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SENSITIVITY ANALYSIS IN ZIKA VIRUS DYNAMICS AND A MODEL DISCREPANCY APPROACH

SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON
NONLINEAR DYNAMICS

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Michel Tosin^a, Americo Cunha Jr^a, Flávio Coelho^b
^aUniversidade do Estado do Rio de Janeiro (UERJ)

^bFundação Getúlio Vargas (FGV)

Núcleo de Modelagem e Experimentação Computacional (NUMERICO)



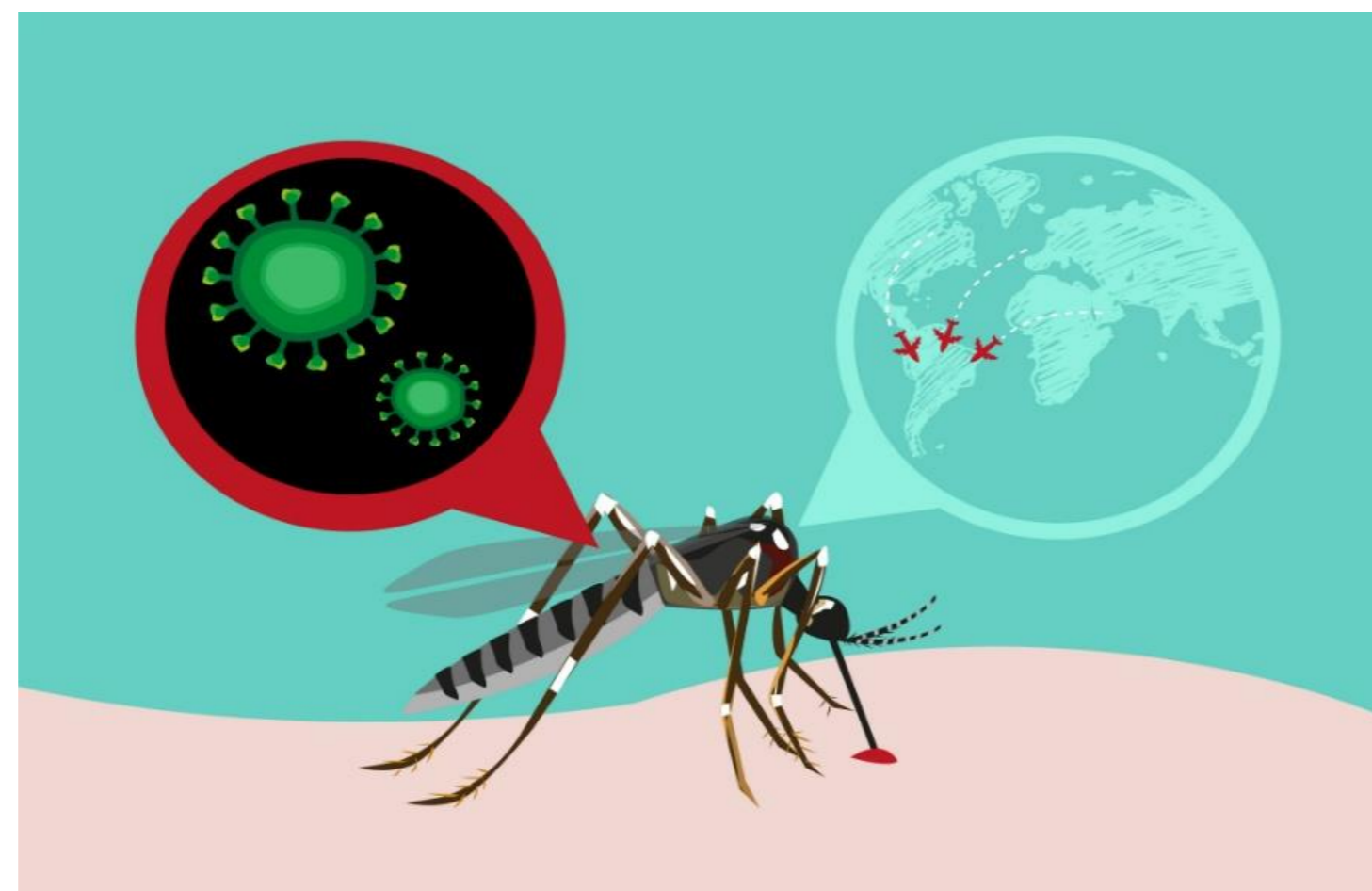
ESCOLA POLITÉCNICA DA
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INTRODUCTION

- Zika virus: global widespread and connection with congenital diseases;
- 2016: Zika becomes a public health emergency of international concern;
- Main vector: Aedes mosquitoes;
- A validated model can reveal new characteristics of the disease;
- Relations of model parameters are also of interest;

FIGURE 1 - Zika transmission representation.

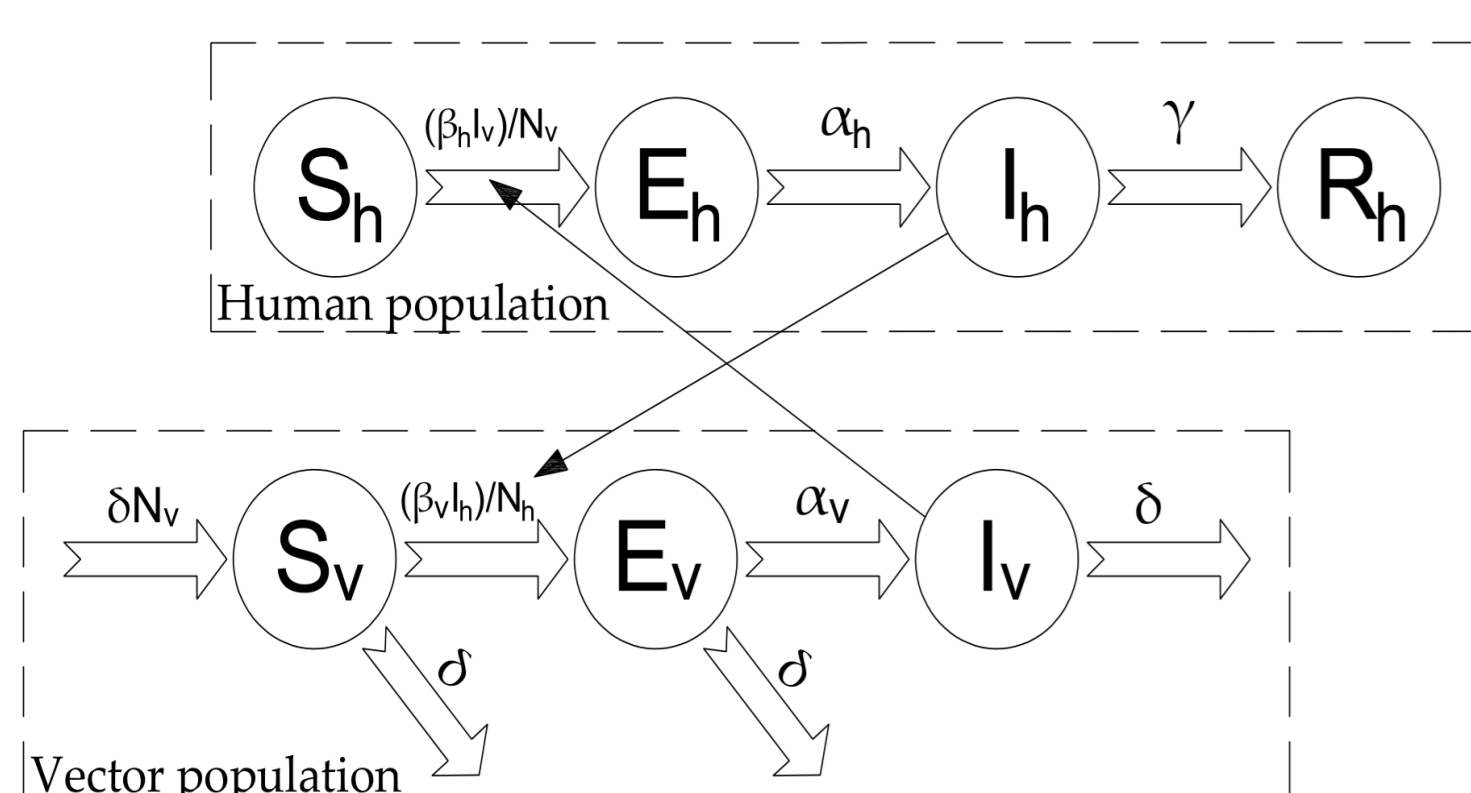


OBJECTIVE

- ◆ Perform sensitivity analysis to compare the parameters' global effect under different scenarios;
- ◆ Develop a statistical framework using Bayesian Inference and Polynomial Chaos Expansion to quantify epidemic model discrepancies;

COMPUTATIONAL MODEL

FIGURE 2 - SEIR-SEI model schematic [1].

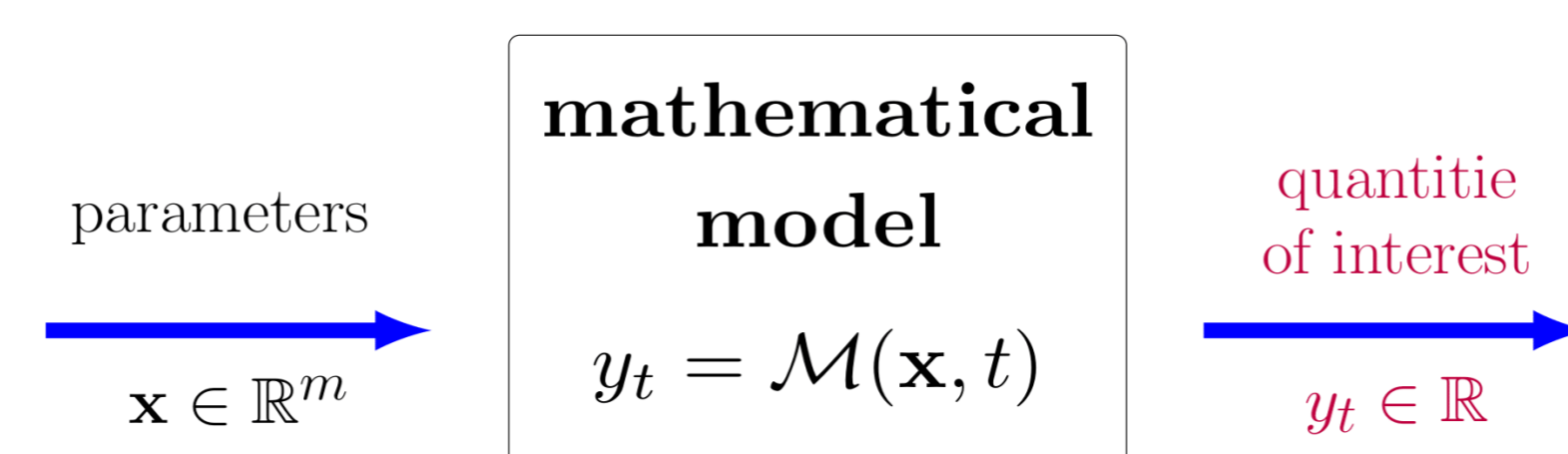


DYNAMICAL SYSTEM:

$$\begin{aligned} \frac{dS_h}{dt} &= -\beta_h S_h \frac{I_v}{N_v}, & \frac{dS_v}{dt} &= \delta N_v - \beta_v S_v \frac{I_h}{N_h} - \delta S_v, \\ \frac{dE_h}{dt} &= \beta_h S_h \frac{I_v}{N_v} - \alpha_h E_h, & \frac{dE_v}{dt} &= \beta_v S_v \frac{I_h}{N_h} - (\alpha_v + \delta) E_v, \\ \frac{dI_h}{dt} &= \alpha_h E_h - \gamma I_h, & \frac{dI_v}{dt} &= \alpha_v E_v - \delta I_v, \\ \frac{dR_h}{dt} &= \gamma I_h, & \frac{dC}{dt} &= \alpha_h E_h. \end{aligned}$$

+ Initial Conditions

FIGURE 3 - Observation operator schematic [2].



QUANTITIES OF INTEREST:

- Cumulative cases of infectious:
 $C(t) = \int_{\tau=0}^t \alpha_h E_h(\tau) d\tau$
- New cases per week:
 $\mathcal{N}_w = C_w - C_{w-1}, w = 1 \dots 52, \mathcal{N}_1 = C_1$

SENSITIVITY ANALYSIS

The Hoeffding-Sobol' decomposition [3] for n iid $X_i \sim \mathcal{U}(0,1)$ gives

$$Y_t = \mathcal{M}_0 + \sum_{1 \leq i \leq n} \mathcal{M}_i(X_i) + \sum_{1 \leq i < j \leq n} \mathcal{M}_{ij}(X_i, X_j) + \dots + \mathcal{M}_{1 \dots n}(X_1 \dots X_n),$$

$$\mathcal{M}_0 = \mathbb{E}[Y_t], \mathcal{M}_i(X_i) = \mathbb{E}[Y_t | X_i] - \mathcal{M}_0, \mathcal{M}_{ij}(X_i, X_j) = \mathbb{E}[Y_t | X_i, X_j] - \mathcal{M}_0 - \mathcal{M}_i - \mathcal{M}_j.$$

Sobol' Indices: interaction effect of inputs in \mathbf{u}

$$S_{\mathbf{u}} = \text{Var} [\mathcal{M}_{\mathbf{u}}(X_{\mathbf{u}})] / \text{Var} [\mathcal{M}(\mathbf{X})]$$

The Polynomial Chaos Expansion [2] of $m \times Y = \mathcal{M}(\mathbf{X})$, for a multivariate orthonormal polynomial Φ_{α} family with y_{α} coefficients

$$Y_t = \sum_{\alpha \in \mathbb{N}^k} y_{\alpha}(t) \Phi_{\alpha}(\mathbf{X}),$$

enables analytic computation of Sobol Indices:

$$S_{\mathbf{u}} = \sum_{\alpha \in \mathcal{A}_{\mathbf{u}}} y_{\alpha}^2 / \sum_{\alpha \in \mathcal{A} \setminus \{0\}} y_{\alpha}^2, \quad \mathcal{A}_{\mathbf{u}} = \{\alpha \in \mathcal{A} : i \in \mathbf{u} \iff \alpha_i \neq 0\}$$

FIGURE 4 - Sobol' indices of the model.

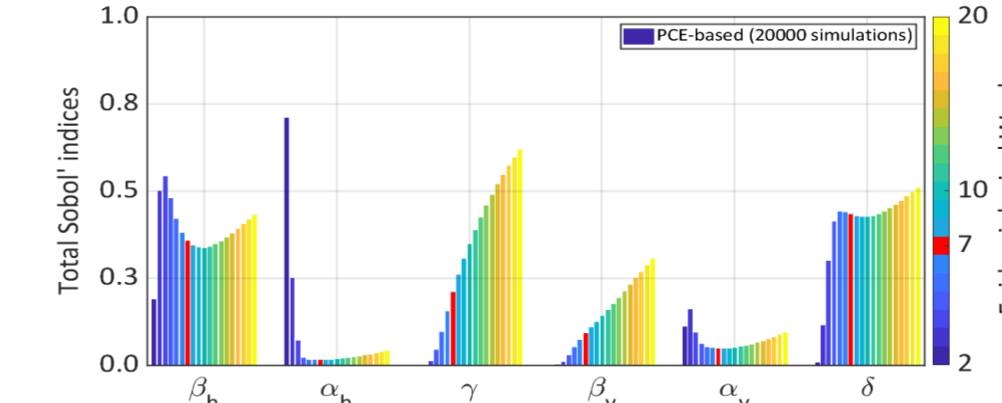


FIGURE 6 - Sobol' indices with α_v

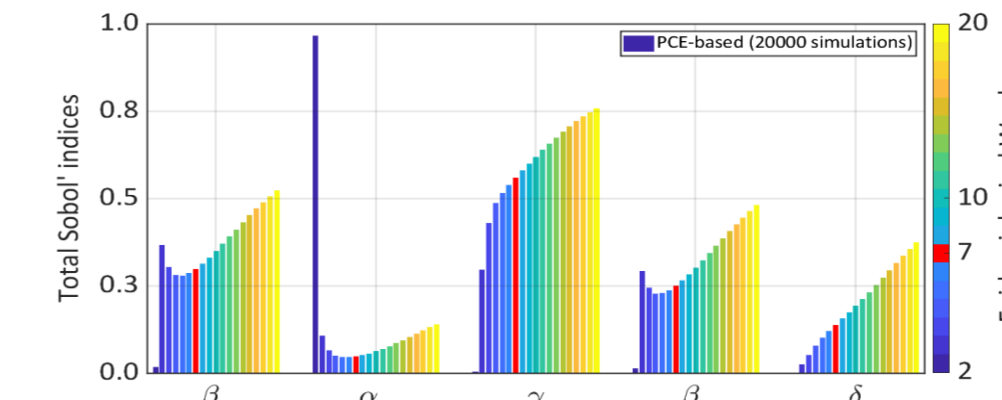


FIGURE 5 - Sobol' indices with α_v constant.

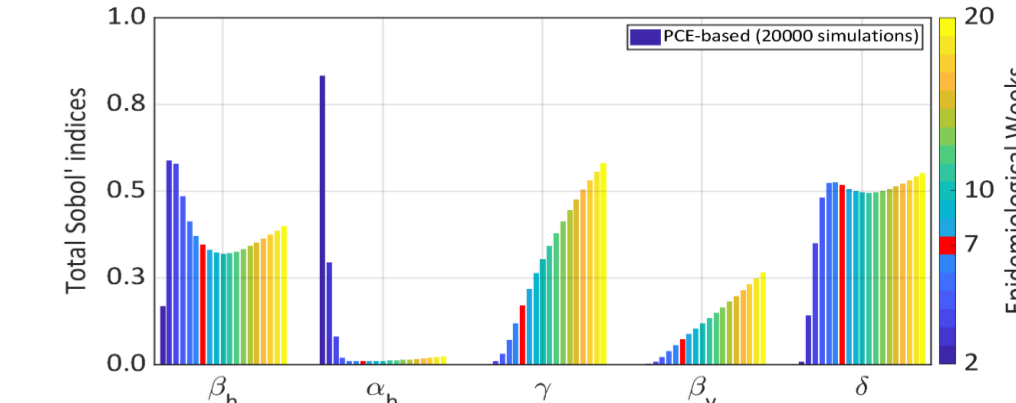
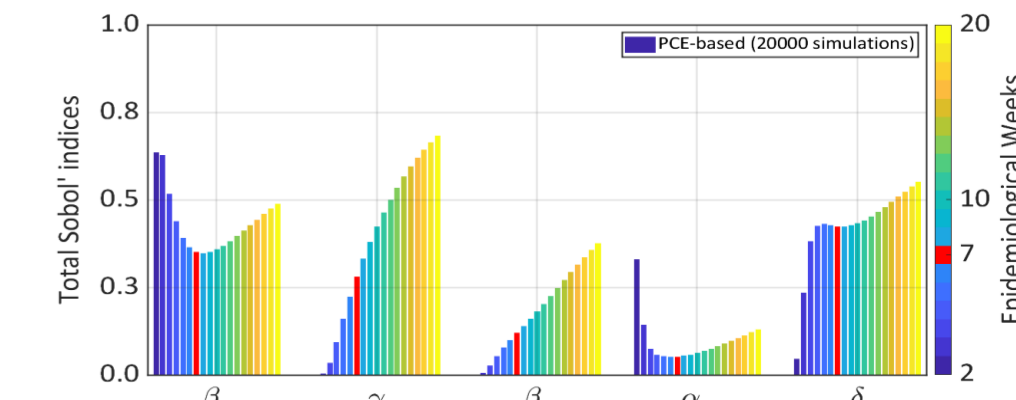


FIGURE 7 - Sobol' indices without α_h



STATISTICAL INFERENCE (ONGOING RESEARCH)

DISCREPANCY CALCULATION:

Suppose a data set $\mathcal{D} = (t_1, y_1^{dat}), (t_2, y_2^{dat}), \dots, (t_{N_d}, y_{N_d}^{dat})$ of measurement y_t of the i -th observation is given by

$$y_i^{dat} = \underbrace{\mathcal{M}(\mathbf{x}, t_i)}_{\text{model}} + \underbrace{\varepsilon_i}_{\text{error}}.$$

Sargsyan, Najm and Ghanem's [4] novel approach to deal with the model discrepancies is to adopt a metamodel structure which lumps the error into the parameters

$$Y^{dat} \approx \mathcal{M}(\mathbf{X}_{\varepsilon}, t), \quad \mathbf{X}_{\varepsilon} = \sum_{\alpha \in \mathcal{I}} \mathbf{X}_{\alpha}(t) \Psi_{\alpha}(\boldsymbol{\xi}),$$

where \mathbf{X}_{α} coefficients are defined as random to be able to be identified by using Bayesian Inference.

BAYESIAN INFERENCE:

- Inference problem become use data information to update the prior probability density function (PDF $\rho(\mathbf{X}_{\alpha})$ defined for \mathbf{X}_{α}). The solution corresponds posterior PDF;
- From Bayes' rule,

$$\rho(\mathbf{X}_{\alpha} | \mathcal{D}) = \frac{\rho(\mathcal{D} | \mathbf{X}_{\alpha}) \rho(\mathbf{X}_{\alpha})}{\rho(\mathcal{D})}.$$

- $\rho(\mathbf{X}_{\alpha} | \mathcal{D})$: posterior distribution
- $\rho(\mathbf{X}_{\alpha})$: prior distribution
- $\rho(\mathcal{D} | \mathbf{X}_{\alpha})$: likelihood function
- $\rho(\mathcal{D})$: evidence

To define a good point of start, the Maximum Entropy Principle is applied to construct the most informative prior distribution.

FINAL REMARKS

- ✓ Comparative results of global Sobol' Indices show how the lack of some parameters can change the sensibility effect of the others;
- ✓ A framework for statistical inference exploring Polynomial Chaos to measure the model discrepancies was presented;
- ✓ In future works, the authors intend explore this new framework to quantify model discrepancy and then improve its predictions;

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