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► **To cite this version:**

Sergey Solodukhin. Logarithmic terms in entropy of Schwarzschild black holes in higher loops. 2022.
hal-02266992

HAL Id: hal-02266992

<https://hal.science/hal-02266992>

Preprint submitted on 21 Jul 2022

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Logarithmic terms in entropy of Schwarzschild black holes in higher loops

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Abstract

The Bekenstein-Hawking (BH) entropy is expected to be modified by certain correction terms in the quantum loop expansion. As is well known the logarithmic terms in the entropy of black holes appear as a one-loop addition to the classical BH entropy. In this note we study the further modifications of the logarithmic terms in the entropy of the Schwarzschild black holes due to higher quantum loops: up to three loops in a general renormalizable theory of gauge fields, scalars and fermions and two loops in quantum gravity. For a large class of field multiplets (including that of the Standard Model) that include graviton and for a certain range in the values of the couplings these modifications manifest themselves in cooling down the black holes at later stages of evaporation and, respectively, in increasing the life time of the black hole. If this picture persists to even higher loops, then the small black holes formed in the early stages of the cosmic evolution do not evaporate completely by now as is predicted in the standard picture. Instead, their long-lived (Planckian mass) remnants are present in abundance in today's Universe.

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Since the works of Bekenstein and Hawking [1] it is generally accepted that the black holes, considered to be exact solutions to General Relativity, are characterized by a certain entropy proportional to the area of the horizon. The Schwarzschild black hole is a solution that is described by a single dimensionful parameter, mass M or horizon radius $r_+ = 2GM$. Classically, it has the entropy

$$S_{BH} = \frac{A_+}{4G} = 4\pi \frac{M^2}{M_{PL}^2}, \quad (1)$$

where $A_+ = 4\pi r_+^2$ is horizon area and we introduced the Planck mass, $M_{PL} = 1/\sqrt{G}$, and the temperature $T_H^{-1} = \partial_M S_{BH} = 8\pi M/M_{PL}^2$. This temperature becomes unbounded as soon as mass M of the black hole decreases. So that the evaporation rate

$$\frac{dM}{dt} \sim -T_H^4 A_+ \quad (2)$$

accelerates for small black holes as $dM/dt \sim -M_{PL}^4/M^2$ and the black hole evaporates in finite time $t_{BH} \sim M_0^3/M_{PL}^4$, where M_0 is the initial mass.

At the quantum level, when matter and/or gravity, are quantized the classical formula (1) gets modified. One way to think about this is to consider the entanglement entropy of the quantum fields. This entropy requires a UV regulator ϵ to be properly defined. The entropy contains both the area law similar to (1) [2] and a logarithmic term, which for a Schwarzschild black hole takes a simple form [3] - [6],

$$S_{ent} = \frac{A_+}{48\pi\epsilon^2} + s_0 \ln \frac{r_+}{\epsilon}, \quad (3)$$

where the pre-factor in front of the logarithmic term is

$$s_0 = \frac{1}{45}(N_0 + \frac{7}{2}N_{1/2} - 13N_1 - \frac{233}{4}N_{3/2} + 212N_2 + 91N_A) \quad (4)$$

for a multiplet of N_0 scalars, $N_{1/2}$ Dirac fermions, N_1 vector fields, N_2 particles of spin 2 (gravitons), $N_{3/2}$ fields of spin 3/2 (gravitino) and N_A rank 2 antisymmetric tensor fields. It should be noted that the existing technique allows us to compute only the UV divergent parts of the entropy. The dependence on r_+ in the logarithmic term then comes from the two facts: that the entropy is a dimensionless quantity and that there is only one dimensionful parameter, r_+ that characterises the geometry. So that the entanglement entropy may only be a function of the combination (r_+/ϵ) . The other remark is that the scalar field contribution in (4) to logarithmic term is the same for any value of non-minimal coupling ξ provided the latter is introduced as $\xi\phi^2 R$.

The quantum entropy (3) can be considered as a one-loop correction to the classical entropy (1). In the total entropy, $S_{BH} + S_{ent}$, the UV divergences then can be absorbed in the

renormalization¹ of the Newton constant, $1/G_{ren} = 1/G + 1/12\pi\epsilon^2$, and the R^2 couplings in the gravitational action, see [9]. So that one finds for the total black hole entropy expressed in terms of the renormalized couplings

$$S_{1-loop} = 4\pi \frac{M^2}{M_{PL}^2} + s_0 \ln M/\mu, \quad (5)$$

where M_{PL} is defined with respect to the renormalized G_{ren} and we omitted the irrelevant constants. Respectively, the temperature is modified,

$$T^{-1} = 8\pi M/M_{PL}^2 + s_0/M. \quad (6)$$

The modification becomes important when the mass of the black hole approaches the Planck mass. Extrapolating this approximation to those masses one finds that the evaporation scenario depends on the sign of s_0 .

If $s_0 = -q < 0$ then there exists a certain mass $M_{min} = \sqrt{8\pi q}M_{PL}$ for which the entropy as function of mass develops a minimum and the temperature becomes infinite (just as for $M = 0$ in the classical case). The black hole evaporates down to this minimal mass in a finite time $t \sim (M_0 - M_{min})^5/(M_0^2 M_{PL}^4)$. Formally extended to region where $M < M_{min}$ the temperature becomes negative there.

On the other hand, if $s_0 > 0$ then the entropy is monotonic function of mass while the temperature develops a maximum at $M_*^2 = s_0 M_{PL}^2/8\pi$, $T_{max} = M_{PL}/\sqrt{32\pi s_0}$. Extrapolating the formulas below this mass the temperature decreases and the black hole cools down so that $T \sim M$ for small M . The evaporation rate (2) then slows down and the black hole evaporates in infinite time. Notice that in the case when $s_0 \gg 1$ the critical mass M_* can be well above the Planck scale. For masses $M > M_*$ the specific heat is negative in a similar way as it follows from the Bekenstein-Hawking entropy [1] and, irrespectively of black holes, is in fact valid for a wide class of gravitationally coupled systems [12]. However, for smaller masses $M < M_*$ the specific heat becomes positive².

The entropy (5) is not automatically positive function of mass that may signal of the breakdown of the one-loop approximation in the region where the entropy becomes negative. The entropy is positive (and the approximation is reliable) for $M > \mu$. For sufficiently small μ (or large values of s_0) the threshold point lies well below the critical point M_* so that the most of the interesting modifications discussed above happen well above the point where the approximation breaks down.

¹There are certain subtleties [10] due to the presence of the non-minimal coupling [11]. These are not, however, essential in the present discussion.

²In the critical point $M = M_*$ the specific heat diverges that makes it quite attractive to interpret this point as a second order phase transition (for a discussion of singular behavior of specific heat in the second order transition see, for instance, [13]). This is an interesting direction to explore that we leave for a separate study.

The other way to obtain entropy (5) is to consider the effective gravitational action,

$$W_{gr} = -\frac{1}{16\pi G} \int R + W_Q \quad (7)$$

where the quantum part W_Q is represented as an expansion in powers of the Riemann curvature and its covariant derivatives. It contains both local and non-local terms. In the one-loop approximation $W_Q = \frac{1}{2} \ln \det \mathcal{D}$, where \mathcal{D} is the operator that governs the quadratic perturbations. The entropy then arises as a response of the gravitational action to a small angle deficit $\delta = 2\pi(1 - \alpha)$ at the horizon Σ , $S = (\alpha\partial_\alpha - 1)W_{gr}|_{\alpha=1}$, see [4] for more details on this formalism. The conical singularity manifests itself in the singular terms in the Riemann curvature concentrated at the horizon Σ , see [14],

$$R_{\mu\nu\alpha\beta} = 2\pi(1 - \alpha)((n_\mu n_\alpha)(n_\nu n_\beta) - (n_\mu n_\beta)(n_\nu n_\alpha))\delta_\Sigma + \dots, \quad (8)$$

where \dots stand for the regular terms in the curvature and n_μ^a , $a = 1, 2$ is a pair of vectors normal to Σ , $(n_\mu n_\nu) = \sum_{a=1}^2 n_\mu^a n_\nu^a$.

Discussing the entropy of the Schwarzschild black holes, for which the Ricci tensor identically vanishes, we need to look only at the terms that contain the Riemann tensor. Focusing on the quadratic terms one finds

$$W_Q = \frac{s_0}{64\pi^2} \int R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \ln \epsilon + \dots \quad (9)$$

Applying formula (8) to (9) and using that $\int_\Sigma R^{\mu\nu\alpha\beta} n_\mu^a n_\nu^b n_\alpha^a n_\beta^b = 8\pi$ one arrives at (3). Notice that $\ln r_+$ term is then restored by the dimensionality arguments. This term, in fact, comes from the non-local (UV finite) part of the quantum action W_Q .

Notice that formula (5) is valid for any, not necessarily conformal, massless field. However, if computed in a 4d CFT the logarithmic term takes the form [8]

$$s_0 = 64\pi^2(C - A) \quad (10)$$

and is, thus, related to the conformal charges A and C that appear in the conformal anomaly, $\langle T \rangle = AE_4 - CW^2$, where E_4 is the Euler density and W^2 is the square of the Weyl tensor. Notice that in certain theories s_0 vanishes. This is so, for instance, for $\mathcal{N} = 4$ super-Yang-Mills theory.

In the entanglement entropy of massive fields there appear new terms, both UV divergent and finite, that are due to mass m . However, most of them go away in the total entropy expressed in terms of the renormalized Newton constant. Such terms appear already in Minkowski space-time, see [10],

$$S_{ent} = c_s \frac{A_+}{48\pi} \int_{\epsilon^2} \frac{ds}{s^2} e^{-ms^2}, \quad (11)$$

where for a scalar $c_0 = 1$ and for a Dirac fermion $c_{1/2} = 2$. The respective renormalization of the Newton constant goes as

$$\frac{1}{G_{ren}} = \frac{1}{G} + c_s \frac{A_+}{12\pi} \int_{\epsilon^2} \frac{ds}{s^2} e^{-ms^2}. \quad (12)$$

In a curved space-time there appear more terms in the entropy that depend on m . However, the logarithmic term in the entropy remains unchanged. This is due to the fact that in the logarithmically UV divergent terms the mass m may appear only in a combination $m^2 R$ that contributes to the renormalization of the Newton constant that is already taken into account in (12). There, however, may appear new UV finite terms in the entropy of the form $f(r_+ m)$ which grow slower than logarithm³. Exact form of those terms is not yet known and is to be determined. In our analysis we shall ignore these terms concentrating our attention on the logarithmic terms only.

The story described up to this point is what we have at the level of one loop, when only the interaction of the matter with the background gravitational field is taken into account. The sign of the logarithmic term in (5) is essential when we discuss the evaporation of the black hole since the logarithmic term becomes important at the later stages of the evaporation when mass of the black hole becomes comparable to the Planck mass. For negative values of s_0 the evaporating black hole reaches, in a finite time, the stage of infinite temperature, just as in the classical case. The further evolution is difficult to predict within the present analysis. Since this configuration corresponds to the minimum of the entropy the system would probably tend to absorb the energy rather than emit it. Conceptually, this situation is not an improvement over the classical case.

On the other hand, in the case of positive s_0 the black hole evaporation is much less violent. As soon as the black hole reaches some maximal temperature its mass then further decreases and the hole starts to quickly cool down. As a result, in this case, the black holes never evaporate completely, at the later times they are present as the long-lived sub-Planckian remnants.

As we see it from (4) the sign of s_0 is not a priori definite, it depends on the multiplet of fields existing in Nature. Although many fields contribute positively to s_0 some of them, as vector fields, contribute negatively. s_0 can be computed in the Standard Model which contains 24 fermions, 12 gauge bosons and 4 scalars (a complex doublet). Provided one graviton is added to this multiplet eq. (4) gives us a positive value, $s_{0,SM} = 16/5$. Note that without the graviton this value would be negative, $-68/45$.

The goal of this note is to investigate how the one-loop approximation (5) is modified when the interactions are turned on and the respective higher quantum loops are taken into account.

³ For instance, one anticipates an exponential term of the form $e^{-r_+^2 m^2}$ which for a large range of parameters (M, m) leads to smaller effects than the logarithmic term.

By interactions here we mean both the interaction of the matter fields and the purely gravitational interaction when particles of spin 2 are considered. In the case of gravity the coupling constant is dimensionful and we expect there to appear corrections to (5) of the form M_{PL}^2/M^2 . Identifying those correction terms one expands the domain of validity of the approximation.

Our strategy is first to look at the logarithmic UV divergent terms in the effective action, then compute the respective UV terms in the entropy using (8) and finally restore the dependence on r_+ using the dimensionality arguments, in the same line as in one loop, (3) and (5). We base our further analysis on the results available in the literature: the instrumental works of Jack and Osborn [20] and Jack [21] - [22] (see also a recent paper [24] where some typos in the previous publications have been corrected) for the matter fields and the classical papers by Goroff and Sagnotti and by van de Ven [29] for gravity. (In the case of gravity the one-loop UV terms were computed in [28] that results in the graviton contribution in (4).) We use these earlier results and compute the corresponding modifications in the entropy. Earlier, it was suggested by Sen [7] that the logarithmic term is a one-loop effect and it does not get renormalized in higher loops. This is valid for a theory with a dimensionful coupling constant such as quantum gravity. In the interacting quantum field theories with dimensionless couplings one expects the possible corrections to the one-loop result and, indeed, as we show below these corrections do appear in the higher loops.

Before proceeding one technical remark is that we use here a regularisation, such as heat kernel or Pauli-Villars, in which the UV regulator ϵ is dimensionful. In the literature one quite often uses the dimensional regularisation, $\epsilon_d = (4 - d)$. The conversion rule between the two regularisations is the following: $\frac{1}{\epsilon_d} = -\ln \epsilon$. Earlier works on entanglement entropy in the interacting field systems include [15] - [19].

Below we consider some examples of interacting quantum field theories.

General renormalizable scalar field theories. Consider a multi-component scalar field ϕ^i , $i = 1, \dots, N_0$ with a general Lagrangian

$$L = \frac{1}{2}(\nabla\phi)^2 + V(\phi), \quad (13)$$

where the potential takes the form

$$V(\phi) = \frac{1}{4!}\lambda_{ijkl}\phi^i\phi^j\phi^k\phi^l + \frac{1}{3!}g_{ijk}\phi^i\phi^j\phi^k + \frac{1}{2}m_{ij}\phi^i\phi^j + \frac{1}{2}\xi_{ij}\phi^i\phi^j R + \dots \quad (14)$$

The corresponding UV divergences up to 3 loops in this theory have been calculated by Jack and Osborn [20] in 1984. Leaving aside the terms linear in Ricci scalar R , that as we discussed above will contribute to the renormalization of the Newton constant we are interested in terms which are quadratic in the Riemann tensor. Those terms in the theory (13) appear in one loop,

the corresponding contribution to the entropy is given by the scalar field part in (5) and in three loops⁴, as was shown by Jack and Osborn⁵,

$$W_Q^{(3)} = \frac{1}{(4\pi)^6} \frac{\mu^{-\epsilon_d}}{\epsilon_d} \frac{1}{2592} \int \lambda_{ijkl} \lambda_{ijkl} (R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} - 2R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2) + \dots, \quad (15)$$

where we keep only terms quadratic in curvature. Notice that (15) represents the renormalization of the Weyl-square term. Applying (8) we compute the corresponding contribution to the entropy of the Schwarzschild black hole and find that s_0 in the three loop approximation reads

$$s_0(\lambda) = \frac{N_0}{45} - \frac{1}{(4\pi)^4} \frac{1}{648} \lambda_{ijkl} \lambda_{ijkl}. \quad (16)$$

This value of s_0 is smaller than the one for the free scalar fields.

Gauge theory coupled to fermions and scalars. A system of a non-abelian gauge field $A = A_\mu^a t^a$ coupled to Dirac fermions ψ and real scalars ϕ carrying the representations of a simple gauge group \mathcal{G} is described by the Lagrangian

$$L = \frac{1}{4g^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{1}{2} (D\phi)^T D\phi + \bar{\psi} \not{D}\psi + \frac{1}{2} \bar{\psi} Y_i \psi \phi^i, \quad (17)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$ and one included the Yukawa couplings Y_i . The two loop computation in this theory was performed in [21], [22], [23] (some typos have been corrected in [24], see also [25] for further developments and [26] for a non-perturbative CFT consideration). As in (15) the appropriate UV divergence comes from the renormalization of Weyl-square term,

$$W_Q^{(2)} = \frac{1}{(4\pi)^4} \frac{\mu^{-\epsilon_d}}{\epsilon_d} \int \left(g^2 \left(\frac{2}{9} C_2^A - \frac{7}{72} C_2^\psi - \frac{1}{18} C_2^\phi \right) N_1 + \frac{1}{96} \text{Tr} Y^2 \right) R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} + \dots, \quad (18)$$

where C_2^A (C_2^ψ and C_2^ϕ) is Casimir operator for gauge fields (respectively for fermions and scalars). In these notations the beta-function for the gauge coupling, $\beta_g = \frac{g^3}{(4\pi)^2} \left(-\frac{11}{3} C_2^A + \frac{2}{3} C_2^\psi + \frac{1}{6} C_2^\phi \right)$ [27].

So that one finds in the two loop approximation that

$$s_0(g, Y) = s_0^{free} - \frac{1}{16\pi^2} \left(g^2 \left(\frac{2}{9} C_2^A - \frac{7}{72} C_2^\psi - \frac{1}{18} C_2^\phi \right) N_1 + \frac{1}{96} \text{Tr} Y^2 \right). \quad (19)$$

We see that the contributions of the gauge fields and of the Yukawa couplings tend to decrease the value of s_0 while the contributions of the matter fields (fermions and scalars) increase it. In the QCD sector $C_2^A = 3$ and $C_2^\psi = n_F$ (number of flavors) and no scalars one has that $\frac{2}{9} C_2^A - \frac{7}{72} C_2^\psi = \frac{48-7n_F}{72}$. It is negative if $n_F \geq 7$. In the Standard Model $n_F = 6$ and, hence,

⁴In two loops the UV term $W_Q^{(2)}$ contains terms at most linear in R , $\int g_{ijk} g_{ijk} R$, that contributes to the renormalization of the Newton constant and does not effect the logarithmic term.

⁵Notice that our definition for W_Q differs by sign from the one used in [20].

$s_0(g) < s_0^{free}$. This observation and that the Yukawa couplings and λ -couplings in (16) make the negative contributions to s_0 appear to indicate that in the Standard Model the value of s_0 is smaller than in the free theory. To see whether this value remains positive (provided we included the graviton's contribution in one loop) requires a more careful analysis including the actual values of all couplings involved⁶.

Quantum Gravity. The one-loop calculation of UV divergences in quantum gravity has been first performed by 't Hooft and Veltman and later confirmed by various authors [28]. For pure gravity the divergent term in the effective action is

$$W_Q^{(1)} = -\frac{1}{(4\pi)^2} \frac{\mu^{-\epsilon_d}}{\epsilon_d} \int \left(\frac{53}{45} E_4 + \frac{7}{20} R_{\mu\nu}^2 + \frac{1}{120} R^2 \right), \quad (20)$$

where E_4 is the Euler density. This term results in graviton's contribution to logarithmic term s_0 presented in (5).

The two loop computation in quantum gravity was performed by Goroff and Sagnotti and later by van de Ven [29],

$$W_Q^{(2)} = -\frac{209}{2880} \frac{\mu^{-\epsilon_d}}{\epsilon_d} \frac{\kappa}{(4\pi)^4} \int R^{\mu\nu\alpha\beta} R_{\alpha\beta\sigma\rho} R^{\sigma\rho}{}_{\mu\nu}, \quad (21)$$

where $\kappa = 32\pi^2 G$. We keep here the term that contains only the Riemann tensor. This term is gauge independent and not vanishes on-shell. In fact, in two loops there may appear more terms than just (21), in particular the products of two Riemann tensors and either Ricci tensor or Ricci scalar (see paper of Goroff and Sagnotti for some of such terms). The complete information on the exact numerical factors with which those terms appear in the effective action is not available in the literature. These terms, however, appear to depend on the gauge and are unlikely to make any contributions to physical quantities such as the entropy.

Concentrating our attention only on the Riemann tensor we see that the one-loop and two-loop results (20) and (21) have the same sign. So that the two-loop correction due to gravitons to s_0 comes with the same sign as the one-loop result. Using (8) one finds that on a conical manifold

$$\kappa \int_{\mathcal{M}_\alpha} R^{\mu\nu\alpha\beta} R_{\alpha\beta\sigma\rho} R^{\sigma\rho}{}_{\mu\nu} = 12\kappa\pi(1-\alpha) \int_\Sigma R_{ab\sigma\rho} R^{\sigma\rho}{}_{ab} + \dots = 192\pi^2(1-\alpha) \frac{\kappa}{r_+^2} + \dots, \quad (22)$$

where \dots stands for terms quadratic in $(1-\alpha)$ and $r_+ = 2GM$ is the horizon radius. With this relation we find

$$S(M) = 4\pi \frac{M^2}{M_{PL}^2} + s(M) \ln \frac{M}{\mu}, \quad s(M) = s_0 + \sigma \frac{M_{PL}^2}{M^2}, \quad \sigma = \frac{209}{480} \quad (23)$$

⁶ In $\mathcal{N} = 4$ $SU(N)$ SYM the C -charge is not expected to renormalise and, hence, the shift in s_0 (19) is expected to vanish. I thank K. Skenderis for this remark.

for the entropy of the Schwarzschild black hole in the two-loop approximation. In agreement with the arguments of [7] there is no higher loop gravity corrections to value of s_0 . However, the two loop computation results in a new σ -term which is negligible for masses much larger than the Planck mass but it gives an important positive contribution for $M \sim M_{PL}$. This σ -term, combined with the standard s_0 -term, can still be regarded as a “logarithmic term” keeping in mind that the coefficient in front of the logarithm now becomes a function of the mass. In the higher gravitational loops there will appear higher powers of M_{PL}^2/M^2 in the function $s(M)$.

Modified temperature and evaporation. With the entropy as function of the mass given by eq.(23) one finds the modified temperature

$$M_{PL}T^{-1} = \frac{1}{x^{3/2}}(8\pi x^2 + s_0x - \sigma \ln \frac{x}{\lambda}), \quad x = M^2/M_{PL}^2, \quad \lambda = \frac{\mu^2 e}{M_{PL}^2} \quad (24)$$

In the complete analysis of the temperature one should study the dependence on three parameters: s_0 , σ and λ . Here we just mention that the presence of the σ -term with $\sigma > 0$ makes the temperature positive for any M even if s_0 is negative. Indeed, for $-s_0 = q > 0$ one finds that the function in the right hand side of (24) is everywhere positive if two conditions are satisfied: 1) $q < 16\pi\lambda$ and 2) $\frac{q^2}{32\pi^2 \ln(16\pi\lambda/q)} < \sigma$. Thus, for sufficiently small values of q the temperature remains positive function of M and it develops a maximum below which the evaporating black hole cools down as soon as its mass decreases. Extrapolating (2) to the process of evaporation with the temperature (24) one finds that at later times the mass of the evaporating black hole falls off as $t^{-1/13}$ that is a much slower rate compared to $t^{-1/5}$ in the case of vanishing σ .

We conclude with some remarks.

1. Strictly speaking, the extrapolation of the 1- and 2-loop results for the entropy to values of mass M approaching the Planck mass goes beyond the validity of the loop approximation and, thus, is outside the domain of applicability. However, the same and even for more reasons could (and should) be said about the applicability of the classical BH formula for the entropy. BH formula is not valid for small black holes of mass $M \sim M_{PL}$ and should be replaced with a better one. We believe that the higher loop results, presented here and yet approximative, show us a certain tendency in the behaviour of $S(M)$ for small M which, likely, will become even stronger in the higher loops when more terms in the function $s(M)$ will be available. The higher loop contributions will appear as the higher order terms in the series with respect to M_{PL}^2/M^2 . It is possible that, provided all loops are included, these terms will sum up to something non-analytic at $M = 0$.

2. The other limitation of our results comes from the fact that here the quantum corrections to the entropy are computed in a (still classical) Ricci flat geometry. In a fully consistent consideration one would have to also analyse the modification of the geometry itself due to the

quantum corrections. On one hand, this is a more difficult problem. On the other, in a non-perturbative analysis of the quantum corrected Einstein equations the black hole may appear to be replaced with a rather different (and yet longer-lived) object as the consideration of [30] shows. These are different indications that the existence of the long-lived compact objects in the complete theory is a likely possibility.

3. The analysis based on the use of the BH entropy plays an important role in the story of primordial black holes (for a review see, for instance, [31]). Namely, it imposes a lower limit on the mass of the primordial black holes since the small ones have enough time to evaporate completely until the present days. Provided one uses the modified version of the entropy with the logarithmic terms present this conclusion is no more in place: the black hole evaporation slows down as soon as mass decreases below the critical mass $M_* \sim \sqrt{s_0} M_{PL}$ that leads to much longer life time of the black holes. In this picture the small primordial black holes do not evaporate completely. Instead, all of them are present in today's Universe as some long-lived Planckian compact objects. From a phenomenological point of view this is a particular case of a more general conclusion of [32] about the much longer life time of black holes for a wide class of possible modifications of the Hawking's formula for the temperature as function of mass M .

Acknowledgements

I would like to thank B. Pioline for a discussion on the logarithmic terms, K. Krasnov for discussions on the loop calculations in the background field formalism, and, especially, M. Shaposhnikov for warm hospitality in his group at EPFL and many inspiring discussions during the initial stages of this project. The present work is supported in part by the ERC-AdG-2015 grant 694896. I also acknowledge the useful communications with I. Jack regarding his papers and with K. Skenderis on renormalization of Weyl-square term in 4d CFTs and many insightful conversations with my colleagues in Tours: M. Chernodub, N. Mohammadi and S. Nicolis.

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