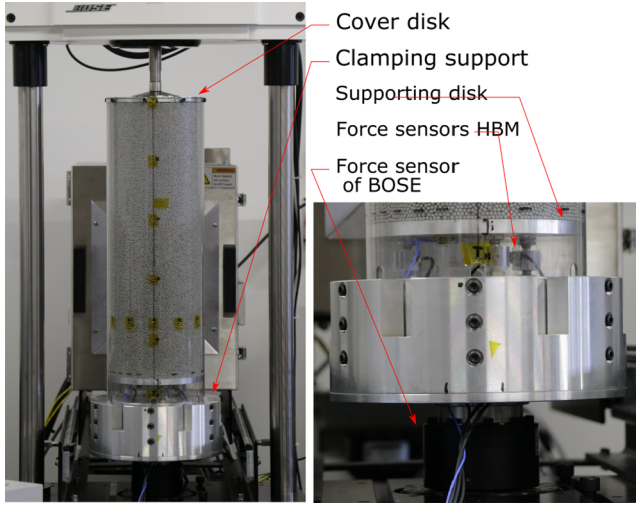


Fig. 5. Experiment setup for compression phase.

### 2.2.3. Equipment setup for vibration test

Our vibratory tests are done with the help of 10kN vibrating table LDS from BRUEL&KJAER. The electromagnetic actuator allows imposing vibrations the frequency of which varies from 5 Hz to 2000 Hz. The whole system including the polycarbonate tank, the grains contained inside the tank and the aluminum clamping support is clamped to vibrating table. A general view on the installation is provided in Fig. 2.

After being assembled to the vibrating table, a set of 12 accelerometers is installed on cylindrical tank wall in order to reconstruct modal shapes during vibration. The mass of each accelerometer is 2 g, which is an important property because this light mass does not influence the global behavior of the analyzed system. 12 accelerometers are put along the circumference in order to capture lobed modes. Moreover, 7 accelerometers are put along the longitudinal direction in order to capture flexural modes. Details on the disposition and labeling of accelerometers are shown on Fig. 6.

Additionally, three accelerometers are located under the plate which support grains contained in the tank in order to verify that this plate is rigid with respect to the other components of this setup in frequency domain. In order to verify that condition, numerical simulations were carried out with COMSOL. The obtained results show that the 1st vibratory mode of the supporting plate is found at 2000 Hz that is out of our interested range (from 150 Hz to 1000 Hz). The setup and numerical result for the supporting plate are shown in Fig. 7

## 3. Experimental procedure and results

In this section, the experimental method and the data acquisition are first introduced. Then, the results obtained from the experiments are presented. Finally, some important aspects concerning the physical complexity of the analyzed system are discussed.

### 3.1. Experimental method and data acquisition

In our experimental tests, the TGM system is vibrated under a random or swept sine excitation control. During the sinusoidal tests, the vibrating table applies a sinusoidal translation in the x direction. The frequency band of interest for these experiments spans from 20 Hz to 1000 Hz with a sweep speed of 0.5 oct/min. and a controlled acceleration amplitude of  $0.5g_{RMS}$ . In order to analyze the results obtained from the vibratory tests, we use two following principle tools: the *frequency response function* (FRF) and the *modal reconstruction*. The modal reconstruction is realized with help of PolyMax module [13] of the LMS software developed by Siemens. By analyzing the relative position and

motion of the accelerometers installed on tank wall, we are able to reconstruct the experimental modal analysis [14].

From the position of accelerometers illustrated in Fig. 6, it is important to notice that the accelerometers labeled from **A** to **L** are used to reconstruct the shape along circumference and the accelerometers labeled from **M** to **R** are used to capture the shape along the length of cylindrical tank.

### 3.2. Experimental results for empty tank

The most important purpose of this experiment is the vibration analysis of TGM system and comparison between experimental and analytical approaches. In order to compare these two approaches, we need first to verify the suitability of the chosen shell theory mentioned in [5,6,15]. By applying the method and data acquisition mentioned in the Section 3.1, first, we are able to identify the resonance peaks during the excitation. Result obtained by averaging the measurement of all accelerometers is shown in Fig. 8. The mean amplitude of vibratory response captured by accelerometers from M to R is shown in Fig. 9. The mean amplitude of vibratory response captured by accelerometers from A to L is shown in Fig. 10.

Then, the modal reconstruction process is realized at each resonance peak with the help of PolyMax tool to identify the modal shape of each natural mode. Finally, by combining the modal shape and the natural frequency of each mode, we are able to obtain the experimental results to compare with analytical and numerical results. It is interesting to notice that both flexural and lobed modes are well captured by the set of accelerometers from A to L (used to capture the lobed modes) and the set of accelerometers from M to R (used to capture the flexion modes).

Fig. 11 shows an example of this process for the identification of the 2nd flexural mode and the lobed mode ( $i = 4, j = 3$ ). It is important to remind that the double index ( $i, j$ ) denotes the lobed mode with  $i$  lobes along the circumference and  $j$  lobes along the length of the tank.

The natural frequency of each identified mode is compared with analytical results obtained with the formulations mentioned in [5,6] and numerical results obtained with the help of COMSOL. Results are shown in Table 3. It is important to notice that the numerical and analytical modelings in this paper are different from the ones presented in [5] due to the different length and Young's modulus of the tanks.

The good agreements between experimental, analytical and numerical approaches that are observed here show not only the suitability of the chosen shell theory discussed in [5] but also the reliability of the experimental method and data acquisition tools.

### 3.3. Experimental results for a tank filled with granular grains

In the experiments realized with the cylindrical tank fully filled with grains, both flexural and lobed modes are the targeted objectives of our analyses. In this section, we will discuss the detectability and the evolution of the natural frequencies as a function of pre-stress. It is important to remind that the vibratory packing mentioned in Section 2.2 is realized at the 1st flexural mode, the frequency of which is theoretically calculated with analytical formulations in [5,6]. With help of these formulations, we determine that the frequency of the 1st flexural mode is equal to 44 Hz and no other modes are found in the range from 0 to 150 Hz. Hence, in the experiments, the random excitation is varied from 150 Hz to 700 Hz in order to avoid the packing frequency. The process presented in the Section 1 is applied for the experiments with the tank fully filled with grains. In general, granular assembly is compressed to different levels of pre-stress. At each level, the excited vibration control in the targeted range with a sweep speed of 0.5oct/min and acceleration amplitude of  $0.5g_{RMS}$  is applied to the base of (Tank-Grains) system. Then, the data acquisition discussed in Section 3.1 is applied in order to achieve the modal shape and natural frequency of each vibratory mode. Finally, the curve of frequency - pre-stress evolution for each mode is constructed and compared with analytical results.



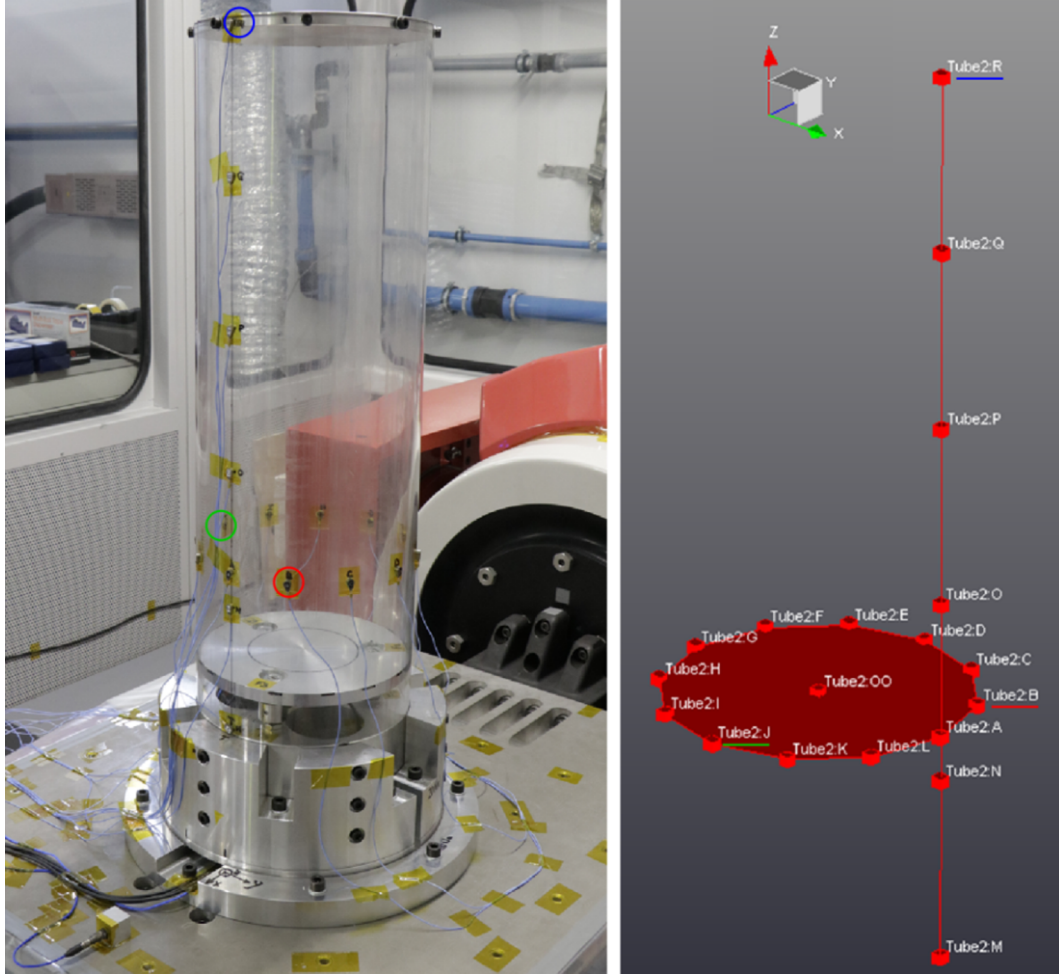


Fig. 6. Accelerometer setup and numeration of accelerometers.

### 3.3.1. Experimental results for flexural modes

First, we analyze the results for flexural modes. By applying the same process as discussed in the Sections 3.1 and 3.2, we are able to determine the resonance peaks found by all accelerometers. Figs. 12–14 show the mean values of vibratory response of the system in which granular medium is compressed with a force equal to 300 N (this force induces a pre-stress within the granular assembly equal to 10150 Pa), captured respectively by all accelerometers, accelerometers from M to R and accelerometers from A to L.

By observing the results obtained in these Figures, there are three peaks found at 215 Hz, 444.1 Hz. With help of modal reconstruction process presented in the Section 3.1, we are able to identify that these peaks correspond respectively to the 2nd, 3rd modes of flexion. In the

search for all detectable modes, the FRF graph is drawn for all the accelerometers. For the accelerometer R on top of the tank, another resonance peak is found at 597 Hz. By the reconstruction tool, this peak corresponds to the 4th flexural mode of the system. The FRF graph for this one is presented in Fig. 15.

One can also observe that each of the resonance peaks of the 2nd and 3rd flexural modes is split into two peaks which are very close to each other. Also by modal reconstruction tools, the modal shapes found at these doubled peaks are respectively very close to the ones of the 2nd and 3rd modes. This phenomenon may possibly be due to the non-perfect isotropy of granular materials embedded in the tank. Theoretically, for one natural frequency, any axial-rotation of one natural mode shape around the cylindrical tank vertical axis  $z$  (cf.

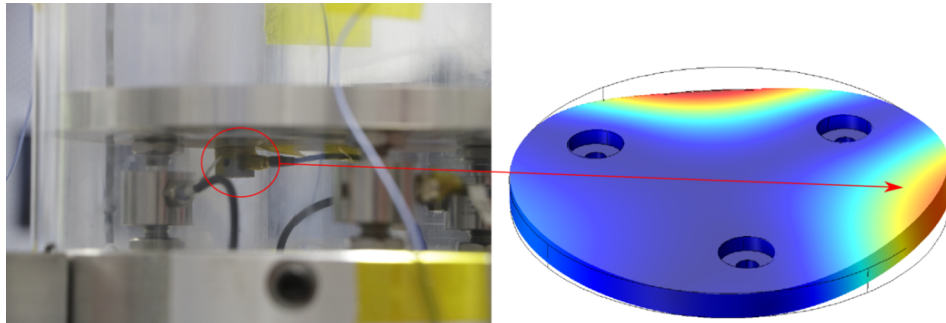


Fig. 7. (a) Accelerometers setup for the supporting plate (b) The 1st modal shape of the supporting disk.

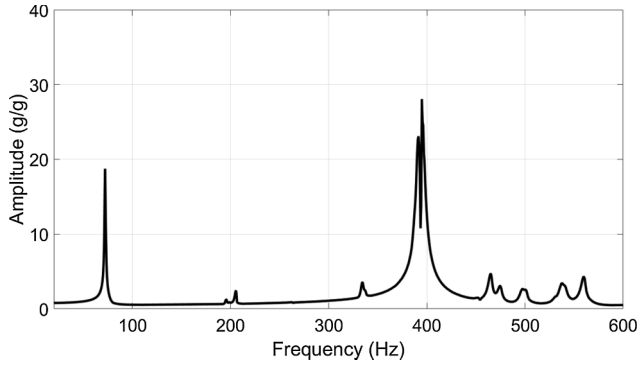


Fig. 8. Average value of frequency response captured by all accelerometers for the test with empty tank.

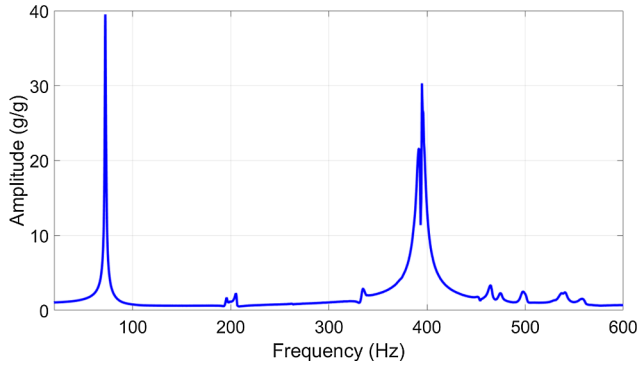


Fig. 9. Average value of frequency response of accelerometers from M to R for the test with empty tank.

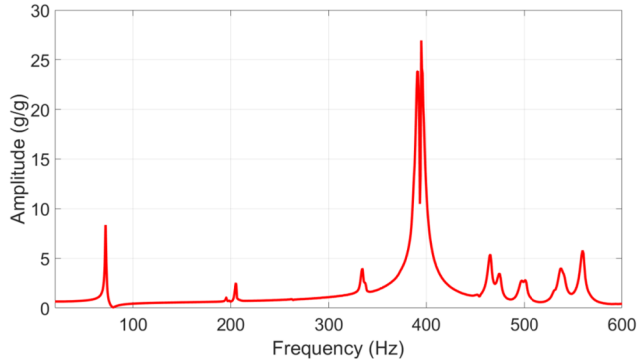


Fig. 10. Average value of frequency response of accelerometers from A to L for the test with empty tank.

Fig. 5) is also the same mode shape of the system. In these experiments, the translation along the axis  $x$  is favored in order to create modal shapes. In the case of fully filled tank, the stiffness of the whole system is slightly different in different directions due to the internal organization and the contact force chains created by embedded grains. By consequence, for a same modal shape, the vibratory frequencies of analyzed structure in different directions are a little bit different. By this conclusion, there may be even more than two similar modal shapes whose the frequencies are very close to each other that can be found by experiments with a tank fully filled with granular materials. This phenomenon also suggests the 1st limit of the proposed homogenization technique that is the granular assembly is considered as an isotropic medium whose mechanical behavior is identical in every direction [5]. Nevertheless, the model and test is relevant for the fundamental vibroacoustic modes.

The same process is repeated with other levels of compression

stress. For the sake of simplicity, the level of pre-stress in granular medium is decreased by imposing a random excitation in the frequency band [50; 100] Hz in one minute. The evolutions of the natural frequency as a function of pre-stress for each vibratory mode are presented on Fig. 16a, b and c for 2nd, 3rd and 4th flexural modes. A polynomial fit of 3rd order is also performed in order to show the evolution tendency.

The natural frequencies of the analyzed vibratory modes increase during the compression. In particular, the natural frequency of the 2nd flexural mode increases from 217 Hz to 235 Hz, the natural frequency of the 3rd flexural mode increases from 465 Hz to 495 Hz and the natural frequency of the 4th flexural mode increases from 655 Hz to 710 Hz. These phenomena agree with the analytical prediction and can be explained by the mathematical formulations presented in [5]. During compression, the effective Young's modulus evolves enormously due to new creations of new contact in granular system and non-linearity of Hertz's contact model [16,24]. As the consequence of this evolution, the global stiffness of the TGM system increases. Therefore, the natural frequencies increase during the compression. More interestingly, by the prediction made in [5,6], the natural frequencies of the flexural mode evolve slightly due to the small contribution of the granular assembly into the global stiffness of the whole analyzed system. However, despite of this predicted sensitivity, the evolution of the *natural frequencies* in term of *pre-stress* is really observed. These results are very encouraging and show the potential of the proposed methodology. One can also observe that natural frequency of three observed flexural modes increases rapidly with the pre-stress increased from 0 to 7 kPa. Then, these natural frequencies increase progressively with respect to pre-stress from 7 kPa to 14 kPa. After this value, the natural frequencies increase rapidly. Thus, the pressurization is represented in a pertinent way.

To have a better view on the natural frequency evolution, we take a look at the FRF of the 2nd flexural mode illustrated in Fig. 17.

As we can observe, the frequency of this natural mode increases slightly from 219 Hz to 224 Hz, 227 Hz and finally 229 Hz while the compressive force is increased from 124 N to 215 N, 385.5 N and 407.5 N. This FRF graph points out evidence which confirms the prediction made by the analytical method. Another interesting phenomenon which is observed in these experiments is that the amplitude of vibration also increases with respect to the pre-stress applied on the granular medium. This phenomenon shows that the dissipation of energy during vibration decreases when the pre-stress applied on the granular medium increases. It is possible to explain that the high valued pre-stress prevents the relative motions between the grains in the granular system. Hence, less energy is dissipated by granular materials during the vibration. With the low value of the applied pre-stresses, the grains can easily move inside the tank, thus the energy given by the excitation is dissipated greatly by this motion inside the granular medium. This observation also suggests the idea that in order to better capture all the vibration modes of a Tank-Granular materials system, it is necessary to impose high pre-stresses on the embedded granular materials, hence, the rigid grain is a better candidate for this new method. It is important to remind that the proposed method is aimed for the modal analysis of cryogenic tanks fully filled with an incompressible fluid by using granular materials.

In order to have quantitative comparison with the analytical results, we attempt to mathematically compute the evolution of *frequency* as a function of *pre-stress* by using the proposed formulations [5]. In those formulas, the macroscopic Poisson's ratio and Young's modulus are determined from the shear modulus and Poisson's ratio of the grains as in [7]

$$\vartheta_G = \frac{2(1 - \vartheta_m)(5 - 10\mu) + 2(5 - 4\vartheta_m)\mu^2}{2(1 - \vartheta_m)(20 - 20\mu) + 4(5 - 4\vartheta_m)\mu^2} \quad (1)$$

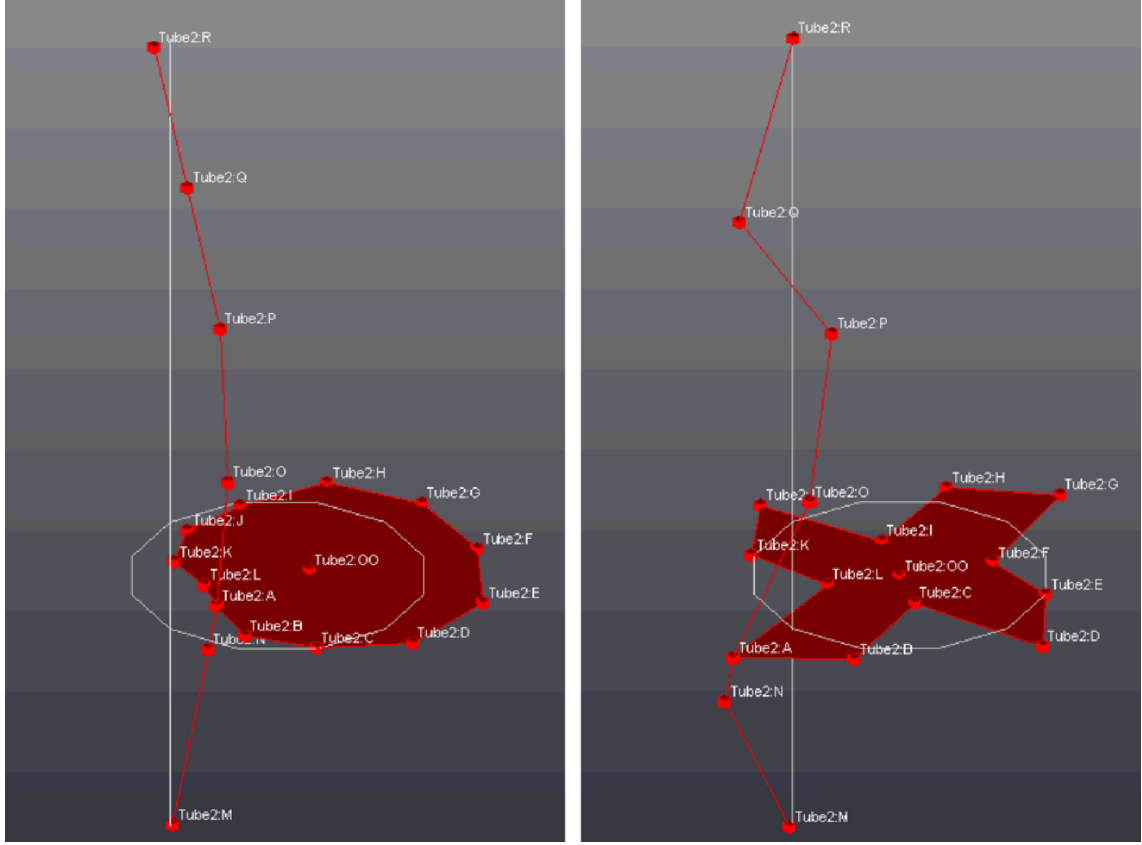


Fig. 11. (from left to right): modal shapes of the 2nd flexural mode and the lobed mode (4,3) captured by the accelerometers.

$$E_G = E_0 \frac{1 - \vartheta_m}{2(1 - \vartheta_m)(1 - \mu) + \frac{4}{5}(5 - 4\vartheta_m)\mu^2} \quad (2)$$

with

$$E_0 = \left( \frac{G_m N \bar{D}^3}{\sqrt{6}(1 - \vartheta_m)} \right)^{\frac{2}{3}} \cdot \sigma_M^{\frac{1}{3}} \quad (3)$$

In these expressions

- $\vartheta_m$ ,  $G_m$  are respectively Poisson's ratio and shear modulus of grains
- $\sigma_M$  is the mean stress defined by  $\sigma_M = \frac{1}{3}tr(\sigma)$
- $\bar{D}$  is the average diameter of grains
- $N$  is the average number of contacts in one unity of volume, this operator is defined by the formula  $N = \frac{Z \cdot n}{2V}$  with  $n$  is the total number of grains,  $V$  is the volume of granular assembly,  $Z$  is the coordination number defined as the average number of contact per particle. This parameter characterizes the local arrangement of grains in the system. we attempt to use the model proposed by Field [22]:

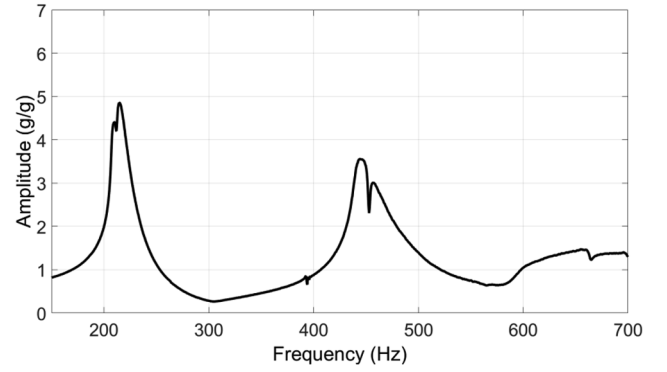


Fig. 12. Average value of frequency response captured by all accelerometers for the test with (Tank – Grains) at 10150 Pa.

$$Z = \frac{12}{1 + e} \quad (4)$$

Table 3

Comparison between experimental, analytical and numerical results of vibration test with empty tank.

			Experiment (Hz)	Analytical (Hz)	Difference (%)	COMSOL (Hz)	Difference (%)
Flexural modes		1	72	70.3	2.4	69.9	3.0
		2	398	401.2	0.8	388	2.6
Lobed modes	$j = 1$	$i = 2$	196	228.0	14.1	200	2.0
		$i = 3$	206	209.7	1.8	208	1.0
		$i = 4$	335	351.0	4.6	337	0.6
		$i = 1$	398	401.2	0.8	388	2.6
	$j = 2$	$i = 2$	468	489.3	4.3	469	0.2
		$i = 3$	340	321.1	5.9	340	0
		$i = 4$	501	490.0	2.3	502	0.2
	$j = 3$						

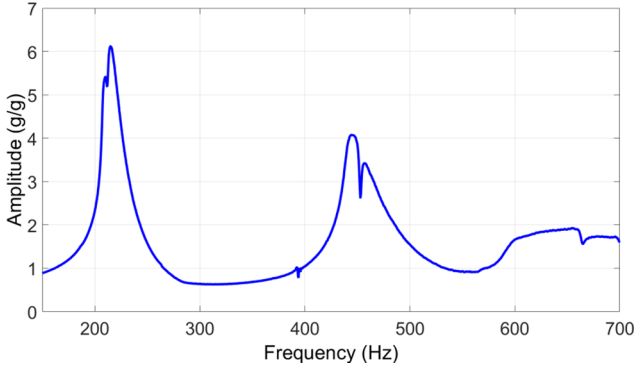


Fig. 13. Average value of frequency response captured by accelerometers from M to R for the test with (Tank – Grains) at 10150 Pa.

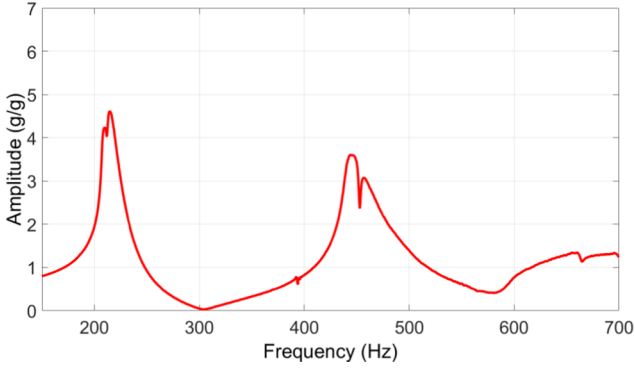


Fig. 14. Average value of frequency response captured by accelerometers from A to L for the test with (Tank – Grains) at 10150 Pa.

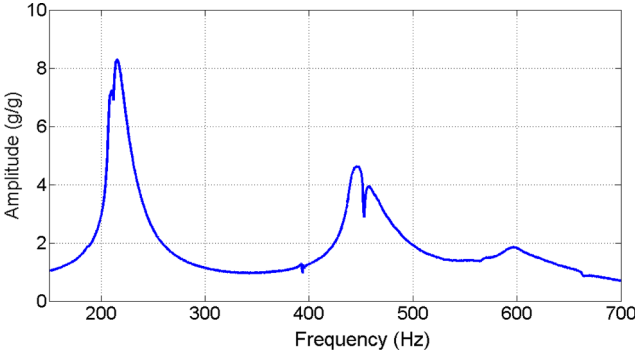


Fig. 15. Frequency response captured by accelerometer from R for the test with (Tank – Grains) at 10150 Pa.

where  $e$  denotes the void ratio of granular medium

–  $\mu$  is the homogenization parameter. This parameter may vary from 0 to 1 and is identified by curve-fitting technique with an oedometric test.

First, we need to identify the parameters for the chosen homogenization model based on the results obtained from oedometric compression. We found that the parameter of homogenization  $\mu = 0.5$  and the Poisson coefficient  $\nu = 0.465$  give the best fit between experimental and analytical results as illustrated in Fig. 18

For flexural modes, the analyzed system is considered as a Timoshenko's beam that is clamped at one of its extremities and carrying a mass of the cover disk at its other free end [5,6]. As we can observe on Figs. 5 and 19, the tank is not fully filled all along its length for the accelerometers installation. This setup makes a difficulty in

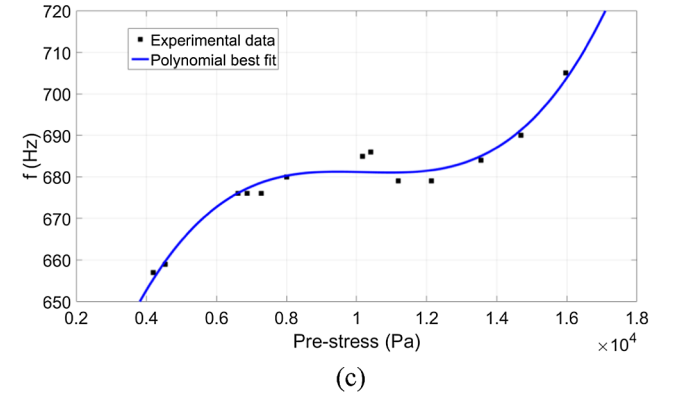
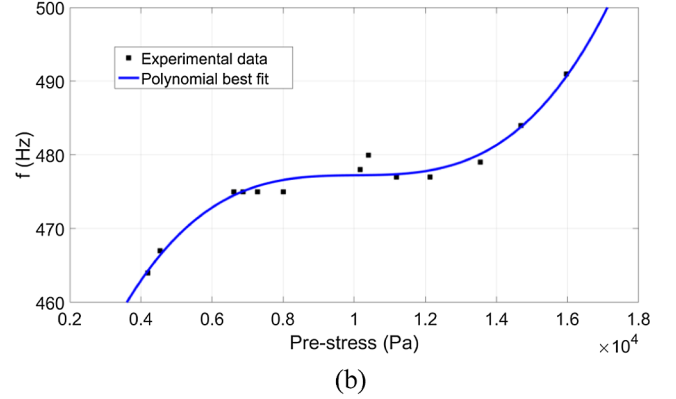
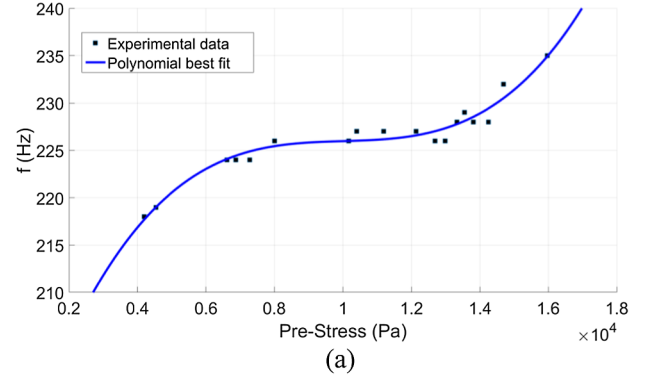


Fig. 16. Evolution of the eigen frequencies of flexural modes as a function of pre stress: (a) 2nd mode (b) 3rd mode (c) 4th mode.

analytical modeling because the contribution of the granular assembly to the global inertia and stiffness is taken into account at every cross-section of the Timoshenko's beam. As we can observe in Fig. 19, there are two different lengths. The first one is measured from surface of vibrating table to the cover disk; this length is denoted  $L_1$  and equal to 0.612 m. The second one is measured from the upper surface of the disk in contact with grains to the cover disk; this length is denoted  $L_2$  and equal to 0.561 m. In this work, the second length ( $L_2 = 0.561$  m) was taken into the analytical modeling

With the help of the analytical formulations [5,6], the evolutions of the natural frequency as a function of axial pre-stress for the flexural modes 2, 3 and 4 are computed and compared with experimental results. Fig. 20a, b and c present the analytical results obtained with the proposed models.

The mean absolute percentage difference is calculated for these three vibratory modes with following formula:

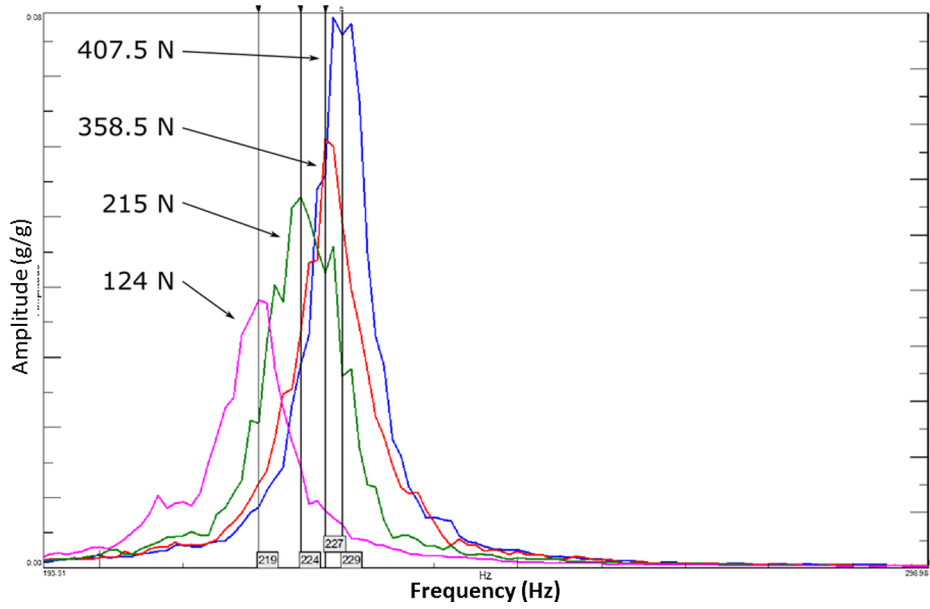


Fig. 17. Frequency response for the 2nd flexural mode at different levels of compression.

$$\text{Difference} = \frac{1}{n} \sum_i^n \frac{|f_{exp}^i - f_{ana}^i|}{f_{exp}^i} \quad (5)$$

where

- $f_{exp}^i$  is natural frequency obtained by experiments
- $f_{ana}^i$  is eigen frequency determined with analytical formulations [5] and [6]
- $n$  is total number of experimental results
- $i$  Denotes the result number “i”

Table 4 presents differences between experimental and analytical results.

We are able to conclude that the errors between two different approaches are very small for the flexural modes 2, 3 and 4. These results experimentally show the potential and suitability of the new proposed methodology

By observing these graphs (an considering the scale) one can deduce that good agreements between experimental and analytical results are

obtained in terms of tendency and quantitative values. The proposed model [5] is able to predict the evolution of natural frequencies of the (Tank and Granular materials) system with respect to pre-stress from 0 to 14 kPa. However, this model is not capable of predicting the evolution of natural frequency when the pre-stress is superior to 14 kPa. The reason for this conclusion is that the elastic yield of this granular material is attained when the applied pre-stress exceeds 14 kPa in oedometric condition. That means, plasticity appears if the pre-stress is superior to 14 kPa. Plasticity may reduce the stiffness (Young's modulus) of granular material. As consequence, the natural frequencies of the vibratory system decrease with respect to the increase of pre-stress. As the matter of fact, the proposed analytical formulations cannot predict the evolution of natural frequencies because only elastic behavior is taken into account of this model. It is also important to notice that plasticity in this material may be due to different mechanisms, such as, deformation, crushing of grains, slip between grains in the medium. To conclude, further research needs to be done in order to understand the physical mechanism behind the plasticity and to well predict the natural frequencies of the TGM system after the elastic yield.

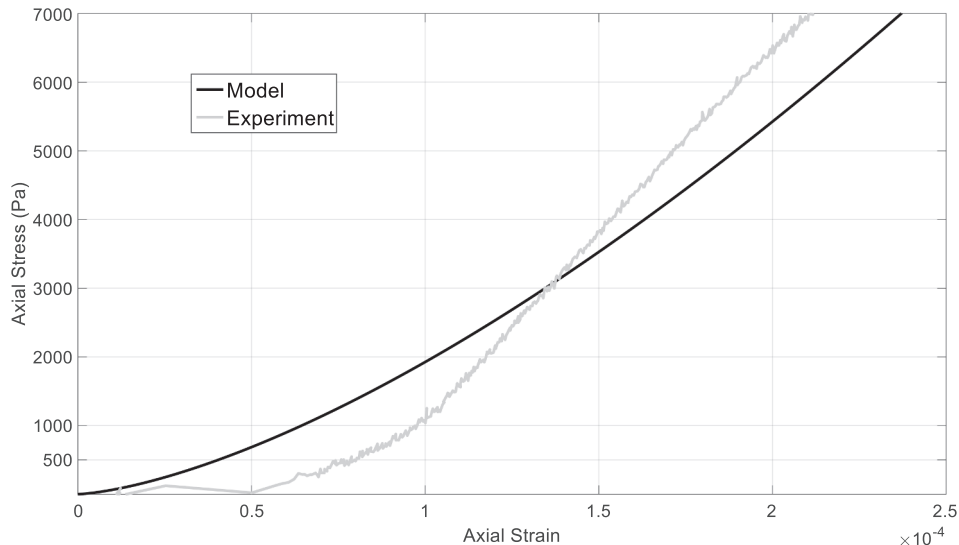


Fig. 18. Modeling the static behavior of granular material by using Emeriault and Cambou's model.



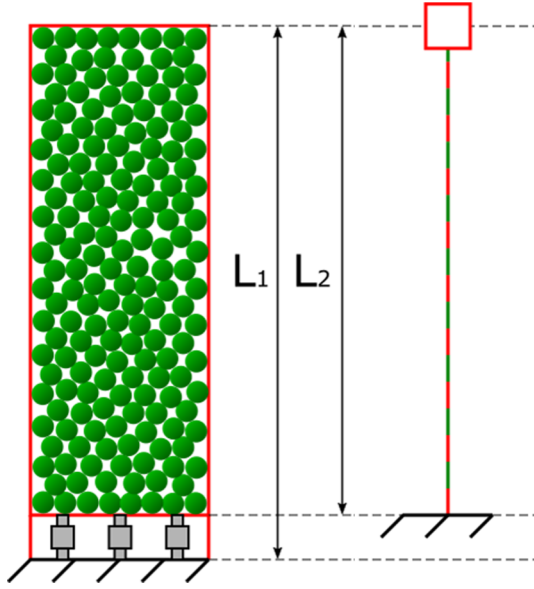


Fig. 19. Analytical models.

The analytical model in which only the part filled with granular material is taken into account gives a good fit between the analytical and experimental approaches for TGM system. It can be explained that the filled part of the tank plays the most decisive role in vibratory behavior of the analyzed system. The gaps between the analytical and experimental results can be explained by the fact that the homogenization model cannot give the perfect fit with the real mechanical behavior of the granular material under oedometric conditions. There are several reasons behind this phenomenon. First, the proposed homogenization technique is developed for granular assemblies made of homogenous solid grains, while our grains are heterogeneous, filled with polystyrene and coated outside with epoxy resin. This complex constitution of the provided composite grains may give to the granular medium a different behavior than the solid grains do. Secondly, the tank is manually filled. This process was done very progressively; however, it is difficult to avoid some localization in the medium. More automatic filling process may be a better solution.

### 3.3.2. Experimental results for lobed modes

By observing the FRF in Figs. 12–14, there are only three detected resonance peaks corresponding to the 2nd, 3rd, and 4th flexural modes. However, by using analytical tools, we are able to predict that the lobed mode (2, 1) should be found at 344 Hz, the lobed mode (3, 2) should be found at 521 Hz. These facts show that, in contrary with empty tank, the lobed modes in the experiments with the filled tank are not easy to be detected. In order to solve this problematic, many other tests are realized. In those tests, the sweep speed is decreased to 0.25 oct/min, the amplitude of excitation increased to 1  $g_{RMS}$ , then 2  $g_{RMS}$  and finally 4  $g_{RMS}$ . In order to increase the detectability, the excitation is targeted in the ranges [70; 160] Hz, [250; 400] Hz and [500; 700] Hz in which, some lobed modes are analytically predicted to be appeared. Despite of all these efforts, it is impossible to detect the lobe modes with this kind of excitation.

## 4. Energy dissipation produced by granular materials

In this section, the non-linear behavior of TGM system obtained from experiments is first presented. Then, several explications for this phenomenon are introduced in order to analyze mechanisms behind this non-linearity. Based on this analysis, some solutions are proposed to detect the lobed modes which are an important objective in this project.

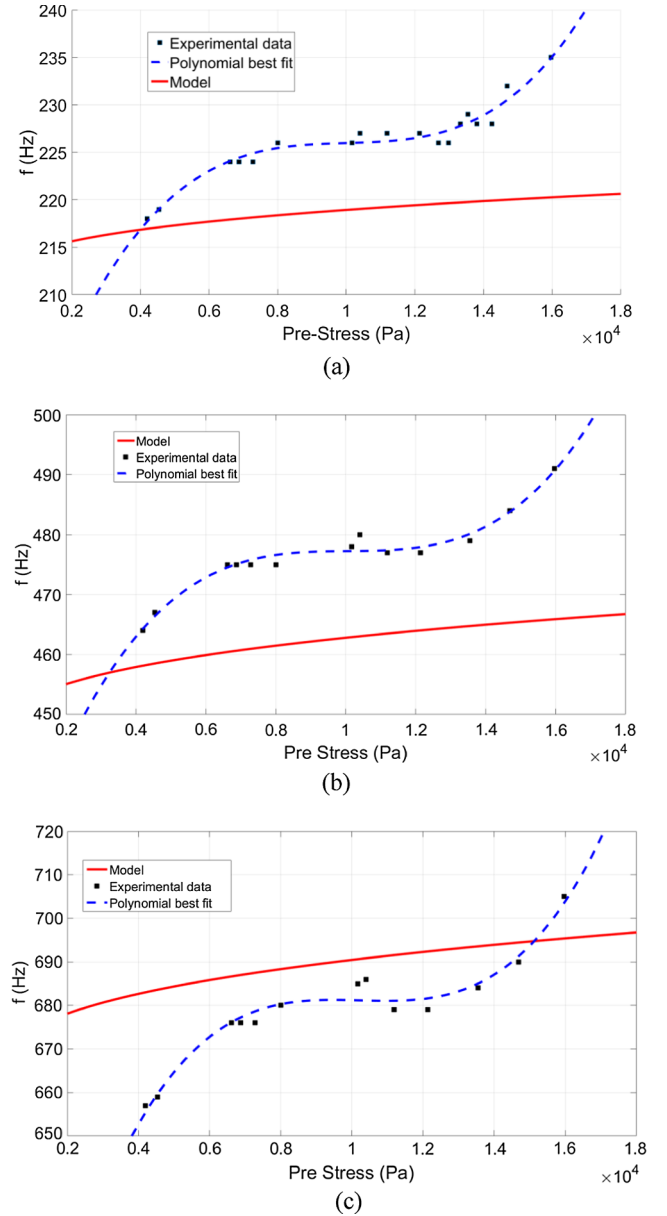


Fig. 20. Modeling the evolution of eigen frequency of flexural modes (a) 2nd mode (b) 3rd mode (c) 4th mode.

Table 4

Difference between experimental and analytical methods for flexural modes 2, 3 and 4.

Flexural mode	Difference (%)
2	3.2
3	3.3
4	5.0

In the search for lobed modes, many experiments carried out with different levels of excitation as previously presented in the Section 3.3.2. No lobed mode can be detected even if the amplitude of excitation is increased enormously. The experimental setup with vibrating table does not really allow exciting the lobed modes in case of tank fully filled with grains because of the dissipation due to embedded material. In the last attempt for detecting the lobed modes, another setup is prepared. In this test, we use point shaker PODERA instead of the vibrating table. Fig. 21 shows a general view on this setup.



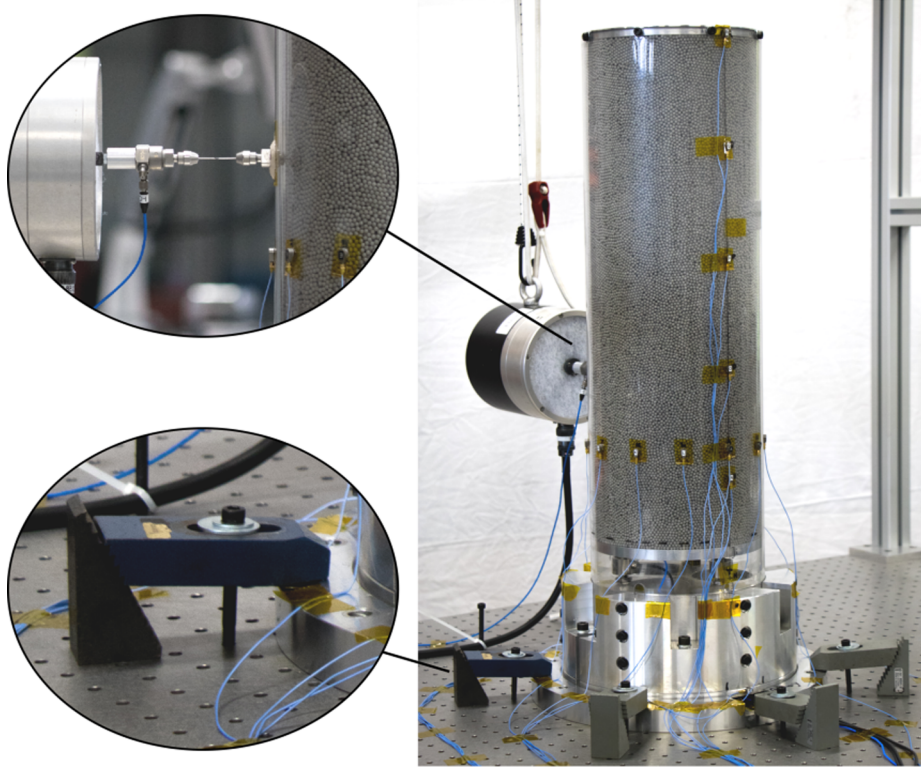


Fig. 21. Experiment setup for vibratory test with point-shaker.

An excitation is applied to one particular point situated at the altitude of 230 mm from the clamped surface of the tank. With the point shaker, a sinusoidal excitation at targeted frequency is applied to structure. Many experimental tests were carried out with different levels of excitation (expressed in tension). With the new experiment setup, it is still not possible to detect the appearance of lobed modes. However, in the search for these natural modes, we discover as described hereafter that the analyzed TGM system has a non-linear behavior during vibration. The non-detectability is due to non-linear damping produced by granular media. This phenomenon along with the viscous behavior of granular materials will be presented and discussed carefully in this section.

The experiments with point shaker are applied for the flexural modes which are clearly identified as presented in the Section 3.3.1. To summarize, experiments were done with tension varied from 0.1 V to 2 V, the frequency increment is 0.05 Hz. Fig. 22 presents the results for the 3rd flexural modes at the pre-stress of 15,220 Pa (corresponding to a compressive force equal to 450 N) measured by the accelerometer B.

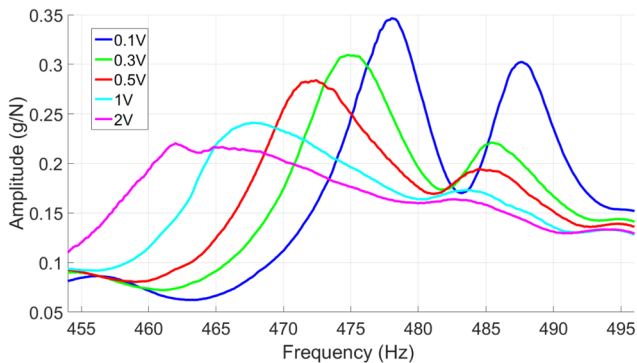


Fig. 22. Frequency response for the 3rd flexural mode at different amplitudes of excitation.

We are able to observe a phenomenon where the frequency and amplification factor decrease while the amplitude of excitation increases respectively from 0.1 V to 2.0 V. The resonance peaks obtained with stronger excitations are less clear than the ones obtained with weaker excitations. This suggests so that the analyzed structure presents a non-linear behavior during vibration due to damping. The damping factor for this dynamic system may be due to several mechanisms such as, viscoelastic damping (due to viscous behavior of material), hysteresis damping (due to friction between grains) and structural damping. All these damping mechanisms may depend on frequency and the modal shape. Hence, the total damping factor for the different modes is different. It suggests the idea to explain why there are the modes which can be detected and the other modes cannot be detected with a same material.

In the reference [6], numerical simulations for the experiment with point shakers were performed with ABAQUS in order to demonstrate the influence of viscoelastic damping on detectability of different modes. Two simulations are carried out, in which, the granular material is considered as a homogeneous continuum. The 1st one is performed with pure elastic material whose Young's modulus is equal to 10 MPa and overall structural damping is 0.15% in the frequency domain. The 2nd one is performed with a viscoelastic material whose complex modulus is expressed in the form:  $G^* = G_s(\omega) + iG_l(\omega)$  in which  $\frac{G_l(\omega)}{G_s(\omega)} = \tan(\varphi)$ . It is important to notice that  $\varphi$  represents the lag between response and loading in DMA (Dynamic Mechanical Analysis). A DMA test on the granular material was also performed in the laboratory and the obtained result is  $\varphi = 1^\circ$  for the frequency range [0, 50] Hz. However, in this simulation, we assume that this value is also valid for the frequency range [0, 600] Hz. The following Fig. 23 presents the results of 2 simulations.

As we can observe, with a pure elastic material, all the resonance peaks from 0 to 600 Hz are well detected (blue line). With a viscoelastic behavior, only the resonance peaks of flexural modes are detected. On contrary, all the resonance peaks of lobed modes have disappeared as

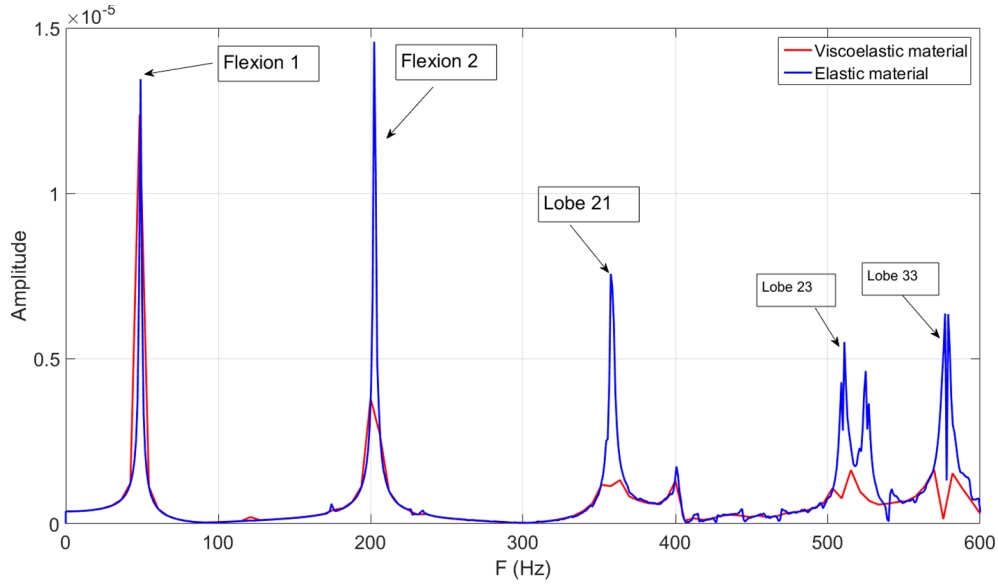


Fig. 23. FRF graphs for vibrating system with elastic and viscoelastic granular materials.

shown in the experimental results. This simulation result puts in evidence that the material viscous behavior has different influences on the detectability of vibration modes.

The hysteresis phenomenon was also observed in literature, such as, beams filled with entangled – cross –linked materials [17], vibratory behavior of mineral, glass wool [18,19], felt pads materials [20] and granular materials [21]. In those studies, the authors pointed out that friction produced by the internal arrangement of the constitutive materials was an important source of energy dissipation during vibration and acts like damping in this vibratory system. The dynamical behavior of the TGM system is similar with the one of a (Beam – Entangled cross – linked fibers) system for which the energy is dissipated by friction between embedded fibers [17]. Due to this similarity, it is possible to deduce that the energy dissipation in the recent analyzed system is produced by the internal friction between grains contained inside the tank. The energy dissipated by intergranular friction maybe very important because of the high angle of repose between grains determined by our industrial partners DynaS+ and Ateca [10]. This value is about  $18.8^\circ$ , which gives the friction coefficient a value of 0.34 ( $k = \tan(\varphi)$ )

## 5. Conclusion

This paper developed and discussed about some experimental aspects of the new methodology for Fluid – Tank vibration analyses by using granular grains as surrogate material. In these experiments, we focused on the evolution of natural frequency of each natural mode as a function of axial pre-stress. In the experimental setup, a system of accelerometers is installed on the tank interfaces in order to capture the modal shape of two families of natural modes: flexural and lobed modes during vibration. Experiments were carried out with help of vibrating table and point shaker under swept sine or random excitation control. The experimental tests were carried out with the tank submitted to different levels of compression. To summarize, the flexural modes are clearly detected and show an evolution of frequency as the results predicted with analytical tools. One can observe the increase of the natural frequencies with respect to compressive stress. Experimental results for the 2nd, 3rd and 4th modes were also compared with analytical results (which were carried out in the present work) and showed a good agreement. This is an encouraging result and demonstrates the potential of the new methodology. From the experimental results, it is necessary to find a homogenization model for granular media composed of heterogeneous grains (in this work, the polystyrene grains is

coated outside by a thin layer made by epoxy resin). By having a better homogenization model, we are able to have a better analytical formulation to predict the evolution of each natural frequency during compression. It is also important to notice that filling process mentioned in Section 2.2.3 created initial localization in granular system because more grains were added at the end of manual filling. For future work, the process combining (manual – vibrating) packing should be done layer by layer in order to avoid this initial non –perfection in granular medium. Thus, analytical formulations are able to predict more precisely the evolution of natural frequencies as a function of pre-stress.

In contrary to flexural modes, it was impossible to detect the lobed modes for the tank filled with grains. In the search for this family of natural modes, we attempted to use different types of excitation by using vibrating table and point shaker. The tests were carried out with different amplitudes of excitation and targeted to some special ranges in which lobed modes were predicted to be appeared but no positive result was obtained. This difficulty is due to the fact that energy given to vibratory structure is dissipated by granular assembly contained inside the tank. There are two mechanisms which play important role in energy dissipation: friction between grains and viscous behavior of granular medium. These two mechanisms also produced non-linear behavior of the analyzed system during vibration. We observed a diminution of vibratory amplitude while increasing the amplitude of excitation. Based on the analyses of dynamical behaviors, a solution of using quasi – frictionless grains may be a potential candidate to solve the remained problem. By using quasi-frictionless grains, it is possible to reduce the energy dissipated by the granular media; hence, the damping factor of the whole system is also reduced. Another solution is that it is necessary to impose large values of pre-stresses on the granular medium in order to prevent the motion of grains inside that granular medium. By consequence, we are able to detect the lobed modes. Also, the utilization of multiple point-shakers set up in different directions to excite targeted lobed modes could be a good solution. For example, we may put 2 point shakers at the middle of the tank and excite in 2 opposing directions to excite the lobed mode (2, 1). The effectiveness of these proposals remains to be demonstrated numerically and experimentally and will be investigated in future works.

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valuable suggestions and comments which have help us improving this paper.

## Appendix A

The purpose of these appendices is to remind some important ideas and conclusions of the previous works related to this paper. The details can be found in [5].

### Homogenization model for granular materials

As already mentioned in the Section 3.3.1, the homogenization proposed by Emeriault and Cambou is chosen in order to model the elastic behavior of granular materials. The authors decided to use this model because it is enable to link the mechanical properties (such as Young's modulus, Poisson ratio), the geometrical properties (diameter) of each individual grain, the total number of grains, the void ratio and the mean stress applied on whole medium to the global mechanical behavior of the granular material. This homogenization model is elaborated with the important assumptions that each grain is rigid, homogeneous, the contact between grains is based on Hertz's model [25]. In this paper and [5], the authors used the isotropic model of the Emeriault-Cambou's homogenization. An anisotropic model can be a better candidate to achieve a better result for the modelling of the mechanical behavior of granular material embedded inside the tank.

An inconvenience of this homogenization is that it is based on the hypothesis that each grain is homogenous. Thus, for the grain constituted of polystyrene and enrobed with a thin layer of resin in this project, it is impossible to fit perfectly the experimental and analytical results. Another disadvantage of this homogenization model is that the frictional coefficient between grains is not explicitly taken into account. However, the dependence of mechanical behavior on friction angle can be expressed implicitly via void ratio. For the media composed of grains whose mechanical and geometrical properties are the same except the friction angle, the void ratio of 2 media are different, hence, their mechanical behaviors are different.

It is also important to remind that only elastic behavior of granular material is the object of this research. Thus, the plastic behavior is not taken into account of the modeling.

### Methodology for modal analysis of cryogenic tank by using granular materials as surrogate materials

The new methodology proposed in [5] is based on the modal equivalency between Tank-Fluid and Tank-Granular material system by applying pre-stress on the embedded granular material. As the details mentioned in [5], the Young's modulus of granular material increases greatly during the compression. On contrary, the density of granular material is almost constant during this process. An example for the evolution of these two variables are presented in Fig. B1a and B1b. Details about this modeling is mentioned in the Section 3.1 of [5]

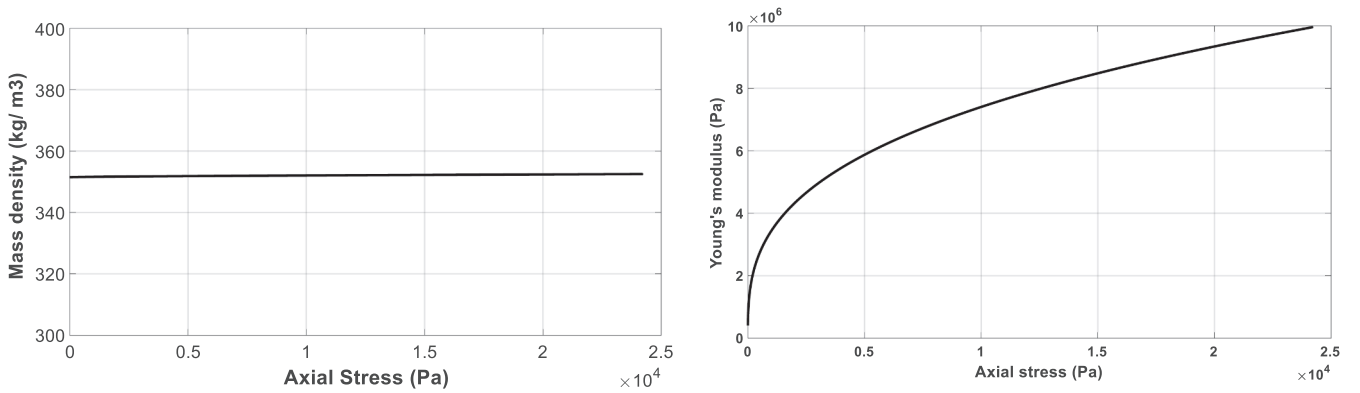


Fig. B1. Evolutions of (a) the mass density and (b) the Young's modulus of granular assembly during the compression test.

The eigenfrequencies and the mode shapes of the empty tank are determined with the help of Donnel-Mushtari's thin shell theory for the lobed modes and Timoshenko's beam theory for the flexural modes. For a tank fully filled with liquid, the embedded fluid plays the role of an added mass, the embedded granular material plays the role of an added mass and added stiffness to the vibrating system. By using the equations already presented in [5], we are able to predict that the eigenfrequency of each vibration mode increases during the compression process due to the large evolution of Young's modulus. Hence, the pre-stress needed to apply on granular medium in order to achieve the modal equivalency between Tank-Fluid and Tank-Granular material is determined as the intersection of two curves: *evolution of eigenfrequency as a function of pre-stress for Tank-Granular system* and *the targeted frequency of Tank-Fluid system*. It is important to notice that the necessary pre-stress is different for different vibrating mode. Fig. B2a and b illustrate the results for the lobed mode  $i = 2, j = 2$  and the flexural mode 1. Details about these calculations are presented in the Sections 3.2 and 3.3 of [5]

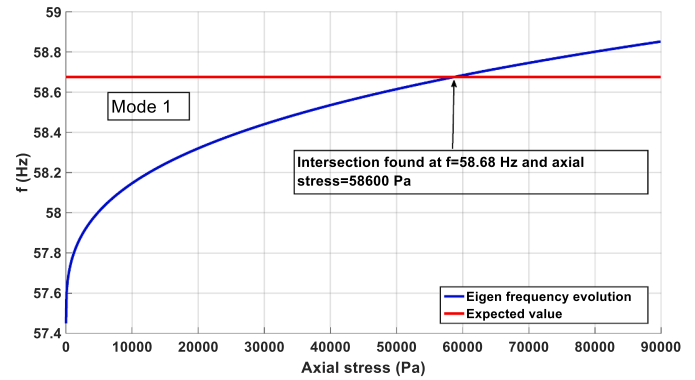
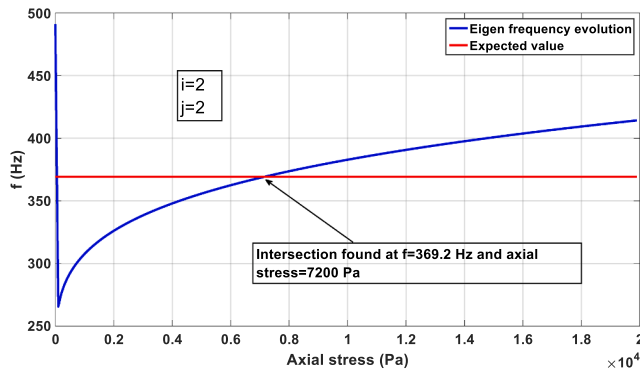


Fig. B2. Evolution of eigen frequencies with respect to axial stress – determination of required axial pre-stress for (a) Lobed modes  $i = 2, j = 2$  (b) Flexural mode 1.

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